basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

CURRICULUM AND ASSESSMENT POLICY STATEMENT (CAPS)

SENIOR PHASE (GRADES 7 – 9)

MATHEMATICS

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SECTION 1
CURRICULUM AND ASSESSMENT POLICY STATEMENT (CAPS)

INTRODUCTION AND BACKGROUND

1.1 Background

The *National Curriculum Statement Grades R – 12 (NCS)* stipulates policy on curriculum and assessment in the schooling sector.

To improve implementation, the National Curriculum Statement was amended, with the amendments coming into effect in January 2012. A single comprehensive Curriculum and Assessment Policy document was developed for each subject to replace Subject Statements, Learning Programme Guidelines and Subject Assessment Guidelines in Grades R - 12.

1.2 Overview

(a) The *National Curriculum Statement Grades R – 12 (January 2012)* represents a policy statement for learning and teaching in South African schools and comprises the following:

- (i) National Curriculum and Assessment Policy Statements for each approved school subject*;*
- (ii) The policy document, National policy pertaining to the programme and promotion requirements of the National Curriculum Statement Grades R – 12; and
- (iii) The policy document, National Protocol for Assessment Grades R – 12 (January 2012).

(b) The *National Curriculum Statement Grades R – 12 (January 2012)* replaces the two current national curricula statements, namely the

- *(i)* *Revised National Curriculum Statement Grades R - 9, Government Gazette No. 23406* of 31 May 2002*,* and
- (ii) *National Curriculum Statement Grades 10 - 12 Government Gazettes, No. 25545 of 6 October 2003* and *No. 27594* of 17 May 2005.

(c) The national curriculum statements contemplated in subparagraphs (a) and (b) comprise the following policy documents which will be incrementally repealed by the *National Curriculum Statement Grades R – 12 (January 2012)* during the period 2012-2014:

- (i) The Learning Area/Subject Statements, Learning Programme Guidelines and Subject Assessment Guidelines for Grades R - 9 and Grades 10 – 12;
- (ii) The policy document, *National Policy on assessment and qualifications for schools in the General Education and Training Band d*, promulgated in *Government Notice No. 124* in *Government Gazette No. 29626* of 12 February 2007;

(iii) The policy document, the *National Senior Certificate: A qualification at Level 4 on the National Qualifications Framework (NQF)*, promulgated in *Government Gazette No.27819* of 20 July 2005;

(iv) The policy document, *An addendum to the policy document, the National Senior Certificate: A qualification at Level 4 on the National Qualifications Framework (NQF), regarding learners with special needs*, published in *Government Gazette, No.29466* of 11 December 2006, is incorporated in the policy document, *National policy pertaining to the programme and promotion requirements of the National Curriculum Statement Grades R – 12; and*

(v) The policy document, *An addendum to the policy document, the National Senior Certificate: A qualification at Level 4 on the National Qualifications Framework (NQF), regarding the National Protocol for Assessment (Grades R – 12)*, promulgated in *Government Notice No.1267* in *Government Gazette No. 29467* of 11 December 2006.

(d) The policy document, *National policy pertaining to the programme and promotion requirements of the National Curriculum Statement Grades R – 12*, and the sections on the Curriculum and Assessment Policy as contemplated in Chapters 2, 3 and 4 of this document constitute the norms and standards of the *National Curriculum Statement Grades R – 12.* It will therefore, in terms of *section 6A* of the *South African Schools Act, 1996 (Act No. 84 of 1996,)* form the basis for the Minister of Basic Education to determine minimum outcomes and standards, as well as the processes and procedures for the assessment of learner achievement to be applicable to public and independent schools.

1.3 General aims of the South African Curriculum

(a) The *National Curriculum Statement Grades R - 12* gives expression to the knowledge, skills and values worth learning in South African schools. This curriculum aims to ensure that children acquire and apply knowledge and skills in ways that are meaningful to their own lives. In this regard, the curriculum promotes knowledge in local contexts, while being sensitive to global imperatives.

(b) The National Curriculum Statement Grades R - 12 serves the purposes of:

- equipping learners, irrespective of their socio-economic background, race, gender, physical ability or intellectual ability, with the knowledge, skills and values necessary for self-fulfilment, and meaningful participation in society as citizens of a free country;
- providing access to higher education;
- facilitating the transition of learners from education institutions to the workplace; and
- providing employers with a sufficient profile of a learner's competences.

(c) The National Curriculum Statement Grades R - 12 is based on the following

principles:

- Social transformation: ensuring that the educational imbalances of the past are redressed, and that equal educational opportunities are provided for all sections of the population;
- Active and critical learning: encouraging an active and critical approach to learning, rather than rote and uncritical learning of given truths;
- High knowledge and high skills: the minimum standards of knowledge and skills to be achieved at each grade are specified and set high, achievable standards in all subjects;
- Progression: content and context of each grade shows progression from simple to complex;
- Human rights, inclusivity, environmental and social justice: infusing the principles and practices of social and environmental justice and human rights as defined in the Constitution of the Republic of South Africa. The National Curriculum Statement Grades R – 12 is sensitive to issues of diversity such as poverty, inequality, race, gender, language, age, disability and other factors;
- Valuing indigenous knowledge systems: acknowledging the rich history and heritage of this country as important contributors to nurturing the values contained in the Constitution; and
- Credibility, quality and efficiency: providing an education that is comparable in quality, breadth and depth to those of other countries.

(d) The National Curriculum Statement Grades R - 12 aims to produce learners that are able to:

- identify and solve problems and make decisions using critical and creative thinking;
- work effectively as individuals and with others as members of a team;
- organise and manage themselves and their activities responsibly and effectively;
- collect, analyse, organise and critically evaluate information;
- communicate effectively using visual, symbolic and/or language skills in various modes;
- use science and technology effectively and critically showing responsibility towards the environment and the health of others; and
- demonstrate an understanding of the world as a set of related systems by recognising that problem solving contexts do not exist in isolation.

(e) Inclusivity should become a central part of the organisation, planning and teaching at each school. This can only happen if all teachers have a sound understanding of how to recognise and address barriers to learning, and how to plan for diversity.

The key to managing inclusivity is ensuring that barriers are identified and addressed by all the relevant support structures within the school community, including teachers, District-Based Support Teams, Institutional-Level Support Teams, parents and

Special Schools as Resource Centres. To address barriers in the classroom, teachers should use various curriculum differentiation strategies such as those included in the Department of Basic Education's *Guidelines for Inclusive Teaching and Learning* (2010).

1.4 Time Allocation

1.4.1 Foundation Phase

(a) The instructional time in the Foundation Phase is as follows:

SUBJECT	GRADE R (HOURS)	GRADES 1-2 (HOURS)	GRADE 3 (HOURS)
Home Language	10	7/8	7/8
First Additional Language		2/3	3/4
Mathematics	7	7	7
Life Skills	**6**	**6**	**7**
▪ Beginning Knowledge	(1)	(1)	(2)
▪ Creative Arts	(2)	(2)	(2)
▪ Physical Education	(2)	(2)	(2)
▪ Personal and Social Well-being	(1)	(1)	(1)
TOTAL	**23**	**23**	**25**

(b) Instructional time for Grades R, 1 and 2 is 23 hours and for Grade 3 is 25 hours.

(c) Ten hours are allocated for languages in Grades R-2 and 11 hours in Grade 3. A maximum of 8 hours and a minimum of 7 hours are allocated for Home Language and a minimum of 2 hours and a maximum of 3 hours for Additional Language in Grades R – 2. In Grade 3 a maximum of 8 hours and a minimum of 7 hours are allocated for Home Language and a minimum of 3 hours and a maximum of 4 hours for First Additional Language.

(d) In Life Skills Beginning Knowledge is allocated 1 hour in Grades R – 2 and 2 hours as indicated by the hours in brackets for Grade 3.

1.4.2 Intermediate Phase

(a) The instructional time in the Intermediate Phase is as follows:

SUBJECT	HOURS
Home Language	6
First Additional Language	5
Mathematics	6
Natural Science and Technology	3,5
Social Sciences	3
Life Skills ▪ Creative Arts ▪ Physical Education ▪ Personal and Social Well-being	**4** (1,5) (1) (1,5)
TOTAL	**27,5**

1.4.3 Senior Phase

(a) The instructional time in the Senior Phase is as follows:

SUBJECT	HOURS
Home Language	5
First Additional Language	4
Mathematics	4,5
Natural Science	3
Social Sciences	3
Technology	2
Economic Management Sciences	2
Life Orientation	2
Arts and Culture	2
TOTAL	**27,5**

1.4.4 Grades 10-12

(a) The instructional time in Grades 10-12 is as follows:

Subject	Time allocation per week (hours)
I. Home Language	**4.5**
II. First Additional Language	**4.5**
III. Mathematics	**4.5**
IV. Life Orientation	**2**
V. A minimum of any three subjects selected from **Group B** Annexure B, Tables B1-B8 of the policy document, *National policy pertaining to the programme and promotion requirements of the National Curriculum Statement Grades R – 12,* subject to the provisos stipulated in paragraph 28 of the said policy document.	**12 (3x4h)**

The allocated time per week may be utilised only for the minimum required NCS subjects as specified above, and may not be used for any additional subjects added to the list of minimum subjects. Should a learner wish to offer additional subjects, additional time must be allocated for the offering of these subjects.

SECTION 2
CURRICULUM AND ASSESSMENT POLICY STATEMENT (CAPS)

DEFINITION, AIMS, SKILLS AND CONTENT

2.1. Introduction

In Section 2, the Foundation Phase Mathematics Curriculum and Assessment Policy Statement (CAPS) provides teachers with a definition of mathematics, specific aims, specific skills, focus of content areas, weighting of content areas

2.2. What is Mathematics?

Mathematics is a language that makes use of symbols and notations to describe numerical, geometric and graphical relationships. It is a human activity that involves observing, representing and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves. It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem-solving that will contribute in decision-making.

2.3. Specific Aims

The teaching and learning of Mathematics aims to develop

- a critical awareness of how mathematical relationships are used in social, environmental, cultural and economic relations
- confidence and competence to deal with any mathematical situation without being hindered by a fear of Mathematics
- an appreciation for the beauty and elegance of Mathematics
- a spirit of curiosity and a love for Mathematics
- recognition that Mathematics is a creative part of human activity
- deep conceptual understandings in order to make sense of Mathematics
- acquisition of specific knowledge and skills necessary for:
    - the application of Mathematics to physical, social and mathematical problems
    - the study of related subject matter (e.g. other subjects)
    - further study in Mathematics.

2.4. Specific Skills

To develop essential mathematical skills the learner should

- develop the correct use of the language of Mathematics
- develop number vocabulary, number concept and calculation and application skills
- learn to listen, communicate, think, reason logically and apply the mathematical knowledge gained
- learn to investigate, analyse, represent and interpret information
- learn to pose and solve problems
- build an awareness of the important role that Mathematics plays in real life situations including the personal development of the learner.

2.5. Focus of Content Areas

Mathematics in the Senior Phase covers five main content areas (or domains).

- Numbers, Operations and Relationships
- Patterns, Functions and Algebra
- Space and Shape (Geometry)
- Measurement
- Data Handling

Each content area contributes towards the acquisition of specific skills. The table below shows the general focus of the content areas as well as the specific focus of the content areas for the Senior Phase.

MATHEMATICS CONTENT KNOWLEDGE		
Content Area	**General content focus**	**Senior Phase specific content focus**
1. Numbers, Operations and Relationships	Development of number sense that includes: • the meaning of different kinds of numbers • relationship between different kinds of numbers • the relative size of different numbers • representation of numbers in various ways • the effect of operating with numbers • the ability to estimate and check solutions.	• Representation of numbers in a variety of ways and moving flexibly between representations • Recognising and using properties of operations with different number systems • Solving a variety of problems, using an increased range of numbers and the ability to perform multiple operations correctly and fluently
2. Patterns, Functions and Algebra	Algebra is the language for investigating and communicating most of Mathematics and can be extended to the study of functions and other relationships between variables. A central part of this content area is for the learner to achieve efficient manipulative skills in the use of algebra. It also focuses on the • description of patterns and relationships through the use of symbolic expressions, graphs and tables; and • identification and analysis of regularities and change in patterns, and relationships that enable learners to make predictions and solve problems.	• Investigation of numerical and geometric patterns to establish the relationships between variables • Expressing rules governing patterns in algebraic language or symbols • Developing algebraic manipulative skills that recognize the equivalence between different representations of the same relationship • Analysis of situations in a variety of contexts in order to make sense of them • Representation and description of situations in algebraic language, formulae, expressions, equations and graphs
3. Space and Shape (Geometry)	The study of Space and Shape improves understanding and appreciation of the pattern, precision, achievement and beauty in natural and cultural forms. It focuses on the properties, relationships, orientations, positions and transformations of two-dimensional shapes and three-dimensional objects.	• Drawing and constructing a wide range of geometric figures and solids using appropriate geometric instruments • Developing an appreciation for the use of constructions to investigate the properties of geometric figures and solids • Developing clear and more precise descriptions and classification categories of geometric figures and solids • Solving a variety of geometric problems drawing on known properties of geometric figures and solids
4. Measurement	Measurement focuses on the selection and use of appropriate units, instruments and formulae to quantify characteristics of events, shapes, objects and the environment. It relates directly to the learner's scientific, technological and economic worlds, enabling the learner to • make sensible estimates; and • be alert to the reasonableness of measurements and results.	• Using formulae for measuring area, perimeter, surface area and volume of geometric figures and solids • Selecting and converting between appropriate units of measurement • Using the Theorem of Pythagoras to solve problems involving right-angled triangles
5. Data handling	Data handling involves asking questions and finding answers in order to describe events and the social, technological and economic environment. Through the study of data handling, the learner develops the skills to collect, organize, represent, analyse, interpret and report data. The study of probability enables the learner to develop skills and techniques for making informed predictions, and describing randomness and uncertainty.	• Posing of questions for investigation • Collecting, summarizing, representing and critically analysing data in order to interpret, report and make predictions about situations • Probability of outcomes include both single and compound events and their relative frequency in simple experiments

2.6. Weighting of content areas

The weighting of Mathematics content areas serves two primary purposes:

- guidance on the time needed to adequately address the content within each content area
- guidance on the spread of content in the examination (especially end–of-year summative assessment).

WEIGHTING OF CONTENT AREAS			
Content Area	**Grade 7**	**Grade 8**	**Grade 9**
Number, Operations and Relations	30%	25%	15%
Patterns, Functions and Algebra	25%	30%	35%
Space and Shape (Geometry)	25%	25%	30%
Measurement	10%	10%	10%
Data handling (Statistics)	10%	10%	10%
	100%	**100%**	**100%**

2.7. Specification of content to show progression

The specification of content shows progression in terms of concepts and skills from Grades 7 - 9 for each Content Area. However, in certain topics the concepts and skills are similar in two or three successive grades. The clarification of content in Section 3 provides guidelines on how progression should be addressed in these cases. The specification of content in Section 2 should therefore be read in conjunction with the clarification of content in Section 3.

SENIOR PHASE OVERVIEW
1. NUMBERS, OPERATIONS AND RELATIONSHIPS

- Progression in Numbers, Operations and Relationships in the Senior Phase is achieved primarily by:
  - development of calculations using whole numbers to calculations using rational numbers, integers and numbers in exponential form
  - development of understanding of different number systems from **natural** and **whole** numbers to **integers** and **rational** numbers, as well as the recognition of **irrational** numbers
  - increasing use of properties of numbers to perform calculations
  - increasing complexity of contexts for solving problems
- Numbers, Operations and Relationships in the Senior Phase consolidates work done in the Intermediate Phase and is geared towards making learners competent and efficient in performing calculations particularly with integers and rational numbers.
- Recognising and using the properties of operations for different numbers provides a critical foundation for work in algebra, when learners work with variables in place of numbers and manipulate algebraic expressions and solve algebraic equations.

CONTENT	GRADE 7	GRADE 8	GRADE 9
1.1 Whole numbers	**Mental calculations** Revise the following done in Grade 6: • Multiplication of whole numbers to at least 12×12 • Multiplication facts for: ○ units and tens by multiples of ten ○ units and tens by multiples of 100 ○ units and tens by multiples of 1000 ○ units and tens by multiples of $10\ 000$ • Inverse operation between multiplication and division **Ordering and comparing whole numbers** • Revise the following done in Grade 6: ○ order, compare and represent numbers to at least 9-digit numbers ○ recognize and represent prime numbers to at least 100 ○ round off numbers to the nearest 5, 10, 100 or 1000 **Properties of whole numbers** • Revise the following done in Grade 6: ○ recognize and use the commutative; associative; distributive properties of whole numbers	**Mental calculations** • Revise multiplication of whole numbers to at least 12×12 **Ordering and comparing whole numbers** • Revise prime numbers to at least 100 **Properties of whole numbers** • Revise: ○ The commutative; associative; distributive properties of whole numbers ○ 0 in terms of its additive property (identity element for addition)	**Properties of numbers** • Describe the real number system by recognising, defining and distinguishing properties of: ○ natural numbers ○ whole numbers ○ integers

	○ recognize and use 0 in terms of its additive property (identity element for addition) ○ recognize and use 1 in terms of its multiplicative property (identity element for multiplication) **Calculations using whole numbers** • Revise the following done in Grade 6, without use of calculators: ○ Addition and subtraction of whole numbers to at least 6 –digit numbers ○ Multiplication of at least whole 4-digit by 2-digit numbers ○ Division of at least whole 4-digit by 2-digit numbers ○ Perform calculations using all four operations on whole numbers, estimating and using calculators where appropriate **Calculation strategies** • Use a range of strategies to perform and check written and mental calculations of whole numbers including: ○ long division ○ adding, subtracting and multiplying in columns ○ estimation ○ rounding off and compensating ○ using a calculator **Multiples and factors** • Revise the following done in Grade 6: ○ multiples of 2-digit and 3-digit whole numbers ○ factors of 2-digit and 3-digit whole numbers ○ prime factors of numbers to at least 100 • List prime factors of numbers to at least 3-digit whole numbers	○ 1 in terms of its multiplicative property (identity element for multiplication) • Recognize the division property of 0, whereby any number divided by 0 is undefined **Calculations using whole numbers** • Revise: ○ calculations using all four operations on whole numbers, estimating and using calculators where appropriate **Calculation strategies** • Use a range of strategies to perform and check written and mental calculations of whole numbers including: ○ long division ○ adding, subtracting and multiplying in columns ○ estimation ○ rounding off and compensating ○ using a calculator **Multiples and factors** • Revise: ○ Prime factors of numbers to at least 3-digit whole numbers ○ LCM and HCF of numbers to at least 3-digit whole numbers, by inspection or factorisation	○ rational numbers ○ irrational numbers **Calculations using whole numbers** • Revise: ○ calculations using all four operations on whole numbers, estimating and using calculators where appropriate **Calculation strategies** • Use a range of strategies to perform and check written and mental calculations of whole numbers including: ○ long division ○ adding, subtracting and multiplying in columns ○ estimation ○ rounding off and compensating ○ using a calculator **Multiples and factors** • Use prime factorisation of numbers to find LCM and HCF

	• Find the LCM and HCF of numbers to at least 3-digit whole numbers, by inspection or factorisation **Solving problems** • Solve problems involving whole numbers, including ○ comparing two or more quantities of the same kind (ratio) ○ comparing two quantities of different kinds (rate) ○ sharing in a given ratio where the whole is given • Solve problems that involve whole numbers, percentages and decimal fractions in financial contexts such as: ○ profit, loss and discount ○ budgets ○ accounts ○ loans ○ simple interest	**Solving problems** • Solve problems involving whole numbers, including ○ comparing two or more quantities of the same kind (ratio) ○ comparing two quantities of different kinds (rate) ○ sharing in a given ratio where the whole is given increase or decreasing of a number in a given ratio • Solve problems that involve whole numbers, percentages and decimal fractions in financial contexts such as: ○ profit, loss, discount and VAT ○ budgets ○ accounts ○ loans ○ simple interest ○ higher purchase ○ exchange rates	**Solving problems** • Solve problems in contexts involving ○ ratio and rate ○ direct and indirect proportion • Solve problems that involve whole numbers, percentages and decimal fractions in financial contexts such as: ○ profit, loss, discount and VAT ○ budgets ○ accounts ○ loans ○ ○ Simple interest ○ higher purchase ○ exchange rates ○ commission ○ rentals ○ compound interest

1.2 Exponents	**Mental calculations** • Determine squares to at least $12^2$and their square roots • Determine cubes to at least 6^3 and cube roots **Comparing and representing numbers in exponential form** • Compare and represent whole numbers in exponential form: $a^b = a \times a \times a \times$ ... for b number of factors **Calculations using numbers in exponential form** • Recognize and use the appropriate laws of operations with numbers involving exponents and square and cube roots • Perform calculations involving all four operations using numbers in exponential form, limited to exponents up to 5, and square and cube roots	**Mental calculations** • Revise: ○ Squares to at least 12^2 and their square roots ○ Cubes to at least 6^3 and their cube roots **Comparing and representing numbers in exponential form** • Revise: ○ compare and represent whole numbers in exponential form ○ compare and represent integers in exponential form ○ compare and represent numbers in scientific notation, limited to positive exponents **Calculations using numbers in exponential form** • Establish general laws of exponents, limited to: ○ natural number exponents ○ $a^m \times a^n = a^{m+n}$ ○ $a^m \div a^n = a^{m-n}$ ○ $(a^m)^n = a^{m \times n}$ ○ $(n \times t)^n = a^n t^n$ ○ $a^0 = 1$ • Recognize and use the appropriate laws of operations using numbers involving exponents and square and cube roots • Perform calculations involving all four operations with numbers that involve the squares, cubes, square roots and cube roots of integers • Calculate the squares, cubes, square roots and cube roots of rational numbers	**Comparing and representing numbers in exponential form** • Revise: ○ compare and represent integers in exponential form ○ compare and represent numbers in scientific notation • Extend scientific notation to include negative exponents **Calculations using numbers in exponential form** • Revise the following general laws of exponents: ○ $a^m \times a^n = a^{m+n}$ ○ $a^m \div a^n = a^{m-n}$ ○ $(a^m)^n = a^{m \times n}$ ○ $(n \times t)^n = a^n t^n$ ○ $a^0 = 1$ • Extend the general laws of exponents to include: ○ integer exponents ○ $a^{-m} = \frac{1}{a^m}$ • Perform calculations involving all four operations using numbers in exponential form, using the laws of exponents

	Solving problems • Solve problems in contexts involving numbers in exponential form.	**Solving problems** • Solve problems in contexts involving numbers in exponential form	**Solving problems** • Solve problems in contexts involving numbers in exponential form, including scientific notation

1.3 Integers	**Counting, ordering and comparing integers**
	• Count forwards and backwards in integers for any interval
	• Recognize, order and compare integers
	
	Calculations with integers
	• Add and subtract with integers
	
	Properties of integers
	• Recognise and use commutative and associative properties of addition and multiplication for integers
	
	Solving problems
	Solve problems in contexts involving addition and subtraction with integers
		Counting, ordering and comparing integers
		• Revise:
		  ○ counting forwards and backwards in integers for any interval
		  ○ recognizing, ordering and comparing integers
		
		Calculations with integers
		• Revise:
		  ○ addition and subtraction with integers
		• Multiply and divide with integers
		• Perform calculations involving all four operations with integers
		• Perform calculations involving all four operations with numbers that involve the squares, cubes, square roots and cube roots of integers
		
		Properties of integers
		• Recognise and use commutative, associative and distributive properties of addition and multiplication for integers
		• Recognize and use additive and multiplicative inverses for integers
		
		Solving problems
		Solve problems in contexts involving multiple operations with integers
			Calculations with integers
			• Revise:
			  ○ perform calculations involving all four operations with integers
			  ○ perform calculations involving all four operations with numbers that involve the squares, cubes, square roots and cube roots of integers
			
			Properties of integers
			• Revise:
			  ○ Commutative, associative and distributive properties of addition and multiplication for integers
			  ○ additive and multiplicative inverses for integers
			
			Solving problems
			Solve problems in contexts involving multiple operations with integers

1.4 Common Fractions	**Ordering, comparing and simplifying fractions**
	• Revise:
	  ○ compare and order common fractions, including specifically tenths and hundredths
	• Extend to thousandths
	
	Calculations with fractions
	• Revise the following done in Grade 6:
	  ○ addition and subtraction of common fractions, including mixed numbers, limited to fractions with the same denominator or where one denominator is a multiple of another
	  ○ finding fractions of whole numbers
	• Extend addition and subtraction to fractions where one denominator is not a multiple of the other
	• Multiplication of common fractions, including mixed numbers, not limited to fractions where one denominator is a multiple of another
	
	Calculation strategies
	• Convert mixed numbers to common fractions in order to perform calculations with them
	• Use knowledge of multiples and factors to write fractions in the simplest form before or after calculations
	• Use knowledge of equivalent fractions to add and subtract common fractions
	
	Solving problems
	• Solve problems in contexts involving common fractions and mixed numbers, including grouping, sharing and finding fractions of whole numbers
	
	Percentages
	• Revise:
	  ○ Find percentages of whole numbers
		Calculations with fractions
		• Revise:
		  ○ addition and subtraction of common fractions, including mixed numbers
		  ○ finding fractions of whole numbers
		  ○ multiplication of common fractions, including mixed numbers
		• Divide whole numbers and common fractions by common fractions
		• Calculate the squares, cubes, square roots and cube roots of common fractions
		
		Calculation strategies
		• Convert mixed numbers to common fractions in order to perform calculations with them
		• Use knowledge of multiples and factors to write fractions in the simplest form before or after calculations
		• Use knowledge of equivalent fractions to add and subtract common fractions
		• Use knowledge of reciprocal relationships to divide common fractions
		
		Solving problems
		• Solve problems in contexts involving common fractions and mixed numbers, including grouping, sharing and finding fractions of whole numbers
		
		Percentages
		• Revise:
		  ○ finding percentages of whole numbers
			Calculations with fractions
			• All four operations with common fractions and mixed numbers
			• All four operations, with numbers that involve the squares, cubes, square roots and cube roots of common fractions
			
			Calculation strategies
			• Convert mixed numbers to common fractions in order to perform calculations with them
			• Use knowledge of multiples and factors to write fractions in the simplest form before or after calculations
			• Use knowledge of equivalent fractions to add and subtract common fractions
			• Use knowledge of reciprocal relationships to divide common fractions
			
			Solving problems
			• Solve problems in contexts involving common fractions, mixed numbers and percentages

	• Calculate the percentage of part of a whole • Calculate percentage increase or decrease of whole numbers • Solve problems in contexts involving percentages **Equivalent forms** • Revise: ○ recognize and use equivalent forms of common fractions with 1-digit or 2-digit denominators (fractions where one denominator is a multiple of the other) ○ recognize equivalence between common fraction and decimal fraction forms of the same number ○ recognize equivalence between common fraction, decimal fraction and percentage forms of the same number	○ calculating the percentage of part of a whole ○ calculating percentage increase or decrease • Calculate amounts if given percentage increase or decrease • Solve problems in contexts involving percentages **Equivalent forms** • Revise equivalent forms between: ○ common fractions (fractions where one denominator is a multiple of the other) ○ common fraction and decimal fraction forms of the same number ○ common fraction, decimal fraction and percentage forms of the same number	**Equivalent forms** • Revise equivalent forms between: ○ common fractions where one denominator is a multiple of another ○ common fraction and decimal fraction forms of the same number ○ common fraction, decimal fraction and percentage forms of the same number
1.5 Decimal fractions	**Ordering and comparing decimal fractions** • Revise: ○ count forwards and backwards in decimal fractions to at least two decimal places ○ compare and order decimal fractions to at least two decimal places ○ place value of digits to at least two decimal places ○ rounding off decimal fractions to at least 1 decimal place • Extend all of the above to decimal fractions to at least three decimal places and rounding off to at least 2 decimal places **Calculations with decimal fractions** • Revise: ○ addition and subtraction of decimal fractions of at least two decimal places	**Ordering and comparing decimal fractions** • Revise: ○ ordering, comparing and place value of decimal fractions to at least 3 decimal places ○ rounding off decimal fractions to at least 2 decimal place **Calculations with decimal fractions** • Revise: ○ addition, subtraction, multiplication and of decimal fractions to at least 3 decimal places	**Calculations with decimal fractions** • Multiple operations with decimal fractions, using a calculator where appropriate • Multiple operations, with numbers that involve the squares, cubes, square roots and cube roots of decimal fractions

	○ multiplication of decimal fractions by 10 and 100 • Extend addition and subtraction to decimal fractions of at least three decimal places • Multiply decimal fractions to include: ○ decimal fractions to at least 3 decimal places by whole numbers ○ decimal fractions to at least 2 decimal places by decimal fractions to at least 1 decimal place • Divide decimal fractions to include: decimal fractions to at least 3 decimal places by whole numbers **Calculation strategies** • Use knowledge of place value to estimate the number of decimal places in the result before performing calculations • Use rounding off and a calculator to check results where appropriate **Solving problems** • Solve problems in context involving decimal fractions **Equivalent forms** • Revise: ○ recognize equivalence between common fraction and decimal fraction forms of the same number ○ recognize equivalence between common fraction, decimal fraction and percentage forms of the same number	○ division of decimal fractions by whole numbers • Extend multiplication to multiplication by decimal fractions not limited to one decimal place • Extend division to division of decimal fractions by decimal fractions • Calculate the squares, cubes, square roots and cube roots of decimal fractions **Calculation strategies** • Use knowledge of place value to estimate the number of decimal places in the result before performing calculations • Use rounding off and a calculator to check results where appropriate **Solving problems** • Solve problems in context involving decimal fractions **Equivalent forms** • Revise equivalent forms between: ○ common fraction and decimal fraction forms of the same number ○ common fraction, decimal fraction and percentage forms of the same number	**Calculation strategies** • Use knowledge of place value to estimate the number of decimal places in the result before performing calculations • Use rounding off and a calculator to check results where appropriate **Solving problems** • Solve problems in context involving decimal fractions **Equivalent forms** Revise equivalent forms between: • common fraction and decimal fraction forms of the same number • common fraction, decimal fraction and percentage forms of the same number

SENIOR PHASE OVERVIEW
2. PATTERNS, FUNCTIONS AND ALGEBRA

- Progression in Patterns, Functions and Algebra is achieved primarily by
  - increasing the range and complexity of
    - relationships between numbers in given patterns
    - rules, formulae and equations for which input and output values can be found
    - equations that can be solved
  - developing more sophisticated skills and techniques for
    - solving equations
    - expanding and simplifying algebraic expressions
    - drawing and interpreting graphs
  - developing the use of algebraic language and conventions.
- In Patterns, Functions and Algebra, learners' conceptual development progresses from
  - an understanding of number to an understanding of variables, where the variables are numbers of a given type (e.g. natural numbers, integers, rational numbers) in generalized form
  - the recognition of patterns and relationships to the recognition of functions, where functions have unique outputs values for specified input values
  - a view of Mathematics as memorized facts and separate topics to seeing Mathematics as interrelated concepts and ideas represented in a variety of equivalent forms (e.g. a number pattern, an equation and a graph representing the same relationship)
- While techniques for solving equations are developed in Patterns, Functions and Algebra, learners also practise solving equations in Measurement and Space and Shape, when they apply known formulae to solve problems.

TOPICS	GRADE 7	GRADE 8	GRADE 9
2.1 Numeric and geometric patterns	**Investigate and extend patterns** • Investigate and extend numeric and geometric patterns looking for relationships between numbers, including patterns ○ represented in physical or diagram form ○ not limited to sequences involving a constant difference or ratio ○ of learner's own creation ○ represented in tables • Describe and justify the general rules for observed relationships between numbers in own words	**Investigate and extend patterns** • Investigate and extend numeric and geometric patterns looking for relationships between numbers , including patterns ○ represented in physical or diagram form ○ not limited to sequences involving a constant difference or ratio ○ of learner's own creation ○ represented in tables ○ represented algebraically • Describe and justify the general rules for observed relationships between numbers in own words or in algebraic language	**Investigate and extend patterns** • Investigate and extend numeric and geometric patterns looking for relationships between numbers , including patterns ○ represented in physical or diagram form ○ not limited to sequences involving a constant difference or ratio ○ of learner's own creation ○ represented in tables ○ represented algebraically • Describe and justify the general rules for observed relationships between numbers in own words or in algebraic language

2.2 Functions and relationships	**Input and output values** • Determine input values, output values or rules for patterns and relationships using ○ flow diagrams ○ tables ○ formulae **Equivalent forms** • Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented ○ verbally ○ in flow diagrams ○ in tables ○ by formulae ○ by number sentences	**Input and output values** • Determine input values, output values or rules for patterns and relationships using ○ flow diagrams ○ tables ○ formulae ○ equations **Equivalent forms** • Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented ○ verbally ○ in flow diagrams ○ in tables ○ by formulae ○ by equations	**Input and output values** • Determine input values, output values or rules for patterns and relationships using ○ flow diagrams ○ tables ○ formulae ○ equations **Equivalent forms** • Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented ○ verbally ○ in flow diagrams ○ in tables ○ by formulae ○ by equations ○ by graphs on a Cartesian plane
2.3 Algebraic expressions	**Algebraic language** • Recognize and interpret rules or relationships represented in symbolic form • Identify variables and constants in given formulae or equations	**Algebraic language** • Revise the following done in Grade 7: ○ recognize and interpret rules or relationships represented in symbolic form ○ identify variables and constants in given formulae or equations • Recognize and identify conventions for writing algebraic expressions • Identify and classify like and unlike terms in algebraic expressions • Recognize and identify coefficients and exponents in algebraic expressions	**Algebraic language** • Revise the following done in Grade 8: ○ recognize and identify conventions for writing algebraic expressions ○ identify and classify like and unlike terms in algebraic expressions ○ recognize and identify coefficients and exponents in algebraic expressions • Recognize and differentiate between monomials, binomials and trinomials

		Expand and simplify algebraic expressions
		Use commutative, associative and distributive laws for rational numbers and laws of exponents
		• Add and subtract like terms in algebraic expressions
		• Multiply integers and monomials by:
		  ○ monomials
		  ○ binomials
		  ○ trinomials
		
		• Divide the following by integers or monomials:
		  ○ Monomials
		  ○ Binomials
		  ○ trinomials
		• Simplify algebraic expressions involving the above operations
		• Determine the squares, cubes, square roots and cube roots of single algebraic terms or like algebraic terms
		• Determine the numerical value of algebraic expressions by substitution
			Expand and simplify algebraic expressions
			• Revise the following done in Grade 8, using the commutative, associative and distributive laws for rational numbers and laws of exponents to:
			  ○ Add and subtract like terms in algebraic expressions
			  ○ Multiply integers and monomials by:
			  ▪ monomials
			  ▪ binomials
			  ▪ trinomials
			  ○ Divide the following by integers or monomials:
			  ▪ Monomials
			  ▪ Binomials
			  ▪ trinomials
			  ○ Simplify algebraic expressions involving the above operations
			  ○ Determine the squares, cubes, square roots and cube roots of single algebraic terms or like algebraic terms
			  ○ Determine the numerical value of algebraic expressions by substitution
			• Extend the above algebraic manipulations to include:
			  ○ Multiply integers and monomials by polynomials
			  ○ Divide polynomials by integers or monomials
			  ○ The product of two binomials
			  ○ The square of a binomial
			
			Factorize algebraic expressions
			• Factorize algebraic expressions that involve:
			  ○ common factors
			  ○ difference of two squares
			  ○ trinomials of the form:
			  ▪ $x^2 + bx + c$
			  ▪ $ax^2 + bx + c$, where a is a common factor.
			○ Simplify algebraic expressions that involve the above factorisation processes.
			○ Simplify algebraic fractions using factorisation.

2.4 Algebraic equations	**Number sentences** • Write number sentences to describe problem situations • Analyse and interpret number sentences that describe a given situation • Solve and complete number sentences by: ○ inspection ○ trial and improvement • Identify variables and constants in given formulae or equations	**Equations** • Revise the following done in Grade 7: ○ set up equations to describe problem situations ○ analyse and interpret equations that describe a given situation ○ solve equations by Inspection ○ determine the numerical value of an expression by substitution. • Extend solving equations to include: ○ using additive and multiplicative inverses ○ using laws of exponents • Use substitution in equations to generate tables of ordered pairs	**Equations** • Revise the following done in Grade 8: ○ set up equations to describe problem situations ○ analyse and interpret equations that describe a given situation ○ solve equations by inspection ○ determine the numerical value of an expression by substitution. • Extend solving equations to include: ○ using additive and multiplicative inverses ○ using laws of exponents ○ using factorisation ○ equations of the form: a product of factors = 0 • Use substitution in equations to generate tables of ordered pairs
2.5 Graphs	**Interpreting graphs** • Analyse and interpret global graphs of problem situations, with special focus on the following trends and features: ○ linear or non-linear ○ constant, increasing or decreasing **Drawing graphs** • Draw global graphs from given descriptions of a problem situation, identifying features listed above	**Interpreting graphs** • Revise the following done in Grade 7: ○ analyse and interpret global graphs of problem situations, with a special focus on the following trends and features: ▪ linear or non-linear ▪ constant, increasing or decreasing • Extend the focus on features of graphs to include: ○ maximum or minimum ○ discrete or continuous **Drawing graphs** • Draw global graphs from given descriptions of a problem situation, identifying features listed above • Use tables or ordered pairs to plot points and draw graphs on the Cartesian plane	**Interpreting graphs** • Revise the following done in Grade 8: ○ analyse and interpret global graphs of problem situations, with a special focus on the following trends and features: ▪ linear or non-linear ▪ constant, increasing or decreasing ▪ maximum or Minimum ▪ discrete or continuous • Extend the above with special focus on the following features of **linear graphs**: ○ *x*-intercept and *y*-intercept ○ gradient **Drawing graphs** • Revise the following done in Grade 8: ○ draw global graphs from given descriptions of a problem situation, identifying features listed above. ○ use tables of ordered pairs to plot points and draw graphs on the Cartesian plane

			• Extend the above with special focus on: ○ drawing **linear graphs** from given equations ○ determining equations from given **linear graphs**

SENIOR PHASE OVERVIEW
3 SPACE AND SHAPE (GEOMETRY)

- Progression in geometry in the Senior Phase is achieved primarily by:
  - investigating new properties of shapes and objects
  - developing from informal descriptions of geometric figures to more formal definitions and classification of shapes and objects
  - solving more complex geometric problems using known properties of geometric figures
  - developing from inductive reasoning to deductive reasoning.
- The geometry topics are much more inter-related than in the Intermediate Phase, especially those relating to constructions and geometry of 2D shapes and straight lines, hence care has to be taken regarding sequencing of topics through the terms.
- In the Senior Phase, transformation geometry develops from general descriptions of movement in space to more specific descriptions of movement in co-ordinate planes. This lays the foundation for analytic geometry in the FET phase.
- Solving problems in geometry to find unknown angles or lengths provides a useful context to practise solving equations.

CONTENT	GRADE 7	GRADE 8	GRADE 9
3.1 Geometry of 2D shapes	**Classifying 2D shapes** • Describe, sort, name and compare triangles according to their sides and angles, focusing on: ○ equilateral triangles ○ isosceles triangles ○ right-angled triangles • Describe, sort, name and compare quadrilaterals in terms of: ○ length of sides ○ parallel and perpendicular sides ○ size of angles (right-angles or not) • Describe and name parts of a circle	**Classifying 2D shapes** • Identify and write clear definitions of triangles in terms of their sides and angles, distinguishing between: ○ equilateral triangles ○ isosceles triangles ○ right-angled triangles • Identify and write clear definitions of quadrilaterals in terms of their sides and angles, distinguishing between: ○ parallelogram ○ rectangle ○ square ○ rhombus ○ trapezium ○ kite	**Classifying 2D shapes** • Revise properties and definitions of triangles in terms of their sides and angles, distinguishing between: ○ equilateral triangles ○ isosceles triangles ○ right-angled triangles • Revise and write clear definitions of quadrilaterals in terms of their sides, angles and diagonals, distinguishing between: ○ parallelogram ○ rectangle ○ square ○ rhombus ○ trapezium ○ kite

	Similar and congruent 2D shapes
	• Recognize and describe similar and congruent figures by comparing:
	  ○ shape
	  ○ size
	
	Solving problems
	• Solve simple geometric problems involving unknown sides and angles in triangles and quadrilaterals, using known properties and definitions.
		Similar and congruent 2D shapes
		• Identify and describe the properties of congruent shapes
		• Identify and describe the properties of similar shapes
		
		Solving problems
		• Solve geometric problems involving unknown sides and angles in triangles and quadrilaterals, using known properties and definitions.
			Similar and congruent triangles
			• Through investigation, establish the minimum conditions for congruent triangles
			• Through investigation, establish the minimum conditions for similar triangles
			
			Solving problems
			• Solve geometric problems involving unknown sides and angles in triangles and quadrilaterals, using known properties of triangles and quadrilaterals, as well as properties of congruent and similar triangles.

3.2 Geometry of 3D objects	**Classifying 3D objects**
	• Describe, sort and compare polyhedra in terms of:
	  ○ shape and number of faces
	  ○ number of vertices
	  ○ number of edges
	
	Building 3D models
	• Revise using nets to create models of geometric solids, including:
	  ○ cubes
	  ○ prisms
		Classifying 3D objects
		• Describe, name and compare the 5 Platonic solids in terms of the shape and number of faces, the number of vertices and the number of edges
		
		Building 3D models
		• Revise using nets to create models of geometric solids, including:
		  ○ cubes
		  ○ prisms
		  ○ pyramids
			Classifying 3D objects
			• Revise properties and definitions of the 5 Platonic solids in terms of the shape and number of faces, the number of vertices and the number of edges
			• Recognize and describe the properties of:
			  ○ spheres
			  ○ cylinders
			
			Building 3D models
			• Use nets to create models of geometric solids, including:
			  ○ cubes
			  ○ prisms
			  ○ pyramids
			  ○ cylinders

3.3 Geometry of straight lines	**Define:**
	• Line segment
	• Ray
	• Straight line
	• Parallel lines
	• Perpendicular lines
		Angle relationships
		• Recognize and describe pairs of angles formed by:
		  ○ perpendicular lines
		  ○ intersecting lines
		  ○ parallel lines cut by a transversal
		
		Solving problems
		• Solve geometric problems using the relationships between pairs of angles described above
			Angle relationships
			• Revise and write clear descriptions of the relationship between angles formed by:
			  ○ perpendicular lines
			  ○ intersecting lines
			  ○ parallel lines cut by a transversal
			
			Solving problems
			• Solve geometric problems using the relationships between pairs of angles described above

3.4 Transformation Geometry	**Transformations** • Recognize, describe and perform translations, reflections and rotations with geometric figures and shapes on squared paper • Identify and draw lines of symmetry in geometric figures **Enlargements and reductions** • Draw enlargements and reductions of geometric figures on squared paper and compare them in terms of shape and size	**Transformations** • Recognize, describe and perform transformations with points on a co-ordinate plane, focusing on: ○ reflecting a point in the Y-axis or X-axis ○ translating a point within and across quadrants • Recognize, describe and perform transformations with triangles on a co-ordinate plane, focusing on the co-ordinates of the vertices when: ○ reflecting a triangle in the x-axis or y-axis ○ translating a triangle within and across quadrants ○ rotating a triangle around the origin **Enlargements and reductions** • Use proportion to describe the effect of enlargement or reduction on area and perimeter of geometric figures	**Transformations** • Recognize, describe and perform transformations with points, line segments and simple geometric figures on a co-ordinate plane, focusing on: ○ reflection in the Y-axis or X-axis ○ translation within and across quadrants ○ reflection in the line $y = x$ • Identify what the transformation of a point is, if given the co-ordinates of its image **Enlargements and reductions** • Use proportion to describe the effect of enlargement or reduction on area and perimeter of geometric figures • Investigate the co-ordinates of the vertices of figures that have been enlarged or reduced by a given scale factor
3.5 Construction of geometric figures	**Measuring angles** • Accurately use a protractor to measure and classify angles in terms of: ○ acute ○ right ○ obtuse ○ straight ○ reflex **Constructions** • Accurately construct geometric figures appropriately using a compass, ruler and protractor, including: ○ angles, to one degree of accuracy	**Constructions** • Accurately construct geometric figures appropriately using a compass, ruler and protractor, including: ○ bisecting lines and angles	**Constructions** • Accurately construct geometric figures appropriately using a compass, ruler and protractor, including bisecting angles of a triangle

	○ circles ○ parallel lines ○ perpendicular lines	○ perpendicular lines at a given point or from a given point ○ triangles ○ quadrilaterals • Construct angles of 30°, 45°, 60° and their multiples without using a protractor **Investigating properties of geometric figures** • By construction, investigate the angles in a triangle, focusing on: ○ the sum of the interior angles of triangles ○ the size of angles in an equilateral triangle ○ the sides and base angles of an isosceles triangle • By construction, investigate sides and angles in quadrilaterals, focusing on: ○ the sum of the interior angles of quads ○ the sides and opposite angles of parallelograms	• Construct angles of 45° , 30° and 60° and their multiples without using a protractor **Investigating properties of geometric figures** • By construction, investigate the angles in a triangle, focusing on the relationship between the exterior angle of a triangle and its interior angles • By construction, investigate sides, angles and diagonals in quadrilaterals, focusing on: ○ the diagonals of rectangles, squares, parallelograms, rhombi and kites ○ exploring the sum of the interior angles of polygons

SENIOR PHASE OVERVIEW
4. MEASUREMENT

- Progression in Measurement is achieved by the selection of shapes and objects in each grade for which the formulae for finding area, perimeter, surface area and volume become more complex.
- The use of formulae in this phase provides a useful context to practise solving equations.
- The introduction of the Theorem of Pythagoras is a way of introducing a formula to calculate the lengths of sides in right-angled triangles. Hence, the Theorem of Pythagoras becomes a useful tool when learners solve geometric problems involving right-angled triangles.
- Measurement disappears as a separate topic in the FET phase, and becomes part of the study of Geometry and Trigonometry.

CONTENT	GRADE 7	GRADE 8	GRADE 9
4.1 Area and perimeter of 2D shapes	**Area and Perimeter** • Calculate the perimeter of regular and irregular polygons • Use appropriate formulae to calculate perimeter and area of: ○ squares ○ rectangles ○ triangles **Calculations and solving problems** • Solve problems involving perimeter and area of polygons • Calculate to at least 1 decimal place • Use and convert between appropriate SI units, including: ○ $mm^2 \leftrightarrow cm^2$ ○ $cm^2 \leftrightarrow m^2$	**Area and perimeter** • Use appropriate formulae to calculate perimeter and area of: ○ squares ○ rectangles ○ triangles ○ circles • Calculate the areas of polygons, to at least 2 decimal places, by decomposing them into rectangles and/or triangles • Use and describe the relationship between the radius, diameter and circumference of a circle in calculations • Use and describe the relationship between the radius and area of a circle in calculations **Calculations and solving problems** • Solve problems, with or without a calculator, involving perimeter and area of polygons and circles • Calculate to at least 2 decimal places • Use and describe the meaning of the irrational number **Pi** (π) in calculations involving circles • Use and convert between appropriate SI units, including: $mm^2 \leftrightarrow cm^2 \leftrightarrow m^2 \leftrightarrow km^2$	**Area and perimeter** • Use appropriate formulae and conversions between SI units, to solve problems and calculate perimeter and area of: ○ polygons ○ circles • Investigate how doubling any or all of the dimensions of a 2D figure affects its perimeter and its area

4.2 Surface area and volume of 3D objects	**Surface area and volume** • Use appropriate formulae to calculate the surface area, volume and capacity of: ○ cubes ○ rectangular prisms • Describe the interrelationship between surface area and volume of the objects mentioned above **Calculations and solving problems** • Solve problems involving surface area, volume and capacity • Use and convert between appropriate SI units, including: ○ $mm^2 \leftrightarrow cm^2$ ○ $cm^2 \leftrightarrow m^2$ ○ $mm^3 \leftrightarrow cm^3$ ○ $cm^3 \leftrightarrow m^3$ • Use equivalence between units when solving problems: ○ $1\ cm^3 = 1\ ml$ ○ $1\ m^3 = 1\ kl$	**Surface area and volume** • Use appropriate formulae to calculate the surface area, volume and capacity of: ○ cubes ○ rectangular prisms ○ triangular prisms • Describe the interrelationship between surface area and volume of the objects mentioned above **Calculations and solving problems** • Solve problems, with or without a calculator, involving surface area, volume and capacity • Use and convert between appropriate SI units, including: ○ $mm^2 \leftrightarrow cm^2 \leftrightarrow m^2 \leftrightarrow km^2$ ○ $mm^3 \leftrightarrow cm^3 \leftrightarrow m^3$ ○ $ml\ (cm)^3 \leftrightarrow l \leftrightarrow kl$	**Surface area and volume** • Use appropriate formulae and conversions between SI units to solve problems and calculate the surface area, volume and capacity of: ○ cubes ○ rectangular prisms ○ triangular prisms ○ cylinders • Investigate how doubling any or all the dimensions of right prisms and cylinders affects their volume
4.3 The Theorem of Pythagoras		**Develop and use the Theorem of Pythagoras** • Investigate the relationship between the lengths of the sides of a right-angled triangle to develop the Theorem of Pythagoras • Determine whether a triangle is a right-angled triangle or not, if the length of the three sides of the triangle are known • Use the Theorem of Pythagoras to calculate a missing length in a right-angled triangle, leaving irrational answers in surd form	**Solve problems using the Theorem of Pythagoras** • Use the Theorem of Pythagoras to solve problems involving unknown lengths in geometric figures that contain right-angled triangles

SENIOR PHASE OVERVIEW
5. DATA HANDLING

- Progression in Data Handling is achieved primarily by:
  - increasing complexity of data sets and contexts
  - reading, interpreting and drawing new types of data graphs
  - becoming more efficient at organizing and summarizing data
  - becoming more critical and aware of bias and manipulation in representing, analysing and reporting data
- Learners should work through at least 1 data cycle for the year – this involves collecting and organizing, representing, analysing, summarizing, interpreting and reporting data. The data cycle provides the opportunity for doing projects.
- All of the above aspects of data handling should also be dealt with as discrete activities in order to consolidate concepts and practise skills. For example, learners need to practise summarizing data presented in different forms, and summaries should be used when reporting data.
- Data handling contexts should be selected to build awareness of social, economic and environmental issues.
- Learners should become sensitized to bias in the collection of data, as well as misrepresentation of data through the use of different scales and different measures of central tendency.
- The following resources provide interesting contexts for data comparison and analysis that can be used in this phase:
  - Census at School – for school based surveys
  - national surveys from Statistics South Africa (StatsSA) – for household and population surveys.
  - international surveys from United Nations (UN Data) – for international social, demographic and environmental surveys. Many other websites may be consulted, especially for health and environmental data.

CONTENT	GRADE 7	GRADE 8	GRADE 9

5.1 Collect, organize and summarize data	**Collect data**
	• Pose questions relating to social, economic, and environmental issues in own environment
	• Select appropriate sources for the collection of data (including peers, family, newspapers, books, magazines), including distinguishing between samples and populations
	• Design and use simple questionnaires to answer questions:
	  ○ with yes/no type responses
	  ○ with multiple choice responses
	
	Organize and summarize data
	• Organize (including grouping where appropriate) and record data using
	  ○ tallies
	  ○ tables
		Collect data
		• Pose questions relating to social, economic, and environmental issues
		• Select appropriate sources for the collection of data (including peers, family, newspapers, books, magazines), including distinguishing between samples and populations
		• Design and use simple questionnaires to answer questions with multiple choice responses
		
		Organize and summarize data
		• Organize (including grouping where appropriate) and record data using
		  ○ tallies
		  ○ tables
			Collect data
			• Pose questions relating to social, economic, and environmental issues
			• Select and justify appropriate sources for the collection of data
			• Select and justify appropriate methods for collecting data
			
			Organize and summarize data
			• Organize numerical data in different ways in order to summarize by determining:
			  ○ measures of central tendency
			  ○ measures of dispersion, including extremes

	○ stem-and-leaf displays • Group data into intervals • Summarize and distinguishing between ungrouped numerical data by determining: ○ mean ○ median ○ mode • Identify the largest and smallest scores in a data set and determine the difference between them in order to determine the spread of the data (range)	○ stem-and-leaf displays • Group data into intervals • Summarize data using measures of central tendency, including: ○ mean ○ median ○ mode • Summarize data using measures of dispersion, including: ○ range ○ extremes	and outliers • Organize data according to more than one criteria
5.2 Represent data	**Represent data** • Draw a variety of graphs by hand/technology to display and interpret data (grouped and ungrouped) including: ○ bar graphs and double bar graphs ○ histograms with given intervals ○ pie charts	**Represent data** • Draw a variety of graphs by hand/technology to display and interpret data including**:** ○ bar graphs and double bar graphs ○ histograms with given and own intervals ○ pie charts ○ broken-line graphs	**Represent data** • Draw a variety of graphs by hand/technology to display and interpret data including: ○ bar graphs and double bar graphs ○ histograms with given and own intervals ○ pie charts ○ broken-line graphs ○ scatter plots
5.3 Analyse, Interpret, and Report data	**Interpret data** • Critically read and interpret data represented in: ○ words ○ bar graphs ○ double bar graphs ○ pie charts ○ histograms **Analyse data** • Critically analyse data by answering questions related to: ○ data categories, including data intervals ○ data sources and contexts ○ central tendencies – (mean, mode, median) ○ scales used on graphs	**Interpret data** • Critically read and interpret data represented in: ○ words ○ bar graphs ○ double bar graphs ○ pie charts ○ histograms ○ broken-line graphs **Analyse data** • Critically analyse data by answering questions related to: ○ data categories, including data intervals ○ data sources and contexts ○ central tendencies – (mean, mode, median) ○ scales used on graphs ○ samples and populations ○ dispersion of data	**Interpret data** • Critically read and interpret data represented in a variety of ways • Critically compare two sets of data related to the same issue **Analyse data** • Critically analyse data by answering questions related to: ○ data collection methods ○ summary of data ○ Sources of error and bias in the data

	Report data • Summarize data in short paragraphs that include ○ drawing conclusions about the data ○ making predictions based on the data ○ identifying sources of error and bias in the data ○ choosing appropriate summary statistics for the data (mean, median, mode, range)	○ error and bias in the data **Report data** • Summarize data in short paragraphs that include ○ drawing conclusions about the data ○ making predictions based on the data ○ identifying sources of error and bias in the data ○ choosing appropriate summary statistics for the data (mean, median, mode, range) ○ the role of extremes in the data	**Report data** • Summarize data in short paragraphs that include ○ drawing conclusions about the data ○ making predictions based on the data ○ making comparisons between two sets of data ○ identifying sources of error and bias in the data ○ choosing appropriate summary statistics for the data (mean, median, mode, range) ○ the role of extremes and outliers in the data
5.4 Probability	**Probability** • Perform simple experiments where the possible outcomes are equally likely and: ○ list the possible outcomes based on the conditions of the activity ○ determine the relative frequency of actual outcomes for a series of trials	**Probability** • Consider a simple situation (with equally likely outcomes) that can be described using probability and: ○ list all the possible outcomes ○ determine the probability of each possible outcome using the definition of probability ○ predict with reasons the relative frequency of the possible outcomes for a series of trials based on probability ○ compare relative frequency with probability and explains possible differences	**Probability** • Consider situations with equally probable outcomes, and: ○ determine probabilities for compound events using two-way tables and tree diagrams ○ determine the probabilities for outcomes of events and predict their relative frequency in simple experiments ○ compare relative frequency with probability and explains possible differences

SECTION 3
CURRICULUM AND ASSESSMENT POLICY STATEMENT (CAPS)

CONTENT CLARIFICATION

3.1. Introduction

- In this chapter, content clarification includes:
    - teaching guidelines
    - suggested sequencing of topics per term
    - suggested pacing of topics over the year.
- Each Content Area has been broken down into topics. The sequencing of topics within terms gives an idea of how content areas can be spread and re-visited throughout the year.
- Teachers may choose to sequence and pace the contents differently from the recommendations in this section. However, cognisance should be taken of the relative weighting and notional hours of the Content Areas for this phase.

3.2. Allocation of teaching time

Time has been allocated in the following way:

- 10 weeks per term, with 4,5 hours for Mathematics per week (10 x 4 x 4,5 hours = 180 hrs per year)
- Between 6 and 12 hours have been allocated for revision and assessment per term
- Therefore, 150 notional hours have been distributed across the Content Areas
- The distribution of time per topic, has taken account of the weighting for the Content Area as specified for the Senior Phase in Section 2.
- The weighting of Content Areas represents notional hours; therefore, the recommended distribution of hours may vary slightly across grades.

Table 3.1: Allocation of teaching time for grade 7

Time allocation per Topic: Grade 7							
TERM 1		**TERM 2**		**TERM 3**		**TERM 4**	
Topic	**Time**	**Topic**	**Time**	**Topic**	**Time**	**Topic**	**Time**
Whole numbers Ratio and finance	**9 hours**	Common fractions Ratio and finance	**9 hours**	Numeric and geometric patterns	**6 hours**	Integers	**9 hours**
Exponents	**9 hours**	Decimal fractions	**9 hours**	Functions and relationships	**3 hours**	Numeric and geometric patterns	**3 hours**
Constructions	**10 hours**	Functions and relationships	**3 hours**	Algebraic expressions	**3 hours**	Functions and relationships	**3 hours**
Geometry of 2D shapes	**10 hours**	Area and perimeter	**7 hours**	Algebraic equations	**3 hours**	Algebraic expressions	**3 hours**
		Volume and surface area	**8 hours**	Graphs	**6 hours**	Algebraic equations	**4 hours**
				Transformation geometry	**9 hours**	Collect, organize and summarize data	**4 hours**
				Geometry of 3D objects	**9 hours**	Represent data	**3 hours**
						Analyse, interpret, report data	**3,5 hours**
						Probability	**4,5 hours**
Revision/ assessment	**7 hours**	**Revision/ assessment**	**9 hours**	**Revision/ assessment**	**6 hours**	**Revision/ assessment**	**8hours**
TOTAL: 45 hours		**TOTAL: 45 hours**		**TOTAL: 45 hours**		**TOTAL: 45 hours**	

Table 3.2: Allocation of teaching time for grade 8

Time allocation per Term: Grade 8							
TERM 1		**TERM 2**		**TERM 3**		**TERM 4**	
Topic	**Time**	**Topic**	**Time**	**Topic**	**Time**	**Topic**	**Time**
Whole numbers	**6 hours**	Algebraic expressions	**9 hours**	Common fractions	**7 hours**	Functions and relationships	**6 hours**
Integers	**9 hours**	Algebraic equations	**3 hours**	Decimal fractions	**6 hours**	Algebraic equations	**3 hours**
Exponents	**9 hours**	Constructions	**8 hours**	Theorem of Pythagoras	**5 hours**	Graphs	**9 hours**
Numeric and geometric patterns	**4,5 hours**	Geometry of 2D shapes	**8 hours**	Area and perimeter	**5 hours**	Transformation geometry	**6 hours**
Functions and relationships	**3 hours**	Geometry of straight lines	**9 hours**	Volume and surface Area	**5 hours**	Geometry of 3D objects	**7 hours**
Algebraic expressions	**4,5 hours**			Collect, organize and summarize data	**4 hours**	Probability	**4,5 hours**
Algebraic equations	**3 hours**			Represent data	**3 hours**		
				Analyse, interpret, report data	**3,5 hours**		
Revision/ Assessment	**6 hours**	**Revision/ Assessment**	**8 hours**	**Revision/ Assessment**	**6,5 hours**	**Revision/ Assessment**	**9,5 hours**
TOTAL: 45 hours		**TOTAL: 45 hours**		**TOTAL: 45 hours**		**TOTAL: 45 hours**	

Table 3.3: Allocation of teaching time for grade 9

Time allocation per Term: Grade 9							
TERM 1		**TERM 2**		**TERM 3**		**TERM 4**	
Topic	**Time**	**Topic**	**Time**	**Topic**	**Time**	**Topic**	**Time**
Whole numbers	**4,5 hours**	Constructions	**9 hours**	Functions and relationships	**5 hours**	Transformation geometry	**9 hours**
Integers	**4,5 hours**	Geometry of 2-D shapes	**9 hours**	Algebraic expressions	**9 hours**	Geometry of 3-D objects	**9 hours**
Common fractions	**4,5 hours**	Geometry of straight-lines	**9 hours**	Algebraic equations	**9 hours**	Collect, organize and summarize data	**4 hours**
Decimal fractions	**4,5 hours**	Theorem of Pythagoras	**5 hours**	Graphs	**12 hours**	Represent data	**3 hours**
Exponents	**5 hours**	Area and Perimeter	**5 hours**	Volume and Surface Area	**5 hours**	Analyse, interpret, report data	**3,5 hours**
Numeric and geometric patterns	**4,5 hours**					Probability	**4,5 hours**
Functions and relationships	**4 hours**						
Algebraic expressions	**4,5 hours**						
Algebraic equations	**4 hours**						
Revision/ Assessment	**5 hours**	**Revision/ Assessment**	**8 hours**	**Revision/ Assessment**	**5 hours**	**Revision/ Assessment**	**12 hours**
TOTAL: 45 hours		**TOTAL: 45 hours**		**TOTAL: 45 hours**		**TOTAL: 45 hours**	

3.3. Clarification notes with teaching guidelines

The tables below provide the teacher with:

- content areas and topics per grade per term;
- concepts and skills per term;
- clarification notes with teaching guidelines; and
- the duration of time allocated per topic in hours.

3.3.1. Clarification of content for Grade 7

TERM 1 – Grade 7				
CONTENT AREA	**TOPICS**	**CONCEPTS and SKILLS**	**SOME CLARIFICATION NOTES or TEACHING GUIDELINES**	**DURATION (in hours)**

Numbers, Operations and Relationships	**1.1 Whole numbers**	**Mental calculations**
		Revise the following done in Grade 6:
		• Multiplication of whole numbers to at least 12×12
		• Multiplication facts for:
		  ○ Units and tens by multiples of ten
		  ○ Units and tens by multiples of 100
		  ○ Units and tens by multiples of 1000
		  ○ Units and tens by multiples of 10 000
		
		• Inverse operation between multiplication and division
			What is different to Grade 6?
			• Prime factors of 3-digit numbers
			• LCM and HCF
			• More complex financial contexts for solving problems
			
			In Grade 7 learners consolidate number knowledge and calculation techniques for whole numbers, developed in the Intermediate Phase.
			
			Mental calculations
			Mental calculations should be used to practice concepts and skills developed through the main lesson, sometimes with smaller number ranges. Learners should not be asked to do random calculations each day. Rather, mental calculations should be used as an opportunity to consolidate four aspects of learners' number knowledge:
			• number facts (number bonds and times tables)
			• calculation strategies (doubling and halving, using multiplication to do division, multiplying and dividing by 10, 100, 1 000
			• multiplying by multiples of 10, 100, 1 000,
			• building up and breaking down numbers, rounding off and compensating etc)
			• number concept (counting, ordering and comparing, place value, odd and even numbers, multiples and factors)
			• properties of numbers (identity elements for addition and multiplication;
			• commutative and associative property for addition and multiplication;
			• inverse operation for multiplication and division; inverse
				9hours.

			operation for addition and subtraction)	
		Ordering and comparing whole numbers Revise the following done in Grade 6: • Order, compare and represent numbers to at least 9-digit numbers • Recognise and represent prime numbers to at least 100 • Rounding off numbers to the nearest 5, 10, 100 or 1000	**Ordering and comparing numbers** Learners should be given a range of exercises such as: • Arrange given numbers from the smallest to the biggest: or biggest to smallest • Fill in missing numbers in ○ a sequence ○ on a number grid ○ on a number line e.g. which whole number is halfway between $471\ 340$ and $471\ 350$. • Fill in $<$, = or $>$ **Examples:** a) $247\ 889 \quad * \quad 247\ 898$ b) $784\ 109 \quad * \quad 785\ 190$	
		Properties of whole numbers Revise the following done in Grade 6: • Recognise and use the commutative; associative; distributive properties with whole numbers • Recognise and use O in terms of its additive property (identity element for addition) • Recognise and use 1 in terms of its multiplicative property (identity element for multiplication)	**Properties of whole numbers** • Revising the properties of whole numbers should be the starting point for work with whole numbers. The properties of numbers should provide the motivation for why and how operations with numbers work. • When learners are introduced to new numbers, such as integers for example, they can again explore how the properties of numbers work for the new set of numbers. • Learners also have to apply the properties of numbers in algebra, when they work with variables in place of numbers. • Learners should know and be able to use the following properties: ○ The commutative property of addition and multiplication: ▪ $a + b = b + a$ ▪ $a \times b = b \times a$ ○ The associative (grouping) property of addition and multiplication:	

		Calculations with whole numbers
		• Revise the following done in Grade 6, without use of calculators:
		  ○ Addition and subtraction of whole numbers to at least 6 –digit numbers
		  ○ Multiplication of at least whole 4-digit by 2-digit numbers
		  ○ Division of at least whole 4-digit by 2-digit numbers
		• Perform calculations with all four operations on whole numbers, estimating and using calculators where appropriate
		
		Calculation strategies
		• Use a range of strategies to perform and check written and mental calculations with whole numbers including:
		  • estimation
		  • adding, subtracting and multiplying in columns
		  • long division
		  • rounding off and compensating
		  • using a calculator
			▪ $(a + b) + c = a + (b + c)$
			▪ $(a \times b) \times c = a \times (b \times c)$
			○ The distributive property of multiplication over addition and subtraction:
			▪ $a(b + c) = (a \times b) + (a \times c)$
			▪ $a(b - c) = (a \times b) - (a \times c)$
			○ Addition and subtraction as inverse operations
			○ Multiplication and division as inverse operations
			○ 0 is the identity element for addition : $t + 0 = t$
			○ 1 is the identity element for multiplication: $t \times 1 = t$
			
			Illustrating the properties with whole numbers
			a) $33 + 99 = 99 + 33 = 132$
			b) $51 + (19 + 46) = (51 + 19) + 46 = 116$
			c) $4(12 + 9) = (4 \times 12) + (4 \times 9) = 48 + 36 = 84$
			d) $(9 \times 64) + (9 \times 36) = 9 \times (64 + 36) = 9 \times 100 = 900$
			e) If $33 + 99 = 132$, then $132 - 99 = 33$ and $132 - 33 = 99$
			f) If $20 \times 5 = 110$, then $110 \div 20 = 5$ and $110 \div 5 = 20$
			
			Calculations with whole numbers
			• Learners should do context free calculations and solve problems in contexts
			• Learners should become more confident in and more independent at mathematics, if they have strategies
			  ○ to check their solutions themselves, e.g. using inverse operations; using calculators
			  ○ to judge the reasonableness of their solutions e.g. estimate by rounding off; estimate by doubling or halving;
			• Adding, subtracting and multiplying in columns, and long division, should only be used to practice number facts and calculation techniques, and hence should be done with familiar and smaller number ranges. For big and unwieldy calculations,	

			learners should be encouraged to use a calculator.	

		Multiples and factors
		• Revise the following done in Grade 6:
		  ○ Multiples of 2-digit and 3-digit whole numbers
		  ○ Factors of 2-digit and 3-digit whole numbers
		  ○ Prime factors of numbers to at least 100
		• List prime factors of numbers to at least 3-digit whole numbers
		• Find the LCM and HCF of numbers to at least 3-digit whole numbers, by inspection or factorisation
			Multiples and factors
			• Practice with finding multiples and factors of whole numbers are especially important when learners do calculations with fractions. They use this knowledge to find the LCM when one denominator is a multiple of another, and also when they simplify fractions or have to find equivalent fractions.
			• Factorising whole numbers lays the foundation for factorisation of algebraic expressions.
			• Using the definition of prime numbers, emphasise that 1 is not classified as a prime number
			
			Examples
			a) The multiples of 6 are $6, 12, 18, 24, \ldots$ or M6 $= \{6; 12; 18; 24; \ldots\}$
			b) LCM of 6 and 18 is 18
			LCM of 6 and 7 is 42
			c) The factors of 24 are $1, 2, 3, 4, 6, 12$ and 24 by inspection and, the prime factors of 24 are 2 and 3
			d) The factors of 140 are $1, 2, 5, 7, 10, 14, 28, 35, 70$ and 140
			e) Determine the HCF of $120;\ 300$ and 900.
			Learners do this by finding the prime factors of the numbers first.
			$120 = 5 \times 3 \times 2^3$. Initially learners may write this as:
			$5 \times 3 \times 2 \times 2 \times 2$
			$300 = 5^2 \times 3 \times 2^2$
			$900 = 5^2 \times 3^2 \times 2^3$
			HCF = $5 \times 3 \times 2^2 = 60$ (Multiply the common prime factors of the three numbers)	

		Solving problems • Solve problems involving whole numbers, including the following types of problems: ○ Compare two or more quantities of the same kind (ratio) ○ Comparing two quantities of different kinds (rate) ○ Share in a given ratio where the whole is given • Solve problems that involve whole numbers, percentages and decimal fractions in financial contexts such as: ○ Profit, loss and discount ○ Budgets ○ Accounts ○ Loans ○ Simple interest	**Solving problems** • Solving problems in contexts should take account of the number ranges learners are familiar with. • Contexts involving ratio and rate, should include speed, distance and time problems. • In financial contexts, learners are **not** expected to use formulae for calculating simple interest.	

Numbers, Operations and Relationships	**1.2 Exponents**	**Mental calculations** • Determine squares to at least 122 and their square roots • Determine cubes to at least 63 and their cube roots **Comparing and representing numbers in exponential form** • Compare and represent whole numbers in exponential form: $a^b = a \times a \times a \times \ldots$ for b number of factors	**What is different to Grade 6?** • Although learners may have encountered square numbers in Grade 6, they were not expected to write these numbers in exponential form. **Comparing and representing numbers in exponential form** • Learners need to understand that in the exponential form a^b, the number is read as '$\boldsymbol{a}$ **to the power** $\boldsymbol{b}$', where a is called the base and b is called the exponent or index. b indicates the number of factors that are multiplied. **Example:** a) $a^3 = a \times a \times a$; b) $a^5 = a \times a \times a \times a \times a$ • Learners can represent any number in exponential form, without needing to compute the value. **Example:** $50 \times 50 \times 50 \times 50 \times 50 \times 50 \times 50 = 50^7$	**9hours.**

			• Make sure learners understand that square roots and cube roots are the inverse operations of squaring and cubing numbers. **Examples:** $3^2 = 9$ therefore $\sqrt{9} = 3$ • Make sure learners understand that any number raised to the power 1 is equal to the number. Example: $m^1 = m$ • At this point, learners do not need to know the rule for raising a number to the power 0. This will only be introduced in Grade 8 when they use other laws of exponents in calculations. • To avoid common misconceptions, emphasize the following with examples: ○ $12^2 = 12 \times 12$ and not 12×2 ○ 1^3 means $1 \times 1 \times 1$ and not 1×3 ○ $100^1 = 100$ ○ $\sqrt{81} = 9$ because $9^2 = 81$ ○ $\sqrt[3]{27} = 3$ because $3^3 = 27$ ○ The square of $9 = 81$, whereas the square root of $9 = 3$ • Learners should use their knowledge of representing numbers in exponential form when simplifying and expanding algebraic expressions and solving algebraic equations.	
		Calculations using numbers in exponential form • Recognize and use the appropriate laws of operations using numbers involving exponents and square and cube roots • Perform calculations involving all four operations using numbers in exponential form, limited to exponents up to 5, and square and cube roots **Solving problems** • Solve problems in contexts involving numbers in exponential form	**Calculations using numbers in exponential form** • Knowing the rules of operations for calculations involving exponents, is important. **Example:** a) $(7-4)^3 = 3^3$ **AND NOT** $7^3 - 4^3$ b) $\sqrt{16+9} = \sqrt{25}$, **AND NOT** $\sqrt{16} + \sqrt{9}$	

Shape and Space (Geometry)	**3.5 Construction of geometric figures**	**Measuring angles** • Accurately use a protractor to measure and classify angles:: ○ $< 90°$ (acute angles) ○ Right-angles ○ $> 90°$ (obtuse angles) ○ Straight angles ○ $> 180°$ (reflex angles) **Constructions** • Accurately construct geometric figures appropriately using compass, ruler and protractor, including: ○ angles, to one degree of accuracy ○ circles ○ parallel lines ○ perpendicular lines	**What is different to Grade 6?** • Measure angles with a protractor • Geometric constructions using a compass, ruler and protractor **Measuring angles** • Learners have to be shown how to place the protractor on the arm of the angle to be measured. • Learners also have to learn how to read the size of angles on a protractor. **Constructions** • Constructions provide a useful context to explore or consolidate knowledge of angles and shapes. • Learners have to be shown how to use a compass to draw circles, although they might have done this in Grade 6. • Learners should be aware that the centre of the circle is at the fixed point of the compass and the radius of the circle is dependent on how wide the compass is opened up. • Make sure learners understand that arcs are parts of the circles of a particular radius. • Initially, learners have to be given careful instructions about how to do the constructions of the various shapes • Once they are comfortable with the apparatus and can do the constructions, they can practise by drawing patterns, for example of circles or parallel lines.	**10 hours**
Shape and Space (Geometry)	**3.1. Geometry of 2D shapes**	**Classifying 2D shapes** • Describe, sort, name and compare triangles according to their sides and angles, focusing on: ○ equilateral triangles ○ isosceles triangles ○ right-angled triangles • Describe, sort, name and compare quadrilaterals in terms of: ○ length of sides ○ parallel and perpendicular sides ○ size of angles (right-angles or not)	**What is different to Grade 6?** • Distinguishing and naming triangles in terms of their sides and angles • Distinguishing quadrilaterals in terms of parallel and perpendicular sides • Distinguishing similar and congruent figures • Using known properties of shapes to solve geometric problems **Triangles** • Learners should be able to distinguish between an equilateral triangle (all the sides are equal), an isosceles triangle (at least two equal sides) and a right-angled triangle (one right-angle).	**10 hours**

			Quadrilaterals • Learners should be able to sort and group quadrilaterals in the following ways: ○ all sides equal (square and rhombus) ○ opposite sides equal (rectangle, parallelogram, square, rhombus) ○ at least one pair of adjacent sides equal (square, rhombus, kite) ○ all four angles right angles (square, rectangle) ○ perpendicular sides (square, rectangle) ○ two pairs of opposite sides parallel (rectangle, square, parallelogram) ○ only one pair of opposite sides parallel (trapezium)	
		• Describe and name parts of a circle	**Circles** • Parts of a circle learners should know: ○ radius ○ circumference ○ diameter ○ chord ○ Segments ○ sectors	
		Similar and congruent 2D shapes • Recognize and describe similar and congruent figures by comparing: ○ shape ○ size	**Similarity and congruency** • Similarity and congruency can be explored with any 2D figures. • Learners should recognize that two or more figures are congruent if they are equal in all respects i.e. angles and sides are equal. • Learners should recognize that two or more figures are similar if they have the same shape, but differ in size i.e. angles are the same, but sides are proportionally longer or shorter.	
		Solving problems • Solve simple geometric problems involving unknown sides and angles in triangles and quadrilaterals, using known properties.	**Solving problems** • At this stage learners can solve simple geometric problems to find unknown sides in equilateral and isosceles triangles, and unknown sides and angles in quadrilaterals. • Learners should be encouraged to give reasons for their solutions.	

			Examples a) If ΔABC is an equilateral triangle, and side AB is $3\ cm$, what is the length of BC? Here learners should answer: $BC = 3\ cm$, because the sides of an equilateral triangle are equal. b) If $ABCD$ is a kite, and $AB = 2{,}5\ cm$ and $BC = 4{,}5\ cm$, what is the length of AD and DC? Learners should use the property for kites, that adjacent pairs of sides are equal, to find the unknown sides.	
REVISION/ASSESSMENT: At this stage learners should have been assessed on: • calculating and solving problems using whole numbers • working with numbers in exponential form • constructing geometric objects • geometry of 2D shapes				**7 hours**

GRADE 7 – TERM 2				
CONTENT AREA	**TOPICS**	**CONCEPTS and SKILLS**	**SOME CLARIFICATION NOTES or TEACHING GUIDELINES**	**DURATION (in hours)**

Numbers, Operations and Relationships	**1.4 Common fractions**	**Ordering, comparing and simplifying fractions**
		• Revise the following done in Grade 6:
		  ○ Compare and order common fractions, including specifically tenths and hundredths
		• Extend to more than thousandths
		
		Calculations using fractions
		• Revise the following done in Grade 6:
		  ○ addition and subtraction of common fractions, including mixed numbers, limited to fractions with the same denominator or where one denominator is a multiple of another
		  ○ finding fractions of whole numbers
		• Extend addition and subtraction to fractions where one denominator is not a multiple of the other
		• Multiplication of common fractions, including mixed numbers, not limited to fractions where one denominator is a multiple of another
		
		Calculation strategies
		• Convert mixed numbers to common fractions in order to perform calculations
		• Use knowledge of multiples and factors to write fractions in the simplest form
			What is different to Grade 6?
			• Compare and order thousandths
			• Multiplication of common fractions
			• Percentage of part of a whole
			• Percentage increase or decrease
			
			In Grade 7 learners also consolidate number knowledge and calculation techniques for common fractions, developed in the Intermediate Phase.
			
			Calculations with fractions
			• Learners should do context free calculations and solve problems in contexts.
			• It is not expected that learners know rules for simplifying fractions or for converting between mixed numbers and fraction forms. Learners should know from working with equivalence, when a fraction is equal to or greater than 1.
			• LCMs have to be found when adding and subtracting fractions of different denominators. Here learners use knowledge of common multiples to find the LCM i.e. what number can both denominators be divided into.
			
			• To simplify fractions, learners use knowledge of common factors i.e. what can divide equally into the numerator and denominator of a fraction. Emphasize that when simplifying, the fractions must remain equivalent.
			
			Example
			$\frac{3}{4} \times \frac{2}{5} = \frac{6}{20} = \frac{3}{10}$
			or
			$\frac{3}{4} \times \frac{2}{5} = \frac{3}{10}$
				9 hours

		before or after calculations • Use knowledge of equivalent fractions to add and subtract common fractions **Solving problems** • Solve problems in contexts involving common fractions and mixed numbers, including grouping, sharing and finding fractions of whole numbers **Percentages** • Revise the following done in Grade 6: ○ percentages of whole numbers • Calculate the percentage of part of a whole • Calculate percentage increase or decrease of whole numbers • Solve problems in contexts involving percentages **Equivalent forms** Revise the following done in Grade 6: • Recognize and use equivalent forms of common fractions with 1-digit or 2-digit	• Learners should recognize that finding a 'fraction ***of*** a whole number' or 'finding a fraction ***of*** a fraction' means multiplying the fraction and the whole number or the fraction with the fraction. • When learners find fractions of whole numbers, the examples can be chosen to result either in whole numbers or fractions or both. • Learners should also use the convention of writing the whole number as a fraction over 1 when multiplying. **Examples** a) Calculate $\frac{4}{5} of\ 20$ Answer: $\frac{4}{5}\ of\ 20 = \frac{4}{5} \times \frac{20}{1} = \frac{4}{1} \times \frac{4}{1} = 16$ **OR** $\frac{4}{5}\ of\ 20 = \frac{4}{5} \times \frac{20}{1} = \frac{80}{5} = 16$ b) Calculate $\frac{2}{3} of\ \frac{5}{6}$ Answer $\frac{2}{3}\ of\ \frac{5}{6} = \frac{2}{3} \times \frac{5}{6} = \frac{1}{3} \times \frac{5}{3} = \frac{5}{9}$ **OR** $\frac{2}{3}\ of\ \frac{5}{6} = \frac{2}{3} \times \frac{5}{6} = \frac{10}{18} = \frac{5}{9}$ **Calculation using percentages** ○ Learners should do context free calculations and solve problems in contexts. ○ When doing calculations using percentages, learners have to use the equivalent common fraction form, which is a fraction with denominator 100. Learners should become familiar with the equivalent fraction and decimal forms of common percentages like a) 25% or $\frac{1}{4}$ or $0{,}25$; b) 50% or $\frac{1}{2}$ or $0{,}5)$ c) 60% or $\frac{3}{5}$ or $0{,}6$. ○ To calculate percentage of part of a whole, or percentage increase or decrease, learners have to learn the strategy of multiplying by $\frac{100}{1}$. It is useful for learners to learn to use calculators for some of these calculations where	

		denominators (fractions where one denominator is a multiple of the other) • Recognize equivalence between common fraction and decimal fraction forms of the same number • Recognize equivalence between common fraction, decimal fraction and percentage forms of the same number	the fractions are not easily simplified. ○ When using calculators, learners can also use the equivalent decimal fraction form for percentages to do the calculations. **Examples:** a) Calculate 60% of $\mathrm{R}105$ Amount = $\frac{3}{5} \times \mathrm{R}105 = \mathrm{R}63$ b) What percentage is $40\mathrm{c}$ of $\mathrm{R}3{,}20$? Percentage = $\frac{40}{320} \times \frac{100}{1} = \frac{100}{8} = 12{,}5\%$ c) Calculate the percentage increase if the price of a bus ticket of R60 is increased to $\mathrm{R}84$. Amount increased = $\mathrm{R}24$. Therefore percentage increase = $\frac{24}{60} \times \frac{100}{1} = 40\%$ d) Calculate the percentage decrease if the price of petrol goes down from 20 cents a litre to 18 cents a litre. Amount decreased = 2 cents. Therefore percentage decrease = $\frac{2}{20} \times \frac{100}{1} = 10\%$	
Numbers, Operations and Relationships	**1.5 Decimal fractions**	**Ordering and comparing decimal fractions** • Revise the following done in Grade 6: ○ count forwards and backwards in decimal fractions to at least two decimal places ○ compare and order decimal fractions of at least two decimal places	**What is different to Grade 6?** ○ Decimal fractions to at least 3 decimal places ○ Rounding off to at least 2 decimal places ○ Multiply and divide decimal fractions by whole numbers ○ Multiply decimal fractions by decimal fractions In Grade 7 learners consolidate number knowledge and calculation techniques for decimal fractions, developed in the Intermediate Phase. **Ordering, counting and comparing decimal fractions** • Counting should not only be thought of as verbal counting. Learners can count in decimal intervals using: ○ structured, semi-structured or empty number lines ○ chain diagrams for counting • Learners should be given a range of exercises such as: ○ arrange given numbers from the smallest to the biggest: or biggest to smallest ○ fill in missing numbers in	9 hours

		○ place value of digits to at least two decimal places ○ rounding off decimal fractions to at least 1 decimal place • Extend all of the above to decimal fractions of at least three decimal places and rounding off to at least 2 decimal places **Calculations using decimal fractions** • Revise the following done in Grade 6: ○ addition and subtraction of decimal fractions of at least two decimal places ○ multiplication of decimal fractions by 10 and 100 • Extend addition and subtraction to decimal fractions of at least three decimal places • Multiply decimal fractions to include: ○ decimal fractions to at least 3 decimal places by whole numbers ○ decimal fractions to at least 2 decimal places by decimal fractions to at least 1 decimal place • Divide decimal fractions to include: ○ decimal fractions to at least 3 decimal places by whole numbers **Calculation strategies** • Use knowledge of place value to estimate the number of decimal places in the result before performing calculations • Use rounding off and a calculator to	▪ a sequence ▪ on a number grid ▪ on a number line ▪ fill in <, = or > **Example:** $0{,}4 * 0{,}04$ • Counting exercises in chain diagrams can be checked using calculators and learners can explain any differences between their answers and those shown by the calculator. **Calculating with decimal fractions** • Learners should do context free calculations and solve problems in contexts. • Learners should estimate their answers before calculating, especially with multiplication by decimal fractions. They should be able to judge the reasonableness of answers relating to how many decimal places and also check their own answers. • Multiplication by decimal fractions should start with familiar numbers that learners can calculate by inspection, so that learners get a sense of how decimal places are affected by multiplication. **Examples**: a) $3 \times 2 = 6$ $0{,}3 \times 2 = 0{,}6$ $0{,}3 \times 0{,}2 = 0{,}06$ $0{,}3 \times 0{,}02 = 0{,}006$ $0{,}03 \times 0{,}02 = 0{,}0006$ etc b) $15 \times 3 = 45$ $1{,}5 \times 3 = 4{,}5$ $0{,}15 \times 3 = 0{,}45$ $0{,}15 \times 0{,}3 = 0{,}045$ $0{,}015 \times 0{,}3 = 0{,}0045$ etc **Equivalence between common fractions and decimal fractions** ○ Learners are not expected to be able to convert any common fraction into its decimal form, merely to see the relationship between tenths, hundredths and thousandths in their decimal forms. ○ Learners should start by rewriting and converting tenths, hundredths and thousandths in common fraction form to decimal fractions. Where	

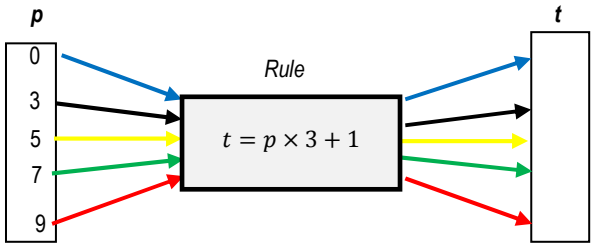
		check results where appropriate
		
		Solving problems
		• Solve problems in context involving decimal fractions
		
		Equivalent forms
		• Revise the following done in Grade 6:
		  ○ recognize equivalence between common fraction and decimal fraction forms of the same number
		  ○ Recognize equivalence between common fraction, decimal fraction and percentage forms of the same number
			denominators of other fractions are factors of 10 e.g. $2, 5$ or factors of 100 e.g $2, 4, 25, 20$ learners can convert these to hundredths using what they know about equivalence.
			○ It is useful to use calculators to help learners convert between common fractions and decimal fractions (here learners will use what they know about the relationship between fractions and division).
			○ Dividing whole numbers by $10, 100, 1\ 000$, etc. can help to build learners' understanding of place value with decimals. This is also useful to do on the calculator – learners can discuss the patterns they see when dividing.
			○ Similarly calculators can be useful tools for learners to learn about patterns when multiplying decimals by $10, 100$ or 1000, etc.	

Patterns, functions and algebra	**2.2 Functions and relationships**	**Input and output values**
		• Determine input values, output values or rules for patterns and relationships using:
		  ○ flow diagrams
		  ○ tables
		  ○ formulae
		
		Equivalent forms
		• Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented
		  ○ verbally
		  ○ in flow diagrams
		  ○ in tables
		  ○ by formulae
		  ○ by number sentences
			What is different to the Intermediate Phase?
			• Finding input or output values using given formulae
			• The rules and number patterns for which learners have to find input or output values are extended to include patterns with integers, square numbers and cubic numbers
			
			Finding input and output values in flow diagrams, tables and formulae should be done more than just once a year. It can be done after number work, to practise properties and operations with numbers and after measurement or geometry to practise solving problems using formulae.
			
			In Term 2 the focus of Functions and Relationships is on practising operations with whole numbers as well as common fractions or decimal fractions as input values, or including common fractions and decimal fractions in the rules for finding output values. In Term 3 the focus of Functions and Relationships is on using formulae and in Term 4, the focus is on practising addition and subtraction of integers.
			
			• In this phase, it is useful to begin to specify whether the input values are natural numbers, or integers or rational numbers. Hence, to find output values, learners should be given the rule/formula as well as the input values.
				3 hours

Flow diagrams are representations of functional relationships. Hence, when using flow diagrams, the correspondence between input and output values should be clear in its representational form i.e. the first input produces the first output, the second input produces the second output, etc.

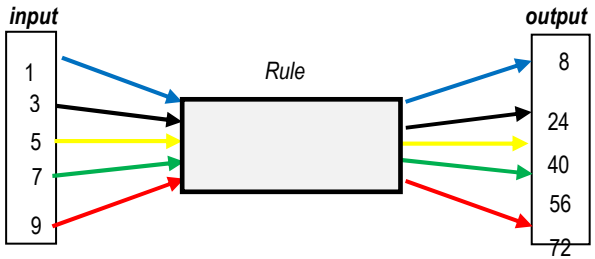
Examples

a) Use the given rule to calculate the values of ***t*** for each value of ***p***, where ***p*** is a natural number.



In this kind of flow diagram, learners can also be asked to find the value of ***p*** for a given value ***t***.

b) Find the rule for calculating the output value for every given input value in the flow diagram below.



In flow diagrams such as these, more than one rule might be possible to describe the relationship between input and output values. The rules are

acceptable if they match the given input values to the corresponding output values.

c) If the rule for finding y in the table below is: $y = 3x - 1$, find y for the given x values:

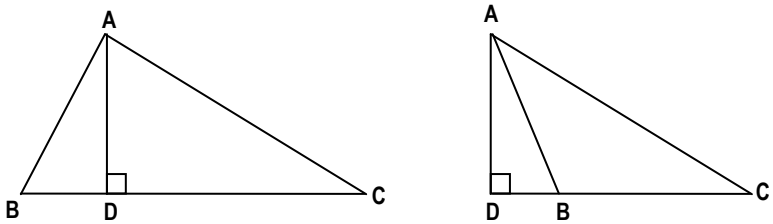
x	0	1	2	5	10	50	100
y							

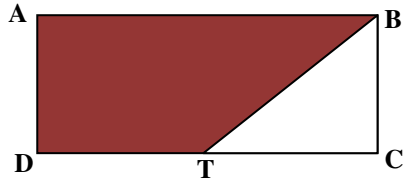
d) Describe the relationship between the numbers in the top row and bottom row in the table. Then write down the value of ***m*** and ***n***.

$\boldsymbol{x}$	1	2	3	4		12	$\boldsymbol{n}$
$\boldsymbol{y}$	5	6	7	8		$\boldsymbol{m}$	34

In tables such as these, more than one rule might be possible to describe the relationship between x and y values. The rules are acceptable if they match the given input values to the corresponding output values. For example, the rule $y = x + 4$ describes the relationship between the given x and y values in the table. To find $\boldsymbol{m}$ and $\boldsymbol{n}$, you have to substitute the corresponding values for x or y into this rule and solve the equation by inspection.

Measurement	**4.1 Area and perimeter of 2D shapes**	**Area and perimeter** • Calculate the perimeter of regular and irregular polygons • Use appropriate formulae to calculate perimeter and area of: ○ squares ○ rectangles ○ rriangles **Calculations and solving problems** • Solve problems involving perimeter and area of polygons • Calculate to at least 1 decimal place	**What is different to Grade 6?** • In Grade 6 learners did not have to use formulae to calculate area and perimeter. • **Formulae** learners should know and use are: ○ perimeter of a square = $4s$ ○ perimeter of a rectangle = $2(l + b)$ or $2l + 2b$ ○ area of a square = l^2 ○ area of a rectangle = $l \times b$ ○ area of a triangle = $\frac{1}{2}(b \times h)$ **Solving equations using formulae** • The use of formulae provides a context to practise solving equations by	**7 hours**

		• Use and convert between appropriate SI units, including: ○ mm2 and cm2 ○ cm2 and m2	inspection. **Example** 1. If the perimeter of a square is $32\ cm$, what is the length of each side? Learners should write this as: $4s = 32$ and solve by inspection by asking: 4 times what will be 32? 2. If the area of a rectangle is $200\ cm^2$, and its length is $50\ cm$, what is its width? Learners should write this as: $50 \times\ b = 200$ and solve by inspection by asking: 50 times what will be 200? **Examples of calculations for area and perimeter** **Calculate:** a) Perimeter of a rectangle which is $24\ cm$ long and $18\ cm$ wide. b) Perimeter of a regular octagon if the length of each side is $17\ cm$. c) Area of ΔABC if $BC\ =\ 12\ cm$ and its height AT = $9\ cm$. d) Perimeter of a square if its area is $225\ cm^2$ **For areas of triangles:** • Make sure learners know that the height of a triangle is a line segment drawn from any vertex perpendicular to the opposite side. **Example:** AD is the height onto base BC of ΔABC.  • Point out that every triangle has 3 bases, each with a related height or altitude. • **For conversions, note**: ○ if $1\ cm\ =\ 10\ mm$ then $1\ cm^2 = 100\ mm^2$ ○ if $1\ m = 100\ cm$ then $1\ m^2 =\ 10\ 000\ cm^2$	

			Examples of solving problems involving perimeter and area. a) Calculate the area of the shaded part in the diagram if ABCD is a rectangle, $AB = 18{,}6\ cm$, DC = 2TC and $BC = 8\ cm$.  b) The area of the floor of the dining room is$18{,}4\ cm^2$. How many square tiles with sides of $20\ cm$ are needed to tile the floor? c) The length of the side of a square is doubled. Will the area of the enlarged square be double or four times that of the original square?	
Measurement	**4.2 Surface area and volume of 3Dobjects**	**Surface area and volume** • Use appropriate formulae to calculate the surface area, volume and capacity of: ○ cubes ○ rectangular prisms • Describe the interrelationship between surface area and volume of the objects mentioned above **Calculations and solving problems** • Solve problems involving surface area, volume and capacity • Use and convert between appropriate SI units, including: ○ mm^2 and cm^2 ○ cm^2 and m^2	**What is different to Grade 6?** • In Grade 6 learners did not have to use formulae to calculate surface area and volume. • **Formulae** learners should know and use: ○ the volume of a prism = the area of the base × the height ○ the surface area of a prism = the sum of the area of all its faces ○ the volume of a cube = l^3 ○ the volume of a rectangular prism = $l \times b \times h$ • **For conversions, note:** ○ if $1\ cm = 10\ mm$ then $1\ cm^3 = 1\ 000\ mm^3$and ○ if $1\ m = 100\ cm$ then $1\ m^3 = 1\ 000\ 000$ or $10^6 cm^3$. ○ an object with a volume of $1\ cm^3$ will displace exactly 1m*l* of water; and ○ an object with a volume of $1m^3$ will displace exactly 1k*l* of water.	**8 hours**

		○ mm^3 and cm^3 ○ cm^3 and m^3 • Use equivalence between units when solving problems: ○ $1\ cm^3 = 1\ ml$ ○ $1\ m^3 = 1\ kl$	• Emphasize that the amount of space inside a prism is called its capacity; and the amount of space occupied by a prism is called its volume. • Investigate the nets of cubes and rectangular prisms in order to deduce formulae for calculating their surface areas.	
REVISION/ASSESSMENT: At this stage learners should be assessed on: • calculating and solving problems with common fractions and decimal fractions • using formulae to find area and perimeter of 2D shapes • using formulae to find volume and surface area of 3D objects				**9 hours**

GRADE 7 – TERM 3				
CONTENT AREA	**TOPICS**	**CONCEPTS and SKILLS**	**SOME CLARIFICATION NOTES or TEACHING GUIDELINES**	**DURATION (in hours)**
Patterns, functions and algebra	**2.1 Numeric and geometric patterns**	**Investigate and extend patterns** • Investigate and extend numeric and geometric patterns looking for relationships between numbers, including patterns: ○ represented in physical or diagram form ○ not limited to sequences involving a constant difference or ratio ○ of learner's own creation ○ represented in tables • Describe and justify the general rules for observed relationships between numbers in own words	**What is different to the Intermediate Phase?** • In the Senior Phase the emphasis is less on merely extending a pattern, and more on describing a general rule for the pattern or sequence and being able to predict unknown terms in a sequence using a general rule. • Investigating number patterns is an opportunity to generalize – to give general algebraic descriptions of the relationship between terms and its position in a sequence and to justify solutions. • The range of number patterns are extended to include patterns integers, square numbers and cubic numbers • As learners become used to describing patterns in their own words, their descriptions should become more precise and efficient with the use of algebraic language to describe general rules of patterns. • It is useful also to introduce the language of 'term in a sequence' in order to distinguish the term from the position of a term in a sequence **Numeric and geometric patterns are done again in Term 4, where patterns can include integers. In this term patterns should be restricted to using whole numbers, numbers in exponential form, common fractions and decimal fractions.**	**6 hours**

Kinds of numeric patterns

- Provide a sequence of numbers, learners have to identify a pattern or **relationship between consecutive terms** in order to extend the pattern.

  Examples

  Give a rule to describe the relationship between the numbers in the sequences below. Use this rule to give the next three numbers in the sequence:

  a) $3;\ 7;\ 11;\ 15;$ … …

  b) $120;\ 115;\ 110;\ 105;$ …. .

  Here learners could identify the constant difference between consecutive terms in order to extend the pattern. These patterns can be described in learners' own words as (a) 'adding 4' or 'counting in $4s$' or 'add 4 to the previous number in the pattern' (b) 'subtracting 5' or 'counting down in $5s$', or 'subtract 5 from the previous number in the pattern'.

  c) $2;\ 4;\ 8;\ 16$ … …

  Here learners could identify the constant ratio between consecutive terms. This pattern can be described in learners' own words as 'multiply the previous number by 2'.

  d) $1;\ 2;\ 4;\ 7;\ 11;\ 16$ … …

  This pattern has neither a constant difference nor constant ratio. This pattern can be described in learners' own words as 'increase the difference between consecutive terms by 1 each time' or 'add 1 more than was added to get the previous term'. Using this rule, the next 3 terms will be $22, 29, 37$.

- Provide a sequence of numbers, learners have to identify a pattern or **relationship between the term and its position in the sequence**. This enables learners to predict a term in a sequence based on the position of that term in the sequence. It is useful for learners to represent these sequences in tables so that they can consider the position of the term.

  Examples:

a) Provide a rule to describe the relationship between the numbers in this sequence: $1;\ 4;\ 9;\ 16$ …. .. Use the rule to find the 10th term in this sequence.

Firstly, learners have to understand that the '10th term' refers to position 10 in

the number sequence. They have to find a rule in order to determine the 10th term, rather than continuing the sequence to the tenth term.

This sequence can be represented in the following table:

Position in sequence	1	2	3	4		10
Term	1	4	9	16		?

Learners should recognize that each term in the bottom row is obtained by squaring the position number in the top row. Thus the 10th term will be '10 squared' or10^2, which is 100. Using the same rule, learners can also be asked what term number or position will 625 be? If the term is obtained by squaring the position number of the term, then the position number can be obtained by finding the square root of the term. Hence, 625 will be the 25th term in the sequence since $\sqrt{625} = 25$

b) Provide a rule to describe the relationship between the numbers in this sequence: 4; 7; 10; 13; …... Use the rule to find the 20th term in the sequence.
If learners consider only the relationship between consecutive terms, then they can continue the pattern ('add 3 to previous number') to the 20th term to find the answer. However, if they look for a relationship or rule between the term and the position of the term, they can predict the answer without continuing the pattern. Using number sentences can be useful to find the rule:

1st term: $4 = 3(1) + 1$
2nd term: $7 = 3(2) + 1$
3rd term: $10 = 3(3) + 1$
4th term: $13 = 3\,(4) + 1$
The number in the brackets corresponds to the position of the term. Hence, the 20th term will be:
$3(20) + 1 = 61$
The rule in learners' own words can be written as '3 × the position of the term $+ 1$'

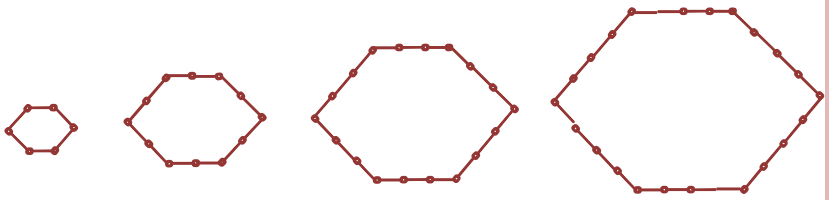
- These types of numeric patterns develop an understanding of functional relationships, in which you have a dependent variable (position of the term) and independent variable (the term itself), and where you have a unique output for any given input value.

Kinds of geometric patterns

- Geometric patterns are number patterns represented diagrammatically. The diagrammatic representation reveals the structure of the number pattern.
- Hence, representing the number patterns in tables, makes it easier for learners to describe the general rule for the pattern.

Example

Consider this pattern for building hexagons with matchsticks. How many matchsticks will be used to build the 10th hexagon?



The rule for the pattern is contained in the structure (construction) of the successive hexagonal shapes:

(1) add on 1 matchstick per side
(2) there are 6 sides, so
(3) add on 6 matchsticks per hexagon as you proceed from a given hexagon to the next one.

For the 2nd hexagon, you have 2×6 matches; for the 3rd hexagon you have 3×6 matches; Using this pattern for building hexagons, the 10th hexagon will have 10×6 matches.

Learners can also use a table to record the number of matches used for each hexagon. This way they can look at the number pattern related to the number of matches used for each new hexagon.

Position of hexagon in pattern	**1**	**2**	**3**	**4**	**5**	**6**	**10**
Number of matches	6	12	18				

			Describing patterns • It does not matter if learners are already familiar with a particular pattern. Their descriptions of the same pattern can be different when they encounter it at different stages of their mathematical development. **Example** The rule for the sequence : $4;\ 7;\ 10;\ 13$ can be described in the following ways: a) add three to the previous term b) $3\,x$ the position of the term $+\ 1$' c) $3(n)\ +\ 1$, where n is the position of the term d) $3(n)\ +\ 1$, where n is a Natural number.	
	2.2 Functions and relationships	**Input and output values** • Determine input values, output values or rules for patterns and relationships using: ○ flow diagrams ○ tables ○ formulae **Equivalent forms** • Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented: ○ verbally ○ in flow diagrams ○ in tables ○ by formulae ○ by number sentences	**Functions and relationships were also done in Term 2, and will be done again in Term 4, focusing on integers. In this term, the focus is on finding output values for given formulae and input values.** **See additional notes and examples in Term 2.** Note, when learners find input or output values for given rules or formulae, they are actually finding the numerical value of algebraic expressions using substitution. **Examples** Use the formula for the area of a rectangle: A = *l x b* to calculate the following: a) The area, if the length is 4 cm and the width is 2 cm b) The length, if the area is 20 cm2 and the width is 4 cm c) The width, if the area is 24 cm2 and the length is 8 cm Learners can write these as number sentences, and solve by inspection.	**3 hours**
	2.3 Algebraic expressions	**Algebraic language** • Recognize and interpret rules or	**What is different to the Intermediate Phase?** This is an introduction to formal algebraic language and is new in the Senior	**3 hours**

		relationships represented in symbolic form • Identify variables and constants in formulae and equations	Phase. The use of symbolic language helps to develop an understanding of variables. **Algebraic expressions are done again in Term 4, where rules and relationships can include integers.** Learners have opportunities to write and interpret algebraic expressions when they write general rules to describe relationships between numbers in number patterns, and when they find input and output values for given rules in flow diagrams, tables and formulae. **Examples** a) What does the rule $2\ x\ n - 1$ mean for the following number sequence: $1;\ 3;\ 5;\ 7;\ 9$ .... Here learners should recognize that $2 \times n - 1$ represents the general term in this sequence, where n represents the position of the term in the sequence. Thus it is the rule that can be used to find any term in the given sequence. b) The relationship between a boy's age (x yrs old) and his mother's age is given as $25 + x$. How can this relationship be used to find the mother's age when the boy is 11 years old? Here learners should recognize that to find the mother's age, they should substitute the boy's given age into the rule $25 + x$. They should also recognize that the given rule means the mother is 25 years older than the boy. See further examples given for functions and relationships.	
	2.4 Algebraic equations	**Number sentences** • Write number sentences to describe problem situations • Analyse and interpret number sentences that describe a given situation • Solve and complete number sentences by: ○ inspection ○ trial and improvement • Determine the numerical value of an expression by substitution.	**What is different to the Intermediate Phase?** The number sentences that learners can solve are extended to include number sentences with integers, square numbers and cubic numbers. Number sentences are used here as a more familiar term for Grade 7 learners than equations. However, the term equation will be used instead of number sentences in later grades. **Algebraic equations are done again in Term 4, where number sentences can include integers.** Learners have opportunities to write, solve and complete number sentences when they write general rules to describe relationships between numbers in number	**3 hours**

			patterns, and when they find input and output values for given rules in flow diagrams, tables and formulae. Rather than use formal algebraic processes, learners solve number sentences by inspection or determine the numerical value of expressions by substitution. In this phase, it is useful when solving equations to begin to specify whether x is a natural number, integer or rational number. This builds learners' awareness of the domain of x. **Examples** a) Solve x if $+4 = 7$, where x is a natural number. (What must be added to 4 to give 7?) b) Solve x if $x + 4 = -7$, where x is an integer. (What must be added to 4 to give-7?) c) Solve x if $2x = 30$, where x is a natural number. (What must be multiplied by 2 to give30?) d) Write a number sentence to find the area of a rectangle with length $4{,}5$ cm and breadth 2 cm. e) If $y = x^2 + 1$, calculate y when $x = 3$	
	2.5 Graphs	**Interpreting graphs** • Analyse and interpret global graphs of problem situations, with special focus on the following trends and features: ○ linear or non-linear ○ constant, increasing or decreasing **Drawing graphs** • Draw global graphs from given descriptions of a problem situation, identifying features listed above	**What is different to the Intermediate Phase?** In the Intermediate Phase learners encountered graphs in the form of data bar graphs and pie charts. This means they do have some experience reading and interpreting graphs. However, in the Senior Phase, they are introduced to line graphs that show functional relationships described in terms of dependent and independent variables. In Grade 7, the focus is on drawing, analysing and interpreting global graphs only. That is, learners do not have to plot points to draw graphs and they focus on the features of the global relationship shown in the graph. **Examples** of contexts for global graphs include: • the relationship between time and distance travelled • the relationship between temperature and time over which it is measured • the relationship between rainfall and time over which it is measured, etc.	**6 hours**
Space and Shape (geometry)	**3.2. Transformation Geometry**	**Transformations** • Recognize, describe and perform translations, reflections and rotations	**What is different to Grade 6** Learners in Grade 7 have to do transformations on squared paper. **Focus of transformations**	**9 hours**

		using geometric figures and shapes on squared paper • Identify and draw lines of symmetry in geometric figures **Enlargements and reductions** • Draw enlargements and reductions of geometric figures on squared paper and compare them in terms of shape and size	• Using squared paper for transformations allows learners to more accurately perform transformations and to compare the shape and size of figures. • Learners should recognize that translations, reflections and rotations only change the position of the figure, and not its shape or size. • They should recognize that the above transformations produce congruent figures. • Learners should recognize that enlargements and reductions change the size of figures by increasing or decreasing the length of sides, but keeping the angles the same, producing similar rather than congruent figures. • Learners should also be able to work out the factor of enlargement or reduction.	

	3.3. Geometry of 3D objects	**Classifying 3D objects**
		• Describe, sort and compare polyhedra in terms of
		  ○ shape and number of faces
		  ○ number of vertices
		  ○ number of edges
		
		Building 3D model
		• Revise using nets to create models of geometric solids, including:
		  ○ cubes
		  ○ prisms
			What is different to Grade 6?
			• Most of this work consolidates what has been done in Grade 6.
			
			Polyhedra
			Examples of sorting or grouping categories:
			• cubes (only square faces)
			• rectangular prisms (only rectangular faces)
			• triangular prisms (only triangular faces)
			• pyramids (square and triangular faces)
			• cylinders (circular and rectangular faces).
			
			Using and constructing nets
			• Using and constructing nets are useful contexts for exploring or consolidating properties of polyhedra.
			• Learners should recognize the nets of different solids.
			• Learners should draw sketches of the nets using their knowledge of shape and number of faces of the solids, before drawing and cutting out the nets to build models.
			• The construction of nets is based on the number and shape of faces of the solids, and do not require measuring of internal angles of polygons.
			• Learners have to work out the relative position of the faces of the nets, and use trial and error to match up the edges and vertices, in order to build the 3D object.
				9 hours

REVISION/ASSESSMENT: At this stage learners should have been assessed on: • numeric and geometric patterns				**6 hours**

• functions and relationships • algebraic expressions • algebraic equations • graphs • transformations • geometry of 3D objects			

GRADE 7 – TERM 4				
CONTENT AREA	**TOPICS**	**CONCEPTS and SKILLS**	**SOME CLARIFICATION NOTES or TEACHING GUIDELINES**	**DURATION (in hours)**

Numbers, Operations and Relationships	**1.3 Integers**	**Counting, ordering and comparing integers**
		• Count forwards and backwards in integers for any interval
		• Recognize, order and compare integers
		
		Calculations using integers
		• Add and subtract using integers
			What is different to Grade 6?
			Integers are new numbers introduced in Grade 7.
			
			Counting, ordering and comparing integers
			• Counting should not only be thought of as verbal counting. Learners can count using:
			  ○ structured, semi-structured or empty number lines
			  ○ chain diagrams for counting
			• Learners should be given a range of exercises such as:
			  ○ arrange given numbers from the smallest to the biggest: or biggest to smallest
			  ○ fill in missing numbers in
			  ▪ a sequence
			  ▪ on a number grid
			  ▪ on a number line
			  ▪ fill in <, = or > **Example:** $-425 \ * \ -450$;
			
			Calculations using integers
			• Start calculations using integers in small number ranges.
			• Develop an understanding that subtracting an integer is the same as adding its additive inverse.
				9 hours

		Properties of integers • Recognize and use commutative and associative properties of integers **Solving problems** • Solve problems in contexts involving addition and subtraction of integers	• **Example**: $7 - 4 = 7 + (-4) = 3$ **OR** $-7 - 4 = -7 + (-4) = -11$ So too, $7 - (-4) = 7 + (+4) = 11$ **OR** $-7 - (-4) = -7 + (+4) = -3$ Here the use of brackets around the integers are useful. **Properties of integers** • Learners should investigate the properties for operations using whole numbers on the set of integers. • These properties should serve as motivation for the operations they can perform using integers. • Learners should see that the commutative property for addition holds for integers e.g. $8 + (-3) = (-3) + 8 = 5$ • Learners should see that they can still use subtraction to check addition or vice versa. **Example:** If $8 + (-3) = 5$, then $5 - 8 = -3$ and $5 - (-3) = 8$ • Learners should see that the associative property for addition holds for integers. **Example:** $(-6) + 4] + (-1) = (-6) + [4 + (-1)] = -3$ • Learners should only explore the distributive property once they can multiply with integers	

Patterns, functions and algebra	**2.1 Numeric and geometric patterns**	**Investigate and extend patterns**
		• Investigate and extend numeric and geometric patterns looking for relationships between numbers, including patterns:
		  ○ represented in physical or diagram form
		  ○ not limited to sequences involving a constant difference or ratio
		  ○ of learner's own creation
		  ○ represented in tables
		• Describe and justify the general rules for observed relationships between numbers in own words
			The focus of numeric patterns in this term should be on practising operations with integers.
			
			See additional notes in Term 3.
				3 hours

	2.2 Functions and relationships	**Input and output values** • Determine input values, output values or rules for patterns and	Functions and relationships in this term should include integers as input or output values, as well as using integers	**3 hours**

		relationships using: ○ flow diagrams ○ tables ○ formulae **Equivalent forms** • Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented: ○ verbally ○ in flow diagrams ○ in tables ○ by formulae ○ by number sentences	in the rules for patterns and relationships. **See additional notes in Terms 2 & 3.**	
	2.3 Algebraic expressions	**Algebraic language** • Recognize and interpret rules or relationships represented in symbolic form • Identify variables and constants in formulae and equations • Identify variables and constants in formulae and equations	Algebraic expressions should include integers in the rules or relationships represented in symbolic form. **See additional notes in Term 3.**	**3 hours**
	2.4 Algebraic equations	**Number sentences** • Write number sentences to describe problem situations • Analyse and interpret number sentences that describe a given situation • Solve and complete number sentences by: ○ inspection ○ trial and improvement • Determine the numerical value of an expression by substitution.	Number sentences should include integers. **See additional notes in Term 3.**	**4 hours**

DATA HANDLING	**5.1. Collect, organize and summarize data**	**Collect data**
		• Pose questions relating to social, economic, and environmental sues in own environment
		• Select appropriate sources for the collection of data (including peers, family, newspapers, books, magazines)
		• Distinguish between samples and populations
		• Design and use simple questionnaires to answer questions:
		  ○ with yes/no type responses
		  ○ with multiple choice responses.
		
		Organize and summarize data
		• Organize (including grouping where appropriate) and record data using
		  ○ tally marks
		  ○ tables
		  ○ stem-and-leaf displays
		• Group data into intervals
		• Summarize and distinguish between ungrouped numerical data by determining:
		  ○ mean
		  ○ median
		  ○ mode
		• Identify the largest and smallest scores in a data set and determine the difference between them in order to determine the spread of the data (range).
			What is different to Grade 6?
			The following are new in Grade 7
			• samples and populations
			• multiple choice questionnaires
			• stem-and-leaf displays
			• grouping data in intervals
			• mean
			• range
			• histograms
			• scales on graphs
			
			Data sets and contexts
			Learners should be exposed to a variety of contexts that deal with social and environmental issues, and should work with given data sets, represented in a variety of ways, that include big number ranges, percentages and decimal fractions. Learners should then practise organizing and summarizing this data, analysing and interpreting the data, and writing a report about the data.
			
			Complete a data cycle
			Learners should complete at least one data cycle for the year, starting with posing their own questions, selecting the sources and method for collecting data, recording the data, organizing the data, representing the data, then analysing, summarizing, interpreting and reporting the data.
			Challenge learners to think about what kinds of questions and data need to be collected to be represented in a histogram, pie chart, or bar graph.
			
			Representing data
			• Drawing pie charts to represent data do not have to be accurately drawn with a compass and protractor, etc. Learners can use any round object to draw a circle, then divide the circle into halves and quarters
				Time for collecting and organizing data: 4 hours

			and eighths if needed, as a guide to estimate the proportions of the circle that need to be shown to represent the data. What is important is that the values or percentages associated with the data, are shown proportionally on the pie chart. • Drawing, reading and interpreting pie charts is a useful context to re-visit equivalence between fractions and percentages, e.g. 25% of the data is represented by a ¼ sector of the circle. • It is also a context in which learners can find percentages of whole numbers e.g. if 25% of 300 learners like rugby, how many (actual number) learners like rugby? • Histograms are used to represent grouped data shown in intervals on the horizontal axis of the graph. Point out the differences between histograms and bar graphs, in particular bar graphs that represent discrete data (e.g. favourite sports) compared to histograms that show categories in consecutive, non-overlapping intervals, (e.g. test scores out of 100 shown in intervals of 10). The bars on bar graphs do not have to touch each other, while in a histogram they have to touch since they show consecutive intervals.	
	5.2. Representing data	**Represent data** • Draw a variety of graphs by hand/technology to display and interpret data (grouped and ungrouped) including: ○ bar graphs and double bar graphs ○ histograms with given intervals ○ pie charts	**Developing critical analysis skills** • Learners should compare the same data represented in different ways e.g. in a pie chart or a bar graph or a table, and discuss what information is shown and what is hidden; they should evaluate what form of representation works best for the given data. • Learners should compare graphs on the same topic but where data has been collected from different groups of people, at different times, in different places or in different ways. Here learners should discuss differences between the data with an awareness of bias related to the impact of data sources and methods of data collection on the interpretation of the data. • Learners should compare different ways of	**Time for representing data: 3 hours**
	5.3. Analysing, Interpreting, summarising and reporting data	**Interpret data** • Critically read and interpret data represented in: ○ words ○ bar graphs ○ double bar graphs ○ pie charts ○ histograms		

		Analyse data
		• Critically analyse data by answering questions related to:
		  ○ data categories, including data intervals
		  ○ data sources and contexts
		  ○ central tendencies – (mean, mode, median)
		  ○ scales used on graphs
		
		Report data
		• Summarize data in short paragraphs that include
		  ○ drawing conclusions about the data
		  ○ making predictions based on the data
		  ○ identifying sources of error and bias in the data
		  ○ choosing appropriate summary statistics for the data (mean, median, mode, range)
			summarising the same data sets, developing an awareness of how data reporting can be manipulated; evaluating which summary statistics best represent the data.
			• Learners should compare graphs of the same data, where the scales of the graphs are different. Here learners should discuss differences with an awareness of how representation of data can be manipulated; they should evaluate which form of representation works best for the given data.
			
			• Learners should be encouraged to write reports on the data in short paragraphs.
				Time for analysing, interpretin g and reporting data: 3,5 hours

	5.4. Probability	**Probability** Perform simple experiments where the possible outcomes are equally likely and • list all the possible outcomes • determine the probability of each possible outcome, using the definition of probability • predict (with reasons) the relative frequency of the possible outcomes for a series of trials based on probability • compare relative frequency with probability and explain possible differences	**Probability experiments** In the Intermediate Phase learners did experiments with coins, dice and spinners. In this grade experiments can be done with other objects, like, different coloured buttons in a bag; choosing specific cards from a deck of cards, etc. **Comparing relative frequency and probability** • The relative frequency is the observed number of successful outcomes for a finite sample of trials. • For example, if you toss a coin 50 times and have 27 heads and 23 tails, define a head as a successful outcome. The relative frequency of heads is: 27/50 = 54% • The probability of a head is 50% (one of two likely outcomes). The difference between the relative frequency of 54% and the probability of 50% is due to small sample size. • The more trials are done, the closer the relative frequency gets to the probability. This can be compared in class by combining results from trials	**4,5 hours**

			done in groups or pairs.	
REVISION/ASSESSMENT: At this stage learners should have been assessed on: • adding and subtracting with integers • collecting, organizing, representing, analysing, summarizing, interpreting and reporting data • probability				**Total time for revision/ assessment for the term 8 hours**

3.3.2. Clarification of content for Grade 8

GRADE 8 – TERM 1				
CONTENT AREA	**TOPICS**	**CONCEPTS and SKILLS**	**SOME CLARIFICATION NOTES or TEACHING GUIDELINES**	**DURATION (in hours)**
Numbers, Operations and Relationships	**1.1 Whole numbers**	**Mental calculations** Revise: • Multiplication of whole numbers to at least 12×12 **Ordering and comparing whole numbers** Revise Prime numbers to at least 100 **Properties of whole numbers** • Revise: • The commutative; associative; distributive properties with whole numbers • 0 in terms of its additive property (identity element for addition) • 1 in terms of its multiplicative property (identify element for multiplication) • Recognise the division property of 0, whereby any number divided by 0 is undefined	**What is different to Grade 7?** In Grade 8 learners consolidate number knowledge and calculation techniques for whole numbers, developed in the Intermediate Phase and Grade 7 **Properties of whole numbers** • Revising the properties of whole numbers should be the starting point for work with whole numbers. The properties of numbers should provide the motivation for why and how operations with numbers work. • When learners are introduced to new numbers, such as integers for example, they can again explore how the properties of numbers work for the new set of numbers. • Learners also have to apply the properties of numbers in algebra, when they work with variables in place of numbers. • Learners should know and be able to use the following properties: ○ The commutative property of addition and multiplication: ▪ $a + b = b + a$ ▪ $a \times b = b \times a$ ○ The associative (grouping) property of addition and multiplication: ▪ $(a + b) + c = a + (b + c)$ ▪ $(a \times b) \times c = a \times (b \times c)$ ○ The distributive property of multiplication over addition and subtraction: ▪ $a(b + c) = (a \times b) + (a \times c)$	6 hours.

		Calculations with whole numbers Revise: • Calculations with all four operations on whole numbers, estimating and using calculators where appropriate **Calculation strategies** • Use a range of strategies to perform and check written and mental calculations with whole numbers including: ○ estimation ○ adding, subtracting and multiplying in columns ○ long division ○ rounding off and compensating ○ using a calculator	▪ $a(b - c) = (a \times b) - (a \times c)$ ○ Addition and subtraction as inverse operations ○ Multiplication and division as inverse operations ○ 0 is the identity element for addition : $t + 0 = t$ ○ 1 is the identity element for multiplication: $t \times 1 = t$ **Illustrating the properties with whole numbers** a) $33 + 99 = 99 + 33 = 132$ b) $51 + (19 + 46) = (51 + 19) + 46 = 116$ c) $4(12 + 9) = (4 \times 12) + (4 \times 9) = 48 + 36 = 84$ d) $(9 \times 64) + (9 \times 36) = 9 \times (64 + 36) = 9 \times 100 = 900$ e) $If\ 33 + 99 = 132, \text{then } 132 - 99 = 33 \text{ and } 132 - 33 = 99$ f) $If\ 20 \times 5 = 110, \text{then } 110 \div 20 = 5 \text{ and } 110 \div 5 = 20$ **Calculations with whole numbers** • Learners should continue to do context free calculations and solve problems in contexts using whole numbers, integers and fractions • Learners should become more confident in and more independent at mathematics, if they have strategies ○ to check their solutions themselves, e.g. using inverse operations; using calculators ○ to judge the reasonableness of their solutions e.g. estimate by rounding off; estimate by doubling or halving; • Adding, subtracting and multiplying in columns, and long division, should only be used to practice number facts and calculation techniques, and hence should be done with familiar and smaller number ranges. For big and unwieldy calculations, learners should be encouraged to use a calculator.	

		Multiples and factors Revise: • Prime factors of numbers to at least 3-digit whole numbers • LCM and HCF of numbers to at least 3-digit whole numbers, by inspection or factorisation	**Multiples and factors** • Learners should continue practising finding multiples and factors of whole numbers. This is especially important when learners do calculations with fractions. They use this knowledge to find the LCM for denominators that are different from each other, and also when they simplify fractions or have to find equivalent fractions. • Factorising whole numbers lays the foundation for factorisation of algebraic expressions. • Using the definition of prime numbers, emphasise that 1 is not classified as a prime number **Examples** a) LCM of 6 and 18 is 18 LCM of 6 and 7 is 42 b) The factors of 24 are $1, 2, 3, 4, 6, 12$ and 24 by inspection. And, the prime factors of 24 are 2 and 3 c) The factors of 140 are $1, 2, 5, 7, 10, 14, 28, 35, 70$ and 140 d) Determine the HCF of $120;\ 300$ and 900. Learners do this by finding the prime factors of the numbers first. $120 = 5 \times 3 \times 2\,3$ Initially learners may write this as: $5 \times 3 \times 2 \times 2 \times 2$ $300 = 52 \times 3 \times 22$ $900 = 52 \times 32 \times 23$ HCF $= 5 \times 3 \times 22 = 60$ (Multiply the common prime factors of the three numbers)	
		Solving problems • Solve problems involving whole numbers, including the following types of problems: ○ Compare two or more quantities of the same	**Solving problems** • Solving problems in contexts should take account of the number ranges learners are familiar with. • Contexts involving ratio and rate should include speed, distance and time problems. • In financial contexts, learners are not expected to use formulae for calculating simple	

		kind (ratio) ○ Comparing two quantities of different kinds (rate) ○ Share in a given ratio where the whole is given ○ Increase or decrease of a number in a given ratio • Solve problems that involve whole numbers, percentages and decimal fractions in financial contexts such as: ○ Profit, loss, discount and VAT ○ Budgets ○ Accounts ○ Loans ○ Simple interest ○ Higher Purchase ○ Exchange rates	interest.	
	1.3 Integers	**Counting, ordering and comparing integers** • Revise: ○ counting forwards and backwards in integers	**What is different to Grade 7?** • Multiply and divide with integers • All four operations with integers • All four operations with squares, cubes, square and cube roots of integers In Grade 8 learners consolidate number knowledge and calculation techniques for integers, developed in Grade 7. **Counting, ordering and comparing integers** • Learners should continue practising counting, ordering and comparing integers. Counting should not only be thought of as verbal counting. Learners can count using: ○ structured, semi-structured or empty number lines ○ chain diagrams for counting	Total time for Integers: 9 hours

		for any interval ○ ○ recognising, ordering and comparing integers **Calculations using integers** • Revise: ○ addition and subtraction using integers • Multiply and divide with integers • Perform calculations involving all four operations with integers • Perform calculations involving all four operations with numbers that involve the squares, cubes, square roots and cube roots of integers	• Learners should be given a range of exercises • Arrange given numbers from the smallest to the biggest: or biggest to smallest • Fill in missing numbers in ○ a sequence ○ on a number grid ○ on a number line ○ fill in <, = or > e.g. $-425 * -450$; **Calculations using integers** • Start calculations with integers using small number ranges. • Develop an understanding that subtracting an integer is the same as adding its additive inverse. **Example:** $7 - 4 = 7 + (-4) = 3$ OR $-7 - 4 = -7 + (-4) = -11$ So too, $7 - (-4) = 7 + (+4) = 11$ OR $-7 - (-4) = -7 + (+4) = -3$. Here the use of brackets around the integers are useful. • A useful strategy is to use repeated addition and number patterns to show learners the reasonableness of rules for the resultant sign for multiplication with integers. **Example:** a) Repeated addition of (-3): $(-3) + (-3) + (-3) = -9 = 3 \times (-3)$ b) Repeated addition of (-2): $(-2) + (-2) + (-2) + (-2) = -8 = 4 \times (-2)$ c) Counting down in intervals of 3 from 9: $3 \times 3 = 9$ $3 \times 2 = 6$ $3 \times 1 = 3$ $3 \times 0 = 0$ $3 \times -1 = -3$ $3 \times -2 = ?$ $3 \times -3 = ?$ Hence the rule: a positive integer x a negative integer = a negative integer d) Using the rule that a positive integer x a negative integer = a negative integer, established from examples above, the following pattern can be used: $-1 \times 3 = -3$ $-1 \times 2 = -2$	

			$-1 \times 1 = -1$ $-1 \times 0 = 0$ $-1 \times -1 = 1$ $-1 \times -2 = ?$ $-1 \times -3 = ?$ Hence the rule: a negative integer x a negative integer = a positive integer • Use the inverse operation for multiplication and division to develop a rule for the resultant sign for division with integers. **Example:** a) If $4 \times (-2) = -8$, then $-8 \div 4 = -2$ and $-8 \div (-2) = 4$ b) If $(-1) \times (-3) = 3$, then $3 \div (-1) = -3$ and $3 \div (-3) = -1$ Hence the rules: division of a positive and negative integer equals a negative integer and division of two negative integers equal a positive integer. • Finding the squares, cubes, square roots and cube roots of integers are also opportunities to check that learners know the rules for resultant signs when multiplying integers. • Therefore, make sure that learners understand why you cannot find the square root of a negative integer, and that the square of a negative integer is always positive. **Example:** a) $(-5)2 = (-5) \times (-5) = 25$ b) $(-4)3 = (-4) \times (-4) \times (-4) = -64$ c) $\sqrt[3]{-27} = -3$ because $-3 \times -3 \times -3 = -27$	
		Properties of integers • Recognize and use commutative, associative and distributive properties of integers • Recognize and use additive and multiplicative inverses for integers **Solving problems** • Solve problems in contexts	**Properties of integers** • Learners should investigate the properties for operations with whole numbers on the set of integers. • These properties should serve as motivation for the operations they can perform with integers. • Learners should see that the commutative property for addition and multiplication holds for integers, e.g. $8 + (-3) = (-3) + 8 = 5; 8 \times (-3) = (-3) \times 8 = -24$ • Learners should see that they can still use subtraction to check addition or vice versa, e.g. if $8 + (-3) = 5$, then $5 - 8 = -3$ and $5 - (-3) = 8$ • Learners should see that the associative property for addition holds for integers, e.g. $[(-6) + 4] + (-1) = (-6) + [4 + (-1)] = -3$	

		involving multiple operations with integers	• Learners should see that the inverse operation for multiplication and division holds for integers, e.g. if $5 \times (-6) = -30$, then $-30 \div 5 = -6$ and $-30 \div (-6) = 5$ • Learners should develop the rules, through patterning, for resultant signs when multiplying and dividing integers: $(+5) \times (+5) = (+25)$; $(-5) \times (-5) = (+25)$; $(-5) \times (+5) = (-25)$; $(+25) \div (+5) = (+5)$; $(-25) \div (-5) = (+5)$; $(-25) \div (+5) = (-5)$	
	1.2 Exponents	**Mental calculations** • Revise: ○ Squares to at least 122 and their square roots ○ Cubes to at least 63 and their cube roots **Comparing and representing numbers in exponential form** • Revise: ○ Compare and represent whole numbers in exponential form • Compare and represent integers in exponential form • Compare and represent numbers in scientific notation, limited to positive exponents	**What is different to Grade 7?** • Integers and rational numbers in exponential form • Scientific notation of numbers **Comparing and representing numbers in exponential form** • Learners need to understand that in the exponential form a^b , the number is read as '$\boldsymbol{a}$ ***to the power*** $\boldsymbol{b}$**,** where a is called the base and b is called the exponent or index and b indicates the number of factors that are multiplied. **Example:** $a^3 = a \times a \times a;\ \ a^5 = a \times a \times a \times a \times a$ • Learners can represent any number in exponential form, without needing to compute the value. **Example:** $50 \times 50 \times 50 \times 50 \times 50 \times 50 \times 50 = 50^7$ • Make sure learners understand that square roots and cube roots are the inverse operations of squaring and cubing numbers. **Example:** $3^2 = 9$ therefore $\sqrt{9} = 3$ • Make sure learners understand that any number raised to the power 1 is equal to that particular number **Example:** $m^1 = m$ • Using patterns and their knowledge of multiplication with integers, learners should anticipate the resultant sign of an integer raised to an odd or even power e.g.	Total time for exponents: 9 hours

		Calculations using numbers in exponential form • Establish general laws of exponents, limited to: ○ natural number exponents ▪ $a^m \times a^n = a^{m+n}$ ▪ $a^m \div a^n = a^{m-n}$ *if* $m > n$ ▪ $(a^m)^n = a^{mn}$ ▪ $(a \times t)^n = a^n \times t^n$ ▪ $a^0 = 1$ • Recognize and use the appropriate laws of operations with numbers involving exponents and	$(-15)^4$ will be positive, while (-15^3) will be negative • To avoid common misconceptions, emphasize the following ○ $12^2 = 12 \times 12$ and not 12×2 ○ 1^3 means $1 \times 1 \times 1$ and not 1×3 ○ $100^1 = 100$ ○ $\sqrt{81} = 9$ because $9^2 = 81$ ○ $\sqrt[3]{27} = 3$ because $3^3 = 27$ ○ the square of $9 = 81$, whereas the square root of $9 = 3$ • Learners should use their knowledge of representing numbers in exponential form when simplifying and expanding algebraic expressions and solving algebraic equations. **Laws of exponents** • The laws of exponents should be introduced through a range of numeric examples first, then variables can be used. In other words, the numbers are replaced with letters, but the rules work the same. • The following laws of exponents should be introduced, where $\boldsymbol{m}$ and $\boldsymbol{n}$ are natural numbers and $\boldsymbol{a}$ and $\boldsymbol{t}$ are not equal to $\mathbf{0}$: $\boldsymbol{a^m \times a^n = a^{m+n}}$ **Example** a) $2^3 \times 2^4 = 2^{3+4} = 2^7$ b) $x^3 \times x^4 = x^{3+4} = x^7$ $\boldsymbol{a^m \div a^n = a^{m-n}}$ **if** $\boldsymbol{m} > n$ **Example:** a) $3^5 \div 3^2 = 3^3 = 27$ b) $x^5 \div x^3 = x^2$	

		square and cube roots • Perform calculations involving all four operations with numbers that involve squares, cubes, square roots and cube roots of integers • Calculate the squares, cubes, square roots and cube roots of rational numbers **Solving problems** • Solve problems in contexts involving numbers in exponential form	$(a^m)^n = a^{mn}$ **Example:** a) $(2^3)^2 = 2^6 = 64$ b) $(x^3)^2 = x^6$ $(a \times t)^n = a^n t^n$ **Example:** $(3x^2)^3 = 3^3.x^6 = 27x^6$ $a^0 = 1$ **Examples:** $(37)^0 = 1$; $(4x^2)^0 = 1$ • Make sure learners understand these laws reading from both sides of the equal sign i.e. if the LHS = RHS, then the RHS = LHS. • The law $a^0 = 1$ can be derived by using the law of exponents for division in a few examples. $a^4 \div a^4 = \frac{a.a.a.a}{a.a.a.a} = 1 \therefore a^{4-4} = a^0 = 1$ • Learners should be able to use the laws of exponents in calculations and for solving simple exponential equations as well as expanding or simplifying algebraic expressions. • Look out for the following **<u>common misconceptions</u>** where: ○ learners multiply unlike bases and add the exponent. **Example:** $x^m \times y^n = (xy)^{m+n}$ ○ learners multiply like bases and add the exponents **Example:** $2^5 \times 2^7 = 4^{12}$ instead of the correct answer 2^{12}. ○ learners forget, for example, in squaring a binomial there is a middle term	

			Example: $(x+y)^m = x^m + y^m$ ○ learners confuse adding the exponents and adding the terms **Example:** $x^m + x^n = x^{m+n}$ or x^{mn} **Calculations using numbers in exponential form** • Knowing the rules of operations for calculations involving exponents are important, e.g. ○ $(7-4)^3 = 3^3$ **AND NOT** $7^3 - 4^3$ ○ $\sqrt{16+9} = \sqrt{25}$ **AND NOT** the $\sqrt{16} + \sqrt{9}$ • Learners can also do simple calculations where the numerator and denominator of a fraction are written in exponential form, e.g. $\frac{2^3}{2^2} = \frac{2 \times 2 \times 2}{2 \times 2} = \frac{8}{4} = 2$ • Learners can also find squares, cubes, square roots and cube roots of decimal and common fractions by inspection. **Examples** a) $(0{,}7)^2 = 0{,}49$ b) $(0{,}1)^3 = 0{,}001$ c) $\sqrt{0{,}09} = 0{,}3$ d) $(\frac{3}{4})^2 = \frac{3^2}{4^2} = \frac{9}{16}$ e) $\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$ **Scientific notation** • When writing numbers in scientific notation, learners have to understand the relationship between the number of decimal places and the index of 10. **Examples:** a) $25 = 2{,}5 \times 10^1$ b) $250 = 2{,}5 \times 10^2$ c) $2500 = 2{,}5 \times 10^3$	

			• Scientific notation limited to positive exponents, includes writing very large numbers in scientific notation. **Example:** $25 \text{ million} = 2{,}5 \times 10^6$ • Learners practise writing large numbers in scientific notation, they will realize they have encountered these in Natural Science. It is useful to refer to these contexts when talking about scientific notation.	
Patterns, functions and algebra	**2.1 Numeric and geometric patterns**	**Investigate and extend patterns** • Investigate and extend numeric and geometric patterns looking for relationships between numbers , including patterns: ○ represented in physical or diagram form ○ not limited to sequences involving a constant difference or ratio ○ of learner's own creation ○ represented in tables ○ represented algebraically • Describe and justify the general rules for observed relationships between numbers in own words or in algebraic language	**What is different to Grade 7?** • The range of number patterns are extended to include patterns with multiplication and division with integers, numbers in exponential form • As learners become used to describing patterns in their own words, their descriptions should become more precise and efficient with the use of algebraic language to describe general rules of patterns • It is useful also to introduce the language of 'term in a sequence' in order to distinguish the term from the position of a term in a sequence • Investigating number patterns is an opportunity to generalize – to give general algebraic descriptions of the relationship between terms and their position in a sequence and to justify solutions. **Kinds of numeric patterns** • Given a sequence of numbers, learners have to identify a pattern or **relationship between consecutive terms** in order to extend the pattern. **Examples** Provide a rule to describe the relationship between the numbers in the sequences below. Use this rule to provide the next three numbers in the sequence: a) $-3;\ -7-;\ -11;\ -15;\ \ldots\ \ldots$ Here learners should identify the constant difference between consecutive terms in order to extend the pattern. This pattern can be described in learners' own words as 'adding -4' or 'counting in -4 s' or 'add -4 to the previous number in the pattern'. b) $2;\ -4;\ 8;\ -16;\ 32\ \ldots\ \ldots$ Here learners should identify the constant ratio between consecutive terms. This pattern can be described in learners' own words as 'multiply the previous number by -2'. c) $1;\ 2;\ 4;\ 7;\ 11;\ 16\ \ldots\ \ldots$ This pattern has neither a constant difference nor constant ratio. This pattern can be described in learners' own words as 'increase the difference between	Total time for Numeric and geometric patterns: 4,5 hours

consecutive terms by 1 each time' or 'add 1 more than was added to get the previous term'. Using this rule, the next 3 terms will be $22, 29, 37$.

- Given a sequence of numbers, learners have to identify a pattern or **relationship between the term and its position in the sequence**. This enables learners to predict a term in a sequence based on the position of that term in the sequence. It is useful for learners to represent these sequences in tables so that they can consider the position of the term.

Examples

a) Provide a rule to describe the relationship between the numbers in this sequence: $1;\ 8;\ 27;\ 64;$ ... .. Use this rule to find the 10th term in this sequence.

Firstly, learners have to understand that the '10th term' refers to position 10 in the number sequence. They have to find a rule in order to determine the 10th term, rather than continuing the sequence to the tenth term.

Example

b) This sequence can be re-presented in the following table:

Position in sequence	1	2	3	4		10
Term	1	8	27	64		?

Learners have to recognize that each term in the bottom row is obtained by cubing the position number in the top row. Thus the 10th term will be'10 cubed' or 103, which is 1000. Using the same rule, learners can also be asked what term number or position will 512 be? If the term is obtained by cubing the position number of the term, then the position number can be obtained by finding the cube root of the term. Hence, 512 will be the 8th term in the sequence since $\sqrt[3]{512} = 8$

c) Provide a rule to describe the relationship between the numbers in this sequence:
$4;\ 7;\ 10;\ 13;$ ... Use this rule to find the 20th term in this sequence.

If learners consider only the relationship between consecutive terms, then they have to continue the pattern ('add 3 to previous number') to the 20th term to find the answer. However, if they look for a relationship or rule between the term and the position of the term, they can predict the answer without continuing the pattern. Using

number sentences can be useful to find the rule:

1st term: $4 = 3(1) + 1$
2nd term: $7 = 3(2) + 1$
3rd term: $10 = 3(3) + 1$
4th term: $13 = 3(4) + 1$
The number in the brackets corresponds to the position of the term. Hence, the 20th term will be: $3(20) + 1 = 61$
The rule in learners' own words can be written as '$3 \times$ the position of the term $+ 1$' or $3n + 1$, where n is the position of the term.

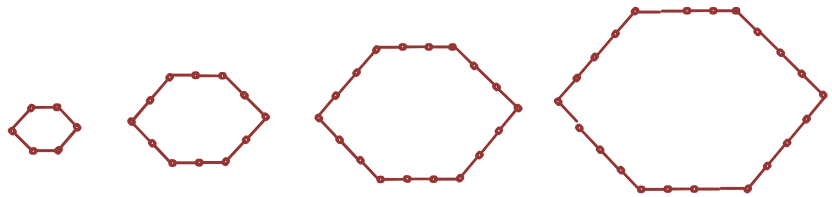
- These types of numeric patterns develop an understanding of functional relationships, in which you have a dependent variable (position of the term) and independent variable (the term itself), and where you have a unique output for any given input value.

Kinds of geometric patterns

- Geometric patterns are number patterns represented diagrammatically. The diagrammatic representation reveals the structure of the number pattern.
- Hence, representing the number patterns in tables makes it easier for learners to describe the general rule for the pattern.

Example

Consider this pattern for building hexagons with matchsticks. How many matchsticks will be used to build the 10th hexagon?



The rule for the pattern is contained in the structure (construction) of the successive hexagonal shapes:
(1) add 1 matchstick per side
(2) there are 6 sides
(3) add 6 matchsticks per hexagon as you proceed from a given hexagon to the next one.

So, for the 2nd hexagon, you have 2×6 matches; for the 3rd hexagon you have 3×6 matches: Using this pattern for building hexagons, the 10th hexagon will have 10×6 matches.

Learners can also use a table to record the number of matches used for each hexagon. This way they can look at the number pattern related to the number of matches used for each new hexagon.

Position of hexagon in pattern	**1**	**2**	**3**	**4**	**5**	**6**	**10**
Number of matches	6	12	18				

Describing patterns

- It does not matter if learners are already familiar with a particular pattern. Their descriptions of the same pattern can be different when they encounter it at different stages of their mathematical development.

  Example
  The rule for the sequence : $4;\ 7;\ 10;\ 13$ can be described in the following ways:

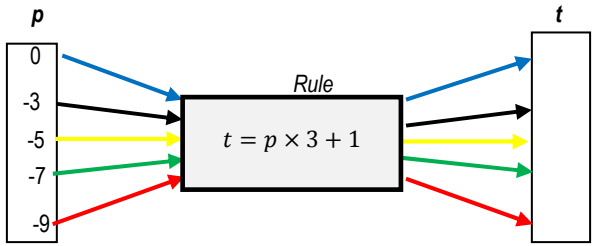
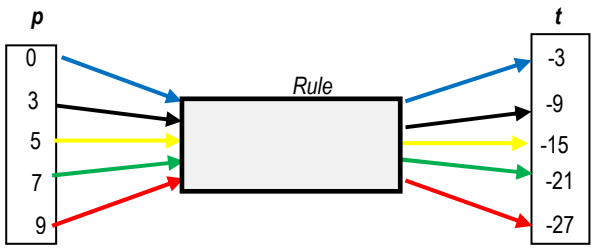
  a) add three to the previous term

  b) $3 \times$ the position of the term $+ 1$

  c) $3(n) + 1$, where n is the position of the term

  d) $3(n) + 1$, where n is a natural number

	2.2 Functions and relationships	**Input and output values** • Determine input values, output values or rules for patterns and relationships using: ○ flow diagrams ○ variables ○ formulae ○ equations	**What is different to Grade 7?** • Finding input or output values using given equations • The rules and number patterns for which learners have to find input and output values are extended to include patterns with multiplication and division of integers and numbers in exponential form • In this phase, it is useful to begin to specify whether the input values are natural numbers, or integers or rational numbers. This builds learners' awareness of the domain of input values. Hence, to find output values, learners should be given the rule/formula as well as the domain of the input values.	Time for Functions and Relationships in this term: 3 hours

		Equivalent forms • Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented: ○ verbally ○ in flow diagrams ○ in tables ○ by formulae ○ by equations	**Functions and relationships will be done again in Term 4. In Term 1 the focus of Functions and relationships is on practising operations with integers, or including integers in the rules for finding output values.** Note that when learners find input or output values for given rules, they are actually finding the numerical value of algebraic expressions using substitution. Flow diagrams are representations of functional relationships. Hence, when using flow diagrams, the correspondence between input and output values should be clear in its representational form i.e. the first input produces the first output, the second input produces the second output, etc. **Examples** a) Use the given rule to calculate the values of $\boldsymbol{t}$ for each value of $\boldsymbol{p}$, where $\boldsymbol{p}$ is a natural number.  In this kind of flow diagram, learners can also be asked to find the value of $\boldsymbol{p}$ for a given $\boldsymbol{t}$ value. b) Find the rule for calculating the output value for every given input value in the flow diagram below.  	

In flow diagrams such as these, more than one rule might be possible to describe the relationship between input and output values. The rules are acceptable if they match the given input values to the corresponding output values.

c) If the rule for finding y in the table below is: $y = -3x - 1$, find y for the given x values:

x	0	1	2	5	10	50	100
y							

d) Describe the relationship between the numbers in the top row and bottom row in the table. Then write down the value of m and n.

x	-2	-1	0	1	2		12	n
y	-5	-4	-3	-2	-1		m	34

In tables such as these, more than one rule might be possible to describe the relationship between x and y values. The rules are acceptable if they match the given input values to the corresponding output values. For example, the rule $y = x - 3$ describes the relationship between the x values and given y values. To find m and n, learners have to substitute the corresponding values for x or y in the rule and solve the equation by inspection.

2.3 Algebraic expressions

Algebraic language

- Revise the following done in Grade 7:
  - recognize and interpret rules or relationships represented in symbolic form
  - identify variables and constants in given formulae and equations
- Recognize and identify

What is different to Grade 7?

- Introduction to conventions of algebraic language
- Manipulating algebraic expressions

Algebraic expressions are done again in Term 2, where the focus is more fully on manipulating algebraic expressions. In this term the focus is on interpreting algebraic expressions and introducing conventions of algebraic language through adding and subtracting like terms.

Learners have opportunities to write and interpret algebraic expressions when they write

Time for algebraic expressions in this term:

4,5 hours

		conventions for writing algebraic expressions • Identify and classify like and unlike terms in algebraic expressions • Recognize and identify coefficients and exponents in algebraic expressions **Expand and simplify algebraic expressions** Use commutative, associative and distributive laws for rational numbers and laws of exponents to: • Add and subtract like terms in algebraic expressions	general rules to describe relationships between numbers in number patterns, and when they find input or output values for given rules in flow diagrams, tables, formulae and equations. **Examples of interpreting algebraic expressions** a) What does the rule 2^n mean for the following number sequence: $2;\ 4;\ 8;\ 16;\ 32$ .... Here learners should recognize that 2^n represents the general term in this sequence, where n represents the position of the term in the sequence. Thus, it is the rule that can be used to find any term in the given sequence. b) The relationship between a boy's age (x yrs old) and his mother's age is given as $25 + x$. How can this relationship be used to find the mother's age when the boy is 11 years old? Here learners should recognize that to find the mother's age, they must substitute the boy's given age into the rule $25 + x$. They should also recognize that the given rule means the mother is 25 years older than the boy. See further examples given for functions and relationships, as well as notes in Term 2.	
	2.4 Algebraic equations	**Equations** • Revise the following done in Grade 7: ○ set up equations to describe problem situations ○ analyse and interpret equations that describe a given situation ○ solve equations by Inspection ○ determine the numerical value of an expression by substitution.	**What is different to Grade 7?** Solving equations using additive and multiplicative inverses as well as laws of exponents **Algebraic equations are done again in Term 2 and Term 4. In this term the focus is on solving equations that involve multiplication and division of integers and numbers in exponential form, by inspection only.** Learners have opportunities to write and solve equations when they write general rules to describe relationships between numbers in number patterns, and when they find input or output values for given rules in flow diagrams, tables and formulae. See further notes in Term 2.	Time for Algebraic equations in this term: 3 hours
REVISION/ASSESSMENT: At this stage learners should have been assessed on: • calculating and solving problems with whole numbers and integers • representing, and calculating with, numbers in exponential form				Total time for revision/assessment for the term

• numeric and geometric patterns • functions and relationships • algebraic expressions • algebraic equations	6 hours

GRADE 8 – TERM 2				
CONTENT AREA	**TOPICS**	**CONCEPTS and SKILLS**	**SOME CLARIFICATION NOTES or TEACHING GUIDELINES**	**DURATION (in hours)**

Patterns, functions and algebra	**2.3 Algebraic expressions**	**Algebraic language**
		• Revise the following done in Grade 7:
		  ○ recognize and interpret rules or relationships represented in symbolic form
		  ○ identify variables and constants in given formulae and equations
		• Recognize and identify conventions for writing algebraic expressions
		• Identify and classify like and unlike terms in algebraic expressions
		• Recognize and identify coefficients and exponents in algebraic expressions
			What is different to Grade 7?
			• Introduction to conventions of algebraic language
			• Manipulating algebraic expressions
			
			Algebraic expressions were also done in Term 1. In this term the focus is on expanding and simplifying algebraic expressions.
			
			Learners have opportunities to write and interpret algebraic expressions when they write general rules to describe relationships between numbers in number patterns, and when they find input or output values for given rules in flow diagrams, tables, formulae and equations.
			
			Examples of interpreting algebraic expressions
			a) What does the rule $2n$ mean for the following number sequence: $2;\ 4;\ 8;\ 16;\ 32$ ....
			Here learners should recognize that $2n$ represents the general term in this sequence, where n represents the position of the term in the sequence. Thus, it is the rule that can be used to find any term in the given sequence.
			b) The relationship between a boy's age (x yrs old) and his mother's age is given as $25 + x$. How can this relationship be used to find the mother's age when the boy is 11 years old?
			Here learners should recognize that to find the mother's age, they have to substitute the boy's given age into the rule $25 + x$. They should also recognize that the given rule means the mother is 25 years older than the boy.
				Time for algebraic expressions in this term:
				
				9 hours

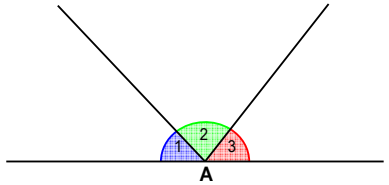
		Expand and simplify algebraic expressions
		Use commutative, associative and distributive laws for rational numbers and laws of exponents to:
		• Add and subtract like terms in algebraic expressions
		• Multiply integers and monomials by:
		  ○ monomials
		  ○ binomials
		  ○ trinomials
		
		• Divide the following by integers or monomials:
		  ○ monomials
		  ○ binomials
		  ○ trinomials
		• Simplify algebraic expressions involving the above operations
		• Determine the squares, cubes, square roots and cube roots of single algebraic terms or like algebraic terms
		• Determine the numerical value of algebraic expressions by substitution
			See further examples given for functions and relationships.
			
			Manipulating algebraic expressions
			• Make sure learners understand that the rule for operating with integers and rational numbers, including laws of exponents, applies equally when numbers are replaced with variables. The variables are numbers of a given type (e.g. whole numbers, integers or rational numbers) in generalized form.
			• When multiplying or dividing expressions, make sure learners understand how the distributive rule works.
			• The associative rule allows for grouping of like terms when adding.
			
			Look out for the following **common misconceptions**:
			• $x + x = 2x$ **AND NOT** $x2$. Note the convention is to write $2x$ rather than $x2$
			• $x^2 + x^2 = 2x^2$ **AND NOT** $2x4$
			• $a + b = a + b$ **AND NOT** ab
			• $(-2x^2)\,3 = -8x\,6$ **AND NOT** $-6x\,5$
			• $-x\,(3x + 1) = -3x^2 - x$ **AND NOT** $-3x^2 + 1$
			• $\frac{6x^2+1}{x^2} = 6 + \frac{1}{x^2}$ **AND NOT** $6 + 1$
			• If $x = 2$ then $-3x^2 = -3(2)2 = -3 \times 4 = -12$ **AND NOT** $(-6)\,2$
			• If $x = -2$ then $-x^2 - x = -(-2)\,2 — 2 = -4 + 2 = -2$ **AND NOT** $4 + 2 = 6$
			• $\sqrt{25x^2 - 9x^2} = \sqrt{16x^2} = 4x$ **AND NOT** $5x - 3x = 2x$
			
			Examples	

			a) Simplify: $2(5+x-x2)-x(3x+1)$ [multiply integer or monomial by polynomial] b) If $x=-2$ determine the numerical value of $3x^2-4x+5$ [using substitution] c) Simplify: $\frac{6x^3-2x^2+4}{2x^2}$ $x\neq 0$ [divide trinomial by monomial; reminder that denominator cannot be 0] d) Simplify: $\frac{8x^3-(-x^2)(2x)}{-x^2}$ $x\neq 0$ [calculations involving multiple operations; remind learners that denominator cannot be 0] e) Determine: $\sqrt{36x^4}$ [square root of monomial] It might help to remind learners that these variables (or x in this case) represent numbers of a particular type – these may be rational, or integers, or perhaps whole numbers; such a reminder also then implies that all the associated rules or properties of these numbers apply here. In the above example, if x is an integer, then $x=a$ or $x=-a$ because $a^4=(-a)^4$	
	2.4 Algebraic equations	**Equations** • Revise the following done in Grade 7: ○ Set up equations to describe problem situations ○ Analyse and interpret equations that describe a given situation ○ Solve equations by: ▪ Inspection ○ Determine the numerical value of an expression by substitution. • Extend solving equations to include: ○ Using additive and multiplicative inverses ○ Using laws of exponents	**What is different to Grade 7?** • Solving equations using additive and multiplicative inverses as well as laws of exponents **Algebraic equations were also done in Term 1. In this term the focus is on solving equations using additive and multiplicative inverses as well as the laws of exponents. Algebraic equations are done again in Term 4.** Learners have opportunities to write and solve equations when they write general rules to describe relationships between numbers in number patterns, and when they find input or output values for given rules in flow diagrams, tables and formulae. **Examples of equations** a) Solve x if $x+6=-9$ To solve the equation: add -6 to both sides of the equation $x+6-6=-9-6 \therefore x=-15$ b) Solve x if $-2x=8$ To solve the equation; divide both sides of the equation by -2: $\frac{-2x}{-2}=\frac{8}{-2} \therefore x=-4$	Time for algebraic equations in this term: 3 hours

			c) Solve x if $-x = -5$ To solve the equation; divide both sides of the equation by -1: $\frac{-x}{-1} = \frac{-5}{-1} \therefore x = 5$ d) Solve x if $3x + 1 = 7$ To solve the equation requires two steps: Add -1 to both sides of the equation: $3x + 1 - 1 = 7 - 1 \therefore 3x = 6$ Then divide both sides of the equation by 3 $\frac{3x}{3} = \frac{6}{3} \therefore x = 2$ e) Provide an equation to find the area of a rectangle with length $2x$ cm and width $2x + 1$ cm. f) If the area of a rectangle is $(4x^2 - 6x)$ cm2, and its width is $2x$ cm, what will be its length in terms of x? g) If $y = x^3 + 1$, calculate y when $x = 4$ h) Thandi is 6 years older than Sophie. In 3 years' time Thandi will be twice as old as Sophie. How old is Thandi now	
Space and shape (geometry)	**3.5 Construction of geometric figures**	**Constructions** • Accurately construct geometric figures appropriately using a compass, ruler and protractor, including: ○ bisecting angles of a triangle • Construct angles of $45°$, $30°$ and $60°$ and their multiples without using a protractor **Investigating properties of geometric figures** • By construction, investigate the angles in a triangle, focusing on the relationship between the exterior angle of a triangle and its interior angles • By construction, explore the minimum conditions for two triangles to be congruent • By construction, investigate sides, angles and diagonals in quadrilaterals,	**What is different to Grade 7?** • Bisecting angles in a triangle • Constructing $30°$ without a protractor • Investigation of new properties of triangles, quadrilaterals and polygons **Constructions** • Constructions provide a useful context to explore or consolidate knowledge of angles and shapes. • Make sure learners are competent and comfortable using a compass and know how to measure and read angle sizes on a protractor. • Revise the constructions of angles if necessary before proceeding with the new constructions. • Start with the constructions of lines, so that learners can first explore angle relationships on straight lines. • When constructing triangles learners should draw on known properties and construction of circles. • Construction of special angles without protractors are done by: ○ bisecting a right-angle to get $45°$ ○ drawing an equilateral triangle to get $60°$	Total Time for constructions of geometric figures: 8 hours

		focusing on the diagonals of rectangles, squares, parallelograms, rhombi and kites • By construction explore the sum of the interior angles of polygons	○ bisecting the angles of an equilateral triangle to get $30°$	
Space and shape (geometry)	**3.1 Geometry of 2D shapes**	**Classifying 2D shapes** • Revise properties and definitions of triangles in terms of their sides and angles, distinguishing between: ○ equilateral triangles ○ isosceles triangles ○ right-angled triangles • Revise and write clear definitions of quadrilaterals in terms of their sides, angles and diagonals, distinguishing between: ○ parallelogram ○ rectangle ○ square ○ rhombus ○ trapezium ○ kite	**What is different to Grade 7?** • Properties of diagonals of quadrilaterals • Minimum conditions for congruent and similar triangles **Triangles** • Constructions serve as a useful context for exploring properties of triangles. See notes on Constructions above. • Properties of triangles learners should know: ○ the sum of the interior angles of triangles = $180°$ ○ an equilateral triangle has all sides equal and all interior angles = $60°$ ○ an isosceles triangle has at least two equal sides and its base angles are equal ○ a right-angled triangle has one angle that is a right-angle ○ the side opposite the right-angle in a right-angled triangle, is called the hypotenuse ○ in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides (Theorem of Pythagoras). ○ the exterior angle of a triangle = the sum of the opposite two interior angles **Quadrilaterals** • Constructions serve as a useful context for exploring properties of triangles. See notes on Constructions above. • The classification of quadrilaterals should include the recognition that: ○ rectangles and rhombi are special kinds of parallelograms ○ a square is a special kind of rectangle and rhombus. • Properties of quadrilaterals learners should know: ○ the sum of the interior angles of quadrilaterals = $360°$ ○ the opposite sides of parallelograms are parallel and equal ○ the opposite angles of parallelograms are equal	Total time for geometry of 2D shapes 8 hours.

			○ the opposite angles of a rhombus are equal ○ the opposite sides of a rhombus are parallel and equal ○ the angles of rectangles and squares are $90°$ ○ a trapezium has one pair of opposite sides parallel ○ a kite has two pairs of adjacent sides equal ○ the diagonals of a square, rectangle, parallelogram and rhombus bisect each other ○ the diagonals of a square, rhombus and kite are perpendicular	
		Similar and congruent triangles • Through investigation, establish the minimum conditions for congruent triangles • Through investigation, establish the minimum conditions for similar triangles	**Congruent triangles** • Constructions are a useful context for establishing the minimum conditions for two triangles to be congruent. See notes on Constructions above. • Conditions for two triangles to be congruent: ○ three corresponding sides are equal (S,S,S) ○ two corresponding sides and the included angle are equal (S,A,S) ○ two corresponding angles and a corresponding side are equal (A,A,S) ○ right-angle, hypotenuse and one other corresponding side are equal (R,H,S) **Similar triangles** • Constructions are a useful context for establishing the minimum conditions for two triangles to be similar. See notes on Constructions above. • Condition for two triangles to be similar: ○ corresponding angles are equal and corresponding sides are proportional	
		Solving problems • Solve geometric problems involving unknown sides and angles in triangles and quadrilaterals, using known properties of triangles and quadrilaterals, as well as properties of congruent and similar triangles	**Solving problems** • Learners can solve geometric problems to find unknown sides and angles in triangles and quadrilaterals, using known definitions as well as angle relationships on straight lines. • For right-angled triangles, learners can also use the Theorem of Pythagoras to find unknown lengths. • Learners should be encouraged to give reasons and justify their solutions for every written statement. • Note that solving geometric problems is an opportunity to practise solving equations.	

			Example:  $\hat{A}_1$, $\hat{A}_2$ and $\hat{A}_3$are three angles on a straight line. $\hat{A}_2 = 75°$, $\hat{A}_3 = 55°$. What is the size of $\hat{A}_1$? Learners can find $\hat{A}_1$ by solving the following equation: $\hat{A}_1 + 75° + 55° + = 180°$ (because the sum of angles on a straight line $= 180°$) $\hat{A}_1 = 180° - 130°$ (add $-55°$ and $-75°$ to both sides of the equation) $\hat{A}_1 = 50°$	
	3.3 Geometry of straight lines	**Angle relationships** • Revise and write clear descriptions of the relationship between angles formed by: ○ perpendicular lines ○ intersecting lines ○ parallel lines cut by a transversal **Solving problems** • Solve geometric problems using the relationships between pairs of angles described above	**What is different to Grade 7?** • Learners revise and write clear descriptions of angle relationships on straight lines **Angle relationships learners should know:** • the sum of the angles on a straight line add up to $180°$ • vertically opposite angles are equal • if parallel lines are cut by a transversal, then corresponding angles are equal • if parallel lines cut by a transversal, then alternate angles are equal The above angles have to be identified and named by learner **Solving problems** • Learners can solve geometric problems to find unknown angles using the angle relationships above, as well as other known properties of triangles and quadrilaterals. • Learners should be encouraged to give reasons and justify their solutions for every written statement. • Note that solving geometric problems is an opportunity to practise solving equations. **Example:** $\hat{A}$, $\hat{B}$ and $\hat{C}$are three angles on a straight line. $\hat{A} = 55°$, $\hat{B} = 75°$. What is the size of $\hat{C}$?	Total time for geometry of straight lines: 9 hours

			Learners can find $\hat{C}$ by solving the following equation: $55° + 75° + \hat{C} = 180°$ (because the sum of angles on a straight line $= 180°$) $\hat{C} = 180° - 130°$ (add $-55°$ and $-75°$ to both sides of the equation) $\hat{C} = 50°$	
REVISION/ASSESSMENT: At this stage learners should have been assessed on: • algebraic expressions • algebraic equations • constructing geometric objects • geometry of 2D shapes • geometry of straight lines				Total time for revision/assessment for the term 8 hours

GRADE 8 – TERM 3				
CONTENT AREA	**TOPICS**	**CONCEPTS and SKILLS**	**SOME CLARIFICATION NOTES or TEACHING GUIDELINES**	**DURATION (in hours)**

Numbers, operations and relationships	**1.4 Common fractions**	**Calculations using fractions**
		• Revise:
		  ○ addition and subtraction of common fractions, including mixed numbers
		  ○ finding fractions of whole numbers
		  ○ multiplication of common fractions, including mixed numbers
		• Divide whole numbers and common fractions by common fractions
		• Calculate the squares, cubes, square roots and cube roots of common fractions
			What is different to Grade 7?
			• Divide by common fractions
			• Squares, cubes, square roots and cube roots of common fractions
			In Grade 8 learners consolidate number knowledge and calculation techniques for common fractions, developed in Grade 7.
			
			Calculations using fractions
			• Learners should continue to do context free calculations and solve problems in contexts.
			  ○ By Grade 8 learners should be comfortable converting mixed numbers to common fractions for calculations.
			  Example
			  $5\frac{1}{2} = \frac{11}{2}$; $6\frac{1}{3} = \frac{19}{3}$
			  ○ To simplify fractions, learners use knowledge of common factors i.e. what can divide equally into the numerator and denominator of a fraction. Emphasize that when simplifying, the fractions must remain equivalent.
				Total time for common fractions:
				
				7 hours

		Calculation strategies
		• Convert mixed numbers to common fractions in order to perform calculations with them
		• Use knowledge of multiples and factors to write fractions in the simplest form before or after calculations
		• Use knowledge of equivalent fractions to add and subtract common fractions
		• Use knowledge of reciprocal relationships to divide common fractions
		
		Solving problems
		• Solve problems in contexts involving common fractions and mixed numbers, including grouping, sharing and finding fractions of whole numbers
		
		Equivalent forms
		• Revise equivalent forms between:
		  ○ common fractions (fractions where one denominator is a multiple of the other)
		  ○ common fraction and decimal fraction forms of the same number
			Addition and subtraction
			• LCMs have to be found when adding and subtracting fractions with different denominators. Here learners use knowledge of common multiples to find the LCM i.e. what number can both denominators be divided into.
			
			Multiplication
			• For multiplication of fractions, learners should be encouraged to simplify fractions by dividing numerators and denominators by common factors.
			• Learners should note the difference between adding or subtracting fractions, and multiplying fractions
			
			Examples
			$\frac{3}{4}+\frac{2}{5}=\frac{15}{20}+\frac{8}{20}=\frac{23}{20}=1\frac{3}{20}$ (using LCM and equivalent fractions)
			
			$\frac{3}{4}\times\frac{2}{5}=\frac{3}{10}$ (divide 2 and 4 by common factor 2)
			
			• Learners should recognize that finding a 'fraction ***of*** a whole number' or 'finding a fraction ***of*** a fraction' means multiplying the fraction and the whole number or the fraction with the fraction.
			• When learners find fractions of whole numbers, the examples can be chosen to result either in whole numbers or fractions or both.
			• Learners should also use the convention of writing the whole number as a fraction over 1 when multiplying.
			Examples
			Find $\frac{4}{5}$ of $20=\frac{4}{5}\times\frac{20}{1}=\frac{80}{5}=16$
			
			Find $\frac{2}{3}$ of $\frac{5}{6}=\frac{2}{3}\times\frac{5}{6}=\frac{10}{18}=\frac{5}{9}$
			
			Division
			• The technique of 'invert and multiply' applies to division in general and not just to division by fractions. Hence, a useful way of making learners comfortable with division by fractions is to start with examples of	

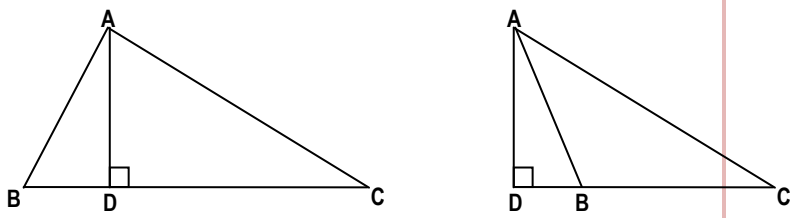
		○ common fraction, decimal fraction and percentage forms of the same number	division by whole numbers. • Learners have to understand that dividing by a number is the same as multiplying by the reciprocal of the number i.e. the reciprocal of n is $\frac{1}{n}$ **Examples**: a) $10 \div 5$ is the same as $10 \times \frac{1}{5} = 2$ (multiply by the reciprocal of 5) b) $10 \div \frac{1}{5} = 10 \times 5 = 50$ (multiply by the reciprocal of $\frac{1}{5}$) This can also be explained by using diagram models for fractions and asking, how many times does $\frac{1}{5}$ fit into 10? We know that 5 fifths fit into 1 whole, so (5×10) fifths will fit into 10 wholes. Hence, $10 \div \frac{1}{5} = 50$ c) $20 \div 4$ is the same as $20 \times \frac{1}{4} = 5$ (multiply by the reciprocal of 4) d) $20 \div \frac{1}{4} = 20 \times 4 = 80$ (multiply by the reciprocal of $\frac{1}{4}$) This can also be explained by using diagram models for fractions and asking, how many times does $\frac{1}{4}$ fit into 20? We know that 4 quarters fit into 1 whole, so (4×20) quarters will fit into 20 wholes. Hence, $20 \div \frac{1}{4} =$ 80 e) Once learners have done a few of the above examples, they can use the technique of multiplying by the reciprocal to divide fractions by fractions: $\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$ (multiply by the reciprocal of $\frac{1}{2}$) **Squares, cubes, square roots and cube roots** • Knowing the rules of operations for calculating squares, cubes, square roots and cube roots of common fractions is important **Examples:** a) $(\frac{3}{4})^2 = \frac{3^2}{4^2} = \frac{9}{16}$ b) $\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$ • Once learners are comfortable doing all the operations with fractions, calculations do not have to be restricted to positive fractions. **Calculation using percentages**	

		Percentages
		• Revise:
		  ○ finding percentages of whole numbers
		  ○ calculating the percentage of part of a whole
		  ○ calculating percentage increase or decrease
		• Calculate amounts if given percentage increase or decrease
		• Solve problems in contexts involving percentages
			• Learners should continue to do context free calculations and solve problems in contexts.
			• When doing calculations using percentages, learners have to use the equivalent common fraction form, which is a fraction with denominator100.
			• Learners should become familiar with the equivalent fraction and decimal forms of common percentages like 25% ($\frac{1}{4}$ or $0,25$); 50% ($\frac{1}{2}$ or $0{,}5$); 60% ($\frac{3}{5}$ or $0{,}6$).
			• To calculate percentage of part of a whole, or percentage increase or decrease, learners have to learn the strategy of multiplying by $\frac{1}{100}$. It is useful for learners to learn to use calculators for some of these calculations where the fractions are not easily simplified.
			• When using calculators, learners can use the equivalent decimal fraction form for percentages to do the calculations.
			
			Examples:
			a) Calculate 60% of $R105$
			Amount $= \frac{60}{100} \times R105 = R63$
			b) What percentage is $40c$ of $R3{,}20$?
			Percentage $= \frac{40}{320} \times \frac{100}{1} = \frac{100}{8} = 12{,}5\%$
			c) Calculate the percentage increase if the price of a bus ticket of $\mathrm{R}60$ is increased to $\mathrm{R}84$.
			Amount increased $=$ $\mathrm{R}24$. Therefore percentage increase
			$= \frac{24}{60} \times \frac{100}{1} = 40\%$
			d) Calculate the percentage decrease if the price of petrol goes down from 20 cents a litre to 18 cents a litre.
			Amount decreased $=$ 2 cents. Therefore percentage decrease
			$= \frac{2}{20} \times \frac{100}{1} = 10\%$	

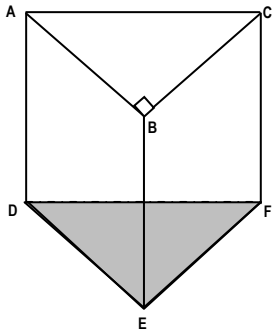
			e) Calculate how much a car will cost if its original price of $R150\ 000$ is reduced by 15%. Calculation involves finding 15% of $R150\ 000$ and then subtracting that amount from the original price. i.e. $\frac{15}{100} \times \frac{150\ 000}{1} = \text{R}22\ 500$ Hence new price of car $= \text{R}150\ 000 - \text{R}22\ 500 = \text{R}127\ 500$	
	1.5 Decimal fractions	**Ordering and comparing decimal fractions** • Revise: ○ Ordering, comparing and place value of decimal fractions to at least 3 decimal places ○ Rounding off decimal fractions to at least 2 decimal place	**What is different to Grade 7?** • Multiplication by decimal fractions not limited to one decimal place • Division of decimal fractions by decimal fractions • Squares, cubes, square roots and cube roots of decimal fractions In Grade 8 learners consolidate number knowledge and calculation techniques for decimal fractions, developed in Grade 7. **Ordering, counting and comparing decimal fractions** • Learners should continue to practise counting, ordering and comparing decimal fractions. Counting should not only be thought of as verbal counting. Learners can count in decimal intervals using: ○ structured, semi-structured or empty number lines ○ chain diagrams for counting • Learners should be given a range of exercises • Arrange given numbers from the smallest to the biggest: or biggest to smallest. • Fill in missing numbers in ○ a sequence ○ on a number grid ○ on a number line ○ fill in $<, =$ or $>$ e.g. $0{,}4 * 0{,}04 * 0{,}004$ • Counting exercises in chain diagrams can be checked using calculators and learners can explain any differences between their answers and those shown by the calculator.	Total time for decimal fractions: 6 hours

		Calculations using decimal fractions
		• Revise:
		  ○ addition, subtraction and multiplication of decimal fractions to at least 3 decimal places
		  ○ division of decimal fractions by whole numbers
		
		• Extend multiplication to include decimal fractions not limited to one decimal place
		• Extend division to include decimal fractions by decimal fractions
		• Calculate the squares, cubes, square roots and cube roots of decimal fractions
		
		Calculation strategies
		• Use knowledge of place value to estimate the number of decimal places in the result before performing calculations
		• Use rounding off and a calculator to check results where appropriate
		
		Solving problems
		• Solve problems in context involving decimal fractions
		
		Equivalent forms
		• Revise equivalent forms between:
		○ common fraction and decimal fraction forms of the same number
		○ common fraction, decimal fraction
			Calculating using decimal fractions
			• Learners should continue to do context free calculations and solve problems in contexts.
			• Learners should estimate their answers before calculating, especially with multiplication and division by decimal fractions. They should be able to judge the reasonableness of answers in respect of how many decimal places and also check their own answers.
			• Multiplication by decimal fractions should start with familiar numbers that learners can calculate by inspection, so that learners get a sense of how decimal places are affected by multiplication.
			Examples:
			a) $3 \times 2 = 6$
			$0{,}3 \times 2 = 0{,}6$
			$0{,}3 \times 0{,}2 = 0{,}06$
			$0{,}3 \times 0{,}02 = 0{,}006$
			$0{,}03 \times 0{,}02 = 0{,}0006\ etc$
			b) $15 \times 3 = 45$
			$1{,}5 \times 3 = 4{,}5$
			$0{,}15 \times 3 = 0{,}45$
			$0{,}15 \times 0{,}3 = 0{,}045$
			$0{,}015 \times 0{,}3 = 0{,}0045\ etc$
			• For division by decimal fractions without calculators, learners have to use their knowledge of multiplication by 10 or multiples of 10 to make the divisor a whole number. Hence start with familiar numbers that learners can calculate by inspection, so that learners get a sense of how decimal places are affected by division.
			Examples:
			a) $54 \div 6 = 9$
			$54 \div 0{,}6 = 540 \div 6 = 90$
			(multiply both numbers by 10 to make the decimal fraction a whole number)
			$54 \div 0{,}06 = 5400 \div 6 = 900$
			(multiply both numbers by 100 to make the decimal fraction a whole number)
			$0{,}54 \div 0{,}06 = 54 \div 6 = 9$
			(multiply both numbers by 100 to make the decimal fraction a whole number)
			b) $125 \div 5 = 25$	

		and percentage forms of the same number	$125 \div 0{,}5 = 1250 \div 5 = 250$ (multiply both numbers by 10 to make the decimal fraction a whole number) $125 \div 0{,}05 = 12500 \div 5 = 2500$ (multiply both numbers by 100 to make the decimal fraction a whole number) $1{,}25 \div 0{,}05 = 125 \div 5 = 25$ (multiply both numbers by 100 to make the decimal fraction a whole number) • For bigger and unfamiliar decimal fractions, learners should use calculators for multiplication and division, but still judge the reasonableness of their solutions. • Similarly, finding squares, cubes, square roots and cube roots for decimal fractions should start with familiar numbers that learners can calculate by inspection. **Examples**: a) $4^2 = 16$ $(0{,}4)^2 = 0{,}4 \times 0{,}4 = 0{,}16$ $(0{,}04)^2 = 0{,}04 \times 0{,}04 = 0{,}0016$ b) $(0{,}1)^3 = 0{,}1\, x\, 0{,}1 \times 0{,}1 = 0{,}001$ c) $\sqrt{0{,}04} = 0{,}2$ • Once learners are comfortable with all the operations using decimal fractions, calculations should not be restricted to positive decimal fractions.	
Measurement	**4.3 The Theorem of Pythagoras**	**Develop and use the Theorem of Pythagoras** • Investigate the relationship between the lengths of the sides of a right-angled triangle to develop the Theorem of Pythagoras • Determine whether a triangle is a right-angled triangle or not, if the length of the three sides of the triangle are known • Use the Theorem of Pythagoras to calculate a missing length in a right-angled triangle, leaving irrational answers in surd form	• The theorem of Pythagoras is new in Grade 8. • It is important that learners understand that the Theorem of Pythagoras applies only to right-angled triangles. • The Theorem of Pythagoras is basically a formula to calculate unknown length of sides in right-angled triangles. • In the FET phase, the Theorem of Pythagoras is crucial to the further study of Geometry and Trigonometry **Examples of solving problems using the Theorem of Pythagoras:** • In ΔABC, $\hat{B} = 90°, AC = 4cm,\ BC = 2cm.$ Calculate the length of AB without using a calculator. Leave the answer in the simplest surd form.	Total time for the Theorem of Pythagoras: 5 hours

	4.1 Area and Perimeter of 2-D shapes	**Area and Perimeter** • Use appropriate formulae to calculate perimeter and area of: ○ squares ○ rectangles ○ triangles • Calculate the areas of polygons, to at least 2 decimal places, by decomposing them into rectangles and/or triangles • Use and describe the relationship between the radius, diameter and circumference of a circle in calculations • Use and describe the relationship between the radius and area of a circle in calculations **Calculations and solving problems** • Solve problems, with or without a calculator, involving perimeter and area of polygons and circles • Calculate to at least 2 decimal places • Use and describe the meaning of the irrational number **Pi (π)** in calculations involving circles • Use and convert between appropriate SI units, including: ○ $mm^2 \leftrightarrow cm^2 \leftrightarrow m^2 \leftrightarrow km^2$	**What is different to Grade 7?** • Areas of polygons by decomposition • Circumference and area of a circle • **Formulae** learners should know and use: ○ perimeter of a square $= 4s$ ○ perimeter of a rectangle $= 2(l + b)$ or $2l + 2b$ ○ area of a square $= l^2$ ○ area of a rectangle $= l \times b$ ○ area of a triangle $=$ ½ $(b \times h)$ ○ diameter of a circle: $d = 2r$ ○ circumference of circle: $c = \pi d$ or $2\pi r$ ○ area of a circle: $A = \pi r^2$ **Solving equations using formulae** • The use of formulae provides a context to practise solving equations by inspection or using additive or multiplicative inverses. **Examples:** 1. If the perimeter of a square is 32 *cm*, what is the length of each side? Learners should write this as: $4s = 32$ and solve by asking: 4 times what will be 32 OR saying $s = \frac{32}{4}$? 2. If the area of a rectangle is $200\ cm^2$, and its length is $50\ cm$, what is its width? Learners must should this as: $50 \times b = 200$ and solve by inspection by asking: 50 times what will be 200 OR saying $b = \frac{200}{50}$? **For areas of triangles:** • Make sure learners know that the height of a triangle is a line segment drawn from any vertex perpendicular to the opposite side. **Example**: AD is the height onto base BC of ΔABC. 	Total time for area and perimeter: 5 hours.

			• Point out that every triangle has 3 bases, each with a related height or altitude. • **For conversions, note**: ○ If $1\ cm\ =\ 10\ mm$ then $1\ cm^2\ =\ 100\ mm^2$ ○ If $1\ m\ =\ 100\ cm$ then $1\ m^2\ =\ 10\ 000\ cm^2$ **Circles** • Make sure learners can identify the centre, radius, diameter and circumference of the circle. • Spend time investigating the relationship between radius, circumference and diameter, so that learners develop a sense of where the irrational number Pi (π) is derived from. • Develop an understanding of π, making sure learners understand that: ○ π represents the value of the circumference divided by the diameter, for any circle ○ π is an irrational number and is given as $3{,}141592654$ correct to 9 decimal places on the calculator ○ $\frac{22}{7}$ or $3{,}14$ are approximate rational values of π in everyday use.	
	4.2 Surface area and volume of 3D objects	**Surface area and volume** • Use appropriate formulae to calculate the surface area, volume and capacity of: ○ cubes ○ rectangular prisms ○ triangular prisms • Describe the interrelationship between surface area and volume of the objects mentioned above **Calculations and solving problems** • Solve problems, with or without a calculator, involving surface area, volume and capacity • Use and convert between appropriate SI units, including: ○ $mm^2 \leftrightarrow cm^2 \leftrightarrow m^2 \leftrightarrow km^2$	**What is different to Grade 7?** • Surface area and volume of triangular prisms • **Formulae** learners should know and use: ○ the volume of a prism = the area of the base x the height ○ the surface area of a prism = the sum of the area of all its faces ○ the volume of a cube $=\ l^3$ ○ the volume of a rectangular prism $=\ l\ \times\ b\ \times\ h$ ○ the volume of a triangular prism $=\ (\frac{1}{2}\ b\ \times\ h)\ \times\ height\ of\ the\ prism$ • **For conversions, note:** ○ if $1cm\ =\ 10mm$ then $1cm^3\ =\ 1\ 000mm^3$ and ○ if $1m\ =\ 100cm$ then $1m^3\ =\ 1\ 000\ 000cm^3$ or 10^6cm^3 ○ an object with a volume of $1cm^3$ will displace exactly $1ml$ of water ○ an object with a volume of $1m^3$ will displace exactly $1kl$ of water.	Total time for surface area and volume: 5 hours

		○ $mm^3 \leftrightarrow cm^3 \leftrightarrow m^3$ ○ $ml\ (cm^3) \leftrightarrow l \leftrightarrow kl$	• Emphasize that the amount of space inside a prism is called its capacity; and the amount of space occupied by a prism is called its volume. • Investigate the nets of cubes and rectangular prisms in order to deduce formulae for calculating their surface areas. **Example of solving problems involving surface area and volume:** • Calculate the volume and surface area of the prism if AB $= 8\ cm$, BC $= 6\ cm$ and CF $= 16\ cm$.  	
Data handling	**5.1 Collect, organize and summarize data**	**Collect data** • Pose questions relating to social, economic, and environmental issues • Select appropriate sources for the collection of data (including peers, family, newspapers, books, magazines) • Distinguish between samples and populations, and suggest appropriate samples for investigation	**What is different to Grade 7?** The following are new in Grade 8 • extremes • broken line graphs • dispersion of data • error and bias in data **Data sets and contexts** Learners should be exposed to a variety of contexts that deal with social and environmental issues, and should work with given data sets, represented in a variety of ways, that include big number ranges, percentages and decimal fractions. Learners should then practise organizing and summarizing this data, analysing and interpreting the data, and writing a report about the data.	Total time for collecting and organizing data: 4 hours.

		• Design and use simple questionnaires to answer questions: ○ with multiple choice responses **Organize and summarize data** • Organize (including grouping where appropriate) and record data using ○ tallies ○ tables ○ stem-and-leaf displays • Group data into intervals • Summarize data using measures of central tendency, including: ○ mean ○ median ○ mode • Summarize data using measures of dispersion, including: ○ range ○ extremes	**Complete a data cycle** Learners should complete at least one data cycle for the year, starting with posing their own questions, selecting the sources and method for collecting, recording, organizing, representing, analysing, summarizing, interpreting and reporting the data. Challenge learners to think about what kinds of questions and data need to be collected to be represented on a histogram, a pie chart, a bar graph, or a line graph.	
	5.5. Represent data	**Represent data** • Draw a variety of graphs by hand/technology to display and interpret data including**:** ○ bar graphs and double bar graphs ○ histograms with given and own intervals ○ pie charts ○ broken-line graphs	**Representing data** • Drawing pie charts to represent data do not have to be accurately drawn with a compass and protractor, etc. Learners can use any round object to draw a circle, then divide the circle into halves and quarters and eighths if needed, as a guide to estimate the proportions of the circle that need to be shown to represent the data. What is important is that the values or percentages associated with the data are shown proportionally on the pie chart. • Drawing, reading and interpreting pie charts is a useful context to re-visit equivalence between fractions and percentages, e.g. 25% of the data is represented by a ¼ sector of the circle. • It is a context in which learners can find percentages of whole numbers e.g. if 25% of 300 learners like rugby, how many (actual number) learners like rugby? • Histograms are used to represent grouped data shown in intervals on the horizontal axis of the graph. Point out the differences between	Total time for representing data: 3 hours

			histograms and bar graphs, in particular bar graphs that represent discrete data e.g. favourite sports, compared to histograms that show categories in consecutive, non-overlapping intervals, e.g. test scores out of 100 shown in intervals of 10. The bars on bar graphs do not have to touch each other, while in a histogram they have to touch since they show consecutive intervals. • Broken-line graphs refer to data graphs that represent data points joined by a line and are not the same as straight line graphs that are drawn using the equation of the line. • Broken-line graphs are used to represent data that changes continuously over time, e.g. average daily temperature for a month. Each day's temperature is represented with a point on the graph, and once the whole month has been plotted, the points are joined to show a broke- line graph. • Broken-line graphs are useful to read 'trends' and patterns in the data, for predictive purposes e.g. will the temperatures go up or down in the next month.	
	5.6. Analyse, Interpret and report data	**Interpret data** • Critically read and interpret data represented in: ○ words ○ bar graphs ○ double bar graphs ○ pie charts ○ histograms ○ broken-line graphs **Analyse data** • Critically analyse data by answering questions related to: ○ data categories, including data intervals ○ data sources and contexts ○ central tendencies – (mean, mode, median) ○ scales used on graphs ○ samples and populations	**Developing critical analysis skills** • Learners should compare the same data represented in different ways e.g. in a pie chart or a bar graph or a table, and discuss what information is shown and what is hidden; they should evaluate what form of representation works best for the given data. • Learners should compare graphs on the same topic but where data has been collected from different groups of people, at different times, in different places or in different ways. Here learners should discuss differences between the data with an awareness of bias related to the impact of data sources and methods of data collection on the interpretation of the data. • Learners should compare different ways of summarizing the same data sets, developing an awareness of how data reporting can be manipulated; they should evaluate which summary statistics best represent the data. • Learners should compare graphs of the same data, where the scales of the graphs are different. Here learners should discuss differences with an awareness of how representation of data can be manipulated; they should evaluate which form of representation works best for the given data.	Total time for analysing, interpreting and summarising data: 3,5 hours

		○ dispersion of data ○ error and bias in the data **Report data** • Summarize data in short paragraphs that include ○ drawing conclusions about the data ○ making predictions based on the data ○ identifying sources of error and bias in the data ○ choosing appropriate summary statistics for the data (mean, median, mode, range) ○ the role of extremes in the data	• Learners should compare data on the same topic, where one set of data has extremes, and discuss differences with an awareness of the effect of the extremes on the interpretation of the data, in particular, extremes affect the range. • Learners should be encouraged to write reports on the data in short paragraphs.	
REVISION/ASSESSMENT: At this stage learners should have been assessed on: ▪ calculating and solving problems with common fractions and decimal fractions ▪ the Theorem of Pythagoras ▪ area and perimeter of 2D shapes ▪ surface area and volume of 3D objects				Total time for revision/assessment for the term 6,5 hours.

GRADE 8 – TERM 4				
CONTENT AREA	**TOPICS**	**CONCEPTS and SKILLS**	**SOME CLARIFICATION NOTES or TEACHING GUIDELINES**	**DURATION (in hours)**
Patterns, functions and algebra	**2.2 Functions and relationships**	**Input and output values** • Determine input values, output values or rules for patterns and relationships using: o flow diagrams o tables o formulae o equations **Equivalent forms** • Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented: o verbally o in flow diagrams o in tables o by formulae o by equations	Functions and relationships were also done in Term 1. In this term the focus is on using formulae to find output values from given input values, as well as equivalent forms of descriptions of the same relationship. **See further notes and examples in Term 1.** **Example** Use the formula for the area of a rectangle: $A = l \times b$ to calculate the following: a) The area, if the length is $4{,}5\ cm$ and the width is $2{,}5\ cm$ b) The length, if the area is $240\ cm^2$ and the width is $4\ cm$ c) The width, if the area is $14\ cm^2$ and the length is $3{,}5\ cm$ Learners can write these as number sentences, and solve by inspection.	Time for Functions and Relationships in this term: 6 hours
	2.3 Algebraic equations	**Equations** • Revise the following done in Grade 7: o set up equations to describe problem situations o analyse and interpret equations that describe a given situation o solve equations by inspection o determine the numerical value of an expression by substitution	Algebraic equations were also done in Terms 1 and 2. In this term the focus is on using substitution in equations to generate tables of ordered pairs. **See further notes and examples in Terms 1 and 2.**	Time for Algebraic equations in this term: 3 hours

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<td>
- Extend solving equations to include:
  - using additive and multiplicative inverses
  - using laws of exponents
- Use substitution in equations to generate tables of ordered pairs
</td>
<td>
Examples of generating ordered pairs
a) Complete the table below for x and y values for the equation : $y = -3x + 2$
<table>
<tr><td>x</td><td>-3</td><td>-1</td><td>0</td><td></td><td></td></tr>
<tr><td>y</td><td></td><td></td><td></td><td>-4</td><td>-10</td></tr>
</table>
b) Complete the table below for x and y values for the equation: $y = x^2 - 2$
<table>
<tr><td>x</td><td>-3</td><td>-2</td><td>0</td><td></td><td></td></tr>
<tr><td>y</td><td></td><td></td><td></td><td>-2</td><td>2</td></tr>
</table>
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<td></td>
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<td>2.5 Graphs</td>
<td>
Interpreting graphs
- Revise the following done in Grade 7:
  - Analyse and interpret global graphs of problem situations, with a special focus on the following trends and features:
    - linear or non-linear
    - constant, increasing or decreasing
- Extend the focus on features of graphs to include:
  - maximum or minimum
  - discrete or continuous

Drawing graphs
- Draw global graphs from given descriptions of a problem situation, identifying features listed above
- Use tables of ordered pairs to plot points and draw graphs on the Cartesian plane
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<td>
What is different to Grade 7?
- New features of global graphs: maximum and minimum; discrete and continuous
- Plotting points to draw graphs

Examples of contexts for global graphs include:
- the relationship between time and distance travelled
- the relationship between temperature and time over which it is measured
- the relationship between rainfall and time over which it is measured, etc.

Examples of drawing graphs by plotting points

a) Complete the table of ordered pairs below for the equation: $y = x + 3$
<table>
<tr><td>x</td><td>-4</td><td>-3</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>y</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>
Now, plot the above co-ordinate points on the Cartesian plane. Join
</td>
<td>Total time for graphs:

9 hours</td>
</tr>
</table>

<table>
<tr>
<td></td>
<td></td>
<td></td>
<td>points to form a graph.

b) Complete the table of ordered pairs below for the equation: $y = x^2 + 3$

<table>
<tr><td>x</td><td>-4</td><td>-3</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>y</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Now, plot the above co-ordinate points on the Cartesian plane. Join points to form a graph.</td>
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<tr>
<td>Space and shape (geometry)</td>
<td>3.4 Transformation geometry</td>
<td>Transformations
• Recognize, describe and perform transformations with points on a co-ordinate plane, focusing on:
○ reflecting a point in the y-axis or x-axis
○ translating a point within and across quadrants
• Recognize, describe and perform transformations with triangles on a co-ordinate plane, focusing on the co-ordinates of the vertices when:
○ reflecting a triangle in the x-axis or y-axis
○ translating a triangle within and across quadrants
○ rotating a triangle around the origin

Enlargements and reductions
• Use proportion to describe the effect of enlargement or reduction on area and perimeter of geometric figures</td>
<td>What is different to Grade 7?
• Transformations are done on a co-ordinate plane
• Co-ordinates of points and vertices

Co-ordinate plane
• Doing transformations on the co-ordinate plane is an opportunity to practise plotting points with ordered pairs, and links up with drawing algebraic graphs.
• Learners have to learn how to plot points on the co-ordinate plane and read the co-ordinates of points off the x-axis and y-axis. This is also done with algebraic graphs.
• Learners have to know the convention for writing ordered pairs $(x; y)$
• Point out the differences between the axes in the four quadrants.

Focus of transformations
• Doing transformations on a co-ordinate plane focuses attention on the co-ordinates of points and vertices of shapes.
• Learners should recognize that translations, reflections and rotations only change the position of the figure, and not its shape or size.
• Learners should recognize that the above transformations produce congruent figures.
• Learners do not have to learn general rules for the transformations at this stage, but should explore the way the co-ordinates of points change when performing different transformations with lines or shapes.
• Learners should recognize that enlargements and reductions change</td>
<td>Total time for Transformations:

6 hours</td>
</tr>
</table>

			the size of figures by increasing or decreasing the length of sides, but keeping the angles the same, produces similar rather than congruent figures. • Learners should also be able to work out the factor of enlargement or reduction of a figure. **Examples of transformation problems** • Plot point $A(4;3)$ and A' , its image, after reflection in: a) the x-axis b) the y-axis. • Write down the co-ordinates of T' if $T(-2;3)$ is translated 4 units downwards. • The perimeter of square $ABCD = 48cm$. a) Write down the perimeter of the square if the length of each side is doubled. b) Will the area of the enlarged square be twice or four times that of the original square?	
	3.1. Geometry of 3D objects	**Classifying 3D objects** • Describe, name and compare the 5 Platonic solids in terms of the shape and number of faces, the number of vertices and the number of edges **Building 3D models** • Revise using nets to make models of geometric solids, including: ○ cubes ○ prisms ○ pyramids	**What is different to Grade 7?** • Naming and comparing Platonic solids • Nets of pyramids **Platonic solids** • Platonic solids are a special group of polyhedra that have faces that are congruent regular polygons. • There are only 5 Platonic solids: ○ Tetrahedron ○ Hexahedron (cube) ○ Octahedron ○ Dodecahedron ○ Icosahedrons • The name of each Platonic solid is derived from its number of faces. • Platonic solids provide an interesting context in which to investigate the relationship between the number of faces, vertices and edges. By listing these properties for all the Platonic solids, learners can investigate the pattern that emerges, to come up with the general rule: $V - E + F = 2$, where $V = number\ of\ vertices$; $E = number\ of\ edges$; $F = faces$	Total time for geometry of 3D objects: 7 hours

			Using and constructing nets • Using and constructing nets are useful contexts for exploring or consolidating properties of polyhedra. • Learners should recognize the nets of different solids. • Learners should make sketches of the nets using their knowledge of the shape and number of faces of the solids, before drawing and cutting out the nets to build models. • Since learners have more knowledge about the size of angles in equilateral triangles, and can measure angles, their constructions of nets should be more accurate. • Learners have to work out the relative position of faces of the nets in order to build the 3D model.	
Data handling	**5.4 Probability**	**Probability** • Consider a simple situation (with equally likely outcomes) that can be described using probability and**:** ○ list all the possible outcomes ○ determine the probability of each possible outcome using the definition of probability ○ predict, with reasons, the relative frequency of the possible outcomes for a series of trials based on probability ○ compare relative frequency with probability and explain possible differences	**Probability experiments** In the Intermediate Phase and Grade 7 learners did probability experiments with coins, dice and spinners. In Grade 8 doing actual trials of experiments become less important, and learners should consider probability for hypothetical events e.g. the probability of white as a successful outcome on a roulette table, or the probability of getting a Coca Cola at the shop if you know what the total number of drinks is that they stock and how many cans of Coca Cola they have. **Comparing relative frequency and probability** • The relative frequency is the observed number of successful outcomes for a finite sample of trials. • For example, if you toss a coin 50 times, the results are 27 heads and 23 tails. Define a head as a successful outcome. The relative frequency of heads is: $\frac{27}{50} = 54\%$ • The probability of a head is 50% (one of two likely outcomes). The difference between the relative frequency of 54% and the probability of 50% is due to small sample size. • The more trials you do, the closer the relative frequency gets to the probability. This can be compared in class by combining results from trials done in groups or pairs.	Total time for probability: 4,5 hours

REVISION/ASSESSMENT: At this stage learners should have been assessed on: ▪ functions and relationships ▪ algebraic equations ▪ graphs ▪ transformation geometry ▪ geometry of 3D objects ▪ probability	Total time for revision/assessment for the term 9, 5 hours

3.3.3. Clarification of content for Grade 9

GRADE 9 – TERM 1				
CONTENT AREA	**TOPICS**	**CONCEPTS and SKILLS**	**SOME CLARIFICATION NOTES or TEACHING GUIDELINES**	**DURATION (in hours)**
Numbers, operations and relationships	**1.1 Whole numbers**	**Properties of numbers** • Describe the real number system by recognising, defining and distinguishing properties of: ○ natural numbers (excludes 0) ○ whole numbers (includes 0) ○ integers ○ rational numbers ○ irrational numbers **Calculations with whole numbers** Revise: • Calculations with all four operations on whole numbers, estimating and using calculators where appropriate **Calculation strategies** Use a range of strategies to perform and check written and mental calculations with whole numbers including: • estimation • adding, subtracting and multiplying in columns • long division • rounding off and compensating • using a calculator **Multiples and factors** Use prime factorisation of numbers to find LCM and HCF	**What is different to Grade 8?** In Grade 9 learners consolidate number knowledge and calculation techniques for whole numbers, developed in Grade 8. • The focus in Grade 9 should be on developing an understanding of different number systems and the properties of operations that apply for different number systems. • The contexts for solving problems should be more complex and varied, involving whole numbers, integers and rational numbers. Financial contexts are especially rich in this regard. • Learners should be given a clear indication of when the use of calculators is permissible or not. Calculators should be used routinely for calculations with big numbers and where knowledge of number facts or concepts are not explicitly assessed. However, guard against learners becoming dependent on calculators for all calculations. Calculators remain a useful tool for checking solutions. • Competency in finding multiples and factors, and prime factorisation of whole numbers, remains important for developing competency in factorising algebraic expressions and solving algebraic equations. **Properties of numbers** • By distinguishing the properties of different number systems, learners should recognize that natural numbers is a subset of whole numbers, which in turn is a subset of integers, which in turn is a subset of rational numbers. All of these numbers form part of the real number system. • Note that 0 may sometimes by included in the set of natural numbers. • Learners should recognize the following distinguishing features of the number systems: ○ ***integers*** extend the ***natural*** and ***whole number systems*** by including the operation $a - b$, where $a < b$. ○ ***rational numbers*** extend the set of ***integers*** by including the operation $\frac{a}{b}$ where $a < b$	Total time for whole numbers 4, 5 hours.

		Solving problems
		• Solve problems in contexts involving
		  ○ ratio and rate
		  ○ direct and indirect proportion
			○ ***rational numbers*** are defined as numbers that can be written in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$
			○ since ***integers*** are a subset of ***rational numbers***, every integer, can be expressed as a rational number $\frac{a}{b}$
			○ ***irrational numbers*** are numbers that cannot be expressed as a ***rational numbers*** in the form $\frac{a}{b}$
			○ **Pi** $(\boldsymbol{\pi})$ is an ***irrational number***, even though we use $\frac{22}{7}$ or $3{,}14$ as rational number approximations for π in calculations
			
			Ratio and rate problems
			• Include problems involving speed, distance and time. Learners should be familiar with the following formulae for these calculations:
			a) $speed = \frac{distance}{time}$
			b) $distance = speed \times time$
			c) $time = \frac{distance}{speed}$
			• Speed is usually given as constant speed or average speed.
			• Make sure learners recognize and are able to convert correctly between units for time and distance.
			
			Examples
			a) A car travelling at a constant speed travels $60\ km$ in 18 minutes. How far, travelling at the same constant speed, will the car travel in 1 hour 12 minutes?
			b) A car travelling at an average speed of $100\ km/h$ covers a certain distance in 3 hours 20 minutes. At what constant speed must the car travel to cover the same distance in 2 hours 40 minutes?
			
			Direct and Indirect proportion
			Learners should be familiar with the following relationships:
			• x is directly proportional to y if $\frac{x}{y}$= constant
			• x and y are directly propotional if, as the value of x increases the value of y increases in the same proportion, and as the value of x	

		• Solve problems that involve whole numbers, percentages and decimal fractions in financial contexts such as: ○ profit, loss, discount and VAT ○ budgets ○ accounts and loans ○ simple interest and higher purchase ○ exchange rates and commission ○ rentals ○ compound interest	decreases the value of y decreases in the same proportion • The direct proportional relationship is represented by a straight line graph • x is indirectly or inversely proportional to y if $x \times y =$ a constant, . In other words $y = \frac{c}{x}$ • x and y are indirectly proportional if, as the value of x increases the valu and as the value of x decreases the value of y increases • an indirect proportional relationship is represented by a non-linear curve **Financial contexts** • Once learners have done sufficient calculations for simple and compound interest through repeated calculations, they could use given formulae for these calculations. **Examples** a) Calculate the simple interest on $\mathrm{R}600$ at 7% p.a for 3 years using the formula $SI = \frac{P.n.r}{100}$ or $SI = P.n.i$ for $i = \frac{r}{100}$. b) $R800$ invested at $r\%$ per annum simple interest for a period of 3 years yields $R168$. Calculate the value of r. c) How long will it take for $R3000$ invested at 6% per annum simple interest to grow to $R4260$? d) Temoso borrowed $R500$ from the bank for 3 years at 8% p.a. compound interest. Without using a formula, calculate how much Temoso owes the bank at the end of three years. e) Use the formula $A = (1 + \frac{r}{100})^n$ to calculate the compound interest on a loan of $R3450$ at $6{,}5\%$ per annum for 5 years.	
	1.3 Integers	**Calculations with integers** • Revise: ○ perform calculations involving all four operations with integers ○ perform calculations involving all	**What is different to Grade 8?** In Grade 9 learners consolidate number knowledge and calculation techniques for integers, developed in Grade 8. In Grade 9, learners work with integers mostly as coefficients in algebraic	Total time for Integers: 4,5 hours

		four operations with numbers that involve the squares, cubes, square roots and cube roots of integers
		
		Properties of integers
		• Revise:
		  ○ commutative, associative and distributive properties of integers
		  ○ additive and multiplicative inverses for integers
		Solving problems
		• Solve problems in contexts involving multiple operations with integers
			expressions and equations. They are expected to be competent in performing all four operations with integers and using the properties of integers appropriately where necessary.	

	1.4 Common fractions	**Calculations using fractions** • All four operations with common fractions and mixed numbers • All four operations, with numbers that involve the squares, cubes, square roots and cube roots of common fractions **Calculation strategies** • Convert mixed numbers to common fractions in order to perform calculations with them • Use knowledge of multiples and factors to write fractions in the simplest form before or after calculations • Use knowledge of equivalent fractions to add and subtract common fractions • Use knowledge of reciprocal relationships to divide common fractions **Solving problems** Solve problems in contexts involving common fractions, mixed numbers and percentages **Equivalent forms** • Revise equivalent forms between:	**What is different to Grade 8?** In Grade 9 learners consolidate number knowledge and calculation techniques for common fractions, developed in Grade 8. In Grade 9, learners work with common fractions mostly as coefficients in algebraic expressions and equations. They are expected to be competent in performing multiple operations using common fractions and mixed numbers, applying properties of rational numbers appropriately. They are also expected to recognize and use equivalent forms for common fractions appropriately in calculations and when simplifying algebraic fractions.	Total time for common fractions: 4,5 hours

		○ common fractions with denominators multiples of each other ○ common fraction and decimal fraction forms of the same number ○ common fraction, decimal fraction and percentage forms of the same number		
	1.5 Decimal fractions	**Calculations with decimal fractions** • Multiple operations with decimal fractions, using a calculator where appropriate • Multiple operations, with or without brackets, with numbers that involve the squares, cubes, square roots and cube roots of decimal fractions **Calculation strategies** • Use knowledge of place values to estimate the number of decimal places in the result before performing calculations • Use rounding off and a calculator to check results where appropriate **Solving problems** Solve problems in context involving decimal fractions **Equivalent forms** • Revise equivalent forms between: ○ common fraction and decimal fraction forms of the same number ○ common fraction, decimal fraction and percentage forms of the same number	**What is different to Grade 8?** In Grade 9 learners consolidate number knowledge and calculation techniques for decimal fractions, developed in Grade 8. In Grade 9, learners work with decimal fractions mostly as coefficients in algebraic expressions and equations. They are expected to be competent in performing multiple operations using decimal fractions and mixed numbers, applying properties of rational numbers appropriately. They are also expected to recognize and use equivalent forms for decimal fractions appropriately in calculations.	Total time for decimal fractions: 4,5 hours
	1.2 Exponents	**Comparing and representing numbers in exponential form** • Revise:	**What is different to Grade 8?** • Additional laws of exponents involving integer exponents • Scientific notation for numbers, including negative exponents	Total time for exponents:

		○ compare and represent integers in exponential form ○ compare and represent numbers in scientific notation • Extend scientific notation to include negative exponents **Calculations using numbers in exponential form** • Revise the following general laws of exponents: ○ $a^m \times a^n = a^{m+n}$ ○ $a^m \div a^n = a^{m-n}$ ○ $(a^m)^n = a^{m\times n}$ ○ $(a \times t)^n = a^n t^n$ ○ $a^0 = 1$ • Extend the general laws of exponents to include: ○ integer exponents ○ $a^{-m} = \frac{1}{a^m}$ • Perform calculations involving all four operations with numbers in exponential form	In Grade 9 learners consolidate number knowledge and calculation techniques for exponents, developed in Grade 8. **Laws of exponents** • The laws of exponents should be introduced through a range of numeric examples first, then variables can be used. • The following laws of exponents should be known, where $\boldsymbol{m}$ and $\boldsymbol{n}$ are integers and $\boldsymbol{a}$ and $\boldsymbol{t}$ are not equal to $\mathbf{0}$: $a^m \times a^n = a^{m+n}$ **Examples** a) $2^3 \times 2^4 = 2^{3+4}$ $= 2^7$ $= 128$ b) $x^3 \times x^4 = x^{3+4}$ $= x^7$ $(a^m)^n = a^{m\times n}$ **Example** $(2^3)^2 = 2^{3\times 2}$ $= 2^6$ $= 64$ $a^m \div a^n = a^{m-n}$ **Examples** a) $3^5 \div 3^2 = 2^{5-2}$ $= 3^3$ $= 27$ b) $x^5 \div x^2 = x^{5-2}$ $= x^3$ $(a \times t)^n = a^n t^n$ **Example** $(3x^2)^3 = 3^3 x^6$ $= 27x^6$	5 hours

		Solving problems
		• Solve problems in contexts involving numbers in exponential form, including scientific notation
			$a^0 = 1$
			
			Examples
			a) $(37)^0 = 1$
			b) $(4x^2)^0 = 1$
			
			$a^{-m} = \frac{1}{a^m}$
			
			Examples
			a) $5^{-3} = \frac{1}{5^3} = \frac{1}{75}$
			b) $7^3 \div 7^5 = 7^{-2} = \frac{1}{7^2} = \frac{1}{49}$
			
			• Make sure learners understand these laws reading from both sides of the equal sign i.e. if the LHS = RHS, then the RHS = LHS
			• The law *a0* = 1 can be derived by using the law of exponents for division in a few examples e.g. $a^4 \div a^4 = \frac{a \times a \times a \times a}{a \times a \times a \times a} = 1 \therefore a^{4-4} = a^0 = 1$
			• Learners should be able to use the laws of exponents in calculations and for solving simple exponential equations as well as expanding and simplifying algebraic expressions.
			• Look out for the following **<u>common misconceptions</u>** where:
			  ○ Learners multiply unlike bases and add the exponents
			  Example: $x^m \times y^n = (xy)^{m+n}$
			  $2^5 \times 2^7 = 4^9$ instead of the correct answer 2^9
			  ○ Learners forget the middle term of the binomial e.g. $(x+y)^m = x^m + y^m$
			  ○ Learners confuse adding the exponents and adding the terms, e.g. $x^m + x^n = x^{m+n}$
			  ○ Learners confuse the exponent of the variable and the coefficient e.g. $2x^{-3} = \frac{1}{2x^3}$ instead of the correct answer $2\frac{1}{x^2}$
			Calculations and simple equations using numbers in exponential form
			• The calculations and equations should provide opportunities to apply the laws of exponents and should not be unduly complex.
			Examples
			a) Calculate: $2^{-1} \times 6^3 \times 3^{-2}$	

			b) Simplify: $(-2x^2)(-2x)^{-2}$ c) Solve x: $3^x = 9$ d) Solve x: $2^x = \frac{1}{4}$ e) $5^{x+1} = 1$ **Scientific notation** • When writing numbers in scientific notation, learners have to understand the relationship between the number of decimal places and the index of 10. **Example:** $25 = 2{,}5 \times 10^1$; and $250 = 2{,}5 \times 10^2$ • Scientific notation that extends to negative exponents includes writing very small numbers in scientific notation. **Example:** 25 millionth $= 2{,}5 \times 10^{-5}$ • Learners practise writing small and large numbers in scientific notation, which they might already have encountered in Natural Science. It is useful to refer to these contexts when discussing scientific notation. • Calculations can be done with or without a calculator. **Examples** a) Calculate: $2{,}6 \times 10^5 \times 9 \times 10^7$ without using a calculator and give answer in scientific notation. b) Write in scientific notation: $0{,}00053$ c) Calculate: $5{,}8 \times 10^{-4} + 2{,}3 \times 10^{-5}$ without using a calculator.	
Patterns, functions and algebra	**2.1 Numeric and geometric patterns**	**Investigate and extend patterns** • Investigate and extend numeric and geometric patterns looking for relationships between numbers including patterns: ○ represented in physical or diagram form ○ not limited to sequences involving a constant difference or ratio ○ of learner's own creation ○ represented in tables ○ represented algebraically • Describe and justify the general rules for	**What is different to Grade 8?** Learners consolidate work involving numeric and geometric patterns done in Grade 8. • Investigating number patterns is an opportunity to generalize – to give general algebraic descriptions of the relationship between terms and thier position in a sequence and to justify solutions. **Kinds of numeric patterns** • Given a sequence of numbers, learners have to identify a pattern or **relationship between consecutive terms** in order to extend the pattern. **Examples** Provide a rule to describe the relationship between the numbers in the	Total time for numeric and geometric patterns 4, 5 hours

		observed relationships between numbers in own words or in algebraic language	sequences below. Use this rule to give the next three numbers in the sequence: a) $-1;\ -1{,}5;\ -2;\ -2{,}5\ \ldots\ldots$ Here learners should identify the constant difference between consecutive terms in order to extend the pattern. This pattern can be described in learners' own words as 'adding $-0{,}5$' or 'counting in $-0{,}5s$' or 'add $-0{,}5$ to the previous number in the pattern'. b) $2;\ -1;\ 0{,}5;\ -0{,}25;\ 0{,}125\ \ldots\ldots$ Here learners should identify the constant ratio between consecutive terms. This pattern can be described in learners' own words as 'multiply the previous number by $-0{,}5$'. c) $1;\ 0;\ -2; -5; -9; -14\ \ldots.$ This pattern has neither a constant difference nor constant ratio. This pattern can be described in learners' own words as 'subtract 1 more than was subtracted to get the previous term'. Using this rule, the next 3 terms will be $-20, -27, -35$. • Given a sequence of numbers, learners have to identify a pattern or **relationship between the term and its position in the sequence**. This enables learners to predict a term in a sequence based on the position of that term in the sequence. It is useful for learners to represent these sequences in tables so that they can consider the position of the term. **Examples** a) Provide a rule to describe the relationship between the numbers in this sequence: $2;\ 5;\ 10;\ 17\ \ldots.$ Use your rule to find the 10th term in this sequence. Firstly, learners have to understand that the '10th term' refers to position 10 in the number sequence. They have to find a rule in order to determine the 10th term, rather than continuing the sequence up to the tenth term.	

This sequence can be represented in the following table:

Position in sequence	1	2	3	4		10
Term	2	5	10	17		?

Learners have to recognize that each term in the bottom row is obtained by squaring the position number in the top row and adding 1. Thus the 10th term will be 10 squared$+1$' or $10^2 + 1$ which is 101. Using the same rule, learners can also be asked what term number or position will 626 be? If the term is obtained by squaring the position number of the term and adding 1, then the position number can be obtained by subtracting 1, then finding the square root of the term. Hence, 626 will be the 25th term in the sequence since $626 - 1 = 625$ and $\sqrt{625} = 25$.

b) Provide a rule to describe the relationship between the numbers in this sequence: $-2;\ -5;\ -8;\ -11. \ldots$ Use this rule to find the 20th term in this sequence.

If learners consider only the relationship between consecutive terms, then they can continue the pattern ('add -3 to previous number') up to the 20th term to find the answer. However, if they look for a relationship or rule between the term and the position of the term, they can predict the answer without continuing the pattern. Using number sentences can be useful to find the rule:

1st term: $-2 = -3(1) + 1$

2nd term: $-5 = -3(2) + 1$

3rd term: $-8 = -3(3) + 1$

4th term: $-11 = -3\,(4) + 1$

The number in the brackets corresponds to the position of the term. Hence, the 20th term will be:
$-3(20) + 1 = -59$

The rule in learners' own words can be written as '$-3\ \times$ the position of the term$+\ 1$' or $-3n+1$, where n is the position of the term.

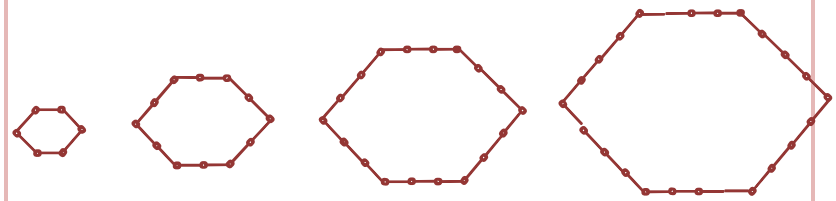
- These types of numeric patterns develop an understanding of functional relationships, in which there is a dependent variable (position of the term) and an independent variable (the term itself), and where you have a unique output for any given input value.

Kinds of geometric patterns

- Geometric patterns are number patterns represented diagrammatically. The diagrammatic representation reveals the structure of the number pattern.
- Hence, representing the number patterns in tables, makes it easier for learners to describe the general rule for the pattern.

Example

Consider this pattern for building hexagons using matchsticks. How many matchsticks will be used to build the 10th hexagon? Provide an expression to describe the general term for this number sequence.



The rule for the pattern is contained in the structure (construction) of the successive hexagonal shapes:
(1) add on 1 matchstick per side
(2) there are 6 sides, so
(3) add on 6 matchsticks per hexagon as you proceed from a given hexagon to the next one.
So, for the 2nd hexagon, you have $2\ \times\ 6$ matches; for the 3rd hexagon you have $3\ \times\ 6$ matches. Using this pattern for building hexagons, the 10th hexagon will have $10\ \times\ 6$ matches.

Learners can also use a table to record the number of matches used for each hexagon. This way the number pattern is related to the number of matches used for each new hexagon.

Position of hexagon in pattern	1	2	3	4	5	6	10
Number of matches	6	12	18				

The nth term for this sequence can be written as $6n$ or $6 + (n - 1)6$

2.2 Functions and relationships

Input and output values

- Determine input values, output values or rules for patterns and relationships using:
  - flow diagrams
  - tables
  - formulae
  - equations

Equivalent forms

- Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented:
  - verbally
  - in flow diagrams
  - in tables
  - by formulae
  - by equations
  - by graphs on a Cartesian plane

What is different to Grade 8?

Learners consolidate work with input and output values done in Grade 8. They should continue to find input or output values in flow diagrams, tables, formulae and equations.

Functions and relationships are done again in Term 3.

In this phase, it is useful to start specifying whether the input values are natural numbers, or integers or rational numbers. This builds learners' awareness of the domain of input values. Hence, to find output values, learners should be given the rule/formula as well as the domain of the input values.

Learners should begin to recognize equivalent representations of the same relationships shown as an equation, a set of ordered pairs in a table or on a graph.

Examples

a) If the rule for finding y in the table below is: $y = \frac{1}{2}x + 1$, determine the values of y for the given x values:

x	0	1	2	4	10	50	100
y							

<table>
<tr>
<td></td>
<td></td>
<td></td>
<td>b) Describe the relationship between the numbers in the top row and those in the bottom row
in the table. Then write down the value of $\boldsymbol{m}$ and $\boldsymbol{n}$.

<table>
<tr><td>$\boldsymbol{x}$</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td></td><td>12</td><td>$\boldsymbol{n}$</td></tr>
<tr><td>$\boldsymbol{y}$</td><td>-7</td><td>-5</td><td>-3</td><td>-1</td><td>1</td><td></td><td>$\boldsymbol{m}$</td><td>27</td></tr>
</table>

In tables such as these, more than one rule might be possible to describe the relationship between x and y values. The rules are acceptable if they match the given input values to the corresponding output values. For example, the rule $y = 2x - 3$ describes the relationship between the given values for x and y. To find $\boldsymbol{m}$ and $\boldsymbol{n}$, learners have to substitute the corresponding values for x or y into the rule and solve the equation by inspection.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.4</td>
<td>Algebraic expressions</td>
<td>Algebraic language
• Revise the following done in Grade 8:
○ Recognize and identify conventions for writing algebraic expressions
○ Identify and classify like and unlike terms in algebraic expressions
○ Recognize and identify coefficients and exponents in algebraic expressions
• Recognize and differentiate between monomials, binomials and trinomials

Expand and simplify algebraic expressions
• Revise the following done in Grade 8, using the commutative, associative and distributive laws for rational numbers and laws of exponents to:
○ add and subtract like terms in algebraic expressions
○ multiply integers and monomials by:
▪ monomials</td>
<td>What is different to Grade 8?
• Algebraic manipulations which include:
○ multiply integers and monomials by polynomials
○ divide polynomials by integers or monomials
○ the product of two binomials
○ the square of a binomial

Algebraic expressions are done again in Term 3. In this term the focus is on expanding and simplifying algebraic expressions. In Term 3 the focus is on factorizing expressions.

Manipulating algebraic expressions
• Make sure learners understand that the rules for operating with integers and rational numbers, including laws of exponents, apply equally when numbers are replaced with variables. The variables are numbers of a given type (e.g. integers or rational numbers) in generalized form.
• When multiplying or dividing expressions, make sure learners understand how the distributive rule works.</td>
<td>Time for algebraic expressions in this term:

4,5 hours</td>
</tr>
</table>

		▪ binomials ▪ trinomials ○ divide the following by integers or monomials: ▪ monomials ▪ binomials ▪ trinomials ○ Simplify algebraic expressions involving the above operations ○ Determine the squares, cubes, square roots and cube roots of single algebraic terms or like algebraic terms ○ Determine the numerical value of algebraic expressions by substitution • Extend the above algebraic manipulations to include: ○ multiply integers and monomials by polynomials ○ divide polynomials by integers or monomials ○ the product of two binomials ○ the square of a binomial	• The associative rule allows for grouping of like terms when adding. Look out for the following **common misconceptions**: • $x + x = 2x$ **<u>and NOT</u>** x^2. Note the convention is to write $2 \times$ rather than $\times 2$ • $x^2 + x^2 = 2x^2$ **<u>and NOT</u>** $2x^4$ • $a + b = a + b$ **<u>and NOT</u>** ab • $(-2x^2)^3 = -8x^6$ **<u>and NOT</u>** $-6x^5$ • $-x(3x + 1) = -3x^2 - x$ **<u>and NOT</u>** $-3x^2 + 1$ • $\frac{6x^2+1}{x^2} = 6 + \frac{1}{x^2}$ **<u>and NOT</u>** $6 + 1$ • If $x = 2$ then $-3x^2 = -3(2)^2 = -3 \times 4 = 12$ **<u>and NOT</u>**$(-6)^2$ • If $x = -2$ then $-x^2 - x = -(-2)^2 - (-2) = -4 + 2 = -2$ **<u>and NOT</u>** $4 + 2 = 6$ • $\sqrt{25x^2 - 9x^2} = \sqrt{16x^2} = 4x$ **<u>and NOT</u>** $5x - 3x = 2x$ • $(x + 2)^2 = x^2 + 4x + 4$ **<u>and NOT</u>** $x^2 + 4$ **Examples of expanding and simplifying expressions** a) Simplify:$-3(x^3 + 2x^2 - x) - x^2(3x + 1)$ [multiply integer and monomial by polynomial] b) Determine/expand:$(x + 2)(x - 3)$ [multiply binomial by binomial] c) Determine/expand: $(x + 2)(x - 2)$ [multiply binomial by binomial] d) Determine/expand:$(x + 3)^2$ [multiply out a perfect square] e) Simplify:$2(x - 3)^2 - 3(x + 1)(2x - 5)$ [multiple calculations involving	

			product of binomials] f) If $x = -2$ determine the numerical value of $3x^2 - 4x + 5$ [using substitution] f) Simplify: $\frac{6x^4-8x^3-2x^2+4}{2x^2}$ for $x \neq 0$ [divide polynomial by monomial; remind learners that denominator cannot be 0] g) Simplify: $\frac{8x^3-(-x^3)(2x)}{-x^2}$ for $x \neq 0$ [calculations involving multiple operations; remind learners that denominator cannot be 0] h) Determine: $\sqrt{36x^4}$ [square root of monomial] It might help to remind learners that these variables (or x in this case) represent numbers of a particular type – these may be rational, or integers, or perhaps whole numbers; such a reminder also then implies that all the associated rules or properties of these numbers apply here. So, in the above example, if x is an integer, then $x = a$ or $x = -a$ because $a^4 = (a)^4$	
	2.5 Algebraic equations	**Equations** • Revise the following done in Grade 8: ○ Set up equations to describe problem situations ○ Analyse and interpret equations that describe a given situation ○ Solve equations by: ▪ inspection ▪ using additive and multiplicative inverses ▪ using laws of exponents ○ Determine the numerical value of an expression by substitution. ○ Use substitution in equations to generate tables of ordered pairs ○ Extend solving equations to include:	**What is different to Grade 8?** • Solving equations using factorization • Solving equations of the form: a product of factors $= 0$ **Algebraic equations are done again in Term 3. In this term the focus is on consolidating solving equations using additive and multiplicative inverses and the laws of exponents. In Term 3, the focus is on solving equations after factorizing as well as generating tables of ordered pairs for linear equations.** Learners have opportunities to write and solve equations when they write general rules to describe relationships between numbers in number patterns, and when they find input or output values for given rules in flow diagrams, tables and formulae. In Grade 9, learners can be given equations where they have to expand,	Time for algebraic equations in this term: 4 hours

		▪ using factorisation ▪ equations of the form: a product of factors$= 0$	simplify or factorize expressions first, before solving the equation. For equations of the form: a product of two factors = 0, learners have to understand that if the product of two factors equals 0, then at least one of the factors must be equal to 0. Hence to solve the equation, each factor must be written as an equation equal to 0, and therefore more than one solution for x is possible. When working with algebraic fractions, learners must be reminded that the denominator cannot equal 0, so any value of x that makes the denominator 0 cannot be a solution to the equation. **Examples of equations** a) Solve x if $3(x-2) = x+2$ $3x - 6 = x + 2$ (expand LHS first) $3x - x - 6 = x - x + 2$ (add $-x$ to both sides of the equation) $\therefore 2x - 6 = 2$ $2x - 6 + 6 = 2 + 6$ (add 6 to both sides of the equation) $\therefore 2x = 8$ $\frac{2x}{2} = \frac{8}{2}$ (divide both sides of the equation by 2) $x = 4$ b) Solve x if $(x-1)(x+3) = 0$ $x - 1 = 0$ **or** $x + 3 = 0$ (at least one factor must be equal to 0) Thus $x = 1$ (add -1 to both sides of the equation) or $x = -3$ (add -3 to both sides of the equation) c) Solve x if $\frac{x}{3} + \frac{2x-1}{4} = 1$ $4x + 3(2x-1) = 12$ (multiply each term on both sides of the equation by the LCM, 12) $x + 6x - 3 = 12$ (expand expression on LHS) $10x = 15$ (add 3 to both sides of the equation, and add like terms to the LHS) $x = \frac{2}{3}$ (divide both sides of the equation by 10) d) If $y = 2x^2 + 4x + 3$, calculate y when $x = -2$ e) Thandi is 6 years older than Sophie. In 3 years' time Thandi will be twice as old as Sophie. How old is Thandi now?	

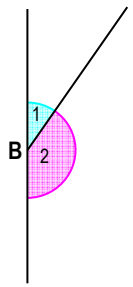
REVISION/ASSESSMENT: At this stage learners should have been assessed on: • the properties of different number systems • calculating and solving problems with whole numbers, integers, common fractions and decimal fractions, numbers in exponential form • numeric and geometric patterns • functions and relationships • algebraic expressions • algebraic equations	Total time for revision/assessment for the term 7 hours

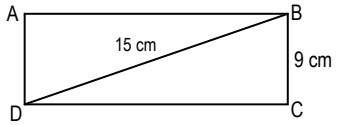
GRADE 9 – TERM 2				
CONTENT AREA	**TOPICS**	**CONCEPTS and SKILLS**	**SOME CLARIFICATION NOTES or TEACHING GUIDELINES**	**DURATION (in hours)**

Space and shape (geometry)	**3.5 Construction of geometric figures**	**Constructions**
		• Accurately construct geometric figures appropriately using a compass, ruler and protractor, including bisecting angles of a triangle
		• Construct angles of $45°$, $30°$ and $60°$ and their multiples without using a protractor
		
		Investigating properties of geometric figures
		• By construction, investigate the angles in a triangle, focusing on the relationship between the exterior angle of a triangle and its interior angles
		• By construction, explore the minimum conditions for two triangles to be congruent
		• By construction, investigate sides, angles and diagonals in quadrilaterals, focusing on the diagonals of rectangles, squares, parallelograms, rhombi and kites
			What is different to Grade 8?
			• Bisecting angles in a triangle
			• Constructing $30°$ without a protractor
			• Investigation of new properties of triangles, quadrilaterals and polygons
			
			Constructions
			• Constructions provide a useful context to explore or consolidate knowledge of angles and shapes.
			• Make sure learners are competent and comfortable in the use of a compass and know how to measure and read angle sizes on a protractor
			• Revise the constructions of angles if necessary, before proceeding with the new constructions.
			• Start with the constructions of lines, so that learners can first explore angle relationships on straight lines.
			• When constructing triangles learners should draw on known properties and construction of circles.
			• Construction of special angles without protractors are done by:
			  ○ bisecting a right-angle to get $45°$
			  ○ drawing an equilateral triangle to get $60°$
			  ○ bisecting the angles of an equilateral triangle to get $30°$
				Total time for constructions of geometric figures:
				
				9 hours

		• By construction explore the sum of the interior angles of polygons		
Space and shape (geometry)	**3.1 Geometry of 2D shapes**	**Classifying 2D shapes** • Revise properties and definitions of triangles in terms of their sides and angles, distinguishing between: ○ equilateral triangles ○ isosceles triangles ○ right-angled triangles • Revise and write clear definitions of quadrilaterals in terms of their sides, angles and diagonals, distinguishing between: ○ parallelogram ○ rectangle ○ square ○ rhombus ○ trapezium ○ kite	**What is different to Grade 8?** • Properties of diagonals of quadrilaterals • Minimum conditions for congruent and similar triangles **Triangles** • Constructions serve as a useful context for exploring properties of triangles. See notes on Constructions above. • Properties of triangles learners should know: ○ the sum of the interior angles of triangles $= 180^{\circ}$ ○ an equilateral triangle has all sides equal and all interior angles $= 60^{\circ}$ ○ an isosceles triangle has at least two equal sides and its base angles are equal ○ a right-angled triangle has one angle that is a right-angle ○ the side opposite the right-angle in a right-angled triangle, is called the hypotenuse ○ in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides (Theorem of Pythagoras). ○ the exterior angle of a triangle = the sum of the opposite two interior angles **Quadrilaterals** • Constructions serve as a useful context for exploring properties of triangles. See notes on constructions above. • The classification of quadrilaterals should include the recognition that: ○ rectangles and rhombi are special kinds of parallelograms ○ a square is a special kind of rectangle and rhombus. **Properties of quadrilaterals learners should know:** ○ the sum of the interior angles of quadrilaterals $= 360^{\circ}$ ○ the opposite sides of parallelograms are parallel and equal ○ the opposite angles of parallelograms are equal ○ the opposite angles of a rhombus are equal ○ the opposite sides of a rhombus are parallel and equal ○ the angles of rectangles and squares are 90° ○ a trapezium has one pair of opposite sides parallel	Total time for geometry of 2D shapes 9 hours

			○ a kite has two pairs of adjacent sides equal ○ the diagonals of a square, rectangle, parallelogram and rhombus bisect each other ○ the diagonals of a square, rhombus and kite are perpendicular	
		Similar and congruent triangles • Through investigation, establish the minimum conditions for congruent triangles • Through investigation, establish the minimum conditions for similar triangles	**Congruent triangles** • Constructions are a useful context for establishing the minimum conditions for two triangles to be congruent. See notes on Constructions above. • Conditions for two triangles to be congruent: ○ three corresponding sides are equal (S,S,S) ○ two corresponding sides and the included angle are equal (S,A,S) ○ two corresponding angles and a corresponding side are equal (A,A,S) ○ right-angle, hypotenuse and one other corresponding side are equal (R,H,S) **Similar triangles** • Constructions are a useful context for establishing the minimum conditions for two triangles to be similar. See notes on Constructions above. • Condition for two triangles to be similar: corresponding angles are equal and corresponding sides are proportional	
		Solving problems Solve geometric problems involving unknown sides and angles in triangles and quadrilaterals, using known properties of triangles and quadrilaterals, as well as properties of congruent and similar triangles.	**Solving problems** • Learners can solve geometric problems to find unknown sides and angles in triangles and quadrilaterals, using known definitions as well as angle relationships on straight lines. • For right-angled triangles, learners can also use the Theorem of Pythagoras to find unknown lengths. • Learners should be encouraged to give reasons and justify their solutions for every written statement. • Note that solving geometric problems is an opportunity to practise solving equations. **Example**: In ΔABC, $\hat{A} = x$, angle $\hat{B} = 50°$ and angle $\hat{C} = 80°$. What is the size of $\hat{A}$? Learners can solve x in the following equation:	

			$x + 50° + 80° = 180°$ (because the sum of the angles in a triangle = 180°) $x = 180° - 130°$ (add $-50°$ and $-80°$ to both sides of the equation) $x = 50°$, hence $\hat{A} = 50°$	
	3.3 Geometry of straight lines	**Angle relationships** • Revise and write clear descriptions of the relationship between angles formed by: ○ perpendicular lines ○ intersecting lines ○ parallel lines cut by a transversal **Solving problems** Solve geometric problems using the relationships between pairs of angles described above.	**What is different to Grade 8?** Learners revise and write clear descriptions of angle relationships on straight lines **Angle relationships learners should know:** • The angle of a straight line is 180°. • For perpendicular lines, adjacent supplementary angles $= 180°$. • For intersecting lines, adjacent supplementary angles $= 180°$ and vertically opposite angles are equal. • For parallel lines cut by a transversal, corresponding and alternate angles are equal. • The above angles have to be identified and named by learners. **Solving problems** • Learners can solve geometric problems to find unknown angles using the angle relationships above, as well as other known properties of triangles and quadrilaterals. • Learners should be encouraged to give reasons and justify their solutions for every written statement. • Note that solving geometric problems is an opportunity to practise solving equations. **Example:**  $\hat{B}_1$ and $\hat{B}_2$ are two angles on a straight line. $\hat{B}_1 = 35°$. What is the size of $\hat{B}_2$? Learners can find $\widehat{B_2}$ by solving the following equation: $35° + \hat{B}_2 = 180°$ (because the sum of angles on a straight line $= 180°$) $\hat{B}_2 = 180° - 35°$ (add $-35°$ to both sides of the equation) $\hat{B}_2 = 145°$	Total time for geometry of straight lines 9 hours

Measurement	**4.3 The Theorem of Pythagoras**	**Solve problems using the Theorem of Pythagoras** Use the Theorem of Pythagoras to solve problems involving unknown lengths in geometric figures that contain right-angled triangles	• The Theorem of Pythagoras was introduced in Grade 8 • It is important that learners understand that the Theorem of Pythagoras applies only to right-angled triangles. • The Theorem of Pythagoras is basically a formula to calculate unknown length of sides in right-angled triangles. • In particular, the Theorem of Pythagoras can be the first step in calculations of perimeters or areas of composite figures, when one of the figures is a right-angled triangle with an unknown length. See example below. • In the FET phase, the Theorem of Pythagoras is crucial to the further study of Geometry and Trigonometry. **Examples of solving problems using the Theorem of Pythagoras:** $ABCD$ is a rectangle where $BD = 15\ cm$ and $BC = 9\ cm$. Determine the: a) perimeter of $ABCD$ b) area of $ABCD$.  	Total time for the Theorem of Pythagoras: 5 hours
Measurement	**4.1 Area and perimeter of 2-D shapes**	**Area and perimeter** • Use appropriate formulae and conversions between SI units, to solve problems and calculate perimeter and area of: ○ polygons ○ circles' • Investigate how doubling any or all of the dimensions of a 2D figure affects its perimeter and its area **Breadth**	**What is different to Grade 8?** • The calculations are the same as in Grade 8, but learners can find perimeters and areas of more composite and complex figures • Polygons can include trapeziums, parallelograms, rhombi and kites • **Formulae** learners should know and use: Perimeter of a square: $P = 4s$ Perimeter of a rectangle: $P = 2(l + b) or\ P = 2l + 2b$ Area of a square: $A = l^2$ Area of a rectangle: $A = length\ x\ breadth$	Total time for Area and Perimeter 5 hours

Area of a rhombus:	$A = length \times height$
Area of a kite:	$A = \frac{1}{2}(diagonal_1 \times diagonal_2)$
Area of a parallelogram:	$A = base \times height$
Area of a trapezium	$A = \frac{1}{2}(sum\ of\ parallel\ sides) \times height$
Area of a triangle:	$A = \frac{1}{2}(b \times h)$
Diameter of a circle:	$d = 2r$
Circumference of circle:	$c = \pi d\ \ or\ \ c = 2\pi r$
Area of a circle:	$A = \pi r^2$

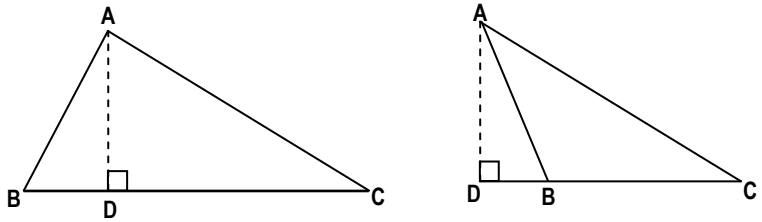
Solving equations using formulae

The use of formulae provides a context to practise solving equations by inspection or using additive or multiplicative inverses.

Example:

a) If the perimeter of a square is $32\ cm$, what is the length of each side? Learners should write this as:
$4s = 32$ and solve by asking: 4 times what will be 32? **or** saying $s = \frac{32}{4}$.

b) If the area of a rectangle is $200\ cm^2$, and its length $is\ 50\ cm$, what is its width? Learners should write this as:
$50 \times b = 200$ and solve by inspection by asking: 50 times what will be 200? **or** saying $b = \frac{200}{50}$

			For areas of triangles: • Make sure learners know that the height of a triangle is a line segment drawn from any vertex perpendicular to the opposite side. **Example**: AD is the height onto base BC of ΔABC.  • Point out that every triangle has 3 bases, each with a related height or altitude. • **For conversions**: ○ If $1\ cm = 10\ mm$ then $1\ cm^2 = 100\ mm^2$ ○ If $1\ m = 100\ cm$ then $1\ m^2 = 10\ 000\ cm^2$	
REVISION/ASSESSMENT: At this stage learners should have been assessed on: • constructing geometric objects • geometry of 2D shapes • geometry of straight lines • the Theorem of Pythagoras • area and perimeter of 2D shapes				Total time for revision/assessment for the term 8 hours

GRADE 9 – TERM 3				
CONTENT AREA	**TOPICS**	**CONCEPTS and SKILLS**	**SOME CLARIFICATION NOTES or TEACHING GUIDELINES**	**DURATION (in hours)**
Patterns, functions & algebra	**2.2 Functions and relationships**	**Input and output values** • Determine input values, output values or rules for patterns and relationships using: ○ flow diagrams ○ tables ○ formulae ○ equations **Equivalent forms** • Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented: ○ verbally ○ in flow diagrams ○ in tables ○ by formulae ○ by equations ○ by graphs on a Cartesian plane	Functions and relationships were also done in Term 1. The focus in this term is on finding output values for given equations, and recognising equivalent forms between different descriptions of the same relationship. **See additional notes and examples in Term 1.**	Time for Functions and relationships in this term: 5 hours
	2.3 Algebraic expressions	**Algebraic language** • Revise the following done in Grade 8: ○ recognize and identify conventions for writing algebraic expressions ○ identify and classify like and unlike terms in algebraic expressions ○ recognize and identify coefficients and exponents in algebraic expressions • Recognize and differentiate between monomials, binomials and trinomials	Algebraic expressions were also done in Term 1. The focus in this term is on factorizing expressions. **See additional notes and examples in Term 1.** **Factorizing expressions** • Make sure learners understand that factorizing is the reverse of expanding an expression through multiplication e.g. Expand: $2x(x+3) = 2x^2 + 6$ Factorize: $2x^2 + 6 = 2x(x+3)$ • Note that 1 and -1 are common factors of every expression e.g.	Total time for algebraic expressions: 9 hours

		Expand and simplify algebraic expressions • Revise the following done in Grade 8, using the commutative, associative and dstributive laws for rational numbers and laws of exponents to: ○ add and subtract like terms in algebraic expressions ○ multiply integers and monomials by: ▪ monomials ▪ binomials ▪ trinomials ○ divide the following by integers or monomials: ▪ monomials ▪ binomials ▪ trinomials ○ simplify algebraic expressions involving the above operations ○ determine the squares, cubes, square roots and cube roots of single algebraic terms or like algebraic terms ○ determine the numerical value of algebraic expressions by substitution • Extend the above algebraic manipulations to include: ○ multiply integers and monomials by polynomials ○ divide polynomials by integers or monomials ○ the product of two binomials	Factorize: $a - 4b = 1(a - 4b)$ Factorize: $4b - a = -1(a - 4b)$ **Examples of expressions with common factors that can be factorized** a) $6a^4 - 4a^2$ b) $ax - bx + 2a - 2b$ c) $2x(a - b) - 3(a - b)$ d) $2x(a - b) - 3(b - a)$ e) $(a + b)^2 - 5(a + b)$ **Examples of expressions with a difference of two squares that can be factorized** a) $25a^2$=1 b) $a^4 - b^4$ c) $9(a + b)^2 - 1$ d) $3x^3 - 27$ **Examples of algebraic fractions that can be factorized** a) $\frac{2x+6y}{x+3y}$ b) $\frac{3x-3y}{6x-6y}$ c) $\frac{9a^2-1}{3a+1}$ **Examples of trinomials that can be factorized** a) $x^2 + 5x + 6$ b) $x^2 - 5x + 6$ c) $x^2 - x - 6$ d) $x^2 - 6x + 9$ e) $2x^2 + 10x + 12$	

		○ the square of a binomial **Factorize algebraic expressions** • Factorize algebraic expressions that involve: ○ common factors ○ difference of two squares ○ trinomials of the form: ▪ $x^2 + bx + c$ ▪ $ax^2 + bx + c$, where a is a common factor • Simplify algebraic expressions that involve the above factorisation processes • Simplify algebraic fractions using factorisation		
	2.4 Algebraic equations	**Equations** • Revise the following done in Grade 8: ○ set up equations to describe problem situations ○ analyse and interpret equations that describe a given situation ○ solve equations by: ▪ inspection ▪ using additive and multiplicative inverses ▪ using laws of exponents ○ determine the numerical value of an expression by substitution. ○ use substitution in equations to generate tables of ordered pairs • Extend solving equations to include: ○ using factorisation ○ equations of the form: a product of factors $= 0$	Algebraic equations were done in Term 1. The focus in this term is on solving equations using factorization, and equations with a product of factors. This term also focuses on using equations to generate tables of ordered pairs. **See additional notes and examples in Term 1.** **Examples of equations** a) Solve x if $x^2 - 3x = 0$ $x(x-3) = 0$ (factorize LHS) $x = 0$ or $x - 3 = 0$ (at least one factor $= 0$) Therefore, $x = 0$ or $x = 3$ (add 3 to both sides of the second equation) b) Solve x if $x^2 - 25 = 0$ $(x+5)(x-5) = 0$ (factorize the difference of two squares on LHS) $x + 5 = 0$ or $x - 5 = 0$ (at least one factor $= 0$) Therefore, $x = -5$ (add -5 to both sides of the equation) or $x = 5$ (add 5 to both sides of the equation)	Time for algebraic equations in this term: 9 hours

c) Write an equation to find the volume of a rectangular prism with length $2x$ cm; width $(2x + 1)$ cm; and height $(2x + 3)$ cm

d) If $y = 2x^2 + 4x + 3$, calculate y when $x = -2$

e) Thandi is 6 years older than Sophie. In 3 years' time Thandi will be twice as old as Sophie. How old is Thandi now?

Examples of generating ordered pairs

a) Complete the table below for x and y values for the equation: $y = 2x^2 - 3$

x	-2	-1	0	1	2
y					

b) Complete the table below for x and y values for the equation: $y = x^2 - 2$

x	-3	-2	0		
y				-2	2

2.5 Graphs

Interpreting graphs

- Revise the following done in Grade 8:
  - analyse and interpret global graphs of problem situations, with a special focus on the following trends and features:
    - linear or non-linear
    - vonstant, increasing or decreasing

What is different to Grade 8?

- x-intercept, y-intercept and gradient of linear graphs
- Draw linear graphs from given equations
- Determine equations of linear graphs

Learners should continue to analyse and interpret graphs of problem situations.

Investigating linear graphs

- To sketch linear graphs from given equations, learners should first draw up a table of ordered pairs, that includes the intercept points $(x; 0)$ and $(0; y)$, and then plotting the points.
- Learners should investigate gradients by comparing $\frac{vertical\ change}{horizontal\ change}$ between any two points on a straight line graph.
- Learners should also investigate the relationship between the value of the

Total time for graphs:

12 hours

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▪ maximum or Minimum
▪ discrete or continuous
• Extend the above with special focus on the following features of linear graphs:
○ x-intercept and y-intercept
○ gradient

Drawing graphs
• Revise the following done in Grade 8:
○ draw global graphs from given descriptions of a problem situation, identifying features listed above.
○ use tables of ordered pairs to plot points and draw graphs on the Cartesian plane
▪ Extend the above with a special focus on :
○ drawing linear graphs from given equations
○ determine equations from given linear graphs
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gradient and the coefficient of x in the equation of a straight line graph.
• Learners should compare y-intercepts of linear graphs to the value of the constant in the equation of the straight line graph.

Examples of linear graphs
a) Sketch and compare the graphs of : $y = 4$ and $x = 4$
b) Sketch and compare the graphs: $y = x$ and $y = -x$
c) Sketch and compare the graphs of: $y = 2x$; $y = 2x + 1$; $y = 2x - 1$
d) Sketch and compare the graphs of: $y = 3x$; $y = 4x$; $y = 5x$
e) Sketch the graph of $y = -3x + 2$, using the table method
f) Determine the equation of the straight line passing through the following points:
<table>
<tr><td>x</td><td>-4</td><td>-3</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>y</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
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<td>Measurement</td>
<td>4.2 Surface area and volume of 3D objects</td>
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Surface area and volume
• Use appropriate formulae and conversions between SI units to solve problems and calculate the surface area, volume and capacity of:
○ cubes
○ rectangular prisms
○ triangular prisms
○ cylinders
• Investigate how doubling any or all the dimensions of right prisms and cylinders affects the volume
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What is different to Grade 8?
• Surface area and volume of cylinders

• Formulae learners should know and use:
○ the volume of a prism = the area of the base × the height
○ the surface area of a prism = the sum of the area of all its faces
○ the volume of a cube = l^3
○ the volume of a rectangular prism = $l \times b \times h$
○ the volume of a triangular prism = $(\frac{1}{2}b \times h) \times$ height of the prism
○ the volume of a cylinder = $(\pi r^2) \times$ the height of the cylinder

• For conversions, note:
○ if $1\ cm = 10\ mm$ then $1\ cm^3 = 1\ 000\ mm^3$; and
○ if $1\ m = 100\ cm$ then $1\ m^3 = 1\ 000\ 000$ or $10^6\ cm^3$.
○ an object with a volume of $1\ cm^3$ will displace exactly $1\ ml$ of water;
</td>
<td>Total time for surface area and volume:

5 hours</td>
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			and ○ an object with a volume of $1\ m^3$ will displace exactly $1\ kl$ of water. **Examples of solving problems involving surface area and volume of cylinders** • Calculate the volume of a cylinder, without using a calculator if its diameter is $28cm$, its height is $30cm$ and $\pi = \frac{22}{7}$ • Calculate the surface area of a cylinder, correct to 2 decimal places, if its height is $65cm$ and the circumference of its base is $47{,}6cm$.	
REVISION/ASSESSMENT: At this stage learners should have been assessed on: ▪ functions and relationships ▪ algebraic expressions ▪ Algebraic equations ▪ graphs ▪ aolume and surface area				Total time for revision/assessment for the term 5 hours

GRADE 9 – TERM 4				
CONTENT AREA	**TOPICS**	**CONCEPTS and SKILLS**	**SOME CLARIFICATION NOTES or TEACHING GUIDELINES**	**DURATION (in hours)**
Space and shape (geometry)	**3.4 Transformation Geometry**	**Transformations** • Recognize, describe and perform transformations with points, line segments and simple geometric figures on a co-ordinate plane, focusing on: ○ reflection in the y −axis or x −axis ○ translation within and across	**What is different to Grade 8?** • Reflection in the line $y = x$ • Identify transformations from co-ordinate points of the image • Co-ordinates of vertices **Co-ordinate plane** • Doing transformations on the co-ordinate plane is an opportunity to practise reading and plotting points with ordered pairs, and links to drawing algebraic graphs. • Make sure learners know how to plot points on the co-ordinate plane and can read the co-ordinates of points off the x −axis and y −axis. • Make sure learners know the convention for writing ordered pairs $(x;\ y)$ • Point out the differences between the axes in the four quadrants. **Focus of transformations**	Total time for transformations: 9 hours

		quadrants ○ reflection in the line $y = x$ • Identify what the transformation of a point is, if given the co-ordinates of its image **Enlargements and reductions** • Use proportion to describe the effect of enlargement or reduction on area and perimeter of geometric figures • Investigate the co-ordinates of the vertices of figures that have been enlarged or reduced by a given scale factor	• Doing transformations on a co-ordinate plane focuses attention on the co-ordinates of points and vertices of shapes. • Learners should recognize that translations, reflections and rotations only change the position of the figure, and not its shape or size. • Learners should recognize that the above transformations produce congruent figures. • Learners should begin to see patterns in terms of the co-ordinate points, for the different transformations, such as: ○ for translations to the right or left, the x −value changes and y −value stays the same ○ for translations up or down, the y −value changes and the x −value stays the same ○ for reflections in the y −axis, the x −value changes sign and the y −value stays the same ○ for reflections in the x −axis, the y −value changes sign and the x −value stays the same ○ for reflections in the line $y = x$, the x −value and y −value are interchanged. • Learners should recognize that enlargements and reductions change the size of figures by increasing or decreasing the length of sides, but keeping the angles the same, produces similar rather than congruent figures. • Learners should also be able to work out the factor of enlargement or reduction of a figure.	
	3.2 Geometry of 3D objects	**Classifying 3D objects** • Revise properties and definitions of the 5 Platonic solids in terms of the shape and number of faces, the number of vertices and the number of edges • Recognize and describe the properties of: ○ spheres ○ cylinders	**What is different to Grade 8?** • Properties of spheres and cylinders • Nets of cylinders **Platonic solids** • Properties of Platonic solids should be revised • Platonic solids are a special group of polyhedra that have faces that are congruent regular polygons. • There are only 5 Platonic solids: ○ tetrahedron ○ hexahedron (cube) ○ octahedron ○ dodecahedron	Total time for geometry of 3-D objects: 9 hours

		Building 3D models
		• Use nets to create models of geometric solids, including:
		  ○ cubes
		  ○ prisms
		  ○ pyramids
		  ○ cylinders
			○ icosahedrons
			• The name of each Platonic solid is derived from its number of faces.
			• Platonic solids provide an interesting context in which to investigate the relationship between the number of faces, vertices and edges. By listing these properties for all the Platonic solids learners can investigate the pattern that emerges, to come up with the general rule:
			$V - E + F = 2, where\ V = vertices;\ E = edges;\ F = faces$
			
			Using and constructing nets
			• Using and constructing nets are useful contexts for exploring or consolidating properties of polyhedra.
			• Learners should recognize the nets of different solids.
			• Learners should make sketches of the nets using their knowledge of the the shape and number of faces of the solids, before drawing and cutting out the nets to build models.
			• Since learners have more knowledge about the size of the internal angles of polygons, and can measure angles, their constructions of nets should be more accurate.
			• Learners have to work out the relative position of faces of the nets in order to build the 3D model.	

Data handling	**5.1 Collect, organize and summarize data**	**Collect data** • Pose questions relating to social, economic, and environmental issues • Select and justify appropriate sources for the collection of data • Distinguish between samples and populations, and suggest appropriate samples for investigation • Select and justify appropriate methods for collecting data	**What is different to Grade 8?** • Organizing data according to more than one criteria • Outliers • Scatter plots **Data sets and contexts** Learners should be exposed to a variety of contexts that deal with social and environmental issues, and should work with given data sets, represented in a variety of ways, that include big number ranges, percentages and decimal fractions. Learners should then practise organizing and summarizing this data, analysing and interpreting the data, and writing a report about the data. **Complete a data cycle** Learners should complete at least one data cycle for the year, starting with posing their own questions, selecting the sources and method for collecting, recording, organizing, representing, then analysing, summarizing, interpreting	Total time for collecting and organizing data: 4 hours

		Organize and summarize data
		• Organize numerical data in different ways in order to summarize by determining**:**
		  ○ measures of central tendency
		  ○ measures of dispersion
		• Organize data according to more than one criteria
			and reporting on the data. Challenge learners to think about what kinds of questions and data need to be collected to be represented on a histogram, a pie chart, a bar graph, a line graph or a scatter plot.	

	5.2 Represent data	**Represent data**
		• Draw a variety of graphs by hand/technology to display and interpret data including:
		  ○ bar graphs and double bar graphs
		  ○ histograms with given and own intervals
		  ○ pie charts
		  ○ broken-line graphs
		  ○ scatter plots
			Representing data
			• Pie charts to represent data do not have to be accurately drawn with a compass and protractor, etc. Learners can use any round object to draw a circle, then divide the circle into halves and quarters and eighths if needed, as a guide to estimate the proportions of the circle that need to be shown to represent the data. What is important is that the values or percentages associated with the data, are shown proportionally on the pie chart.
			• Drawing, reading and interpreting pie charts is a useful context to re-visit equivalence between fractions and percentages, e.g. 25% of the data is represented by a ¼ sector of the circle.
			• It is also a context in which learners can find percentages of whole numbers e.g. if 25% of 300 learners like rugby, how many (actual number) learners like rugby?
			• Histograms are used to represent grouped data shown in intervals on the horizontal axis of the graph. Point out the differences between histograms and bar graphs, in particular bar graphs that represent discrete data (e.g. favourite sports) compared to histograms that show categories in consecutive, non-overlapping interval, (e.g. test scores out of 100 shown in intervals of 10). The bars on bar graphs do not have to touch each other, while in a histogram they have to touch since they show consecutive intervals.
			• Broken-line graphs refer to data graphs that represent data points joined by a line and are not the same as straight line graphs that are drawn using the equation of the line.
			• Broken-line graphs are used to represent data that changes continuously over time, e.g. average daily temperature for a month. Each day's temperature is represented by a point on the graph, and once the whole month has been plotted, the points are joined to show a broke- line graph.
				Total time for representing data:
				
				3 hours

			• Broken-line graphs are useful to read 'trends' and patterns in the data, for predictive purposes e.g. Will the temperatures go up or down in the next month? • A scatter plot is used to represent data that involves two different criteria and the graph is used to look at the relationship between the two criteria. e.g. How does the performance of learners in Mathematics compare to their performance in English? Each point on the graph represents the results of one learner in Mathematics and English. After all the results have been plotted, you can compare the relationship between performance in English and Mathematics for all the learners i.e. if they score high in Mathematics, do they also score high in English? or, if they score high in Mathematics do they score low in English, or is there no relationship between what they score on Mathematics to what they score in English? • The scatter plot allows one to see trends and make predictions, as well as identify outliers in the data.	
	5.3 Analyse, Interpret and report data	**Interpret data** • Critically read and interpret data represented in a variety of ways. • Critically compare two sets of data related to the same issue **Analyse data** • Critically analyse data by answering questions related to: o Data collection methods o Summary statistics of data o Sources of error and bias in the data **Report data** • Summarize data in short paragraphs that include o drawing conclusions about the data o making predictions based on the data o making comparisons between two sets of data	**Developing critical analysis skills** • Learners should compare the same data represented in different ways e.g. in a pie chart or a bar graph or a table, and discuss what information is shown and what is hidden; they should evaluate which form of representation works best for the given data. • Learners should compare graphs on the same topic but where data has been collected from different groups of people, at different times, in different places or in different ways. Here learners should discuss differences between the data with an awareness of bias related to the impact of data sources and methods of data collection on the interpretation of the data. • Learners should compare different ways of summarizing the same data sets, developing an awareness of how data reporting can be manipulated; they should evaluate which summary statistics best represent the data. • Learners should compare graphs of the same data, where the scales of the graphs are different. Here learners should discuss differences with an awareness of how representation of data can be manipulated; they should evaluate which form of representation works best for the given data. • Learners should compare data on the same topic, where one set of data has extremes or outliers, and discuss differences with an awareness of the effect of the extremes or outliers on the interpretation of the data. In particular, extremes affect the range and outliers which are identified on	Total time for analysing, interpreting, summarizing and reporting data: 3,5 hours

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identifying sources of error and bias in the data
choosing appropriate summary statistics for the data (mean, median, mode, range)
the role of extremes and outliers in the data

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scatter plots.
• Learners should be encouraged to write reports on the data in short paragraphs.
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<td>5.4 Probability</td>
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Probability

• Consider situations with equally probable outcomes, and:

determine probabilities of compound events using two-way tables and tree diagrams
determine the probabilities of outcomes of events and predict their relative frequency in simple experiments
compare relative frequency with probability and explain possible differences

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Probability experiments
In Grades 8 and 9 probability experiments are less important, and learners should consider probability for hypothetical events. In Grade 9, learners have to consider outcomes of compound events and use two-way tables and tree diagrams to work out the probability of an outcome.

Probability of compound events
For example, what is the probability of a woman giving birth to two boys after each other? A two- way table can be used:
<table>
<tr><td rowspan="2">1st birth</td><td colspan="2">2nd birth</td></tr>
<tr><td>Boy</td><td>Girl</td></tr>
<tr><td>Boy</td><td>BB</td><td>BG</td></tr>
<tr><td>Girl</td><td>GB</td><td>GG</td></tr>
</table>

Two boys after each other (BB) is 1 of 4 possible outcomes, so the probability of two boys is 1 out of 4, or25%.
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<td>Total time for probability:

4,5 hours</td>
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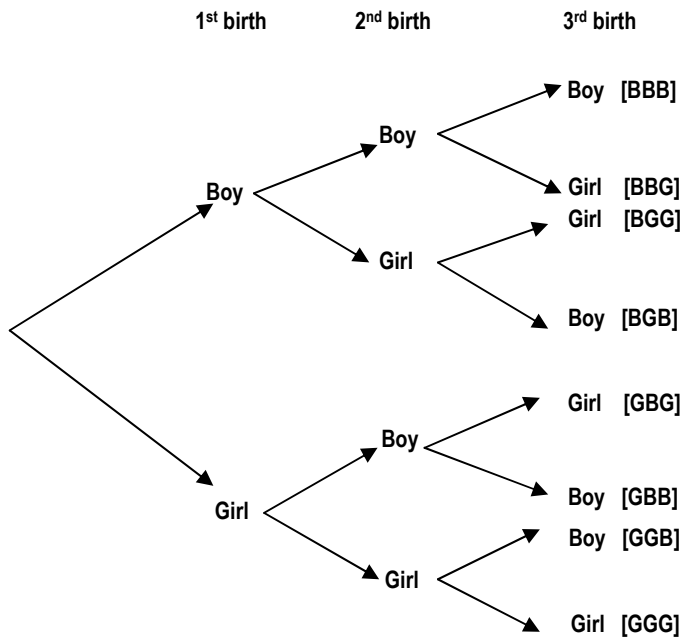
How does this probability change if you ask what is the probability of giving birth to three boys after each other? A tree diagram can then be used:



Total outcomes 8

Three boys after each other, [BBB], is 1 out of 8 possible outcomes, hence the probability of three boys after each other is $\frac{1}{8}$ or$12{,}5\%$..

REVISION/ASSESSMENT:
At this stage learners should have been assessed on:

- transformation geometry
- geometry of 3D objects
- collecting, organizing, representing, analysing, summarizing, interpreting and reporting data
- probability

Total time for revision/assessment for the term

12 hours

SECTION 4
CURRICULUM AND ASSESSMENT POLICY STATEMENT (CAPS)
SENIOR PHASE

ASSESSMENT

4.1. Introduction

Assessment is a continuous planned process of identifying, gathering and interpreting information regarding the performance of learners, using various forms of assessment. It involves four steps: generating and collecting evidence of achievement; evaluating this evidence; recording the findings and using this information to understand and thereby assist the learner's development in order to improve the process of learning and teaching. Assessment should be both informal and formal. In both cases regular feedback should be provided to learners to enhance their learning experience. This will assist the learner to achieve the minimum performance level of 40% to 49% required in Mathematics for promotion purposes.

4.2. Types of assessment

The following types of assessment are very useful in Mathematics; as a result teachers are encouraged to use them to serve the purpose associated with each.

Baseline assessment: Mathematics teachers who might want to establish whether their learners meet the basic skills and knowledge levels required to learn a specific Mathematics topic will use baseline assessment. Knowing learners' level of proficiency in a particular Mathematics topic enables the teacher to plan her/his Mathematics lesson appropriately and to pitch it at the appropriate level. Baseline assessment, as the name suggests, should therefore be administered prior to teaching a particular Mathematics topic. The results of the baseline assessment should not be used for promotion purposes.

Diagnostic assessment: It is not intended for promotion purposes but to inform the teacher about the learner's Mathematics problem areas that have the potential to hinder performance. Two broad areas form the basis of diagnostic assessment: content-related challenges where learners find certain difficulties to comprehend, and psycho-social factors such as negative attitudes, Mathematics anxiety, poor study habits, poor problem-solving behaviour, etc. Appropriate interventions should be implemented to assist learners in overcoming these challenges early in their school careers.

Formative assessment: Formative assessment is used to aid the teaching and learning processes, hence assessment ***for*** learning. It is the most commonly used type of assessment because it can be used in different forms at any time during a Mathematics lesson, e.g. short class works during or at the end of each lesson, verbal questioning during the lesson. It is mainly informal and should not be used for promotion purposes. The fundamental distinguishing characteristic of formative assessment is constant feedback to learners, particularly with regard to learners' learning processes. The information provided by formative assessment can also be used by teachers to inform their methods of teaching.

Summative assessment: Contrary to the character of formative assessment, summative assessment is carried out after the completion of a Mathematics topic or a cluster of related topics. It is therefore referred to as assessment ***of*** learning since it is mainly focusing on the product of learning. The results of summative assessment are recorded and used for promotion purposes. The forms of assessment presented in Table 4.1 are examples of summative assessment.

4.3. Informal or daily assessment

Assessment for learning has the purpose of continuously collecting information on learner performance that can be used to improve their learning.

Informal assessment is a daily monitoring of learners' progress. This is done through observations, discussions, practical demonstrations, learner-teacher conferences, informal classroom interactions, etc. Informal assessment may be as simple as stopping during the lesson to observe learners or to discuss with learners how learning is progressing. Informal assessment should be used to provide feedback to learners and to inform planning for teaching, but need not be recorded. It should not be seen as separate from the learning activities taking place in the classroom.

Self-assessment and peer assessment actively allow learners to assess themselves. This is important as it allows learners to learn from, and reflect on their own performance. The results of the informal daily assessment tasks are not formally recorded unless the teacher wishes to do so. The results of daily assessment tasks are not taken into account for promotion purposes.

4.4. Formal assessment

Formal assessment comprises School-Based Assessment (SBA) and External Assessment. Formal assessment tasks are marked and formally recorded by the teacher for promotion purposes. All Formal Assessment tasks are subject to moderation for the purpose of quality assurance and to ensure that appropriate standards are maintained. The SBA component may take various forms. However, **tests, examinations, projects, assignments** and **investigations** are recommended for Mathematics. The Senior Phase Mathematics minimum formal programme of assessment tasks are outlined in Table 4.1

Table 4.1 Minimum requirements for formal assessment: Senior Phase Mathematics

	Forms of Assessment	Minimum Requirements per term				Number of Tasks per Year	Weighting
		Term 1	Term 2	Term 3	Term 4		
SBA	Tests	1	1	1		3	**40%**
	Examination		1			1	
	Assignment	1		1	1	3	
	Investigation		1		1	2	
	Project			1		1	
	Total	**2**	**3**	**3**	**2**	**10***	
Final Examination		End of the year				1	**60%**

*To be completed before the final examination at the end of the year

Tests and **examinations** are individualised assessment tasks and should be carefully designed to ensure that learners demonstrate their full potential in Mathematics content. The questions should be carefully spread to cater for different cognitive levels of learners. Tests and examinations are predominantly assessed using a memorandum.

The Assignment, as is the case with tests and examinations, is mainly an individualised task. However, it is generally an extended piece of work that may be completed at school or at home. It can be a collection of past questions, but should focus on more demanding work as any resource material can be used, which is not the case in a task that is done in class under supervision.

Projects are used to assess a range of skills and competencies. Through projects, learners are able to demonstrate their understanding of different Mathematics concepts and apply them in real-life situations. Caution should, however, be exercised not to give projects that are above learners' cognitive levels. The assessment criteria should be clearly indicated on the project specification and should focus on the Mathematics involved and

not on duplicated pictures and facts copied from reference material. Good projects contain the collection and display of real data, followed by deductions that can be substantiated.

Investigation promotes critical and creative thinking. It can be used to discover rules or concepts and may involve inductive reasoning, identifying or testing patterns or relationships, drawing conclusions, and establishing general trends. To avoid having to assess work which is copied without understanding, it is recommended that whilst initial investigation could be done at home, the final write-up should be done in class, under supervision, without access to any notes. Investigations are assessed with rubrics, which can be specific to the task, or generic, listing the number of marks awarded for each skill. These skills include:

- organizing and recording ideas and discoveries using, for example, diagrams and tables.
- communicating ideas with appropriate explanations
- calculations showing clear understanding of mathematical concepts and procedures.
- generalizing and drawing conclusions,

The forms of assessment used should be appropriate to the age and cognitive level of learners. The design of these tasks should cover the content of the subject and designed to achieve the broad aims of the subject. Appropriate instruments, such as rubrics and memoranda, should be used for marking. Formal assessments should cater for a range of cognitive levels and abilities of learners as shown in Table 4.2:

Table 4.2 Cognitive levels

DESCRIPTION AND EXAMPLES OF COGNITIVE LEVELS		
Cognitive levels	**Description of skills to be demonstrated**	**Examples**
Knowledge ($\approx 25\%$)	• Estimation and appropriate rounding of numbers • Straight recall • Identification and direct use of correct formula • Use of mathematical facts • Appropriate use of mathematical vocabulary	1. Estimate the answer and then calculate with a calculator: $\frac{62\,816}{325+279}$ **[Grade 7]** 2. Use the formula $A = \pi r^2$ to calculate the area of a circle if the diameter is equal to 10 cm. **[Grade 8]** 3. Write down the y-intercept of the function $y = 2x + 1$ **[Grade 9]**
Routine procedures ($\approx 45\%$)	• Perform well-known procedures • Simple applications and calculations which might involve many steps • Derivation from given information may be involved • Identification and use (after changing the subject) of correct formula • Generally similar to those encountered in class	1. Determine the mean of 5 Grade 7 learners' marks if they have respectively achieved 25; 40; 21; 85; 14 out of 50. **[Grade 7]** 2. Solve x in $x - 6 = 9$ **[Grade 8]** 3. R600 invested at r% per annum for a period of 3 years yields R150 interest. Calculate the value of r if $I = \frac{P.n.r}{100}$. **[Grade 9]**
Complex procedures ($\approx 20\%$)	• Problems involving complex calculations and/or higher order reasoning • Investigate elementary axioms to generalize them into proofs for straight line geometry, congruence and similarity • No obvious route to the solution • Problems not necessarily based on real world contexts • Making significant connections between different representations • Require conceptual understanding	1. Mr Mnisi pays R75 for a book which he marks up to provide 20% profit. He then sells it for cash at 4% discount. Calculate the selling price. **[Grade 7]** 2. A car travelling at a constant speed travels 60 km in 18 minutes. How far, travelling at the same constant speed, will the car travel in 1 hour 12 minutes? **[Grade 8]** 3. Use investigation skills to prove that the angles on a straight line are supplementary. **[Grade 9]**
Problem solving ($\approx 10\%$)	• Unseen, non-routine problems (which are not necessarily difficult) • Higher order understanding and processes are often involved	1. The sum of three consecutive numbers is 87. Find the numbers. **[Grade 7]** 2. Mary travels a distance of x km in 6 hours if she travels at an average speed of 20

	• Might require the ability to break the problem down into its constituent parts	km/hour on her bicycle. What should be her average speed if she wants to cover the same distance in 5 hours? **[Grade 8]** 3. The combined age of a father and son is 84 years old. In 6 years' time the father will be twice as old as the son was 3 years ago. How old are they now? **[Grade 9]**

4.5. Recording and reporting

Recording is a process in which the teacher documents the level of a learner's performance in a specific assessment task. It indicates the learner's progress towards the achievement of the knowledge as prescribed in the National Curriculum and Assessment Policy Statements. Records of learner performance should provide evidence of the learner's conceptual progression within a grade and her/his readiness to be promoted to the next grade. Records of learner performance should also be used to verify the progress made by teachers and learners in the teaching and learning process.

Reporting is a process of communicating learner performance to learners, parents, schools, and other stakeholders. Primary schooling is a critical period for the acquisition of foundational Mathematics skills and conceptual knowledge. Reporting of learner performance is therefore essential and should not be limited to the quarterly report card. Other methods of reporting should be explored, e.g. parents' meetings, school visitation days, parent-teacher conferences, phone calls, letters. These extreme, but worthwhile modalities will ensure that any underperformance is communicated promptly and appropriate measures of intervention are implemented collaboratively by teachers and parents. Formal reporting is done on a 7-point rating scale (see Table 4.3)

Table 4.3: Scale of achievement for the National Curriculum Statement Grades 7 - 9

RATING CODE	DESCRIPTION OF COMPETENCE	PERCENTAGE
7	Outstanding achievement	80 – 100
6	Meritorious achievement	70 – 79
5	Substantial achievement	60 – 69
4	Adequate achievement	50 – 59
3	Moderate achievement	40 – 49
2	Elementary achievement	30 – 39
1	Not achieved	0 – 29

4.6. MODERATION OF ASSESSMENT

Moderation refers to the process that ensures that the assessment tasks are fair, valid and reliable. Moderation should be carried out internally at school and/or externally at district, provincial and national levels. Given that the promotion of learners in the Senior Phase is largely dependent upon the SBA (which contributes 40%); the moderation process should be intensified to ensure that:

- learners are not disadvantaged by the invalid and unreliable assessment tasks,
- quality assessment is given and high but achievable standards are maintained.

4.7. General

This document should be read in conjunction with:

- [National Protocol of Assessment] *An addendum to the policy document, the National Senior Certificate: A qualification at Level 4 on the National Qualifications Framework (NQF), regarding the National Protocol for Assessment (Grades R – 12).*

- Subject specific Exam Guidelines as contained in the draft policy document: **National policy pertaining to the programme and promotion requirements of the National Curriculum Statement, Grades R – 12.**