education

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SUBJECT GUIDELINES

MATHEMATICS

NQF Level 3

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INTRODUCTION

A. What is Mathematics?

Reader's Digest Oxford Complete Wordfinder defines Mathematics as "the abstract science of number, quantity and space studied in its own right".

Mathematics enables creative and logical reasoning about problems in the physical and social world and in the context of Mathematics itself. Through mathematical problem solving, students can understand the world and use that understanding in their daily lives.

Knowledge in the mathematical sciences is constructed through the establishment of descriptive, numerical and symbolic relationships. The Mathematics Subject Outcomes and Assessment Standards are designed to allow all Further Education and Training (FET) (Vocational) students to develop into citizens who confidently deal with mathematics as and when it impinges on their daily lives, their community and the world in general. Mathematics in the Further Education and Training band also provides a platform to link with Higher Education.

B. Why is Mathematics important as a Fundamental?

The Mathematics programme (NQF Level 2 – 4) empowers students to:

- Communicate appropriately using descriptions in words, graphs, symbols, tables and diagrams.
- Use mathematical process skills to identify, pose and solve problems creatively and critically.
- Organise, interpret and manage authentic activities in substantial mathematical ways that demonstrate responsibility and sensitivity to personal and broader societal concerns.
- Work collaboratively in teams and groups to enhance mathematical understanding.
- Collect, analyse and organise quantitative data to evaluate and comment on conclusions.
- Engage responsibly with quantitative arguments relating to local, national and global issues.

C. The link between Mathematics Learning Outcomes and the Critical and Developmental Outcomes

The Mathematics Learning Outcomes provide a platform for students to achieve the seven Critical Outcomes and five Developmental Outcomes.

The Critical Outcomes require students to be able to:

- Identify and solve problems and make decisions using critical and creative thinking.
- Work effectively with others as members of a team, group, organisation and community.
- Organise and manage themselves and their activities responsibly and effectively.
- Collect, analyse, organise and critically evaluate information.
- Communicate effectively using visual, symbolic and/or language skills in various modes.
- Use science and technology effectively and critically showing responsibility towards the environment and the health of others.
- Demonstrate an understanding of the world as a set of related systems by recognising that problem-solving contexts do not exist in isolation.

The Developmental Outcomes require students to be able to:

- Reflect on and explore a variety of strategies to learn more effectively.
- Participate as responsible citizens in the life of local, national and global communities.
- Be culturally and aesthetically sensitive across a range of societal contexts.
- Develop entrepreneurial opportunities.

D. Factors that contribute to achieving Mathematics Learning Outcomes

Students in the Further Education and Training band who are interested in the subject or who intend to follow a career path requiring Mathematics will, while ensuring that they are mathematically literate, work towards being able to:

- Completely use mathematical process skills, such as making conjectures, proving assertions and modelling situations.
- Calculate confidently and competently with and without calculators and use rational and irrational numbers with understanding.
- Competently produce useful equivalents for algebraic expressions and use such equivalents appropriately and with confidence.
- Use Mathematics to critically investigate and monitor the financial aspects of personal and community life and political decisions.
- Work with a wide range of patterns and transformations (translations, rotations, reflections) of functions and solve related problems.
- Describe, represent and analyse shape and space in two and three dimensions using various approaches in geometry (synthetic, analytic, transformational) and trigonometry in an interrelated or connected manner.
- Collect and use data to establish basic statistical and probability models, solve related problems and critically consider representations provided or conclusions reached.
- Use and understand the principles of differential calculus to determine the rate of change of a range of simple, non-linear functions and solve simple optimisation problems.
- Solve problems involving sequences and series in real-life and mathematical situations.
- Use and understand basic principles of integration to calculate volume and area.
- Use and understand other sets of numbers, such as complex numbers, to solve non-real equations particularly with reference to electrical problems.
- Solve non-routine, unseen problems using mathematical principles and processes.
- Investigate historical aspects of the development and use of mathematics in various cultures.
- Use available technology (the minimum being a scientific calculator) in calculations and in the development of models.

These mathematical skills and process abilities will, where possible, be embedded in contexts that relate to HIV/AIDS, human rights, indigenous knowledge systems and political, economic, environmental and inclusivity issues.

1 DURATION AND TUITION TIME

This is a one year instructional programme comprising 200 teaching and learning hours. The subject may be offered on a part-time basis provided all of the assessment requirements set out hereunder are adhered to.

Learners with special education needs (LSEN) must be catered for in a way that eliminates barriers to learning.

2 SUBJECT LEVEL OUTCOMES

- Manipulate mathematical statements involving complex numbers.
- Investigate, analyse, describe and represent a wide range of functions and solve related problems.
- Analyze and justify properties of shape in 2-dimensional and 3-dimensional space.
- Interpret data to establish statistical and probability models to solve related problems.
- Apply mathematical knowledge and skills to plan personal finances and investigate opportunities for entrepreneurship.

3 ASSESSMENT REQUIREMENTS

3.1 Internal assessment (25 percent)

All internal assessments must be finalised by an assessor with at least a certificate of competence.

3.1.1 Theoretical Component

The theoretical component will form 25 percent of the internal assessment.

Assessment of this component should takethe form of tests and assignments. One test and one assignment per month are recommended.

3.1.2 Practical Component

The practical component will form 75 percent of the internl assessment.

The practical component in Mathematics includes all exercises and applications, tests and assignments done by the learner. All practical components must beindicated in a Portflio of Evidence (PoE). One test and one assignment are recommended per month.

- **Some examples of practical assessments include, but are not limited to:**

  A. Practical exercise work and applications to contextual problems.
  B. Presentations (lectures, demonstrations, group discussions and activities, practical work, observation, role play, self activity, judging and evaluation)
  C. Use of aids
  D. Exhibitions
  E. Visits
  F. Research

- **Evidence in practical assessments**

All evidence pertaining to evaluation of practical work must be reflected in the learners' Portfolio of Evidence (PoE). The tools and instruments constructed and used for the purpose of conducting such assessments must be clear from evidence contained in the PoE.

3.1.3 Processing of Internal assessment mark for the year

A year mark out of 100 is calculated by adding the marks of the theoretical component and the practical component of the internal continuous assessment.

3.1.4 Moderation of internal assessment mark

Internal assessment is subjected to both internal and external moderation procedures as contained in the *National Examinations Policy for Further Education and Training College Programmes*.

3.2 External assessment (75 percent)

A national examination is conducted annually in October or November each year by means of two papers set externally and marked and moderated internally. For level 3 there will be two 3 hour examination papers. The content covered in each is described below.

Details in respect of External Assessment are contained in *Assessment Guidelines: Mathematics Level 3*.

4 WEIGHTED VALUES OF TOPICS

Paper 1

TOPICS	WEIGHTED VALUE
1. Number and Relationships between numbers	33,3%
2. Algebraic expressions, functions and relations	33,3%
3. Differential Calculus	33,3%
TOTAL	**100**

Paper 2

TOPICS/THEMES	WEIGHTED VALUE
4. Statistical analysis and data collection	33,3%
5. Geometry and trigonometry	33,3%
6. Financial mathematics	33,3%
TOTAL	**100**

5 CALCULATION OF FINAL MARK

Continuous assessment: Student's mark/100 x 25/1 = a mark out of 25 (a)
Examination mark: Student's mark/100 x 75/1= a mark out of 75 (b)
Final mark: (a) + (b) = a mark out of 100

All marks are systematically processed and accurately recorded to be available as hard copy evidence for, amongst others, purposes of moderation and verification.

6 PASS REQUIREMENTS

The learner must obtain minimum of 30 percent to pass the subject. A pass will be condoned at 25% if it is the only subject preventing the learner from going to level 4.

7 SUBJECT AND LEARNING OUTCOMES

On completion of Mathematics Level 3, the student should have covered the following topics:

Topic 1: Complex Numbers
Topic 2: Functions
Topic 3: Space shape and orientation
Topic 4: Statistical and Probability models
Topic 5 Financial Mathematics

7.1 Topic 1: Complex Numbers

7.1.1 Subject Outcome 1: Represent complex numbers in a form appropriate to the context.

Learning Outcomes
Students should be able to:

- Represent complex numbers in standard and polar form.
- Construct Argand diagrams.

7.1.2 Subject Outcome 2: Perform operations on complex numbers.

Learning Outcomes:
Students should be able to:
- Perform addition, subtraction, multiplication and division on complex numbers in standard and polar form.
- Use De Moivre's theorem to raise complex numbers to powers.
- Execute algorithms correctly and appropriately to do calculation

7.1.3 Subject Outcome 3: Solve problems using complex numbers.

Learning Outcomes:
Students should be able to:
- Represent complex numbers in a form appropriate to the context.
- Perform operations of various forms of complex numbers consistently.
- Use complex numbers to solve problems which cannot be solved using the real number system.

7.2 Topic 2: Functions

7.2.1 Subject Outcome 1: Use a variety of techniques to sketch and interpret information from graphs of functions.

Learning Outcomes:
Students should be able to:
- Work with several types of functions including:

$$y = \sin(kx); y = \cos(kx); y = \tan(kx); y = \sin(x + p); y = \cos(x + p);$$

$$y = \tan(x + p); y = a(x + p)^2; y = a(x + p)^2 + q; y = ax^2 + bx + c; y = \frac{a}{(x + p)} + q;$$

$$y = ab^{x+p} + q, (b \rangle 0); y = ax^3 + bx^2 + cx + d$$

Note: The cubic function should only be done once differential calculus has been done.

- Generate graphs (including those stated above) by means of point by point plotting, supported by available technology.
- Generalize the effects of the parameters *k, p, a* and *q* on the graphs of the above mentioned functions.
- Identify and use the following characteristics to sketch graphs of the above mentioned functions: domain and range, intercepts with axes, turning points minima and maxima, asymptotes, shape and symmetry, periodicity and amplitude, average gradient or rate of change, and intervals on which the function increases or decreases.
- Understand and be able to describe the discrete and continuous nature of graphs.

7.2.2 Subject Outcome 2: Manipulate and simplify algebraic expressions.

Learning Outcomes
Students should be able to:
- Manipulate algebraic expressions by completing the square.
- Simplify algebraic fractions with binomial denominators.

7.2.3 Subject Outcome 3: Solve algebraic equations and inequalities.

Students should be able to:
Learning Outcomes
Students should be able to:
- Solve quadratic equations by factorization, completion of the square and by using the quadratic formula.
- Solve quadratic inequalities in one variable and be able to graph the solution.
- Solve simultaneous equations in two unknowns algebraically and graphically; where the one equation is linear and the other equation is quadratic

7.2.4 Subject Outcome 4: Use mathematical models to investigate problems that arise in a real life context.

Learning Outcomes
Students should be able to:
- Make, demonstrate and prove the validity of conjectures.
- Express mathematical generalizations of situations and justify them.
- Use various representations to interpolate and extrapolate.
- Describe a situation by interpreting a graph especially focussing on trends and pertinent features.
- Draw graphs from descriptions of situations which focus on trends and pertinent features.
- Express generalizations in symbolic form appropriate to the situation.
- Support conjectures with acceptable arguments. Claims where generalizations are not possible must be supported by coherent reasons.
- Investigate the average gradient between two points on a curve numerically to develop an intuitive understanding of the concept of the gradient of a curve at a point.

7.2.5 Subject Outcome 5: Use mathematical models to investigate linear programming problems.

Learning Outcomes
Students should be able to:
- Optimize a function in two variables, subject to one or more linear constraints by creating a feasible region and conducting a boundary search.
- Solve a system of linear equations in order to find the vertices of a feasible region.

7.2.6 Subject Outcome 6: Determine the limits of functions through an intuitive understanding of limits.

Learning Outcomes
Students should be able to:
- Determine limits of functions intuitively.
- Distinguish between the value of a function at a particular point and the limit of that function at that point.

7.2.7 Subject Outcome 7: Determine the derivative from first principle.

Learning Outcomes
Students should be able to:
- Interrelate in meaningful way graphical, numerical and symbolic representations of the process of determining the derivative in an intuitive way.
- Apply first principles correctly.
- Do the necessary algebraic manipulations accurately.
- Use notation appropriately.

7.2.8 Subject Outcome 8: Find derivatives using the rules for differentiations.

Learning Outcomes
Students should be able to:
- Use correctly the constant, sum, difference, product, quotient and chain rules for differentiation.
- Use notation appropriately.

7.2.9 Subject Outcome 9: Use derivatives to solve problems of both mathematical and real life situations.

Learning Outcomes
Students should be able to:
- Differentiate correctly.
- Find the tangent to the graph at a point correctly.
- Solve maxima and minima problems about real-life situations.
- Draw graphs including those of cubic functions, using differentiation, maxima and minima, and interpret graphs.

7.3 Topic 3: Space, shape orientation

7.3.1 Subject Outcome 1: Use and learn formula for area and volume.

Learning Outcomes:
Students should be able to:
- Determine and calculate the surface area and volume of right pyramids, right cones, spheres and combinations of these geometric objects.

7.3.2 Subject Outcome 2: Investigate necessary and sufficient conditions for similarity of polygons.

Learning Outcomes
Students should be able to:
- Carry out an investigation to prove the Mid-point theorem.
- Carry out investigations and prove that equilateral triangles are similar.
- Carry out investigations and prove that triangles with sides in proportion are similar.
- Carry out investigations and prove Pythagoras' Theorem using similar triangles.

7.3.3 Subject Outcome 3: Use the Cartesian co-ordinate system to derive and apply equations.

Learning Outcomes:
Students should be able to:
- Use the Cartesian co-ordinate system to derive the equation of a line through two given points.
- Use the Cartesian co-ordinate system to derive the equation of a line parallel or perpendicular to another line.
- Use the Cartesian co-ordinate system to derive and use the inclination of a line.

7.3.4 Subject Outcome 4: Investigate, generalise and apply the effect of transformations on the co-ordinates.

Learning Outcomes:
Students should be able to:
- Investigate, generalize and apply the effect on the co-ordinates of the point (x; y) after rotation about the origin through an angle of 90°or 180°.
- Investigate, generalize and apply the effect on the vertices $(x_1; y_1), (x_2; y_2) ..... (x_n; y_n)$ of a polygon after enlargement through the origin by a constant factor k.

7.3.5 Subject Outcome 5: Derive and use special angles in calculations.

Learning Outcomes:
Students should be able to:
- Find values of the trigonometric functions of 30°, 45° and 60° using the definitions of trigonometric functions and leaving the answers in surd form.
- Use these special angles in calculations.

7.3.6 Subject Outcome 6: Derive and use quotient and squares trigonometric identities.

Learning Outcomes:
Students should be able to:
- Simplify expressions using these trigonometric identities.
- Prove more complicated identities using these identities.

7.3.7 Subject Outcome 7: Derive and use reduction formula to simplify trigonometric expressions.

Learning Outcomes:
Students should be able to:
- Derive reduction formula for the six trigonometric equations for: (0° ± Θ), (90° ± Θ), (180° ± Θ), (270° ± Θ), and (360°± Θ).
- Determine the general solution of trigonometric equations.

7.3.8 Subject Outcome 8: Establish and use the area, sine and cosine rules to solve real life problems.

Learning Outcomes
Students should be able to:
- Construct geometric and trigonometric models.
- Solve problems in 2- and 3- dimensions using the area, sine and cosine rules.
- Perform an investigative project on the history of Geometry and Trigonometry in different cultures through history.

7.4 Topic 4: Statistical and Probability Models

7.4.1 Subject Outcome 1: Calculate and represent measures of central tendency and dispersion in univariate numerical data.

Learning Outcomes
Students should be able to:
- Calculate and represent measures of central tendency and dispersion in the following 4 ways: five number summary including maximum, minimum and quartiles, box and whisker diagrams, ogives, calculation and graphical representation using histograms and frequency polygons of the variance and standard deviation manually (for small sets of data) and using available technology like a computer (for large sets of data).

7.4.2 Subject Outcome 2: Pose questions, collect and organise data.

Learning Outcomes
Students should be able to:
- Identify situations or issues that can be dealt with through statistical methods.
- Use appropriate and efficient methods to collect, record and organize data.
- Establish data samples of adequate size and which are representative of the population.

7.4.3 Subject Outcome 3: Analyse and interpret data using various techniques.

Learning Outcomes
Students should be able to:
- Make graphical representations and numerical summaries which are consistent with the data, and clear and appropriate to the situation and target audience.
- Take a position on an issue by comparing different representations of data.
- Do calculations and use statistics in a way that is appropriate to the problem that is posed.
- Justify and apply statistics to answer questions about problems.
- Discuss new questions that arise from the modelling of data.

7.5 Topic 5: Financial Mathematics

7.5.1 Subject Outcome 1: Apply mathematical knowledge and skills to plan personal finances and investigate opportunities for entrepreneurship.

Learning Outcomes
Students should be able to:
- Complete Income and Expenditure sheets and budgets for personal and social club finances.
- Simple and compound interest are used to make sense of and define a variety of situations.

8 RESOURCE NEEDS FOR THE TEACHING OF MATHEMATICS - LEVEL 3

8.1 Physical resources

- Soft cover spring file for portfolios
- Scientific calculators
- Graph paper
- Textbook or workbook
- Computer stiffy or memory stick
- Computer and printing facilities
- Geometric sets
- Chalk and chalkboards

- Paper
- Overhead Projectors
- Current newspapers and information about financial packages from banks and investment companies.
- Internet access or access to a good library or resource centre.
- Models

8.2 Human resources

NQF Level 4 Mathematics or equivalent with a teaching qualification is required to teach level 3 Mathematics

8.3 Other resorces

- Workbooks