



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

CURRICULUM AND ASSESSMENT POLICY STATEMENT

(CAPS)

MATHEMATICS GRADES 10-12

FINAL DRAFT

SECTION 1

NATIONAL CURRICULUM AND ASSESSMENT POLICY STATEMENT FOR MATHEMATICS GRADES 10-12

1.1 Background

The *National Curriculum Statement Grades R – 12 (NCS)* stipulates policy on curriculum and assessment in the schooling sector.

To improve its implementation, the National Curriculum Statement was amended, with the amendments coming into effect in January 2011. A single comprehensive Curriculum and Assessment Policy document was developed for each subject to replace the old Subject Statements, Learning Programme Guidelines and Subject Assessment Guidelines in Grades R - 12.

The amended *National Curriculum Statement Grades R - 12: Curriculum and Assessment Policy (January 2011)* replaces the *National Curriculum Statement Grades R - 9 (2002)* and the *National Curriculum Statement Grades 10 - 12 (2004)*.

1.2 Overview

- (a) The *National Curriculum Statement Grades R – 12 (January 2011)* represents a policy statement for learning and teaching in South African schools and comprises the following:
 - (i) Curriculum and Assessment Policy documents for each approved school subject as listed in the policy document *National Senior Certificate: A qualification at Level 4 on the National Qualifications Framework (NQF)*; and
 - (ii) The policy document *National Senior Certificate: A qualification at Level 4 on the National Qualifications Framework (NQF)*.
- (b) The *National Curriculum Statement Grades R – 12 (January 2011)* should be read in conjunction with the following documents:
 - (i) *An addendum to the policy document, the National Senior Certificate: A qualification at Level 4 on the National Qualifications Framework (NQF), regarding the National Protocol for Assessment Grade R – 12*, published in the *Government Gazette*, No. 29467 of 11 December 2006; and
 - (ii) *An addendum to the policy document, the National Senior Certificate: A qualification at Level 4 on the National Qualifications Framework (NQF), regarding learners with special needs*, published in the *Government Gazette*, No.29466 of 11 December 2006.
- (c) The Subject Statements, Learning Programme Guidelines and Subject Assessment Guidelines for Grades R - 9 and Grades 10 - 12 are repealed and replaced by the *Curriculum and Assessment Policy documents for Grades R – 12 (January 2011)*.
- (d) The sections on the Curriculum and Assessment Policy as contemplated in Chapters 2, 3 and 4 of this document constitute the norms and standards of the *National Curriculum Statement Grades R – 12* and therefore, in terms of section 6A of the *South African Schools Act, 1996 (Act No. 84 of 1996)*, form the basis for the Minister of Basic Education to determine minimum outcomes and standards, as well as the processes and procedures for the assessment of learner achievement to be applicable to public and independent schools.

1.3 General aims of the South African Curriculum

- (a) The *National Curriculum Statement Grades R - 12* gives expression to what is regarded to be knowledge, skills and values worth learning. It will ensure that learners acquire and apply knowledge and skills in ways that are

meaningful to their own lives. In this regard, the curriculum promotes the idea of grounding knowledge in local contexts, while being sensitive to global imperatives.

(b) The National Curriculum Statement Grades R - 12 serves the purposes of:

- equipping learners, irrespective of their socio-economic background, race, gender, physical ability or intellectual ability, with the knowledge, skills and values necessary for self-fulfilment, and meaningful participation in society as citizens of a free country;
- providing access to higher education;
- facilitating the transition of learners from education institutions to the workplace; and
- providing employers with a sufficient profile of a learner's competences.

(c) The National Curriculum Statement Grades R - 12 is based on the following principles:

- Social transformation; ensuring that the educational imbalances of the past are redressed, and that equal educational opportunities are provided for all sections of our population;
- Active and critical learning; encouraging an active and critical approach to learning, rather than rote and uncritical learning of given truths;
- High knowledge and high skills; the minimum standards of knowledge and skills to be achieved at each grade are specified and sets high, achievable standards in all subjects;
- Progression; content and context of each grade shows progression from simple to complex;
- Human rights, inclusivity, environmental and social justice; infusing the principles and practices of social and environmental justice and human rights as defined in the Constitution of the Republic of South Africa. The National Curriculum Statement Grades 10 – 12 (General) is sensitive to issues of diversity such as poverty, inequality, race, gender, language, age, disability and other factors;
- Valuing indigenous knowledge systems; acknowledging the rich history and heritage of this country as important contributors to nurturing the values contained in the Constitution; and
- Credibility, quality and efficiency; providing an education that is comparable in quality, breadth and depth to those of other countries.

(d) The National Curriculum Statement Grades R - 12 aims to produce learners that are able to:

- identify and solve problems and make decisions using critical and creative thinking;
- work effectively as individuals and with others as members of a team;
- organise and manage themselves and their activities responsibly and effectively;
- collect, analyse, organise and critically evaluate information;
- communicate effectively using visual, symbolic and/or language skills in various modes;
- use science and technology effectively and critically showing responsibility towards the environment and the health of others; and
- demonstrate an understanding of the world as a set of related systems by recognising that problem solving contexts do not exist in isolation.

(e) Inclusivity should become a central part of the organisation, planning and teaching at each school. This can only happen if all teachers have a sound understanding of how to recognise and address barriers to learning, and how to

plan for diversity.

1.4 Time Allocation

1.4.1 Foundation Phase

(a) The instructional time for subjects in the Foundation Phase is as indicated in the table below:

Subject	Time allocation per week (hours)
I. Home Language	6
II. First Additional Language	4 (5)
III. Mathematics	7
IV. Life Skills	6
• Beginning Knowledge	1 (2)
• Arts and Craft	2
• Physical Education	2
• Health Education	1

(b) Instructional time for Grades R, 1 and 2 is 23 hours. For Grade 3, First Additional Language is allocated 5 hours and Beginning Knowledge is allocated 2 hours as indicated by the hours in brackets in the table above.

1.4.2 Intermediate Phase

(a) The table below shows the subjects and instructional times in the Intermediate Phase.

Subject	Time allocation per week (hours)
I. Home Language	6
II. First Additional Language	5
III. Mathematics	6
IV. Science and Technology	3.5
V. Social Sciences	3
VI. Life Skills	4
• Creative Arts	1.5
• Physical Education	1.5
• Religion Studies	1

1.4.3 Senior Phase

(a) The instructional time in the Senior Phase is as follows:

Subject	Time allocation per week (hours)
I. Home Language	5
II. First Additional Language	4
III. Mathematics	4.5
IV. Natural Sciences	3
V. Social Sciences	3
VI. Technology	2
VII. Economic Management Sciences	2
VIII. Life Orientation	2
IX. Arts and Culture	2

1.4.4 Grades 10-12

(a) The instructional time in Grades 10-12 is as follows:

Subject	Time allocation per week (hours)
I. Home Language	4.5
II. First Additional Language	4.5
III. Mathematics	4.5
IV. Life Orientation	2
V. Three Electives	12 (3x4h)

The allocated time per week may be utilised only for the minimum required NCS subjects as specified above, and may not be used for any additional subjects added to the list of minimum subjects. Should a learner wish to offer additional subjects, additional time must be allocated for the offering of these subjects.

SECTION 2

CURRICULUM AND ASSESSMENT POLICY STATEMENT FOR MATHEMATICS (FET)

2.1 What is Mathematics?

Mathematics is the study of [quantity](#), [structure](#), [space](#) and [change](#). [Mathematicians](#) seek out [patterns](#), formulate new [conjectures](#), and establish axiomatic systems by [rigorous deduction](#) from appropriately chosen [axioms](#) and [definitions](#).¹ Mathematics is a distinctly human activity practiced by all cultures, for thousands of years

Mathematical problem solving enables us to understand the world (physical, social and economical) around us, and, most of all, to teach us to think creatively.

The main topics in the Mathematics (FET) Curriculum

1. Functions
2. Number patterns, sequences, series
3. Finance, growth and decay
4. Algebra
5. Differential calculus
6. Probability
7. Euclidean geometry and mensuration
8. Analytical geometry
9. Trigonometry
10. Statistics

2.2 Specific Aims

IMPORTANT GENERAL PRINCIPLES WHICH APPLY ACROSS ALL GRADES

1. **Mathematical modeling** is an important focal point of the curriculum. Real life problems should be incorporated into all sections whenever appropriate. Examples used should be realistic and not contrived.
2. **Investigations** provide the opportunity to develop in learners the ability to be methodical, to generalize, make conjectures and try to justify or prove them. It needs to be understood that learners need to reflect on the processes and not be concerned only with getting the answer/s. Examples of investigations which lend themselves to these processes are given on pages 14 and 39.
3. **Appropriate approximation** and rounding skills should be taught so that the impression is not gained that all answers which are either irrational numbers or recurring decimals should routinely be given correct to two decimal places.
4. **The history of mathematics** should be incorporated into projects and tasks wherever possible. The aim of the inclusion of some history is to show mathematics as a human creation which is hotly contested and still developing.
5. **Contextual problems** should include issues relating to health, social, economic, cultural, scientific, political and environmental issues whenever possible.
6. Teaching should not be limited to “**how**” but should feature the “**when**” and “**why**” of problem types. Finding the mean and standard deviations of a set of data has little relevance unless learners have a good grasp of why and when such calculations might be useful.
7. **Mixed ability** teaching requires teachers to challenge the most able learners and at the same time provide remedial support for those for whom mathematics is difficult. An appendix of challenging questions is provided at the end of the document. Teachers need to design questions to rectify misconceptions that are exposed by tests and examinations.

¹ See the Wikipedia definition in which “truth” is used instead of “axiomatic systems.” “Truth” does not indicate conformity to perceived physical reality as is usually assumed when this definition is used without reference to the philosophy of mathematics. Axiomatics is one approach to establishing mathematical truth. The Euclidean geometry topic in grade 12 is an example of an axiomatic system.

8. **Problem solving** and **cognitive development** should be central to all mathematics teaching. Learning procedures and proofs without a good understanding of why they are important will leave learners ill-equipped to use their knowledge in later life.

2.3 Time allocation for Mathematics: 4 hours and 30 minutes or six forty five minute periods per week in grades 10, 11 and 12.

2.4 Overview of topics and cognitive levels

1. FUNCTIONS					
Grade 10			Grade 11		Grade 12
10.1.1	Work with relationships between variables in terms of numerical, graphical, verbal and symbolic representations of functions and convert flexibly between these representations (tables, graphs, words and formulae). Include the linear and quadratic polynomial functions, the exponential function and the simple rational function.	11.1.1	Extend Grade 10 work on the relationships between variables in terms of numerical, graphical, verbal and symbolic representations of functions and convert flexibly between these representations (tables, graphs, words and formulae) Include the linear and quadratic polynomial functions, the exponential function and the simple rational function.	12.1.1	Introduce the formal definition of a function and extend Grade 11work on the relationships between variables in terms of numerical, graphical, verbal and symbolic representations of functions and convert flexibly between these representations (tables, graphs, words and formulae) Include the linear and quadratic polynomial functions, the exponential function and the simple rational function.
10.1.2	Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures and hence generalise the effect of the parameter which results in a vertical shift and that which results in a vertical stretch and /or a reflection about the x axis.	11.1.2	Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures and hence generalise the effects of the parameter which results in a horizontal shift and that which results in a horizontal stretch and/or reflection about the y axis.	12.1.2	The inverses of prescribed functions and the fact that in the case of many-to-one functions the domain has to be restricted if the inverse is to be a function.
10.1.3	Problem solving and graph work involving the prescribed functions.	11.1.3	Problem solving and graph work involving the prescribed functions.	12.1.3	Problem solving and graph work involving the prescribed functions (including the logarithmic function).
2. NUMBER PATTERNS, SEQUENCES AND SERIES					
10.2.1	Investigate number patterns (including, but not limited to, those where there is a constant difference between consecutive terms in a number pattern, and the general term is therefore linear).	11.2.1	Investigate number patterns (including, but not limited to, those where there is a constant second difference between consecutive terms in a number pattern, and the general term is therefore quadratic).	12.2.1	Identify and solve problems involving number patterns, including but not limited to arithmetic and geometric sequences and series.

3. FINANCE, GROWTH AND DECAY					
10.3.1	Use simple and compound growth formulae $A = P(1+ni)$ and $A = P(1+i)^n$ to solve problems (including interest, hire purchase, inflation, population growth and other real life problems).	11.3.1	Use simple and compound decay formulae $A=P(1-ni)$ and $A = P(1-i)^n$ to solve problems (including straight line depreciation and depreciation on a reducing balance). Link to work on functions.	12..3.1	(a) Calculate the value of n in the formulae $A = P(1+i)^n$ and $A = P(1-i)^n$ (b) Apply knowledge of geometric series to solve annuity and bond repayment problems.
10.3.2	The implications of fluctuating foreign exchange rates.	11.3.2	The effect of different periods of compounding growth and decay (including effective and nominal interest rates).	12.3.2	Critically analyse different loan options.
4. ALGEBRA					
10.4.1	(a) Identify rational numbers and convert between terminating or recurring decimals and the form : $\frac{a}{b}$ $a \in \mathbb{Z}; b \in \mathbb{Z}$ (b) Show that simple surds are not rational.	11.4.1	The extension of the number system to include non-real numbers.		
10.4.2	(a) Simplify expressions using the laws of exponents for integral exponents. (b) Establish between which two integers any simple surd lies. (c) Round rational and irrational numbers to an appropriate degree of accuracy.	11.4.2	(a) Simplify expressions using the laws of exponents for rational exponents. (b) Add, subtract, multiply and divide simple surds. (c) Error margins.	12.4.2	Demonstrate an understanding of the definition of a logarithm and any laws needed to solve real life problems.

10.4.3	Manipulate algebraic expressions by: • multiplying a binomial by a trinomial; • factorising trinomials; • factorising by grouping in pairs; simplifying , adding and subtracting algebraic fractions with monomial denominators.	11.4.3	Manipulate algebraic expressions by • writing quadratic functions in the completed square form; • simplifying algebraic fractions with binomial denominators.	12.4.3	Factorise third degree polynomials (including examples which require the factor theorem).
10.4.4	• Solve: • linear equations • quadratic equations by factorisation • literal equations (changing the subject of formulae) • exponential equations (accepting that the laws of exponents hold for real exponents and solutions are not necessarily integral or even rational). • linear inequalities in one variable and illustrate the solution graphically • linear equations in two variables simultaneously (numerically, algebraically and graphically)	11. 4.4	Solve: • quadratic equations (by factorisation, by completing the square, and by using the quadratic formula); • quadratic inequalities in one variable and interpret the solution graphically; • equations in two unknowns, one of which is linear the other quadratic, algebraically or graphically.		
5. DIFFERENTIAL CALCULUS					
10.5.1	Investigate the average rate of change of a function between two values of the independent variable demonstrating an understanding of average rate of change over different intervals.	11. 5.1	Investigate numerically the average gradient between two points on a curve and develop an intuitive understanding of the concept of the gradient of a curve at a point	12. 5.1	(a) An intuitive understanding of the concept of a limit. (b) Differentiation of specified functions from first principles. (c) Use of the specified rules of differentiation. (d) The equations of tangents to graphs. (e) Sketch graphs of cubic functions. Practical problems involving optimisation and rates of change (including the calculus of motion).

6. PROBABILITY					
10.6.1	(a) Compare the relative frequency of an outcome with the theoretical probability of the outcome. (b) Venn diagrams as an aid to solving probability problems. (c) Mutually exclusive events and complementary events. (d) The identity for any two events A and B: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$	11.6.1	(a) Dependent and independent events. (b) Venn diagrams contingency tables and tree diagrams as aids to solving probability problems (where events are not necessarily independent).	12.6.1	(a) Generalise and use the fundamental counting (multiplication) principle. (b) Probability problems using the fundamental counting principle and other techniques.
7. EUCLIDEAN GEOMETRY AND MENSURATION					
10.7.1	(a) Investigate and form conjectures about the properties of special triangles, quadrilaterals and other polygons. Try to validate or prove conjectures using any logical method (Euclidean, co-ordinate or transformation geometry from Grade 9) disprove false conjectures by producing counter-examples (b) investigate alternative definitions of various polygons (including the isosceles, equilateral and right-angled triangle, the kite, parallelogram, rectangle, rhombus and square)	11.7.1	(a) Revise Grade 9 and 10 work on the necessary and sufficient conditions for polygons to be similar (b) Prove (accepting results established in earlier grades): ○ that a line drawn parallel to one side of a triangle divides the other two sides proportionally (and the Mid-point Theorem as a special case of this theorem) ○ that equiangular triangles are similar ○ that triangles with sides in proportion are similar ○ the Pythagorean Theorem by similar triangles	12.7.1	(a) Investigate and prove the theorems of the geometry of circles as a mini-axiomatic system. (b) Solve circle geometry problems, providing reasons for statements when required.
10.7.2	Solve problems involving volume and surface area of solids studied in earlier grades and combinations of those objects to form more complex shaped solids.				

8. TRIGONOMETRY					
10.8.1	Definitions of the trigonometric functions $\sin \theta$, $\cos \theta$ and $\tan \theta$ in right angled triangles.	11.8.1	(a) derives and uses the values of the trigonometric functions (in surd form where applicable) of 30° , 45° and 60° (b) derives and uses the identities: $\tan \theta = \frac{\sin \theta}{\cos \theta};$ $\sin^2 \theta + \cos^2 \theta = 1$ (c) derives the reduction formulae (d) determines the general solution of trigonometric equations (e) establishes the sine cosine and area rules	12.8.1	Proof and use of the compound angle and double angle identities:
10.8.2	Problems in 2-dimensions by using the above trigonometric functions and by constructing and interpreting geometric and trigonometric models.	11.8.2	Problems in 2 - dimensions by constructing and interpreting geometric and trigonometric models	12.8.2	Problems in two and three dimensions by constructing and interpreting geometric and trigonometric models.
10.8.3	The definitions of $\sin \theta$, $\cos \theta$ and $\tan \theta$ for any angle θ in terms of x , y and r and graphs of $y = \sin \theta$, $y = \cos \theta$ and $y = \tan \theta$ for $-360^\circ \leq \theta \leq 360^\circ$.				
10.8.4	The effects of a and q in the graphs of $y = a \sin \theta + q$, $y = a \cos \theta + q$ and $y = a \tan \theta + q$	11.8.4	The effects of the parameters b and p in the graphs of $y = \sin(b\theta + p)$, $y = \cos(b\theta + p)$ and $y = \tan(b\theta + p)$		
9. ANALYTICAL GEOMETRY					
10.9.1	Represent geometric figures on a Cartesian co-ordinate system, and derive and apply, for any two points $(x_1; y_1)$ and $(x_2; y_2)$, a formula for calculating: - the distance between the two points - the gradient of the line segment joining the	11.9.1	Use a Cartesian co-ordinate system to derive and apply : - the equation of a line through two given points - the equation of a line through one point and parallel or perpendicular to a given	12.9.1	Use a two-dimensional Cartesian co-ordinate system to derive and apply: - the equation of a circle (any centre) - the equation of a tangent to a circle given a point on the circle.

	points the co-ordinates of the mid-point of the line segment joining the points		line the inclination of a line		
10. STATISTICS					
10.10.1	(a) Collect, organise and interpret univariate numerical data in order to determine - measures of central tendency (mean, median, mode) of grouped data and know which is the most appropriate under given conditions - measures of dispersion: percentiles, quartiles, deciles, interquartile and semi-inter-quartile range (b) Frequency polygons	11.10.1	(a) Represent measures of central tendency and dispersion in univariate numerical data by: - five number summary (maximum, minimum quartiles) - box and whisker diagrams - ogives - calculating the variance and standard deviation of sets of data manually (for small sets of data) and using available technology (for larger sets of data) and representing results graphically. (b) Represent bivariate numerical data as a scatter plot and suggest intuitively and by simple investigation whether a linear, quadratic or exponential function would best fit the data	12.10.1	a) Use of available technology to calculate the linear regression line which best fits a given set of bivariate numerical data b) Use of available technology to calculate the correlation co-efficient of a set of bivariate numerical data and make relevant deductions.
10.10.2	Sources of bias by those displaying statistics	11.10.2	Errors in measurement	12.10.2	Suitable sampling from a population understanding the importance of sample size in predicting the mean and standard deviation of a population.
10.10.3	Identify outliers.	11.10.3	Skewed data in box and whisker diagrams and frequency polygons.	12.10.3	Data which is normally distributed around the mean.

2.5 Approximate weighting of teaching time devoted to identified topics

Topic	Grade 10	Grade 11	Grade 12
Functions	20%	20%	12%
Number patterns, sequences, series	5%	5%	8%
Finance, growth and decay	5%	5%	5%
Algebra	10-15%	10-15%	8%
Differential calculus			12%
Probability	5%	5%	5%
Euclidean geometry and mensuration	10-15%	10-15%	10-15%
Analytical geometry	10%	10%	10%
Trigonometry	20-25%	20-25%	20-25%
Statistics	5%	5%	5%

Chapter 3: ANNUAL TEACHING PLAN

1. The examples discussed in the Clarification Column in the annual teaching plan which follows are by no means a complete representation of all the material to be covered in the curriculum. They only serve as an indication of some questions on the topic at different cognitive levels.
Text books and other resources should be consulted for a complete treatment of all the material.
2. The cognitive levels of examples refer to the descriptors in Chapter 4 on Assessment. These examples are not absolute. What is a complex procedure in one grade becomes routine or even knowledge in a higher grade (or even later in a year). Questions which are indicated as being problem solving cease to be “problem solving” once a learner has been taught how to solve that kind of problem. So teaching the techniques involved in the solution of the most demanding questions in the previous year’s examination paper is unlikely to prepare candidates for the higher order questions that will be asked that year, but there is no better preparation for becoming a problem solver than being given plenty of challenging questions to tackle.
3. The order of topics is not prescriptive but it is recommended that care be taken to ensure that in the first two terms, some of the topics 1 to 6 As well as some of the topics 7 to 10 are taught so that assessment is balanced.

GRADE 10: TERM 1			
			Where an example is given, the cognitive demand is suggested: knowledge (K), routine procedure (R), complex procedure (C) or problem solving (P)
2.5	Algebraic Expressions	1. Multiplication of a binomial by a trinomial. 2. Factorisation to include types taught in grade 9 and : • trinomials • grouping in pairs 3. Simplification of algebraic fractions using factorisation. 4. Addition and subtraction of algebraic fractions with monomial denominators.	Examples to illustrate the different cognitive levels involved in factorisation: 1. Factorise fully: 1.1 $m^2 - 2m + 1$ Since learners must recognize the simplest perfect squares : K 1.2 $2x^2 - x - 3$ Since this type is routine and appears in all texts: R 1.3. $\frac{y^4}{2} - \frac{13y^2}{2} + 18$ Since one is required to work with fractions and identify when the expression has been “fully factorised”: C 2. Show that $n^2 + n$ is even for all $n \in \mathbb{Z}$ and $n^3 - n$ is divisible by 6 for all $n \in \mathbb{Z}$ Since it is not obvious (unless one has seen these questions before) that factorising the general form is the key to showing that the first expression always has an even factor and the second always has an even factor and has a factor of 3: P
2	Exponents	1. Revise laws of exponents learnt in Grade 9 where $x, y > 0; m, n \in \mathbb{Z}$ $x^m \times x^n = x^{m+n}$ $x^m \div x^n = x^{m-n}$	Common misconceptions are that • $x^m \times y^n = (xy)^{m+n}$

		$(x^m)^n = x^{mn}$ $x^m \times y^m = (xy)^m$ $x^{-n} = \frac{1}{x^n}$ $x^0 = 1$ 2. Use the laws of exponents to simplify expressions and solve equations, accepting that the rules also hold for $m, n \in \mathbb{Z}$.	Learners multiply unlike bases <u>and</u> add the exponents $(x + y)^m = x^m + y^m$ They forget that in squaring a binomial, for example, there is a middle term. $x^m + x^n = x^{m+n}$ or x^{mn} They confuse adding the exponents and adding the terms or reverse the rule and multiply the exponents of like bases to add. Examples: Solve for x $2^x = 0,125$ Since 0,125 should be known to be 2^{-3}; K $3 \times 5^x = 75$ A simple two step procedure is involved: R $2^x = 30$ (correct to 2 decimal places by trial and improvement) This requires conceptual understanding: identifying first the two integers between which the variable lies, then refining successive approximations: C By the end of the year this will probably have become routine. $\frac{9^x - 1}{3^x + 1} = 8$ Assuming this type of question has not been taught, spotting that the numerator can be factorised as a difference of squares requires insight: P The equation can also be solved by multiplying both sides by the denominator and then factorizing the resulting equation as a quadratic.
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1	Numbers and patterns	- The definition of a rational number as a number which can be written in the form $\frac{a}{b}$ where $a \in \mathbb{Z}$ and $b \in \mathbb{Z}$. - Non real numbers are encountered when trying to solve equations like $x^2 = -1$. - Patterns: Investigate number patterns (including but not limited to those where there is a constant difference between consecutive terms in a number pattern, and the general term is therefore linear).	Learners should be able to prove that any recurring decimal is rational and that surds like $\sqrt{2}$ are irrational. A common misconception is that $\pi = \frac{22}{7}$ and is therefore rational. (It is known that π is an irrational number). Examples: - Determine the 5th and the nth terms of the number pattern 10 ; 7 ; 4 ; 1 ; ... R since there is an algorithmic approach to answering such questions. - If the pattern MATHSMATHSMATHS... is continued in this way, what will the 267th letter be? P since it is not immediately obvious how one should proceed (unless similar questions have been tackled)
2	Equations and Inequalities	- Revise the solution of linear equations. - Solve quadratic equations (by factorisation). - Solve simultaneous linear equations. - Solve literal equations (Changing the subject of a formula). - Solve linear inequalities.	Examples: - Solve for x: $\frac{2x-3}{3} - 3x = \frac{2x}{6}$ (R) - Solve for m: $2m^2 - m = 1$ (R) - Solve for r in terms of V, π and h: $V = \pi r^2 h$ (R) - Solve for x: $-1 \leq 2 - 3x < 8$ (C)
3	Trigonometry	- Introduce trigonometry using the basic concept of the similarity of triangles. - Solve two dimensional problems in right angled triangles. - Solve simple trigonometric equations for angles between 0° and 90°	Comment: It is important to stress that - trigonometric ratios are independent of the lengths of the sides of a triangle and depend only on the angles; - doubling a ratio has a different effect from doubling an angle. For example, generally $2 \sin \theta \neq \sin 2\theta$; - the domain of $y = a \tan \theta + q$ for $-360^\circ \leq \theta \leq 360^\circ$ excludes the values of θ for which the tan ratio is not defined.

	4. Define the trigonometric functions $y = \sin \theta, \quad y = \cos \theta \text{ and}$ $y = \tan \theta \text{ (in terms of } x, y \text{ and } r) \text{ for}$ $\theta \text{ any angle.}$ 5. Plot the graphs of $y = \sin \theta$, $y = \cos \theta \text{ and } y = \tan \theta \text{ for}$ $\theta \in [-360^\circ; 360^\circ] \text{ using a}$ calculator. 6. Study the effect of a and q on the basic graphs of: $y = a \sin \theta + q$ $y = a \cos \theta + q$ $y = a \tan \theta + q$	Comment: The fact that the effects of the parameters a and q are the same on the graphs of all functions should be stressed once the algebraic functions have also been studied. Examples: - Determine the length of the hypotenuse of a right triangle ABC, where $\hat{B} = 90^\circ$, $\hat{A} = 30^\circ$ and $AB = 10 \text{ cm}$. (K) - Sketch the graph of $y = -\sin x + \frac{1}{2}$ for $x \in [-90^\circ; 90^\circ]$ (C)
Assessment Term 1: 1 Investigation or assignment or project (only one project or investigation in a year) (at least 50 marks) Example of an investigation:		

Imagine a cube of white wood which is dipped into red paint so that the surface is red, but the inside still white.

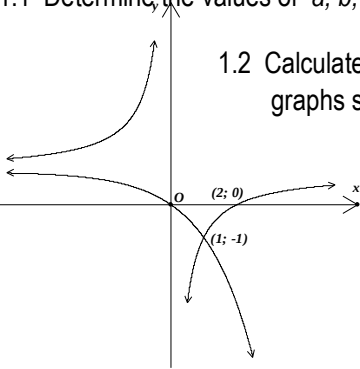
If one cut is made, parallel to each face of the cube then there will be 8 smaller cubes. Each of the smaller cubes will have 3 red faces and 3 white faces.

Investigate the number of smaller cubes which will have 3, 2, 1 and 0 red faces if $2/3/4 \dots n$ cuts are made parallel to each face.

This task provides the opportunity to investigate, tabulate results, make conjectures and justify or prove them.

2. At least one 1 hour test (at least 50 marks). Make sure all topics are tested.

Two or three tests of at least 40 minutes would probably be better. Care needs to be taken to ask questions at all four cognitive levels: approximately between 25% knowledge, approximately 45% routine procedures, 20% complex procedures and 10% problem solving.

GRADE 10: TERM 2			
Weeks	Topic	Curriculum statement	Clarification
4	Functions	1. The concept of a function. Work with relationships between variables using tables, graphs, words and formulae. Convert flexibly between these representations. 2. Point by point plotting of basic graphs of $y = x^2$, $y = \frac{1}{x}$ and $y = b^x$; $b > 0$ and $b \neq 1$ to discover shape, domain, range, asymptotes, axes of symmetry and turning point and intercepts on the axes (where applicable). Notice that the graph of $y = x$ should be known from Grade 9. 3. Investigate the effect of a and q in the graphs of $y = a \cdot f(x) + q$, where $f(x) = x$, $f(x) = x^2$, $f(x) = \frac{1}{x}$ and $f(x) = b^x$, $b > 0$, $b \neq 1$. 4. Sketch graphs, finding the equations of graphs, average gradient, interpretation of graphs. 5. Discrete and continuous graphs in some practical applications.	Comments: 1. A formal definition of a function follows in Grade 12. At this level it is enough to investigate the way (unique) output values depend on how input values vary. The terms independent (input) and dependent (output) variables might be useful. 2. After summaries have been compiled about basic features of prescribed graphs and the effects of parameters a and q have been investigated: a: a vertical stretch (and/or a reflection about the x axis) and q a vertical shift, the following examples might be appropriate: 1. Sketched below are graphs of $y = \frac{a}{x} + b$ and $y = p \cdot q^x + k$. The horizontal asymptote of both graphs is the line $y = 1$. 1.1 Determine the values of a, b, p, q and k. (C) 1.2 Calculate the average gradient of each of the graphs sketched between $x = 1$ and $x = 2$ (R)  Notice that average gradient is the gradient of the secant of a curve between two points (not the average of a number of gradients in the specified interval) 2. Two men do a job in 9 days. Draw a graph to illustrate the number of men required to do the job in a different number of days. Would it make sense for this graph to be continuous? Why or why not? (C)
No of Weeks	Topic	Curriculum statement	Clarification
3	Euclidean Geometry	1. Revise basic results established in	Comments: 1. Triangles are similar if their angles coincide, or if the ratios of their

		earlier grades regarding lines, angles and polygons, especially the similarity and congruence of triangles. 2. Investigate special polygons: the kite, parallelogram, rectangle, rhombus, square and trapezium. Make conjectures about the properties of the sides, angles, diagonals and areas of these polygons. Prove these conjectures.	sides coincide: Triangles ABC and DEF are similar if $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$. They are also similar if $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$. 2. It must be explained that a single counter example disproves a conjecture but that numerous specific examples supporting a conjecture do not constitute a general proof. Example: In quadrilateral KITE, $KI = KE$ and $IT = ET$. The diagonals intersect at M. Prove that: 1. $IM = ME$ and 2. KT is perpendicular to IE. C: Since it is not obvious that one must first prove $\triangle KIT \equiv \triangleKET$
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3	Mid-year examinations		
Assessment term 2: 1. Revision assignment/s (at least 25 marks) 2. Mid-year examination (at least 125 marks) One paper of $2\frac{1}{2}$ hours (125 marks) or Two papers; one $1\frac{1}{2}$ hours (75 marks) and the other 1 hour (50 marks) Paper 1: $1\frac{1}{2}$ hours, 75 marks made up as follows: general algebra ± 15 , equations and inequalities ± 15 , functions and graphs ± 35 and numbers and number patterns ± 15 Paper 2: 1 hour, 50 marks made up as follows: geometry ± 25 and trigonometry ± 25 .			

GRADE 10: TERM 3			
Weeks	Topic	Curriculum statement	Clarification
2	Analytical Geometry	Derive and apply for any two points $(x_1; y_1)$ and $(x_2; y_2)$ the formulae for calculating the: 1. distance between the two given points; 2. gradient of the line segment joining the two points (and hence Identify parallel and perpendicular lines); 3. coordinates of the mid-point of the line segment joining the two points.	Example: Consider the points $P(2;5)$ and $Q(-3;1)$ in the Cartesian plane. 1.1 Calculate the distance PQ. (K) 1.2 Find the coordinates of R if $M(-1;0)$ is the mid-point of PR. (R) 1.3 Determine the coordinates of S if PQRS is a parallelogram. (C) 1.4 Is PQRS a rectangle? Why or why not? (R)
2	Finance, growth and decay	1. Use the simple and compound growth formulae $(A = P(1 + in))$ and $(A = P(1 + i)^n)$ to solve problems, including interest, hire purchase, inflation, population growth and other real life problems. 2. The implications of fluctuating foreign exchange rates.	Example: How long will it take a population of 1,2 million to double if it is increasing at a rate of 12% p.a.? (C) This is considered complex because the value of n must be found by trial and improvement from the compound growth formula. A sensible answer (rounded to the nearest year) should be expected: months and days would not be sensible. Comment: An understanding must be developed of the fact that foreign exchange influences petrol price, imports, exports and overseas travel.

Weeks	Topic	Curriculum statement	Clarification																		
2	Statistics	1. Revision of stem and leaf plots and histograms with equal intervals and extension to histograms with unequal intervals. 2. Measures of central tendency in grouped data: calculation of approximate mean of grouped data and Identification of modal interval and interval in which the median lies. 3. Frequency polygons. 4. Revision of range as a measure of dispersion and extension to include percentiles, quartiles, interquartile and semi-interquartile range.	Comment: Notice that the areas (and not the heights) of the bars of histograms with unequal intervals are proportional to the frequencies of the data items in the specified intervals. Example: 1. The mathematics marks of 200 grade 10 learners at a school can be summarised as follows: <table><thead><tr><th>Percentage obtained</th><th>Number of candidates</th></tr></thead><tbody><tr><td>0-19</td><td>4</td></tr><tr><td>20-29</td><td>10</td></tr><tr><td>30-39</td><td>37</td></tr><tr><td>40-49</td><td>43</td></tr><tr><td>50-59</td><td>36</td></tr><tr><td>60-69</td><td>26</td></tr><tr><td>70-79</td><td>24</td></tr><tr><td>80-100</td><td>20</td></tr></tbody></table> 1. Draw a frequency polygon of the data. 2. Calculate the approximate mean mark for the examination. 3. Identify the interval in which each of the following data items lie: 3.1 the median; 3.2 the lower quartile; 3.3 the upper quartile. 3.4 the thirtieth percentile.	Percentage obtained	Number of candidates	0-19	4	20-29	10	30-39	37	40-49	43	50-59	36	60-69	26	70-79	24	80-100	20
Percentage obtained	Number of candidates																				
0-19	4																				
20-29	10																				
30-39	37																				
40-49	43																				
50-59	36																				
60-69	26																				
70-79	24																				
80-100	20																				

(R)

Weeks	Topic	Curriculum statement	Clarification
1	Euclidean Geometry	1. Investigate alternative definitions of various polygons (including the isosceles, equilateral and right angled triangle and the kite, parallelogram, rectangle, rhombus and square).	Comment: The generally accepted definition of a parallelogram is a quadrilateral in which both pairs of opposite sides are parallel. But if it is given that both pairs of opposite angles of a quadrilateral are equal, then it can be proved that both pairs of opposite sides are parallel. So “ a quadrilateral with both pairs of opposite angles equal “ is an alternative definition of a parallelogram. Example: Prove that ‘a quadrilateral in which the diagonals bisect each other at right angles’ is an alternative definition of a rhombus. (R)
2	Trigonometry	Problems in two dimensions.	Example: 1. Two flagpoles are 30 m apart. The one has height 10 m, while the other has height 15 m. Two tight ropes connect the top of each pole to the foot of the other. How high do the two ropes intersect above the ground? What if the poles were a different distance apart? (P)
1	Mensuration	The calculation of volume and surface areas of shapes made by combining prisms and pyramids, cylinders, hemispheres and cones to form more complex solids. Problems should also involve finding missing dimensions from sufficient information.	Example: The height of a cylinder is 10 cm, and the radius of the circular base is 2 cm. A hemisphere is attached to one end of the cylinder and a cone of height 2cm to the other end. Calculate the volume and surface area of the solid, correct to the nearest cm^3 and cm^2 respectively. (R)

Assessment term 3:

1. Investigation/project or assignment (at least 50 marks)

Example of a project: Collecting data for a survey and analyzing the data in order to find an answer to a question relevant to social issues of concern to adolescents

2. At least one 1 hour test (at least 50 marks) covering all topics in approximately the ratio of the allocated teaching time.

GRADE 10: TERM 4

No of Weeks	Topic	Curriculum statement	Clarification
1	Probability	1. The use probability models to compare the relative frequency of events with the theoretical probability. 2. The use of Venn diagrams to solve probability problems, deriving and applying the following for any two events A and B in a sample space S: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ A and B are mutually exclusive if $P(A \text{ and } B) = 0$ A and B are complementary if they are mutually exclusive and $P(A) + P(B) = 1$. Then $P(A) = P(\text{not } A) = 1 - P(A)$	Comment: It generally takes a very large number of trials before the relative frequency of a coin falling heads up when tossed approaches 0,5. Example: A study was done to how effective three different drugs, A, B and C were in relieving headache pain. Over the period covered by the study, 80 patients were given the chance to use all three drugs. The following results were obtained: 40 reported relief from drug A 35 reported relief from drug B 40 reported relief from drug C 21 reported relief from both drugs A and C 18 reported relief from drugs B and C 68 reported relief from at least one of the drugs 7 reported relief from all three drugs. 1. Record this information in a Venn diagram. (C) 2. How many of the subjects got relief from none of the drugs? (K) 3. How many subjects got relief from drugs A and B but not C? (R) 4. What is the probability that a randomly chosen subject got relief from at least two of the drugs? (R)
3	Revision		Comment: The value of working on good past papers cannot be over-emphasised.
3	Examinations		

Assessment term 4:

1. Revision assignment/s (at least 50 marks)

2. Examination

Paper 1: 2 hours (100 marks made up as follows: ± 10 on numbers and number patterns, ± 25 on general algebra, equations and inequalities, ± 35 on functions, ± 10 on exponents, ± 10 on finance and ± 10 on probability).

Paper 2: 2 hours (100 marks made up as follows: ± 45 on trigonometry, ± 15 on analytical geometry, ± 5 on transformation geometry, ± 25 on Euclidean geometry, volume and area, and ± 10 on data handling).

3. Year mark (term 1: 20, term 2: 40, term 3: 30, term 4: 10) makes up 25% of the promotion mark.

4. Promotion mark: [year mark (out of 100)] + [examination mark (out of 200) $\times 1,5$]

GRADE 11: TERM 1			
No of Weeks	Topic	Curriculum statement	Clarification
			Where an example is given, the cognitive demand is suggested: knowledge (K), routine procedure (R), complex procedure (C) or problem solving (P) Generally complex manipulation for its own sake will not be examined in the NSC. This does not mean such questions could not be set as a challenge or handled as part of classwork if this does not result in other sections being neglected.
2.5	Algebraic Expressions	- Completing the square to determine the maximum or minimum value of a quadratic expression. - The simplification of algebraic fractions with binomial denominators.	Example 1. I have 12 metres of fencing. What are the dimensions of the largest rectangular area I can enclose with this fencing by using an existing wall as one side? Hint: let the length of the equal sides of the rectangle be x metres and hence form an expression for the area of the rectangle. (C) (Without the hint this would probably be problem solving)
3	Equations and Inequalities.	- Quadratic equations (by factorization, by completing the square and by using the quadratic formula. - Quadratic and rational inequalities in one unknown. - Equations in two unknowns, one of which is linear and the other quadratic.	Comment: solving by completing the square should be done only to show where the quadratic formula comes from. Solution of complicated examples like $6x^2 - 2px - 3p^2 = 0$ by completing the square should not be asked. Examples: 1. Solve for x : $x^2 \leq 4$ (R) 2. Solve for x : $\frac{x}{2} \leq \frac{2}{x}$ (C) 3. Two machines, working together, take 2 hours 24 minutes to complete a job. Working on its own, the one machine takes 2 hours longer than the other to complete the job. How long does the slower machine take? (P)

No of Weeks	Topic	Curriculum statement	Clarification
2	Numbers and Patterns	1. Properties of real and non-real numbers. 2. Simplify expressions and solve equations using the laws of exponents for rational exponents where $x^{\frac{p}{q}} = \sqrt[q]{x^p}; \quad x \geq 0$ 4. Add, subtract, multiply and divide simple surds. 5. Patterns: Investigate number patterns (including but not limited to those where there is a constant second difference between consecutive terms in a number pattern, and the general term is therefore quadratic).	Examples: 1. Show that $x^2 + x + 1 = 0$ has no real roots. (K) 2. Determine the value of $9^{\frac{3}{2}}$ (R) 3. Simplify: $(3 + \sqrt{2})(3 - \sqrt{2})$ (R) 4. In the first stage of the World Cup Soccer Finals there are teams from four different countries in each group. Each country in a group plays every other country in the group once. How many matches are there for each group in the first stage of the finals? How many games would there be if there were five teams in each group? Six teams? n teams? (P)
2	Analytical geometry	Derive and apply: 1. the equation of a line through two given points. 2. the equation of a line through one point and parallel or perpendicular to a given line. 3. the inclination of a line.	Example: Given the points $A(2; 5)$, $B(-3; -4)$ and $C(4; -2)$, determine the : 1. equation of the line AB; (R) 2. size of $\hat{A}$ (C)

No of Weeks	Topic	Curriculum statement	Clarification
1	Simple and Compound Decay and Finance	1. Use simple and compound decay formulae: $A = P(1 - in)$ and $A = P(1 - i)^n$ to solve problems (including straight line depreciation and depreciation on a reducing balance). 2. The effect of different periods of compounding growth and decay, including nominal and effective interest rates.	Examples: 1. The value of a piece of equipment depreciates from R10 000 to R5 000 in four years. What is the rate of depreciation if calculated on the : 1.1 straight line method; (R) 1.2 reducing balance? (C) 2. Which is the better investment over a year or longer: 10,5% p.a. compounded daily or 10,55% p.a. compounded monthly? (R) 3. R50 000 is invested in an account which offers 8%p.a. interest compounded quarterly for the first 18 months. The interest then changes to 6% p.a. compounded monthly. Two years after the money is invested, R10 000 is withdrawn. How much will be in the account after 4 years? (C) Comment: stress the importance of not working with rounded answers but of using the maximum accuracy afforded by the calculator right to the final answer when rounding might be appropriate.

Assessment Term 1:

- 1 An Investigation or an assignment or a project (a maximum of one project in a year) (at least 50 marks)
Notice that an assignment is generally an extended piece of work undertaken at home.

Example of an assignment: Ratios and equations in two variables.

(This assignment brings in an element of history which could be extended to requiring the collection of a picture or two of ancient paintings and architecture which are in the shape of the Golden Rectangle).

Task 1

If $2x^2 - 3xy + y^2 = 0$ then $(2x - y)(x - y) = 0$ so $x = \frac{y}{2}$ or $x = y$. Hence the ratio $\frac{x}{y} = \frac{1}{2}$ or $\frac{x}{y} = 1$.

In the same way find the possible values of the ratio $\frac{x}{y}$ if it is given that $2x^2 - 5xy + y^2 = 0$

Task 2:

Most paper is cut to internationally agreed sizes: A0, A1, A2, ...A7 with the property that the A1 sheet is half the size of the A0 sheet and has the same shape as the A0 sheet, the A2 sheet is half the size of the A1 sheet and has the same shape and so on.

Find the ratio of the length (x) to the breadth (y) of A0, A1, A2, ...A7 paper (in simplest surd form).

Task 3

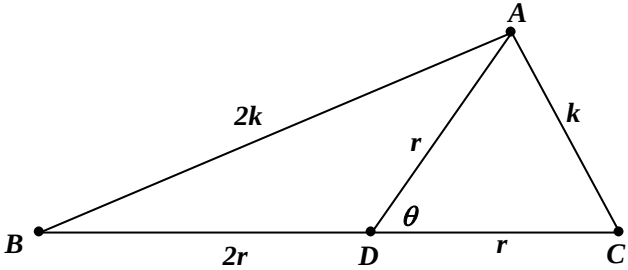
The golden rectangle has been recognised through the ages as being aesthetically pleasing. It can be seen in the architecture of the Greeks, in sculptures and in Renaissance paintings. Any golden rectangle with length x and breadth y has the property that when a square the length of the shorter side (y) is cut from it, another rectangle with the same shape is left. The process can be continued indefinitely, producing smaller and smaller rectangles. Using this information, calculate the ratio $x : y$ in surd form.

2. At least one 1 hour test (at least 50 marks). Two or three tests would be better. Make sure all topics are tested.

Two or three tests of at least 40 minutes would probably be better. Care needs to be taken to ask questions at all four cognitive levels: approximately between 25% knowledge, approximately 45% routine procedures, 20% complex procedures and 10% problem solving.

GRADE 11: TERM 2

No of Weeks	Topic	Curriculum statement	Clarification
5	Trigonometry.	1. Investigate the effect of the parameter k in the graphs of the function. $y = \sin(kx)$, $y = \cos(kx)$ and $y = \tan(kx)$. 2. Investigate the effect of the parameter p in the graphs of the functions $y = f(x) = \sin(x + p)$, $y = f(x) = \cos(x + p)$ and $y = f(x) = \tan(x + p)$ 3. Draw sketch graphs of $y = a(\sin/\cos/\tan)(kx + p) + q$. 4. Derive and use the sin/cos/tan of 30°, 45° and 60°. 5. Derive and use the identities 5.1 $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\theta \neq k.90^\circ; k \in \mathbb{Z}$ and 5.2 $\sin^2 \theta + \cos^2 \theta = 1$	Two parameters at a time can be varied in tests or examinations, for example: Sketch $y = -\frac{1}{2}\sin(x + 30^\circ)$ (C) $y = \cos(2x - 120^\circ)$ (C) Common misconception: the graph of $y = \cos(2x - 120^\circ)$ is a horizontal shift of the graph of $y = \cos 2x$ of 120° to the right. In fact it shifts 60° to the right. Examples: 1. Prove that $\frac{1}{\tan \theta} + \tan \theta = \frac{\tan \theta}{\sin^2 \theta}$ for all $\theta \neq k.90^\circ, k \in \mathbb{Z}$ (R) 2. Simplify $\frac{\cos(180^\circ - x)\sin(x - 90^\circ) - 1}{\tan^2(540^\circ + x)\sin(90^\circ + x)\cos(-x)}$ (R)

	Trigonometry Continued	6. Derive and use the following reduction formulae : 6.1 $\sin/\cos(90^\circ \pm \theta)$ 6.2 $\sin/\cos/\tan(180^\circ \pm \theta)$ 6.3 $\sin/\cos/\tan(360^\circ \pm \theta)$ 6.4 $\sin/\cos/\tan(-\theta)$ 7. Determine for which values of a variable an identity is not valid. 8. Determine the general solutions of trigonometric equations. 9. Establish, prove and apply the sine, cosine and area rules. 10. Solve problems in two dimensions by using the sine, cosine and area rules and by constructing and interpreting geometric and trigonometric models.	3. In $\triangle ABC$, D is on BC, $\hat{ADC} = \theta$, $DA = DC = r$, $BD = 2r$, $AC = k$ and $BA = 2k$  Show that $\cos\theta = \frac{1}{4}$ (P)
3	Mid-year examinations		
Assessment term 2: 1. Revision assignment/s (at least 50 marks) 2. Mid-year examination: Paper 1: 2 hours (100 marks made up as follows: general algebra ± 20, equations and inequalities ± 40, numbers and number patterns ± 20, finance, growth and decay ± 20) Paper 2: 2 hours (100 marks made up as follows: analytical geometry ± 30, trigonometry ± 70).			

GRADE 11: TERM 3

[illegible]

		similar triangles.	
No. of weeks	Topic	Curriculum Statement	Clarification
2	Probability	- Revision of techniques used in solving probability problems: tree diagrams, Venn diagrams, contingency tables, the addition rule for mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B)$, the complementary rule: $P(\text{not } A) = 1 - P(A)$ and the identity : $P(A \text{ or } B) \equiv P(A) + P(B) - P(A \text{ and } B)$ - Dependent and independent events and the product rule for independent events: $P(A \text{ and } B) = P(A) \times P(B)$ - The use of tree diagrams for the probability of consecutive or simultaneous events which are not necessarily independent.	Examples: $P(A) = 0,45$, $P(B) = 0,3$ and $P(A \text{ or } B) = 0,615$. Are the events A and B mutually exclusive, independent or neither mutually exclusive nor independent? (R) - What is the probability of throwing at least one six in four rolls of a regular six sided die? (C)

Assessment term 3:

1. Project) or investigation (at least 50 marks) (only one in a year
Example of project: Collect the heights of at least 50 sixteen year old girls and at least 50 sixteen year old boys. Group your data appropriately and use these two sets of grouped data to draw frequency polygons of the heights of boys and of girls, in different colours, on the same sheet of graph paper.
Identify the modal intervals, the intervals in which the medians lie and the approximate means as calculated from the frequencies of the grouped data. By how much does the approximate mean height of your sample of sixteen year old girls differ from the actual mean?
Comment on the symmetry of the two frequency polygons and any other aspects of the data which are illustrated by the frequency polygons.
2. Assignment (at least 50 marks)
3. Test/s (at least 50 marks): preferably more than one.

GRADE 11: TERM 4

No. of weeks	Topic	Curriculum Statement	Clarification																										
2	Statistics	1. Five number summary (maximum, minimum and quartiles) and box and whisker diagram. 2. Ogives (cumulative frequency curves). 3. Variance and standard deviation of both grouped and ungrouped data. 4. Symmetric and skewed data. 5. Bivariate numerical data. Scatterplots and consideration of models that might model the relationship (if one exists) in the scatterplot.	Example: The table below shows the number of cases of swine flu reported over a 12 week period: <table><tr><th>Week number</th><th>Number of reported cases of swine flu</th></tr><tr><td>1</td><td>14</td></tr><tr><td>2</td><td>19</td></tr><tr><td>3</td><td>21</td></tr><tr><td>4</td><td>39</td></tr><tr><td>5</td><td>72</td></tr><tr><td>6</td><td>70</td></tr><tr><td>7</td><td>125</td></tr><tr><td>8</td><td>176</td></tr><tr><td>9</td><td>170</td></tr><tr><td>10</td><td>291</td></tr><tr><td>11</td><td>331</td></tr><tr><td>12</td><td>437</td></tr></table> 1. Draw a scatterplot of the data. (R) 2. Compare the exponential model $S = 10 \times (1,3)^n$ with the quadratic model $S = 3n^2 + 11$. Decide which is better and why. (C)	Week number	Number of reported cases of swine flu	1	14	2	19	3	21	4	39	5	72	6	70	7	125	8	176	9	170	10	291	11	331	12	437
			Week number	Number of reported cases of swine flu																									
1	14																												
2	19																												
3	21																												
4	39																												
5	72																												
6	70																												
7	125																												
8	176																												
9	170																												
10	291																												
11	331																												
12	437																												
2	Euclidean Geometry	Solving problems on similarity.																											
3	Revision																												
3	Examinations																												

Assessment term 4:

1. Revision assignment/s (at least 50 marks))

2. Examination (300 marks).

Paper 1: 3 hours (150 marks made up as follows: ± 20 on numbers and number patterns, ± 50 on general algebra, equations and inequalities, ± 50 on functions, ± 10 on finance growth and decay, ± 20 on probability).

Paper 2: 3 hours (150 marks made up as follows: ± 60 on trigonometry, ± 25 on analytical geometry, ± 35 on Euclidean geometry, ± 10 on volume and area, ± 20 on statistics).

3. Promotion mark: [year mark (reduced to a mark out of 100)] + [examination mark (out of 300)]

GRADE 12: TERM 1

No of Weeks	Topic	Curriculum statement	Clarification
			Where an example is given, the cognitive demand is suggested: knowledge (K), routine procedure (R), complex procedure (C) or problem solving (P) In some cases examples of questions where complex manipulation is involved for its own sake are mentioned as not being prescribed. This does not mean such questions could not be set as a challenge or handled as part of classwork if this does not result in other sections being neglected.
3	Functions	- Formal definition of a <i>function</i> - Definition of the <i>inverse function</i>. - Determine and sketch graphs of the inverses of the functions $y = ax + q$ $y = ax^2$ $y = b^x; (b > 0, b \neq 1)$ Focus on the following characteristics: domain and range, intercepts with the axes, turning points, minima, maxima, asymptotes (horizontal and vertical), shape and symmetry, average slope (average rate of change), intervals on which the function increases /decreases.	Examples: - Consider the function $f(x) = 3x - 1$. - Write down the domain and range of f. (R) - Show that f is one-to-one. (R) - Determine the inverse function $f^{-1}(x)$. (R) - Sketch the graphs of the functions f, f^{-1} and $y = x$ on the same set of axis. What do you notice? (R) - Repeat Question 1 for the function $f(x) = -x^2, x \leq 0$. (C) - Investigate the relationship between the Celsius (C) and Fahrenheit (F) scales for measuring temperatures: - Find a function f that converts Fahrenheit to Celsius, i.e., such that $C = f(F)$, and also a function g such that $F = g(C)$. (Hint: $0^\circ\text{C} = 32^\circ\text{F}$ and $100^\circ\text{C} = 212^\circ\text{F}$.) - Sketch the graphs of $y = f(x)$ and $y = g(x)$ using the same set of axis. What do you notice? - What can you say about the point where the graphs of the functions f and g intersect? - Is it true that $f^{-1} = g$? Why, or why not? (P)

			Caution: - Do not confuse the inverse $f^{-1}(x)$ with the reciprocal $\frac{1}{f(x)}$ of the function $f(x)$. For example, for the function $f(x) = \sqrt{x}$, the reciprocal is $\frac{1}{\sqrt{x}}$, while the inverse is $f^{-1}(x) = x^2, x \geq 0$. - Notice that the notation $f^{-1}(x) = \dots$ is used only for one to one functions and must not be used for inverses of many-to-one functions, since in these cases the inverses are not functions.
1	Functions: exponential and logarithmic	- Revision of the exponential function and the exponential laws and graph of the function $y = b^x$ where $b > 0$ and $b \neq 1$. - Definition of a logarithm to the base b (where $b > 0$ and $b \neq 1$). Conversion between logarithmic and exponential form: $x = b^y \Leftrightarrow y = \log_b x$ - Standard logarithm laws for $A, B > 0; b > 0$ and $b \neq 1$: $\log_b AB = \log_b A + \log_b B$	Caution: - Make sure learners know the difference between the two functions $y = b^x$ and $y = x^b$ where b is a positive (constant) real number. - Manipulation involving the logarithmic laws will not be examined. For example there will be no questions in the NSC examination asking candidates to simplify; $\frac{\log a^2 - \log b^2}{\log \frac{a}{b}}$ Examples: - Solve for x: $75(1,025)^{x-1} = 300$ (R) - Let $f(x) = a^x$. - Determine a if the graph of f goes through the point $\left(2; \frac{25}{16}\right)$ (R) - Determine the function $f^{-1}(x)$. (R) - For which values of x is $f^{-1}(x) > -1$? (C) - Determine the function $h(x)$ if the graph of h is the reflection of the graph

		$\log_b \frac{A}{B} = \log_b A - \log_b B$ $\log_b (A)^n = n \cdot \log_b A$ $\log_b A = \frac{\log_a A}{\log_a b}$ Limited application of the rules as needed in solving real life problems. 4. The graph of the function $y = \log_b x$ for both the cases $0 < b < 1$ and $b > 1$.	of f through the y-axis. (C) e. Determine the function $k(x)$ if the graph of k is the reflection of the graph of f through the x-axis. (C) f. Determine the function $p(x)$ if the graph of p is obtained by shifting the graph of f two units to the left. (C) g. Write down the domain and range for each of the functions f, f^{-1}, h, k and p. (R) h. Represent all these functions graphically. (R)
3	Patterns, Sequences, Series	1. Number patterns, including arithmetic and geometric sequences and series. 2. Sigma notation 3. Proof and application of the formulae for the sum of arithmetic and geometric series: a. $S_n = \frac{n}{2}[2a + (n-1)d]$ b. $S_n = \frac{a(r^n - 1)}{r - 1}; (r \neq 1)$ c. $S_\infty = \frac{a}{1 - r}; (-1 < r < 1)$	Examples: 1. a. Write down the first five terms of the sequence with general term $T_k = \frac{1}{3k - 1}$ (K) b. Calculate $\sum_{k=0}^3 (3k - 1)$ (K) 2. Determine the 5th term of the geometric sequence of which the 8th term is 6 and the 12th term is 14. (C) 3. Determine the largest value of n such that $\sum_{i=1}^n (3i - 2) < 2000$ (R) 4. Show that $0,9999... = 1$. (P)

2	Finance, growth and decay	1. Annuities and bond repayments as applications of geometric series. 2. The use of logarithms to calculate n in the formulae $A = P(1+i)^n$ and $A = P(1-i)^n$ 3. Loan options.	Comment: The two annuity formulae: $F = \frac{x((1+i)^n - 1)}{i}$ and $P = \frac{x(1 - (1+i)^{-n})}{i}$ only hold when payment commences one period from the present and end after n periods. Some teachers prefer to solve all annuity questions using the geometric series formula $S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$ Examples: 1. Given that a population increased from 120 000 to 214 000 in 10 years, at what annual (compound) rate was the population growing? (R) 2. In order to buy a car, John takes out a loan of R25000 from the bank. The bank charges an annual interest rate of 11%, compounded monthly. The installments start a month after he received the money from the bank. 2.1 Calculate his monthly installments if he has to pay back the loan over a period of 5 years. (R) 2.2 Determine the outstanding balance of his loan after two years (immediately after the 24th installment). (C)
2	Trigonometry	Compound angle identities: $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$ $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$ $\sin 2\alpha = 2 \sin \alpha \cos \alpha$ $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$ $= 2 \cos^2 \alpha - 1 = 1 - 2 \sin^2 \alpha$	Examples: 1. Solve for $x \in [-180^\circ; 180^\circ]: \sin 2x + \cos x = 0$ (R) 2. Prove: that $\frac{1 + \sin 2x}{\cos 2x} = \frac{\cos x + \sin x}{\cos x - \sin x}$ (C)

Assessment Term 1:

1. Investigation or project. ((only one per year) (at least 50 marks)

Example of an investigation which revises the sine, cosine and area rules:

Grade 12 Investigation: Polygons with 12 Matches

- How many different triangles can be made with a perimeter of 12 matches?
- Which of these triangles has the greatest area?
- What regular polygons can be made using all 12 matches?
- Investigate the areas of polygons with a perimeter of 12 matches in an effort to establish the maximum area that can be enclosed by the matches.
- Any extensions or generalisations that can be made, based on this task, will enhance your investigation. But you need to strive for quality rather than simply producing a large number of trivial observations.

Assessment:

The focus of this task is on mathematical processes.

Some of these processes are: specialising, classifying, comparing, inferring, estimating, generalising, making conjectures, validating, proving and communicating mathematical ideas.

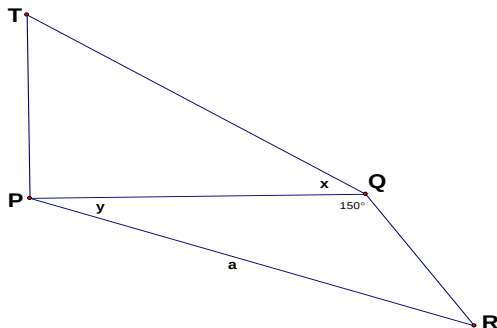
Marks will be awarded as follows:

- 40% for communicating your ideas and discoveries, assuming the reader has not come across the task before. The appropriate use of diagrams and tables will enhance your communication.
- 35% for the effective consideration of special cases.
- 20% for generalising, making conjectures and proving or disproving these conjectures.
- 5% for presentation: neatness and visual impact.

2. Assignment. (at least 50 marks)

3. Test. (at least 50 marks)

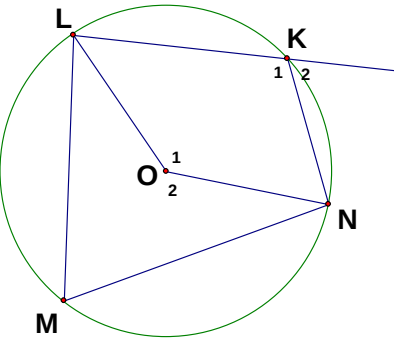
GRADE 12: TERM 2

No of Weeks	Topic	Curriculum statement	Clarification
2	Trigonometry continued	1. Problems in two and three dimensions.	Examples:  1. TP is a building. It's foot, P, and the points Q and R are in the same horizontal plane. From Q the angle of elevation to the top of the building is x°. Furthermore, $\hat{PQR} = 150^\circ$, $\hat{QPR} = y^\circ$ and the distance between P and R is a meters. Prove that $TP = a \tan x(\cos y - \sqrt{3} \sin y)$ (C) 2. In $\triangle ABC$, $AD \perp BC$. Prove that: $a = b \cos C + c \cos B$ where $a = BC$; $b = AC$ and $c = AB$. $\frac{\cos B}{\cos C} = \frac{c - b \cos A}{b - c \cos A}$ (on the condition that $\hat{C} \neq 90^\circ$). $\tan A = \frac{a \sin C}{b - a \cos C}$ (on the condition that $\hat{A} \neq 90^\circ$). $a + b + c = (b + c) \cos A + (c + a) \cos B + (a + b) \cos C$. (P)
1	Functions: polynomials	1. Define the n -th degree polynomial with real coefficients. Using long division, find the quotient and remainder when a first degree	Comment: The Remainder Theorem is proved only as an introduction to the Factor Theorem. Once proved, the Factor theorem is used only to factorise polynomial

		polynomial is divided into another polynomial of degree one or higher. 2. Proof of the : (a) Remainder Theorem; (b) Factor Theorem. 3. Factorise third degree polynomials.	functions of degree three or higher. The Remainder Theorem will not be examined.. Teachers who have time could ask such questions as homework assignments. Examples of question types that will <u>not</u> be examined in the NSC: 1. Determine the quotient and remainder when the polynomial $a(x) = x^3 - 2x^2 + 5x - 1$ is divided by the polynomial $b(x) = 5x + 3$ (R) 2. If $a(x) = x^5 - 2x^3 + px - 1$ is divided by $x - 1$, the remainder is $-\frac{1}{2}$. Determine the value of p. (P) Examples of question types that <u>could</u> be examined in the NSC: 1. Solve for x : $x^3 + 8x^2 + 17x + 10 = 0$ (R) 2. Show that the graph of $y = f(x) = x^3 + 2x^2 + x + 2$ only cuts the x axis once. (C)
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3	Differential Calculus	1. An intuitive understanding of the limit concept . 2. Use limits to define the derivative of a function f at a point a : $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}.$ Generalise to the derivative of f at any point x in the domain of f , i.e., define the derivative function $f'(x)$ of the function $f(x)$. Understand intuitively that $f'(a)$ is the slope of the tangent to the graph of f at the point with x-coordinate a . 3. Using the definition 3.1 find the derivative function $f'(x)$ of each of the following functions, where c is a constant: (a) $f(x) = c$ (b) $f(x) = x$; (c) $f(x) = x^2$; (d) $f(x) = x^3$; (e) $f(x) = \frac{1}{x}$ to support the rule that $\frac{d(x^n)}{dx} = n \cdot x^{n-1}$ 3.2 find the derivative function of : (a) $k \cdot f(x)$ where $f(x)$ is any of the functions listed in 3.1 a to e and k is a constant; (b) the sum or difference of the	Clarification: Differentiation from first principles will be examined on any of the types of functions described in 3.1 and 3.2. Examples: 1. Determine the following limits, if they exist: (a) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 - 4x + 4}$ (b) $\lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$ (R) 2. In each of the following cases, find the derivative of the function $f(x)$ at the point $x = -1$, using the definition of the derivative: (a) $f(x) = x^2 + 2$ (b) $f(x) = x^2 - 2$ (c) $f(x) = -x^3 + 1$ (d) $f(x) = -\frac{2}{x} + 1$ Explain why the answers in (a) and (b) should be the same. Caution: Care should be taken not to apply the sum rule for differentiation (4(a)) in a similar way to products: a. Determine $\frac{d}{dx} ((x+1)(x-1)) = \frac{d}{dx} (x^2 - 1)$. b. Determine $\frac{d}{dx} (x+1) \times \frac{d}{dx} (x-1)$. c. Write down your observation. 3. Use differentiation rules to do the following:
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		functions in 3.1 a, b and/or c 4. To differentiate more advanced functions, use the formula $\frac{d}{dx}(ax^n) = anx^{n-1}$ (for any real number n) together with the rules $\frac{d}{dx}[f(x) \pm g(x)]$ (a) $= \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$ and (b) $\frac{d}{dx}[kf(x)] = k \frac{d}{dx}[f(x)]$ (k a constant) 5. Find equations of tangents to graphs of functions. 6. Sketch graphs of cubic polynomial functions using differentiation to determine stationary points and points of inflection. Also, determine the x-intercepts of the graph using the factor theorem and other techniques. 7. Solve practical problems concerning optimisation and rates of change, including calculus of motion.	(a) Determine $f'(x)$ if $f(x) = (x+2)^3$. (R) (b) Determine $f'(x)$ if $f(x) = \frac{(x+2)^3}{\sqrt{x}}$. (C) (c) Determine $\frac{dy}{dt}$ if $y = (t+1)^3(t-1)$. (C) (d) Determine $f'(\theta)$ if $f(\theta) = (\theta^{3/2} - 3\theta^{-1/2})^2$. (C) 3. Determine the equation of the tangent to the graph of $y = (2x+1)^2(x+2)$ where $x = \frac{3}{4}$. (P) 4. Sketch the graph of $y = -x^3 + 4x^2 - x$ by: (a) finding the intercepts with the axes; (b) finding maxima, minima, points of inflection; (c) looking at the behaviour of the function as $x \rightarrow \infty$ and as $x \rightarrow -\infty$. (P) 5. The radius of the base of circular cylindrical can is x cm, and the volume is 430 cm^3. (a) Determine the height of the can in terms of x; (b) Determine the area of the material needed to manufacture the can (that is, determine the total surface area of the can) in terms of x; (c) Determine the value of x for which the least amount of material is needed to manufacture such a can. If the cost of the material is R500 per m^2, what is the cost of the cheapest can, (labour excluded)? (P)
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2	Euclidean Geometry: circles, chords and cyclic quadrilaterals (a mini axiomatic system)	Accept the following as axioms: • results established in earlier grades; • the tangent to a circle is perpendicular to the radius, drawn to the point of contact and then investigate and prove the theorems of the geometry of circles: • The line drawn from the centre of a circle perpendicular to a chord bisects the chord (and its converse); • The perpendicular bisector of a chord passes through the centre of the circle; • The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre); • Angles subtended by a chord at the circle on the same side of the chord are equal (and its converse); • The opposite angles of a cyclic quadrilateral are supplementary (and its converse); • Two tangents drawn to a circle from the same point outside the circle are equal in length; • The tangent-chord theorem (and its converse). The use of the above theorems to prove riders.	Examples: 1. (C) AB and CD are two chords of a circle with centre O. M is on AB and N is on CD such that $OM \perp AB$ and $ON \perp CD$. Also, $AB = 50$ mm, $OM = 40$ mm and $ON = 20$ mm. Determine the radius of the circle and the length of CD.  1. O is the centre of the circle above and $\hat{O}_1 = 2x$. - Determine $\hat{O}_2$ and $\hat{M}$ in terms of x. - Determine $\hat{K}_1$ and $\hat{K}_2$ in terms of x. - Determine $\hat{K}_1 + \hat{M}$. What do you notice? - Write down your observation regarding the measures of $\hat{K}_2$ and $\hat{M}$. (R)
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			2. O is the centre of the circle above and MPT is a tangent. Also, $OP \perp MT$. Determine, with reasons, x, y and z. (C) 3. Given: $AB = AC$, $AP \parallel BC$ and $A_2 = B_2$. Prove that: (a) PAL is a tangent to circle ABC; (b) AB is a tangent to circle ADP. (P)
2	Tests/ examinations		

Assessment term 2:

1. Assignment (at least 50 marks)

2. Test (at least 100 marks)

or

Examination (300 marks)

GRADE 12: TERM 3

No of Weeks	Topic	Curriculum statement	Clarification
3	Analytical geometry	- The equation $(x - a)^2 + (y - b)^2 = r^2$ defines a circle with radius r and midpoint $(a; b)$. - Calculation of the equation of a tangent a given circle.	Examples: - Determine the equation of the circle with midpoint $(-1;2)$ and radius $\sqrt{6}$. (K) - Determine the equation of the circle which has the line segment with endpoints $(5;3)$ and $(-3;6)$ as diameter. (R) - Determine the equation of a circle with a radius of 6 units, and intersects the x-axis at $(-2;0)$ and the y-axis at $(0;3)$. How many such circles are there? (P) - Determine the equation of the tangent that touches the circle $x^2 - 2x + y^2 + 4y = 5$ at the point $(-2;-1)$. (C) - The line $y = x + 2$ intersects the circle $x^2 + y^2 = 20$ at A and B. - Determine the co-ordinates of A and B. (R) - Determine the length of chord AB. (K) - Determine the co-ordinates of M, the midpoint of AB. (K) - Show that $OM \perp AB$, where O is the origin. (C) - Determine the equations of the tangents to the circle at the points A and B. (C) - Determine the co-ordinates of the point C where the two tangents in (e) intersect. (C) - Verify that $CA = CB$. (R) - Determine the equations of the two tangents to the circle, both parallel to the line $y = -2x + 4$. (P)
No of Weeks	Topic	Curriculum statement	Clarification

1	Statistics	1. Symmetric and skewed data. 2. Identification of normal distribution of data. 3. Sampling. 4. Scatter plot of bivariate data and intuitive choice of function of best fit supported by available technology. 5. The least squares method for linear regression. 6. Regression functions and correlation in bivariate data using available technology.	Example: The following table contains the Mathematics and Physical Science marks of 11 learners in Grade 12: <table><tr><td>Learner</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td></tr><tr><td>Maths % (x)</td><td>44</td><td>54</td><td>78</td><td>23</td><td>98</td><td>78</td><td>44</td><td>35</td><td>85</td><td>76</td><td>40</td></tr><tr><td>Physics % (y)</td><td>51</td><td>52</td><td>63</td><td>32</td><td>89</td><td>87</td><td>56</td><td>43</td><td>78</td><td>76</td><td>33</td></tr></table> a) Find the least squares regression line $\hat{y} = a + bx$ (K) b) Draw a scatter plot of the data on graph paper. (R) c) Show the regression line on the scatter plot. (R) d) What would be the estimated Physics mark of a learner whose Mathematics mark is 70%? (K) e) Determine the correlation coefficient r and explain what the value means. (R) f) Discuss the statement: "A learner in this class performs well in Physical Science if he/she performs well in Mathematics." (P)	Learner	1	2	3	4	5	6	7	8	9	10	11	Maths % (x)	44	54	78	23	98	78	44	35	85	76	40	Physics % (y)	51	52	63	32	89	87	56	43	78	76	33
Learner	1	2	3	4	5	6	7	8	9	10	11																												
Maths % (x)	44	54	78	23	98	78	44	35	85	76	40																												
Physics % (y)	51	52	63	32	89	87	56	43	78	76	33																												
2	Counting and probability	1. Revise: • Dependent and independent events. • The product rule for independent events: $P(A \text{ and } B) = P(A) \times P(B)$. • The sum rule for mutually exclusive events A and B: $P(A \text{ or } B) = P(A) + P(B)$ • The identity: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ • The complementary rule: $P(\text{not } A) = 1 - P(A)$ 2. The fundamental counting principle: derive the multiplication rule. 3. Probability problems using Venn diagrams, trees, two-way contingency	Comment: Permutations (where order matters, including examples where some items are identical) are implied by the fundamental counting principle, but not combinations (where order doesn't matter) except where solutions are easily obtained by using the complementary rule: $P(\text{at least one}) = 1 - P(\text{none})$. Examples: 1. How many three character codes can be formed if the first character must be a letter and the remaining two digits? (K) 2. What is the probability that a random arrangement of the letters BAFANA starts and ends with an 'A'? (R) 3. A drawer contains twenty envelopes. Eight of the envelopes each contain five blue and three red sheets of paper. The other twelve envelopes each contain six blue and two red sheets of paper. An envelope is chosen at random. A sheet of paper is chosen at random from it. What is the probability that this sheet of paper is red? (C)																																				

GRADE 12: TERM 4

No of Weeks	Topic	Curriculum statement	Clarification
3	Revision		
4	Examinations		
Assessment Term 4: Final examination: Paper 1: 150 marks: 3 hours Patterns and sequences ±25 Finance, growth and decay ±15 Functions and graphs ±35 Algebra and equations ±25 Calculus ±35 Probability ±15 Paper 2: 150 marks: 3 hours Euclidean geometry ±40 Analytical geometry ±40 Statistics and regression ±20 Trigonometry ±50			

Section 4: ASSESSMENT

4.1 Guidelines

4.1.1 Cognitive Levels

The four cognitive levels used to guide all assessment tasks is based on those suggested in the TIMSS study of 1999.

Descriptors for each level and the approximate percentages of tasks, tests and examinations which should be at each level are given below:

Cognitive levels	Description of skills to be demonstrated	Examples
Knowledge 25%	- Estimation and appropriate rounding of numbers - Proofs of prescribed theorems and derivation of formulae - Straight recall - Identification and direct use of correct formula on the information sheet (no changing of the subject) - Use of mathematical facts - Appropriate use of mathematical vocabulary	1. Write down the domain of the function $y = f(x) = \frac{3}{x} + 2$ (Grade 10) 2. Prove that the angle $\hat{AOB}$ subtended by arc AB at the centre O of a circle is double the size of the angle $\hat{ACB}$ which the same arc subtends at the circle. (Grade 12)
Routine procedures 45%	- Perform well known procedures - Simple applications and calculations which might involve many steps - Derivation from given information may be involved - Identification and use (after changing the subject) of correct formula - Generally similar to those encountered in class.	1. Solve for x : $x^2 - 5x = 14$ (Grade 10) 2. Determine the general solution of the equation $2 \sin(2x - 30^\circ) + 1 = 0$ (Grade 11)
Complex procedures 20%	- Problems involve complex calculations and/or higher order reasoning - There is often not an obvious route to the solution - Problems need not be based on a real world context - Could involve making significant connections between different representations - Require conceptual understanding	1. What is the average speed covered on a round trip to and from a destination if the average speed going to the destination is 100 km/h and the average speed for the return journey is 80 km/h ? (Grade 11) 2. Differentiate $\frac{(x+2)^2}{\sqrt{x}}$ (Grade 12)
Problem solving 10%	- Unseen, non-routine problems (which are not necessarily difficult) - Higher order understanding and processes are often involved - Might require the ability to break the problem down into its constituent parts	Suppose a piece of wire could be tied tightly around the earth at the equator. Imagine that this wire is then lengthened by exactly one metre and held so that it is still around the earth at the equator. Would a mouse be able to crawl between the wire and the earth? Why or why not? (Any grade)

4.2 Number of assessments per term per year

4.2.1 Diagnostic assessment

Regular assessment and control of routine homework and classwork cannot be used as part of the 25% year mark.

4.2.1 Formal assessment

In Grades 10, 11 and 12 25% of the final promotion mark is a year mark and 75% an examination mark.

All assessment in Grade 10 and 11 is internal whilst in Grade 12 the 25% year mark is internally set and marked and externally moderated and the 75% examination is externally set, marked and moderated.

		GRADE 10		GRADE 11		GRADE 12	
		TASKS	WEIGHT (%)	TASKS	WEIGHT (%)	TASKS	WEIGHT (%)
School-based Assessment	Term 1	Test Project /Investigation/Assignment	15 10	Test Project/Investigation/Assignment	15 10	Test Project /Investigation Assignment	15 10 10
	Term 2	Assignment Examination	10 30	Assignment Examination	10 30	Assignment Examination/Test	10 20
	Term 3	Assignment Test	10 15	Assignment Test	10 15	Assignment Trial Examination/Test	10 25
	Term 4	Assignment	10	Assignment	10		
School-based Assessment mark			100		100		100
School-based Assessment mark (as % of promotion mark)			25%		25%		25%
End-of-year examinations			75%		75%		
Promotion mark			100%		100%		

Note:

- Although the project/investigation is indicated in the first term it could be scheduled in term 2 or 3. Only **ONE** project/investigation should be set per year.
- Tests should be at ONE hour long and at least 50 marks.
- In Grade 12 schools may, subject to special permission, be permitted to write their trial examination in the second term, in which case the weighting of the second and the third term assessments are reversed.

Mark distribution for Mathematics NCS end of yearpapers: Grades 10 - 12			
PAPER 1 : Bookwork maximum 10 marks			
Description	Gr 10	Gr 11	Gr. 12
Algebra and Equations (and inequalities)	± 25	± 25	± 25
Patterns & Sequences	± 15	± 25	± 25
Financial Mathematics	± 15	± 15	± 15
Functions & Graphs	± 30	± 35	± 35
Differential Calculus		± 35	± 35
Probability	± 15	± 15	± 15
TOTAL	100	150	150
PAPER 2 : Bookwork maximum 12 marks			
Description	Gr 10	Gr 11	Gr 12
Statistics	± 15	± 20	± 20
Analytical Geometry	± 25	± 40	± 40
Trigonometry	± 40	± 50	± 50
Euclidean Geometry and Mensuration	± 20	± 40	± 40
TOTAL	100	150	150
Note: - Modelling as a process should be included in all papers, thus contextual questions can be asked in any topic. - Questions will not necessarily be compartmentalised in sections as this table indicates. Various topics can be integrated in the same question.			

4.3 Types of assessment

- 4.3.1. Tests and examinations are assessed using a marking memorandum.
- 4.3.2. Assignments are generally extended pieces of work completed at home. They can be collections of past examination questions, but should focus on the more demanding work as any resource material can be used, which is not the case in a task done in class under strict supervision.
- 4.3.3. The assessment criteria must be clearly indicated on the project specification and should focus on the mathematics involved and not on duplicated pictures and the regurgitation of facts copied from reference material.
The collection and display of real data, followed by deductions that can be substantiated from the data, form good projects.
- 4.3.4. Investigations are set so as to develop the skills of systematic investigation of special cases with a view to establishing general trends, making conjectures and proving them. To avoid having to assess work which is copied without understanding, it is recommended that whilst initial investigation could be done at home, the final write up should be done in class, under supervision, without access to any notes. Investigations are marked with rubrics which can be specific to the task, or generic, listing the number of marks awarded for each skill:

- 40% for communicating your ideas and discoveries, assuming the reader has not come across the task before.
The appropriate use of diagrams and tables will enhance your communication.
- 35% for the effective consideration of special cases.
- 20% for generalising, making conjectures and proving or disproving these conjectures.
- 5% for presentation: neatness and visual impact.

ANNEXURE OF EXTENSION PROBLEMS

The following problems are carefully selected to fall in a category which puts them just outside the ordinary syllabus-type. However, they can be solved with a background in school mathematics. They simply require a little bit more skill and careful thinking (and no calculators!).

1. Sketch the graph of the function $f(x) = \log_x 2$, for $x \in (0,1) \cup (1,\infty)$. Is this function one-to-one?
2. Let a and b be two positive real numbers with $a \leq b$. Let A be the real number such that a, A, b forms an arithmetic sequence and let G be the real number such that a, G, b forms a geometric sequence.
 - a. Write each of A and G in terms of a and b .
 - b. Prove that $A \geq G$.
3. For a real number x , let $|x|$ denote the distance from x to 0 on the number line. In other words, $|x| = x$ if $x \geq 0$ and $|x| = -x$ if $x < 0$. Prove the following:
 - a. $|x - y|$ is the distance between x and y on the real line.
 - b. $|x| \leq a$ (for $a \geq 0$) is equivalent to $-a \leq x \leq a$.
 - c. $|x - b| \leq a$ (for $a \geq 0$) is equivalent to $b - a \leq x \leq b + a$.
 - d. $\sqrt{x^2} = |x|$, for any real number x .
 - e. $|x + y| \leq |x| + |y|$, for all real numbers x and y .

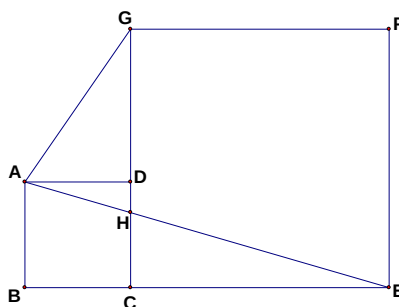
(Note: The non-negative number $|x|$ is usually referred to as the *absolute value* of the real number x .)

4. Find the remainder when the polynomial $x^{100} + x^{99} + x^{98} + \dots + x^2 + x + 1$ is divided by $x^2 - 1$.
5. Show that there does not exist a polynomial $a(x)$ with integral coefficients such that $a(11) = 13$ and $a(13) = 16$.
6. Find a function $F(x)$ such that $F'(x) = x^3 - \sqrt{x} + 3x + 1$. How many such functions $F(x)$ do you think exist?
7. Use the compound angle formulas for sin and cos and prove that

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta} \quad \text{and} \quad \tan(2\alpha) = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}.$$

8. An unbiased die is thrown four times. What is the probability that the sum of the outcomes is divisible by 3?
9. A container contains three identical Blue balls (numbered from 1 to 3), four identical Red balls (numbered from 1 to 4) and five identical Green balls (numbered from 1 to 5). Three balls are selected at random (without replacement). Which is more likely: to select three balls of different colour, or three balls with different numbers?
10. Triangle ABC is a right triangle, with $\hat{B} = 90^\circ$. A square $PQRS$ is inscribed in $\triangle ABC$, with Q on AB , R on BC , and side PS on AC . If $AP = 10$ and $SC = 15$, find the area of square $PQRS$.
11. Show that a perfect cube minus a multiple of 9 can never be equal to 2.
12. A train departs at 09:00 from Johannesburg to Pretoria and maintains a constant speed of 60 km/h. The express train departs at 09:30 (the same day) from Pretoria to Johannesburg, and maintains a constant speed of 100 km/h. The distance between Johannesburg and Pretoria is 60 km. How late do the two trains pass each other?

13. In the following figure,



- ABCD and CEFG are two squares. It is given that the area of triangle HCE is 12 square units. Determine the area of triangle AHG.
14. A hundred men can dig a furrow of 6 km in 60 days, at 12 hours per day. How many hours a day must 75 men work if they want to dig a furrow of 3 km in 48 days?
15. Think about the following statement: "If the temperature in Cape Town is 10 degrees Celsius, and at the same time the temperature in Bloemfontein is 5 degrees Celsius, then it is twice as cold in Bloemfontein as in Cape Town." Does this make sense? Think carefully!
16. Two flagpoles are X metres apart. The one has height 10 m, and the other 15 m. Two tight ropes connect the top of each pole to the foot of the other. The two ropes intersect at a height of 6 m above the ground. Determine X .
17. If $\frac{(a-b)(c-d)}{(b-c)(d-a)} = -\frac{5}{3}$, determine the value of $\frac{(a-c)(b-d)}{(a-b)(c-d)}$.
18. Consider the points $A(0;2)$ and $B(10;-1)$ in the Cartesian plane. Choose points $P(x,0)$ and $Q(x+3,0)$ on the x -axis such that the total length $AP + QB$ is as small as possible. What is this smallest length?
19. Let $x = \sqrt{1 + \frac{1}{1^2} + \frac{1}{2^2}} + \sqrt{1 + \frac{1}{2^2} + \frac{1}{3^2}} + \sqrt{1 + \frac{1}{3^2} + \frac{1}{4^2}} + \cdots + \sqrt{1 + \frac{1}{2010^2} + \frac{1}{2011^2}}$. Determine the integer nearest to x .
20. Consider triangle ABC . Choose points P , Q and R on sides AB , BC and CA respectively, in such a way that $AP = \frac{1}{2}AB$, $BQ = \frac{1}{3}BC$ and $CR = \frac{1}{4}CA$. Determine the ratio of the area of triangle PQR to the area of triangle ABC .
21. You have an unlimited supply of Blue blocks, Green blocks and Red blocks. The Blue blocks and Green blocks have thickness 2 cm, and the Red blocks have thickness 1 cm. In how many ways can these blocks be used to build a tower of height 10 cm? (For example, there is five ways to build a tower of height 3 cm: RRR, BR, RB, GR, RG).
22. Three gamblers, A, B and C, started with sums of money in the ratio $A:B:C = 7:6:5$ and finished with sums of money in the ratio $A:B:C = 6:5:4$, although the total amount of money with which they started, remained the same. One of them won R120. How much did he start with?
23. What is the maximum number of integers that can be selected from the list $1, 2, 3, \dots, 2010$ such that the sum of any two of the selected numbers is not divisible by 7?

24. A runs around a circular track at a constant speed and it takes him 40 seconds to complete one lap. B, running in the opposite direction around the same track (also at a constant speed) meets A every 15 seconds. How long does it take B to complete one lap?
25. A certain type of sheet-glass has the property that if a ray of light shines perpendicularly on it, then 20 % of the light is reflected back, 70 % passes through the glass, and 10 % is absorbed by the glass. Suppose now that a ray of light shines perpendicularly from one side on two of these sheets, parallel to each other. What percentage of the light is absorbed by the two sheets? (This is not as obvious as you might think at first!)
26. Reduce the fraction $\frac{2121212121210}{1121212121211}$ to its simplest form.
27. Suppose that $M \times E = 12$, $N \times S = 30$, $E \times S = 24$, $E \times N = 20$ and $S \times A = 42$, where all unknowns are positive real numbers. Find the value of $M \times E \times N \times S \times A$.
28. Alan, his sister Betty, his son Colin and his daughter Doris play tennis. The best player's twin sibling and the worst player are of opposite sex. The best player and the worst player are the same age. Who is the best player?
29. If a , b and c are positive integers with $abc + ab + bc + ca + a + b + c = 2003$, what is the smallest possible value of abc ?
30. Four friends run down a dusty staircase. Gugu takes two steps at a time, Jim three, John four and Peter five. After they have finished, the only two steps with all four's footprints on, are the first and last steps. How many steps have just one footprint?
31. In a 10 km race, the winner beats John and Peter by 2 km and 4 km respectively. If all runners maintain constant speed, by how many kilometers does John beat Peter?
32. Jenny says: "I am the sixth child in my family, and I have at least as many brothers as I have sisters." Her brother Jim adds: "I have at least twice as many sisters as brothers." How many boys are there in this family?
33. Five integers are added two at a time in all possible ways and the following ten answers are obtained: 27, 29, 32, 33, 36, 38, 40, 43, 45, 49. If the five integers are arranged from small to large, what is the number in the middle?
34. An integer n is chosen at random from the interval $10000000000 \leq n \leq 9999999999$. What is the probability that all the digits of n are distinct?
35. The Adams brothers are more than two but fewer than ten. They are all sheep farmers, and the numbers of sheep in their respective herds are consecutive numbers. The total number of sheep in all their herds is 1972. How many Adams brothers are there?
36. Consider the nine-digit integer $m = 65A321B87$ (in decimal notation), where the digits A and B are unknown. It is known that m is divisible by 99. How many possible values are there for the digit A ?
37. A lucky number is a four-digit positive integer with the property that if you take the number's first two digits and its last two digits, you obtain two two-digit numbers such that twice the product of these two numbers is equal to the lucky number. Find the smallest lucky number.
38. Suppose n is a positive integer. An $n \times n \times n$ cube has some of its faces painted. It is cut into n^3 cubes of size $1 \times 1 \times 1$ and 45 of them are found to have no paint on them. How many faces of the original cube were painted?
39. Let $y = (x - a)^2 + (x - b)^2$, where a and b are constants. For what value of x is y a minimum?