

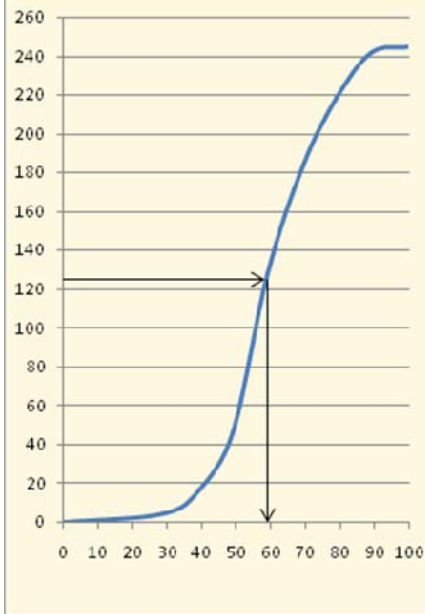


MATHEMATICS PAPER 2 MEMORANDUM

1.1	$M = \left(\frac{-1+3}{2}, \frac{3+(-1)}{2} \right) \checkmark$ $M = (1; 1) \checkmark$	(2)	(diagonals bisect) \checkmark But $EH = EF \checkmark$ $\therefore EFGH$ is a rhombus.	
1.2	Midpoint $FH = \left(\frac{4+(-2)}{2}, \frac{4+(-2)}{2} \right) \checkmark$ $= (1; 1) \checkmark$ \therefore midpoint $FH =$ midpoint EG \therefore lines bisect each other.	(2)	1.4 $m_{EG} = -1$ from above \checkmark $\therefore y = -x + c$ Substitute $(-1; 3): \checkmark$ $3 = -(-1) + c$ $2 = c$ $\therefore y = -x + 2 \checkmark$	(3)
1.3	$m_{EG} = \frac{3-(-1)}{-1-3} = \frac{4}{-4} = -1 \checkmark$ $m_{FH} = \frac{4-(-2)}{4-(-2)} = \frac{6}{6} = 1 \checkmark$ $\therefore m_{EG} \times m_{FH} = -1$ $\Rightarrow EG \perp FH \checkmark$ \therefore using 1.2, diagonals of $EFGH$ bisect at $90^\circ \checkmark$ $\therefore EFGH$ is a rhombus. OR $length_{EH} = \sqrt{(-1-(-2))^2 + (3-(-2))^2}$ $= \sqrt{1+25}$ $= \sqrt{26}$ \checkmark $length_{EF} = \sqrt{(-1-4)^2 + (3-4)^2}$ $= \sqrt{25+1} \checkmark$ $= \sqrt{26}$ \therefore using 1.2, $EFGH$ is a parallelogram	(4)	1.5 $y = -x + 2$ Let $x = \frac{5}{2}$. $y = -\frac{5}{2} + 2 \checkmark$ $y = -\frac{1}{2} \checkmark$ $\therefore \left(\frac{5}{2}; \frac{-3}{4} \right)$ does not lie on the line. \checkmark	(3)
			1.6 $m_{FH} = \frac{4-(-1)}{4-3} = \frac{5}{1} = 5 \checkmark$ $\therefore \tan \alpha = 5$ $A = \tan^{-1}(5) \checkmark$ $= 78,69^\circ \checkmark$	(3)
			1.7 $length_{EG} = \sqrt{(3-(-1))^2 + (-1-3)^2} \checkmark$ $= \sqrt{32}$ $length_{HM} = \sqrt{(1-(-2))^2 + (1-(-2))^2} \checkmark$ $= \sqrt{18}$ $\therefore \text{area } \Delta EGH = \frac{1}{2} \sqrt{18} \times \sqrt{32} \checkmark$	

	$= 12 \text{ units}^2 \checkmark$	(4)
1.8	$G \rightarrow H$ back 5 down 1 $\therefore E \rightarrow P$ is the same \checkmark $\therefore P(-6; 2) \checkmark$ (or equivalent)	(2)
2.1	ΔEDC is right angled at C (tangent, radius) $ED^2 = EC^2 + DC^2 \checkmark$ $13^2 = 12^2 + DC^2$ $25 = DC^2$ $5 = DC \checkmark$	(2)
2.2	$DC^2 = (a - 1)^2 + (2 - (-1))^2 \checkmark$ $25 = a^2 - 2a + 1 + 9 \checkmark$ $0 = a^2 - 2a - 15 \checkmark$ $a = -3; a = 5 \checkmark$ By inspection, for this sketch $a = 5 \checkmark$	(5)
2.3	$m_{DC} = \frac{2 - (-1)}{5 - 1} = \frac{3}{4} \checkmark$ $m_{\text{tangent}} = -\frac{4}{3} \checkmark$ $y = -\frac{4}{3}x + c$ Substitute (1; -1): $-1 = -\frac{4}{3}x + c \checkmark$ $\frac{1}{3} = c \checkmark$ $y = -\frac{4}{3}x + \frac{1}{3}$	(4)
2.4	y -axis is tangent to circle at A. $\therefore AD$ is horizontal $\therefore A(0; 2) \checkmark \checkmark$ (inspection)	(2)
2.5	$(x - a)^2 + (y - b)^2 = c^2 \checkmark$ Substitute (1; -1) and (0; 2): $(0 - 1)^2 + (2 - (-1))^2 = c^2 \checkmark \checkmark$ $10 = c^2 \checkmark$ $(x - 1)^2 + (y + 1)^2 = 10 \checkmark$	(5)
3.1 and 3.2		(10)
3.3	$P(x; y)$ $\rightarrow P'(-x; -y)$ \rightarrow final image $(-\frac{4}{5}x; -\frac{4}{5}y) \checkmark \checkmark$	(2)
3.4	Linear factor $\frac{4}{5}$ \Rightarrow area factor $\frac{16}{25} \checkmark$ $\therefore \text{area} = \frac{16}{25}p \text{ units}^2 \checkmark$	(2)
4.1	$(x \cos \theta - y \sin \theta; x \sin \theta + y \cos \theta) \checkmark$ $= (6 \cos 60^\circ - 3 \sin 60^\circ; 6 \sin 60^\circ + 3 \cos 60^\circ) \checkmark \checkmark$ $= \left(\frac{6 - 3\sqrt{3}}{2}; \frac{3 + 6\sqrt{3}}{2} \right) \checkmark \checkmark$ OR $= \left(3 - \frac{3\sqrt{3}}{2}; 3\sqrt{3} + \frac{3}{2} \right)$	(5)
4.2	$(6; 3) \rightarrow (3; 6) \rightarrow (3; -6) \checkmark \checkmark$	(2)
5.1.1	$\tan \theta = \frac{y}{x} = \frac{b}{a} \checkmark$	(1)
5.1.2	$\cos(-\theta) = \cos \theta = \frac{x}{r} \checkmark \checkmark$ $r = \sqrt{a^2 + b^2}$ (Pythagoras) \checkmark $\therefore \cos(-\theta) = \frac{a}{\sqrt{a^2 + b^2}} \checkmark$	(4)
5.2.1	$\cos 53^\circ$ $= \sin(90^\circ - 53^\circ)$ $= \sin 37^\circ \checkmark$ $= k \checkmark$	(2)
5.2.2	$\sin(-74^\circ)$ $= -\sin 74^\circ$ $= -2 \sin 37^\circ \cos 37^\circ \checkmark \checkmark$ $= -2k \sqrt{1 - k^2} \checkmark$	(4)
5.3.1	LHS $= \frac{\sin a \sin 2a}{\cos a} + \cos 2a \checkmark$ $= \frac{\sin a 2 \sin a \cos a}{\cos a} + 1 - 2 \sin^2 a \checkmark \checkmark$ $= 1 \checkmark$ $= \text{RHS}$	(4)
5.3.2	LHS $= \frac{\sin 234^\circ}{\cos 36^\circ} - \frac{\sin(x - 90^\circ) \cos(90^\circ - 2x)}{\sin(x - 360^\circ)}$ $= \frac{-\sin 54^\circ}{\cos 36^\circ} - \frac{(-\cos x) \sin 2x}{\sin x} \checkmark \checkmark \checkmark \checkmark$ $= \frac{-\sin 54^\circ}{\sin 54^\circ} - \frac{\cos x \cdot 2 \sin x \cos x}{\sin x} \checkmark \checkmark$ $= -1 + 2 \cos^2 x \checkmark$ $= \cos 2x$ $= \text{RHS}$	(8)
5.4	$3 \cos^2 x + 5 \sin x = 3$ $3(1 - \sin^2 x) + 5 \sin x = 3 \checkmark$ $3 - 3 \sin^2 x + 5 \sin x = 3$ $0 = 3 \sin^2 x - 5 \sin x$ $0 = \sin x (3 \sin x - 5) \checkmark$ $\sin x = 0$ or $\sin x = \frac{5}{3} \checkmark \checkmark$ $\therefore x = 0^\circ + n180^\circ$ or x is undefined	(6)

	$(n \in \mathbb{Z}) \checkmark \checkmark$																												
6.1	$\begin{aligned} \widehat{ACB} &= 110^\circ - 50^\circ \\ &= 60^\circ \checkmark \\ \therefore AB^2 &= 150^2 + 260^2 - (2.150.260)\cos 60^\circ \\ &\checkmark \\ AB^2 &= 51\,100 \checkmark \\ \therefore AB &= 226,05 \checkmark \\ AB &= 226 \text{ km} \checkmark \end{aligned}$	(5)																											
6.2.1	$\widehat{CDB} = 180^\circ - (\theta + 30^\circ) \checkmark$	(1)																											
6.2.2	<p>In $\triangle ABC$:</p> $\tan \theta = \frac{p}{CB}$ $CB \tan \theta = p \dots\dots\dots(i) \checkmark$ <p>In $\triangle CBD$:</p> $\frac{CB}{\sin[180^\circ - (\theta + 30^\circ)]} = \frac{8}{\sin \theta} \checkmark$ $\frac{CB}{\sin(\theta + 30^\circ)} = \frac{8}{\sin \theta} \checkmark$ $CB = \frac{8\sin(\theta + 30^\circ)}{\sin \theta} \dots\dots\dots(ii) \checkmark$ <p>Combining (i) and (ii):</p> $p = \frac{8\sin(\theta + 30^\circ)}{\sin \theta} \cdot \tan \theta$ $p = \frac{8\sin(\theta + 30^\circ)}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta} \checkmark$ $p = \frac{8\sin(\theta + 30^\circ)}{\cos \theta} \checkmark$	(6)																											
7.1	$\begin{aligned} \sin 2x &= \cos(x - 45^\circ) \\ \sin 2x &= \sin[90^\circ - (x - 45^\circ)] \\ \sin 2x &= \sin(135^\circ - x) \checkmark \\ \therefore 2x &= 135^\circ - x + n360^\circ \quad (n \in \mathbb{Z}) \checkmark \\ 3x &= 135^\circ + n360^\circ \\ x &= 45^\circ + n120^\circ \checkmark \\ \text{or} \\ 2x &= 180^\circ - (135^\circ - x) + n360^\circ \quad (n \in \mathbb{Z}) \checkmark \\ 2x &= 45^\circ + x + n360^\circ \\ x &= 45^\circ + n360^\circ \checkmark \\ \therefore \text{for } x &\in [-180^\circ; 180^\circ] \\ x &= 45^\circ; 165^\circ; -75^\circ \checkmark \checkmark \checkmark \end{aligned}$	(8)																											
7.2	<p>$\checkmark \checkmark \checkmark$ for g $\checkmark \checkmark \checkmark$ for f</p>	(6)																											
7.3.1	$\begin{aligned} g(x) &\leq f(x) \text{ for } [-180^\circ; 90^\circ] \\ \Rightarrow -180^\circ &\leq x \leq -75^\circ \checkmark \checkmark \checkmark \end{aligned}$	(3)																											
7.3.2	$\frac{f(x)}{g(x)} \text{ undefined} \Rightarrow g(x) = 0 \checkmark$ $\therefore x = -45^\circ \text{ only for } [-180^\circ; 90^\circ] \checkmark$	(2)																											
8.1.1	$\bar{x} = 65,27 \checkmark \checkmark \checkmark$ <p>Using stats mode on calculator or manually</p>	(3)																											
8.1.2	$SD = 8,71 \checkmark \checkmark$	(2)																											
8.1.3	<p>Upper boundary $= 65,27 + 8,71$ $= 73,98 \checkmark$</p> <p>Lower boundary $= 65,27 - 8,71$ $= 56,56 \checkmark$</p> <p>\therefore reject 50; 45; 80 i.e. 3 bags would be rejected \checkmark</p>	(3)																											
8.2.1	<p>Ordered list</p> <p>11 000 12 600 14 200 $Q_1 = 14\,200$ 14 500 15 300 Median = 15 350 15 400 16 500 16 800 $Q_3 = 16\,800$ 18 600 19 600</p> <p>\therefore Minimum = 11 000 \checkmark Lower quartile = 14 200 \checkmark Median = 15 350 \checkmark Upper quartile = 16 800 \checkmark Maximum = 19 600 \checkmark</p>	(5)																											
8.2.2	<p>Scale in 100s</p> <p>$\checkmark \checkmark \checkmark \checkmark$</p>	(3)																											
8.2.3	<p>Maximum per day = $\frac{19\,600}{28} = 700$ patients per day \checkmark</p> <p>Minimum per day = $\frac{11\,000}{28} = 392$ patients per day \checkmark</p>	(2)																											
9.1	<table border="1"> <thead> <tr> <th>Marks</th> <th>F</th> <th>CF</th> </tr> </thead> <tbody> <tr> <td>$20 \leq x \leq 29$</td> <td>4</td> <td>4</td> </tr> <tr> <td>$30 \leq x \leq 39$</td> <td>12</td> <td>16</td> </tr> <tr> <td>$40 \leq x \leq 49$</td> <td>30</td> <td>46</td> </tr> <tr> <td>$50 \leq x \leq 59$</td> <td>82</td> <td>128</td> </tr> <tr> <td>$60 \leq x \leq 69$</td> <td>55</td> <td>183</td> </tr> <tr> <td>$70 \leq x \leq 79$</td> <td>35</td> <td>218</td> </tr> <tr> <td>$80 \leq x \leq 89$</td> <td>24</td> <td>242</td> </tr> <tr> <td>$90 \leq x \leq 100$</td> <td>3</td> <td>245</td> </tr> </tbody> </table> <p>$\checkmark \checkmark$</p>	Marks	F	CF	$20 \leq x \leq 29$	4	4	$30 \leq x \leq 39$	12	16	$40 \leq x \leq 49$	30	46	$50 \leq x \leq 59$	82	128	$60 \leq x \leq 69$	55	183	$70 \leq x \leq 79$	35	218	$80 \leq x \leq 89$	24	242	$90 \leq x \leq 100$	3	245	(2)
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9.2		(4)
9.3	Median at $\frac{245 + 1}{2} = 123$ Median approximately 59 ✓ ✓	(2)
9.4	Data is grouped, so original raw data is lost. \therefore mean or median will be approximate. ✓ ✓	(2)