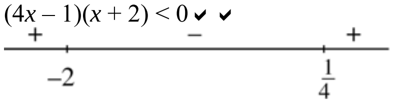
MATHEMATICS PAPER 1 MEMORANDUM

1.1.1 $2x^2 + 7x = 30$
$2x^2 + 7x - 30 = 0$ ✔
$(2x - 5)(x + 6) = 0$ ✔
$\therefore\ x = \frac{5}{2}$ or $x = -6$ ✔ ✔ (4)

1.1.2 $2x(x - 2) - 5 = 0$
$2x^2 - 4x - 5 = 0$ ✔
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{4 \pm \sqrt{(-4)^2 - 4 \times (2)(-5)}}{2 \times 2}$$ ✔
$\therefore x = \frac{4 + \sqrt{56}}{4}$ or $x = \frac{4 - \sqrt{56}}{4}$ ✔ ✔
Or $x = 2{,}87$ or $x = -0{,}87$ (4)

1.1.3 $4x^2 + 7x - 2 < 0$
$(4x - 1)(x + 2) < 0$ ✔ ✔

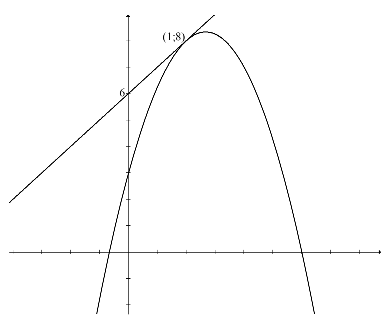


$-2 < x < \frac{1}{4}$ ✔ ✔ (4)

1.2.1 $2x + 6 - y = 0$
$2x + 6 = y$
$y + 3x^2 = 8x + 3$
$y = -3x^2 + 8x + 3$ ✔ (5)

$\therefore 2x + 6 = -3x^2 + 8x + 3$ ✔
$3x^2 - 6x + 3 = 0$
$x^2 - 2x + 1 = 0$
$(x - 1)^2 = 0$ ✔
$\therefore\ x = 1$ ✔
$\therefore\ y = 2x + 6 = 8$ ✔

1.2.2



✔

(1; 8) is the point of contact between the straight line $y = 2x + 6$ and the parabola $y = -3x^2 + 8x + 3$. There is one point of contact.
$\Rightarrow$ line is a tangent to the curve. ✔ ✔ (3)

2.1 $P = \frac{x\left[1 - (1 + i)^{-n}\right]}{i}$ ✔

$$800000 = \frac{7500\left[1-\left(1+\frac{\frac{9}{100}}{12}\right)^{-n}\right]}{\frac{\frac{9}{100}}{12}}$$ ✓ ✓

$$800000 = \frac{7500\left[1-\left(1+\frac{3}{400}\right)^{-n}\right]}{\frac{3}{400}}$$

$800\,000 \times \frac{3}{400} \div 7\,500 = 1 - (\frac{403}{400})^{-n}$

$\frac{20}{25} = 1 - (\frac{403}{400})^{-n}$ ✓

$(\frac{403}{400})^{-n} = \frac{5}{25} = \frac{1}{5}$

$-n = \left(\frac{\log\frac{1}{5}}{\log\frac{403}{400}}\right)$ ✓

$-n = -215,4$ months ✓

∴ It will take 18 years to pay back the loan. ✓ (7)

(Simplifications at various stages are not necessary.)

2.2.1 $A = P(1 - i)^n$ ✓

$A = 1\,500\,000(1 - \frac{18}{100})^5$ ✓

$A = 556\,109,76$ ✓

∴ They are worth R556 109,76. (3)

2.2.2 $A = P(1 + i)^n$ ✓

$A = 1\,500\,000(1 + \frac{4.5}{100})^5$ ✓

$A = 1\,869\,272,91$ ✓

∴ Sinking fund required is

R1 869 272,91 – R556 109,76

= R1 313 163,15 ✓ (4)

2.2.3

$$F = \frac{x\left[(1+i)^n - 1\right]}{i}$$ ✓

$$1313163,15 = \frac{x\left[\left(1+\frac{1}{12}\times 6\%\right)^{5\times 12} - 1\right]}{\frac{1}{12}\times 6\%}$$

✓ ✓

$$x = \frac{1313163,15\times\frac{1}{12}\times 6\%}{\left[\left(1+\frac{1}{12}\times 6\%\right)^{60} - 1\right]}$$ ✓

$x = 18\,821,30465$ ✓

∴ They will need to pay R18 821,30 per month. (5)

3.1.1 Quadratic sequence ✓ (1)

3.1.2

T_0	T_1	T_2	T_3	T_4	T_5
9	9	11	15	21	29
	0	2	4	6	8
	2	2	2	2	

Second difference = 2

$\therefore 2a = 2$

$a = 1$ ✓

By inspection $T_0 = 9$

$\therefore c = 9$ ✓

$\therefore T_n = n^2 + bn + 9$

$T_1 = 1^2 + b + 9 = 9$

$\therefore b = -1$ ✓

$\therefore T_n = n^2 - n + 9$ ✓

(or equivalent method) (4)

3.2 Sammy:

$a = 2 \quad T_4 = a + 3d$ ✓

$= 2 + 3d = 17$

$3d = 15$

$d = 5$ ✓

$\therefore T_{20} = a + 19d$

$= 2 + 19 \times 5$

$= 97$

$\therefore x = 97$ ✓

Solly:

$T_4 = a + 3d = 59$

$T_{20} = a + 19d = 107$ ✓

$16d = 48$

$d = 3$ ✓

$\therefore T_4 = a + 3d = 59$

$a + 9 = 59$

$a = 50$ ✓

$\therefore y = 50$ (6)

4.1.1 2010

109% of 85 000 = R92 650 ✓

109% of 92 650 = R100 988,50 ✓ (2)

4.1.2 GP with $a = 85\,000$

$r = 109\% = 1,09$ ✓

$T_n = ar^{n-1}$ ✓

$T_{15} = 85\,000(1,09)^{15}$ ✓

= R309 611,01 ✓ (4)

4.2 $\sum_{k=0}^{3} x = \frac{2}{5^k}$

$= 2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125}$ ✓ ✓

$= \frac{312}{125}$ (2,496) ✓ (3)

4.3.1 GP with $a = 20$

$r = \frac{3}{4}$

$$S_n = \frac{a\left[1 - r^n\right]}{1 - r}$$ ✓

$$S_n = \frac{20\left[1-\left(\frac{3}{4}\right)^{10}\right]}{1-\frac{3}{4}}$$ ✓

$= 75{,}49$ ✓

$\therefore$ Depth is 75,49 m. (3)

4.3.2 $S_\infty = \dfrac{a}{1-r}$ ✓

$= \dfrac{20}{1-\frac{3}{4}}$ ✓

$= 80$

$s_\infty = 80$ m ✓

$\therefore$ Company will never reach the water. ✓ (4)

5.1.1 $f(x) = \dfrac{-3}{x-1} + 2$

asymptotes at $x = 1$; $y = 2$

y-intercept: $x = 0$ $\therefore$ $f(0) = \dfrac{-3}{0-1} + 2 = 5$

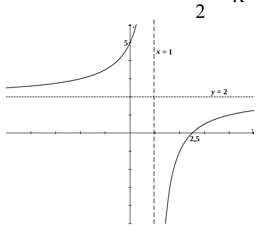
x-intercept: $y = 0$ $\therefore$ $0 = \dfrac{-3}{x-1} + 2$

$\dfrac{3}{x-1} = 2$

$3 = 2x - 2$

$5 = 2x$

$\dfrac{5}{2} = x$



✓ ✓ ✓ ✓ (4)

5.1.2 Axes of symmetry:

$y = x + c$ and $y = -x + c$

Pass through (1; 2) ✓

$\therefore 2 = 1 + c$ $\quad$ $2 = -1 + c$

$1 = c$ $\quad$ $3 = c$

$\therefore$ $y = x + 1$ and $y = -x + 3$ ✓ ✓ (3)

5.2.1 $y = a(x - x_1)(x - x_2)$ ✓

$y = a(x + 1)(x - 3)$ ✓

Substitute (0; –1):

$-1 = a(0 + 1)(0 - 3)$ ✓

$-1 = -3a$

$\frac{1}{3} = a$ ✓

$\therefore$ $p(x) = \frac{1}{3}(x + 1)(x - 3)$

$= \frac{1}{3}(x^2 - 2x - 3)$

$= \frac{1}{3}x^2 - \frac{2}{3}x - 1$ ✓ (5)

5.2.2 $q(x) = \frac{1}{2}b^x - 4$

Substitute (3; 0):

$0 = \frac{1}{2}b^3 - 4$ ✓

$4 = \frac{1}{2}b^3$

$8 = b^3$ ✓

$2 = b$ (2)

5.2.3 Domain: $x \in \Re$ ✓

Range: $y \in \Re$; $y > -4$ ✓ (2)

5.2.4 $q(x) = y = \frac{1}{2} . 2^x - 4$

$q^{-1}(x)$: $x = \frac{1}{2} . 2^y - 4$ ✓

$2x = 2^y - 8$

$2x + 8 = 2^y$ ✓

$\therefore$ $y = \log_2 (2x + 8)$ ✓

$q^{-1}(x) = \log_2 (2x + 8)$ (3)

5.2.5 Domain of $q^{-1}(x)$ = Range of $q(x)$

$= x \in \Re$; $x > -4$ ✓ (1)

5.3.1 $a = -4$ ✓ (1)

5.3.2 $x = 180°$; $x = -180°$ ✓

$x = 0°$ ✓ (2)

5.3.3 $y = \tan (x + 90° + 45°) + 2$ ✓

$y = \tan (x + 135°) + 2$ ✓

or $y = \tan (x - 45°) + 2$

by inspection/reduction (2)

6.1 $f(x) = \frac{1}{2}x^2$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$\therefore f'(x) = \lim_{h\to 0}\frac{\frac{1}{2}(x+h)^2 - \frac{1}{2}x^2}{h}$$ ✓

$$\therefore f'(x) = \lim_{h\to 0}\frac{\frac{1}{2}x^2 + xh + \frac{1}{2}h^2 - \frac{1}{2}x^2}{h}$$ ✓

$$\therefore f'(x) = \lim_{h\to 0}\frac{h(x + \frac{1}{2}h)}{h}$$ ✓

$$\therefore f'(x) = \lim_{h\to 0} x + \frac{1}{2}h$$ ✓

$f(x) = x$ ✓ (5)

6.2.1 $$f(x) = x^4 + \sqrt{x} - \frac{9}{x}$$

$$f(x) = x^4 + x^{\frac{1}{2}} - 9x^{-1}$$ ✓

$$\therefore f'(x) = 4x^3 + \frac{1}{2}x^{-\frac{1}{2}} + 9x^{-2}$$

✓ ✓ (3)

6.2.2 $y = t(t + 1)$

$y = 3x(3x + 1)$ ✓

$y = 9x^2 + 3x$ ✓

$\therefore \dfrac{dy}{dx} = 18x + 3$ ✓ (3)

6.3.1 $A(x) = -\frac{1}{2}x^3 + 12x^2$ (4)

$A(x) = -\frac{3}{2}x^2 + 24x$ ✓

$\therefore$ let $0 = -\frac{3}{2}x^2 + 24x$ ✓

$0 = x^2 - 16x$ ✓
$0 = x(x - 16)$
$x = 0$ or $x = 16$

$\therefore$ Maximum area will be covered after 16 months. ✓

6.3.2 $A'(x) = -\frac{3}{2}x^2 + 24x$

$A'(1) = -\frac{3}{2}(1)^2 + 24(1)$ ✓

$= 22\frac{1}{2}$ ✓

$\therefore$ The rate of growth was $22\frac{1}{2}$ m^2/ month one month after the study had begun. (2)

7.1 $f(x) = x^3 - 4x^2 - 3x + 18$
$f(3) = 27 - (4 \times 9) - (3 \times 3) + 18 = 0$ ✓ (1)

7.2 $0 = x^3 - 4x^2 - 3x + 18$
$0 = (x - 3)(x^2 - x - 6)$ ✓
$0 = (x - 3)(x - 3)(x + 2)$ ✓
$\therefore$ x-intercepts at $x = 3$ and $x = 2$. ✓ ✓ (4)

7.3 $f'(x) = 3x^2 - 8x - 3 = 0$ for turning points ✓
$\therefore$ $(3x + 1)(x - 3) = 0$
$x = -\frac{1}{3}$ and $x = 3$ ✓ ✓

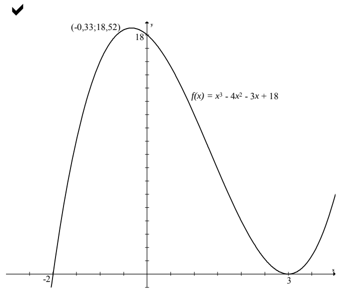
$\therefore f(x) = (-\frac{1}{3})^3 - 4(-\frac{1}{3})^2 - 3(-\frac{1}{3}) + 18$

$= \frac{500}{27}$ $(= 18{,}52)$ ✓

$f(3) = 0$

$\therefore$ Turning points at $(-\frac{1}{3}; \frac{500}{27})$ and $(3; 0)$. ✓ (5)

7.4



✓ ✓ for each of the points (4)

7.5 $f''(x) = 6x - 8 = 0$ ✓
for point of inflection.
$\therefore x = \frac{8}{6} = \frac{4}{3}$ ✓ (2)

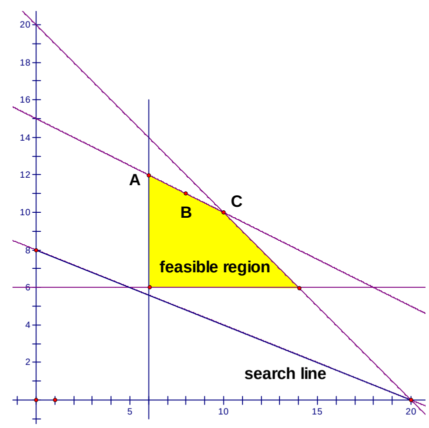
8.1 V = Area of base × Height
$2{,}5 = x^2h$ ✓
$\frac{2.5}{x^2} = h$ ✓ (2)

8.2 Area $= 2(x^2 + 4xh)$ ✓
$= 2x^2 + 8x \times \frac{2.5}{x^2}$ ✓
$= 2x^2 + \frac{20}{x}$ ✓ (3)

8.3 $A = 2x^2 + 20x^{-1}$ ✓
$\frac{dA}{dx} = 4x - 20x^{-2}$ ✓
$4x - \frac{20}{x^2} = 0$ (for minimum) ✓
$\therefore 4x^3 - 20 = 0$
$x^3 = 5$
$x = \sqrt[3]{5} = 1{,}71$ ✓ (5)

9.1 x = no. of guitars of type A
y = no. of guitars of type B
$x + y \leq 20$ ✓
$1\ 500x + 3\ 000y \leq 45\ 000$ ✓
$x \geq 6$ ✓
$y \geq 6$ ✓ (4)

9.2



✓ ✓ ✓ ✓ ✓ (5)

9.3 $P = 400x + 1\ 000y$ ✓ (1)

9.4 $-400x + P = 1\ 000y$
$-\frac{400}{1\ 000}x + \frac{P}{1\ 000} = y$
$-\frac{2}{5}x + \frac{P}{1\ 000} = y$ ✓

$\therefore$ Use search line of slope $-\frac{2}{5}$ … see sketch. ✓
$\therefore$ Maximum profit for $x = 6$ and $y = 12$ at point A. ✓ (3)

9.5 New profit equation:
$P = 500x + 1\ 000y$
$-\frac{500}{1\ 000}x + \frac{P}{1\ 000} = y$
$-\frac{1}{2}x + \frac{P}{1\ 000} = y$ ✓
This line is parallel to one of the borders of the feasible region. Therefore maximum profit occurs at any whole-number point on this line. ✓
i.e. A(6; 12)
B(8; 11) (3)

C(10; 10) ✔