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National Senior Certificate
Grade 12
Mathematics
Paper 2

MEMORANDUM

1.1 $M = \left(\frac{-1+3}{2}; \frac{3+(-1)}{2}\right)$ ✓

$M = (1; 1)$ ✓ (2)

1.2 Midpoint FH $= \left(\frac{4+(-2)}{2}; \frac{4+(-2)}{2}\right)$ ✓

$= (1; 1)$ ✓

∴ midpoint FH = midpoint EG

∴ lines bisect each other. (2)

1.3 $m_{EG} = \frac{3-(-1)}{-1-3} = \frac{4}{-4} = -1$ ✓

$m_{FH} = \frac{4-(-2)}{4-(-2)} = \frac{6}{6} = 1$ ✓

∴ $m_{EG} \times m_{FH} = -1$

⇒ EG⊥FH ✓

∴ using 1.2, diagonals of EFGH bisect at 90° ✓

∴ EFGH is a rhombus.

OR

$length_{EH} = \sqrt{(-1-(-2))^2 + (3-(-2))^2}$

$= \sqrt{1+25}$

$= \sqrt{26}$ ✓

$length_{EF} = \sqrt{(-1-4)^2 + (3-4)^2}$

$= \sqrt{25+1}$

$= \sqrt{26}$ ✓

∴ using 1.2, EFGH is a parallelogram (diagonals bisect) ✓

But EH = EF ✓

∴ EFGH is a rhombus. (4)

1.4 $m_{EG} = -1$ from above ✓

∴ $y = -x + c$ Substitute $(-1; 3)$: ✓

$3 = -(-1) + c$

$2 = c$

∴ $y = -x + 2$ ✓ (3)

1.5 $y = -x + 2$ Let $x = \frac{5}{2}$.

$y = -\frac{5}{2} + 2$ ✓

$y = -\frac{1}{2}$ ✓

∴ $\left(\frac{5}{2}; \frac{-3}{4}\right)$ does not lie on the line. ✓ (3)

1.6 $m_{FH} = \frac{4-(-1)}{4-3} = \frac{5}{1} = 5$ ✓

∴ $\tan \alpha = 5$

$A = \tan^{-1}(5)$ ✓

$= 78{,}69°$ ✓ (3)

1.7 $length_{EG} = \sqrt{(3-(-1))^2 + (-1-3)^2}$
$= \sqrt{32}$ ✓
$length_{HM} = \sqrt{(1-(-2))^2 + (1-(-2))^2}$
$= \sqrt{18}$ ✓
$\therefore$ area $\triangle EGH = \frac{1}{2}\sqrt{18} \times \sqrt{32}$ ✓
$= 5{,}05$ units^2 ✓ (4)

1.8 G $\rightarrow$ H back 5 down 1
$\therefore$ E $\rightarrow$ P is the same ✓
$\therefore$ P(–6; 2) ✓ (or equivalent) (2)

2.1 $\triangle EDC$ is right angled at C (tangent, radius)
$ED^2 = EC^2 + DC^2$ ✓
$13^2 = 12^2 + DC^2$
$25 = DC^2$
$5 = DC$ ✓ (2)

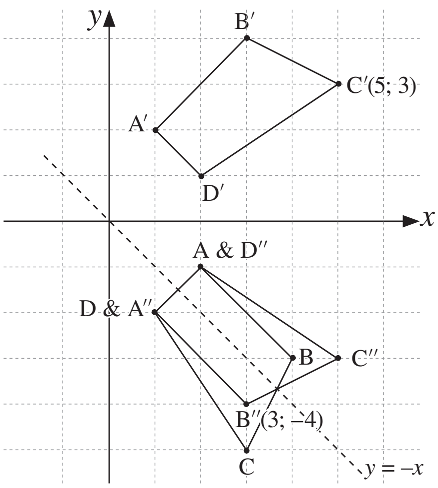
2.2 $DC^2 = (a-1)^2 + (2-(-1)^2$ ✓
$25 = a^2 - 2a + 1 + 9$ ✓
$0 = a^2 - 2a - 15$ ✓
$a = -3; a = 5$ ✓
By inspection, for this sketch $a = 5$. ✓ (5)

2.3 $m_{DC} = \frac{2-(-1)}{5-1} = \frac{3}{4}$ ✓
$m_{tangent} = \frac{-4}{3}$ ✓
$y = \frac{-4}{3}x + c$ Substitute (1; –1):
$-1 = \frac{-4}{3}x + c$ ✓
$\frac{1}{3} = c$ ✓
$y = \frac{-4}{3}x + \frac{1}{3}$ (4)

2.4 y-axis is tangent to circle at A.
$\therefore$ AD is horizontal $\therefore$ A(0; 2) ✓✓ (inspection) (2)

2.5 $(x-a)^2 + (y-b)^2 = c^2$ ✓
Substitute (1; –1) and (0; 2):
$(0-1)^2 + (2-(-1))^2 = c^2$ ✓✓
$10 = c^2$ ✓
$(x-1)^2 + (y+1)^2 = 10$ ✓ (5)

3.1 and 3.2



(10)

3.3 P(x; y)

$\rightarrow$ P′($-x$; $-y$)

$\rightarrow$ final image ($-45x$; $-45y$) ✓✓ (2)

3.4 Linear factor $\frac{4}{5}$

$\Rightarrow$ area factor $\frac{16}{25}$ ✓

$\therefore$ area $= \frac{16}{25}p$ units^2 ✓ (2)

4.1 ($x\cos\theta - y\sin\theta$; $x\sin\theta + y\cos\theta$) ✓

$= (6\cos 60° - 3\sin 60°; 6\sin 60° + 3\cos 60°)$ ✓✓

$= \left(\frac{6 - 3\sqrt{3}}{2}; \frac{3 + 6\sqrt{3}}{2}\right)$ ✓✓

OR

$= \left(3 - \frac{3\sqrt{3}}{2}; 3\sqrt{3} + \frac{3}{2}\right)$ (5)

4.2 (6; 3) $\rightarrow$ (3; 6) $\rightarrow$ (3; –6) ✓✓ (2)

5.1 5.1.1 $\tan\theta = \frac{y}{x} = \frac{a}{b}$ ✓ (1)

5.1.2 $\cos(-\theta) = \cos\theta = \frac{x}{r}$ ✓✓

$r = \sqrt{a^2 + b^2}$ (Pythagoras) ✓

$\therefore \cos(-\theta) = \frac{a}{\sqrt{a^2 + b^2}}$ ✓ (4)

5.2 5.2.1 cos 53°

$= \sin(90° - 53°)$

$= \sin 37°$ ✓

$= k$ ✓ (2)

5.2.2 sin (–74°)

$= -\sin 74°$

$= -2\sin 37°\cos 37°$ ✓✓

$= -2k\sqrt{1 - k^2}$ ✓ (4)

5.3 5.3.1 LHS

$= \frac{\sin \alpha \sin 2\alpha}{\cos \alpha} + \cos 2\alpha$ ✓

$= \frac{\sin \alpha 2\sin \alpha \cos \alpha}{\cos \alpha} + 1 - 2\sin^2\alpha$ ✓✓

$= 1$ ✓

$=$ RHS (4)

5.3.2 LHS

$= \frac{\sin 234°}{\cos 36°} - \frac{\sin (x - 90°)\cos (90° - 2x)}{\sin (x - 360°)}$

$= \frac{-\sin 54°}{\cos 36°} - \frac{(-\cos x)\sin 2x}{\sin x}$ ✓✓ ✓✓

$= \frac{-\sin 54°}{\sin 54°} - \frac{\cos x \cdot 2\sin x\cos x}{\sin x}$ ✓✓

$= -1 + 2\cos^2 x$ ✓

$= \cos 2x$

$=$ RHS (8)

5.4 $3\cos^2 x + 5\sin x = 3$

$3(1 - \sin^2 x) + 5\sin x = 3$ ✓

$3 - 3\sin^2 x + 5\sin x = 3$

$0 = 3\sin^2 x - 5\sin x$

$0 = \sin x\,(3\sin x - 5)$ ✓

$\sin x = 0$ or $\sin x = \frac{5}{3}$ ✓✓

$\therefore x = 0° + n180°$ or x is undefined ✓✓

$(n \in \mathbb{Z})$ (6)

6.1 $A\hat{C}B = 110° - 50°$

$= 60°$ ✓

$\therefore AB^2 = 150^2 + 260^2 - (2.150.260)\cos 60°$ ✓

$AB^2 = 51\ 100$ ✓

$\therefore AB = 226{,}05$ ✓

$AB = 226$ km ✓ (5)

6.2 6.2.1 $C\hat{D}B = 180° - (\theta + 30°)$ ✓ (1)

6.2.2 In △ABC: $\tan\theta = \frac{p}{CB}$

$CB \tan\theta = p$ …………..(i) ✓

In △CBD: $\frac{CB}{\sin [180° - (\theta + 30°)]} = \frac{8}{\sin\theta}$ ✓

$\frac{CB}{(\sin \theta + 30°)} = \frac{8}{\sin\theta}$ ✓

$CB = \frac{8(\sin\theta + 30°)}{\sin\theta}$…………..(ii) ✓

Combining (i) and (ii):

$p = \frac{8\sin(\theta + 30°)}{\sin\theta} \cdot \tan\theta$

$p = \frac{8\sin(\theta + 30°)}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta}$ ✓

$p = \frac{8(\sin\theta + 30°)}{\cos\theta}$ ✓ (6)

7.1 $\sin 2x = \cos (x - 45°)$

$\sin 2x = \sin [90° - (x - 45°)]$

$\sin 2x = \sin (135° - x)$ ✓

$\therefore 2x = 135° - x + n \cdot 360° \ (n \in \mathbb{Z})$ ✓

$3x = 135° + n \cdot 360°$

$x = 45° + n \cdot 120°$ ✓

OR

$2x = 180° - (135° - x) + n \cdot 360° \ (n \in \mathbb{Z})$ ✓

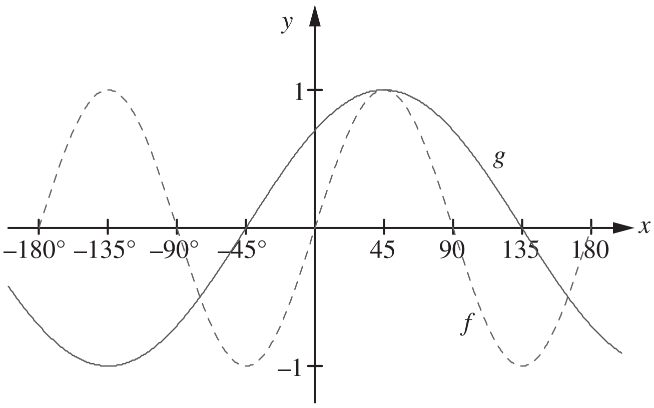
$2x = 45° + x + n \cdot 360°$

$x = 45° + n \cdot 360°$ ✓

$\therefore$ for $x \in [-180°; 180°]$

$x = 45°; 165°; -75°$ ✓✓ ✓ (8)

7.2



✓✓ ✓for g ✓✓ ✓for f (6)

7.3 7.3.1 $g(x) \leq f(x)$ for $[-180°; 90°]$

$\Rightarrow -180° \leq x \leq -75°$ ✓✓ ✓ (3)

7.3.2 $\frac{f(x)}{g(x)}$ undefined $\Rightarrow g(x) = 0$ ✓

$\therefore x = -45°$ only for $[-180°; 90°]$ ✓ (2)

8.1 8.1.1 $\bar{x} = 65{,}27$ ✓✓ ✓

Using stats mode on calculator or manually (3)

8.1.2 $SD = 8{,}71$ ✓✓ (2)

8.1.3 Upper boundary

$= 65{,}27 + 8{,}71$

$= 73{,}98$ ✓

Lower boundary

$= 65{,}27 - 8{,}71$

$= 56{,}56$ ✓

$\therefore$ reject 50; 45; 80

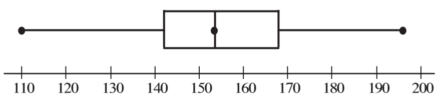
i.e. 3 bags would be rejected ✓ (3)

8.2 8.2.1 Ordered list

11 000
12 600
14 200 Q_1 = 14 200
14 500
15 300
Median = 15 350
15 400
16 500
16 800 Q_3 = 16 800
18 600
19 600

∴ Minimum = 11 000 ✓
Lower quartile = 14 200 ✓
Median = 15 350 ✓
Upper quartile = 16 800 ✓
Maximum = 19 600 ✓ (5)

8.2.2 Scale in 100s



✓✓ ✓✓ (3)

8.2.3 Maximum per day = $\frac{19\,600}{28}$ = 700 patients per day ✓

Minimum per day = $\frac{11\,000}{28}$ = 392 patients per day ✓ (2)

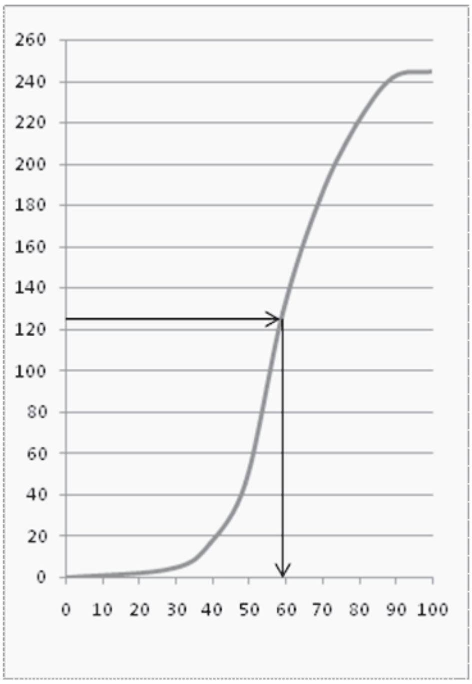
9.1

Marks	F	CF
20 ≤ *x* ≤ 29	4	4
30 ≤ *x* ≤ 39	12	16
40 ≤ *x* ≤ 49	30	46
50 ≤ *x* ≤ 59	82	128
60 ≤ *x* ≤ 69	55	183
70 ≤ *x* ≤ 79	35	218
80 ≤ *x* ≤ 89	24	242
90 ≤ *x* ≤ 100	3	245

✓✓

(2)

9.2



✓✓ ✓✓ (4)

9.3 Median at $\frac{245+1}{2} = 123$

Median approximately 59 ✓✓ (2)

9.4 Data is grouped, so original raw data is lost.

$\therefore$ mean or median will be approximate. ✓✓ (2)