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# National Senior Certificate

## Grade 12

# Mathematics

## Paper 1

# MEMORANDUM

1.1 1.1.1

$$2x^2 + 7x = 30$$
$$2x^2 + 7x - 30 = 0 \checkmark$$
$$(2x - 5)(x + 6) = 0 \checkmark$$
$$\therefore x = 25 \text{ or } x = -6 \checkmark\checkmark$$

(4)

1.1.2

$$2x(x - 2) - 5 = 0$$
$$2x^2 - 4x - 5 = 0 \checkmark$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{4 \pm \sqrt{(-4)^2 - 4 \times (2)(-5)}}{2 \times 2} \checkmark$$
$$\therefore x = \frac{4 + \sqrt{56}}{4} \text{ or } x = \frac{4 - \sqrt{56}}{4} \checkmark\checkmark$$

Or $x = 2{,}87$ or $x = -0{,}87$

(4)

1.1.3

$$4x^2 + 5x - 2 < 0$$
$$(4x - 1)(x + 2) < 0 \checkmark\checkmark$$
$$x < \frac{1}{4} \text{ or } x > -2 \checkmark\checkmark$$

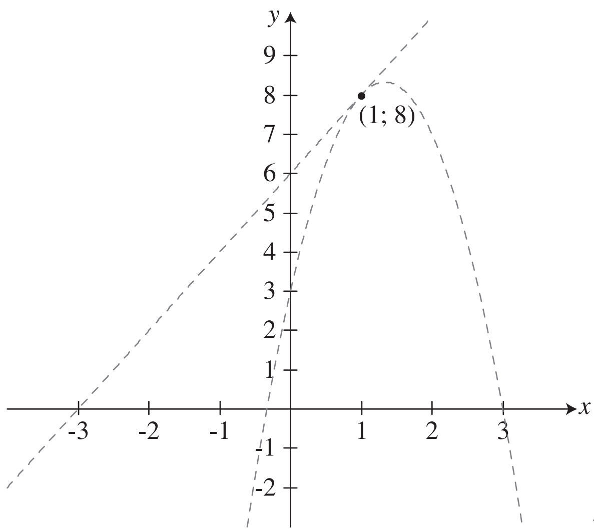
(4)

1.2 1.2.1

$$2x + 6 - y = 0$$
$$2x + 6 = y$$
$$y + 3x^2 = 8x + 3$$
$$y = -3x^2 + 8x + 3 \checkmark$$
$$\therefore 2x + 6 = -3x^2 + 8x + 3 \checkmark$$
$$3x^2 - 6x + 3 = 0$$
$$x^2 - 2x + 1 = 0$$
$$(x - 1)^2 = 0 \checkmark$$
$$\therefore x = 1 \checkmark$$
$$\therefore y = 2x + 6 = 8 \checkmark$$

(5)

1.2.2




✓

(1; 8) is the point of contact between the straight line $y = 2x + 6$ and the parabola $y = -3x^2 + 8x + 3$. There is one point of contact.

$\Rightarrow$ line is a tangent to the curve. ✓✓ (3)

2.1

$$P = \frac{x[1 - (1 + i)^{-n}]}{i} ✓$$

$$800\ 000 = \frac{7\ 500\left[1 - \left(1 + \frac{\frac{9}{100}}{12}\right)^{-n}\right]}{\frac{\frac{9}{100}}{12}} ✓✓$$

$$800\ 000 = \frac{7\ 500\left[1 - \left(1 + \frac{3}{400}\right)^{-n}\right]}{\frac{3}{400}}$$

$$800\ 000 \times \frac{3}{400} \div 7\ 500 = 1 - \left(\frac{403}{400}\right)^{-n}$$

$$\frac{20}{25} = 1 - \left(\frac{403}{400}\right)^{-n} ✓$$

$$\left(\frac{403}{400}\right)^{-n} = \frac{5}{25}$$

$$= \frac{1}{5}$$

$$-n = \left[\frac{\log \frac{1}{5}}{\log \frac{403}{400}}\right] ✓$$

$$-n = -215{,}4 \text{ months} ✓$$

$\therefore$ It will take 17,95 years to pay back the loan.✓

(Simplifications at various stages are not necessary.) (7)

2.2 2.2.1 $A = P(1 - i)^n$ ✓

$A = 1\ 500\ 000\left(1 - \frac{18}{100}\right)^5$ ✓

$A = 556\ 109{,}76$ ✓

$\therefore$ It is worth R556 109,76. (3)

2.2.2 $A = P(1 - i)^n$✓

$A = 1\ 500\ 000\left(1 + \frac{4,5}{100}\right)^5$✓

$A = 1\ 869\ 272,91$✓

$\therefore$ Sinking fund required is:

R1 869 272,91 – R556 109,76 = R1 313 163,15✓ (4)

2.2.3

$$F = \frac{x[(1+i)^2 - 1]}{i}✓$$

$$1\ 313\ 163,15 = \frac{x\left[\left(1 + \frac{\frac{6}{100}}{12}\right)^5 - 1\right]}{\frac{\frac{6}{100}}{12}}✓✓$$

$$\frac{1\ 313\ 163,15 \times 6 \div 100 \div 12}{\left[\left(1 + \frac{\frac{6}{100}}{12}\right)^5 - 1\right]} = x✓$$

$$260\ 019,40 = x✓$$

$\therefore$ They will need to pay R260 019,40 per month. (5)

3.1 3.1.1 Quadratic sequence ✓ (1)

3.1.2

| $T_0$ | $T_1$ | $T_2$ | $T_3$ | $T_4$ | $T_5$ |
|---|---|---|---|---|---|
| 9 | 9 | 11 | 15 | 21 | 29 |

First differences: 0 2 4 6 8

Second differences: 2 2 2 2

Second difference = 2

$\therefore 2a = 2$

$a = 1$✓

By inspection $T_0 = 9$

$\therefore c = 9$✓

$\therefore T_n = n^2 + bn + 9$

$T_1 = 1^2 + b + 9 = 9$

$\therefore b = -1$✓

$\therefore T_n = n^2 - n + 9$✓

(or equivalent method) (4)

3.2 Sammy:

$a = 2 \quad T_4 = a + 3d$ ✓

$= 2 + 3d = 17$

$3d = 15$

$d = 5$ ✓

$\therefore T_{20} = a + 19d$

$= 2 + 19 \times 5$

$= 97$

$\therefore x = 97$ ✓

Solly:

$T_4 = a + 3d = 59$

$T_{20} = a + 19d = 107$ ✓

$16d = 48$

$d = 3$ ✓

$\therefore T_4 = a + 3d = 59$

$a + 9 = 59$

$a = 50$ ✓

$\therefore y = 50$ (6)

4.1 4.1.1 2010

109% of 85 000 = R92 650✓

109% of 92 650 = R100 988,50✓ (2)

4.1.2 GP with $a = 85\ 000$

$r = 109\% = 1{,}09$ ✓

$T_n = ar^{n-1}$ ✓

$T_{15} = 85\ 000(1{,}09)^{15}$ ✓

$= R309\ 611{,}01$ ✓ (4)

4.2 $\sum_{k=0}^{3} x = \frac{2}{5^k}$

$= 2 + 25 + 225 + 2\ 125$ ✓✓

$= \frac{312}{125}\ (2{,}496)$ ✓ (3)

4.3 4.3.1 GP with $a = 20$

$r = \frac{3}{4}$

$S_n = \frac{a[1 - r^n]}{1 - r}$ ✓

$S_n = \frac{20\left[1 - \left(\frac{3}{4}\right)^{10}\right]}{1 - \frac{3}{4}}$ ✓

$= 75{,}49$ ✓

$\therefore$ Depth is 75,49 m. (3)

4.3.2 $S_\infty = \frac{a}{1-r}$ ✓

$= \frac{20}{1-\frac{3}{4}}$ ✓

$= 80$

$s_\infty = 80$ m ✓

$\therefore$ Company will never reach the water. ✓ (4)

5.1 5.1.1 $f(x) = \frac{-3}{x-1} + 2$

asymptotes at $x = 1$; $y = 2$

$y$-intercept: $x = 0$ $\therefore$ $f(0) = \frac{-3}{0-1} + 2 = 5$

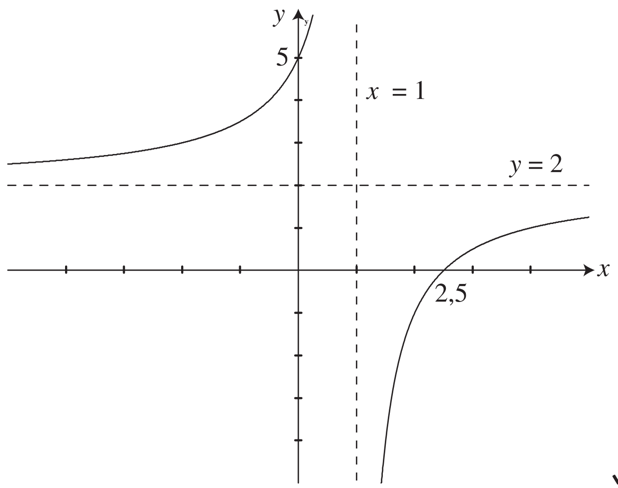
$x$-intercept: $y = 0$ $\therefore$ $0 = \frac{-3}{x-1} + 2$

$\frac{3}{x-1} = 2$

$3 = 2x - 2$

$5 = 2x$

$\frac{5}{2} = x$




✓✓✓✓ (4)

5.1.2 Axes of symmetry:

$y = x + c$ and $y = -x + c$

Pass through (1; 2) ✓

$\therefore$ $2 = 1 + c$ $2 = -1 + c$

$1 = c$ $3 = c$

$\therefore$ $y = x + 1$ and $y = -x + 3$ ✓✓ (3)

5.2 5.2.1 $y = a(x - x_1)(x - x_2)$ ✓

$y = a(x + 1)(x - 3)$ ✓

Substitute (0; –1):

$-1 = a(0 + 1)(0 - 3)$ ✓

$-1 = -3a$

$\frac{1}{3} = a$ ✓

$\therefore\ p(x) = \frac{1}{3}(x + 1)(x - 3)$

$= \frac{1}{3}(x^2 - 2x - 3)$

$= \frac{1}{3}x^2 - \frac{2}{3}x - 1$ ✓ (5)

5.2.2 $q(x) = 12b^x - 4$

Substitute (3; 0):

$0 = \frac{1}{2}b^3 - 4$ ✓

$4 = \frac{1}{2}b^3$

$8 = b^3$ ✓

$2 = b$ (2)

5.2.3 Domain: $x \in \mathbb{R}$ ✓

Range: $y \in \mathbb{R}; y > -4$ ✓ (2)

5.2.4 $q(x) = y = \frac{1}{2} \cdot 2^x - 4$

$q^{-1}(x)$: $x = \frac{1}{2} \cdot 2^y - 4$ ✓

$2x = 2^y - 8$

$2x + 8 = 2^y$ ✓

$\therefore\ y = \log_2 (2x + 8)$ ✓

$q^{-1}(x) = \log_2 (2x + 8)$ (3)

5.2.5 Domain of $q^{-1}(x)$ = Range of $q(x)$

$= x \in \mathbb{R}; x > -4$ ✓ (1)

5.3 5.3.1 $a = -4$ ✓ (1)

5.3.2 $x = 180°; x = -180°$ ✓

$x = 0°$ ✓ (2)

5.3.3 $y = \tan (x + 90° + 45°) + 2$ ✓

$y = \tan (x + 135°) + 2$ ✓

or $y = \tan (x - 45°) + 2$

by inspection/reduction (2)

6.1 $f(x) = \frac{1}{2}x^2$

$f'(x) = \lim_{h \circledR 0} \frac{f(x+h) - f(x)}{h}$

$\therefore f'(x) = \lim_{h \circledR 0} \frac{\frac{1}{2}(x+h)^2 - \frac{1}{2}x^2}{h}$ ✓

$\therefore f'(x) = \lim_{h \circledR 0} \frac{\frac{1}{2}x^2 + xh + \frac{1}{2}h^2 - \frac{1}{2}x^2}{h}$ ✓

$\therefore f'(x) = \lim_{h \circledR 0} \frac{h\left(x + \frac{1}{2}h\right)}{h}$ ✓

$f'(x) = \lim_{h \circledR 0} x + \frac{1}{2}h$ ✓

$f(x) = x$ ✓ (5)

6.2 6.2.1 $f(x) = x^4 + \sqrt{x} - \frac{9}{x}$

$f(x) = x^4 + x^{\frac{1}{2}} - x^{-1}$ ✓

$\therefore f'(x) = 4x^3 + \frac{1}{2}x^{-\frac{1}{2}} - 9x^{-2}$ ✓✓ (3)

6.2.2 $y = t(t + 1)$

$y = 3x(3x + 1)$ ✓

$y = 9x^2 + 3x$ ✓

$\therefore \frac{dy}{dx} = 18x + 3$ ✓ (3)

6.3 6.3.1 $A(x) = -12x^3 + 12x^2$

$A'(x) = -32x^2 + 24x$ ✓

$\therefore 0 = -32x^2 + 24x$ ✓

$0 = x^2 - 16x$ ✓

$0 = x(x - 16)$

$x = 0$ or $x = 16$

$\therefore$ Maximum area will be covered after 16 months.✓ (4)

6.3.2 $A'(x) = -\frac{3}{2}x^2 + 24x$

$A'(1) = -\frac{3}{2}(1)^2 + 24(1)$ ✓

$= 22\frac{1}{2}$ ✓

$\therefore$ The rate of growth was $22\frac{1}{2}$ $m^2$/month one month after the study had begun. (2)

7.1 $f(x) = x^3 - 4x^2 - 3x + 18$

$f(3) = 27 - (4 \times 9) - (3 \times 3) + 18 = 0$ ✓ (1)

7.2 $0 = x^3 - 4x^2 - 3x + 18$

$0 = (x - 3)(x^2 - x - 6)$ ✓

$0 = (x - 3)(x - 3)(x + 2)$ ✓

$\therefore$ $x$-intercepts at $x = 3$ and $x = 2$ ✓✓ (4)

7.3 $f'(x) = 3x^2 - 8x - 3 = 0$ for turning points ✓

$\therefore (3x + 1)(x - 3) = 0$

$x = -\frac{1}{3}$ and $x = 3$✓✓

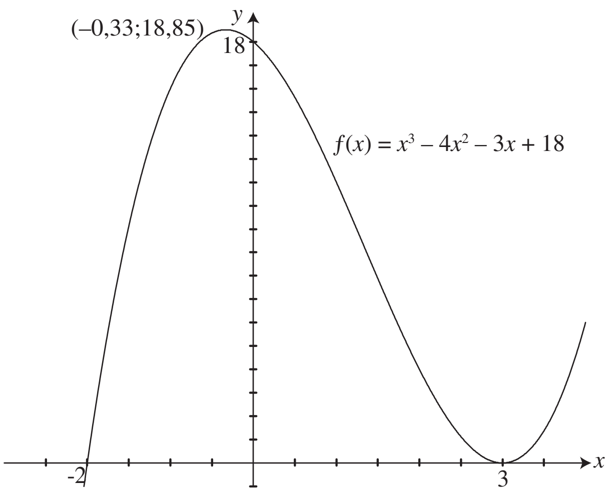
$\therefore f(x) = \left(-\frac{1}{3}\right)^3 - 4\left(-\frac{1}{3}\right)^2 - 3\left(-\frac{1}{3}\right) + 18$

$= \frac{500}{27}$ (= 18,52) ✓

$f(3) = 0$

$\therefore$ Turning points at $\left(-\frac{1}{3}; \frac{500}{27}\right)$ and (3; 0). ✓ (5)

7.4




✓✓ for each of the points (4)

7.5 $f''(x) = 6x - 8 = 0$✓

for point of inflection.

$\therefore x = \frac{8}{6} = \frac{4}{3}$✓ (2)

8.1 $V$ = Area of base × Height

$2{,}5 = x^2h$✓

$\frac{2.5}{x^2} = h$✓ (2)

8.2 Area $= 2x^2 + 4xh$ ✓

$= 2x^2 + 4x \times \frac{2.5}{x^2}$✓

$= 2x^2 + \frac{10}{x}$✓ (3)

8.3 Area $= 2x^2 + 10x^{-1}$✓

Area $= 4x - 10x^{-2}$✓

$4x - \frac{10}{x^2} = 0$ (for minimum) ✓

$4x^3 - 10 = 0$

$x^3 = \frac{10}{4}$

$x = \sqrt[3]{\frac{5}{2}}$✓

$x = 1{,}36$✓ (5)

9.1 $x$ = No. of guitars of type A

$y$ = No. of guitars of type B

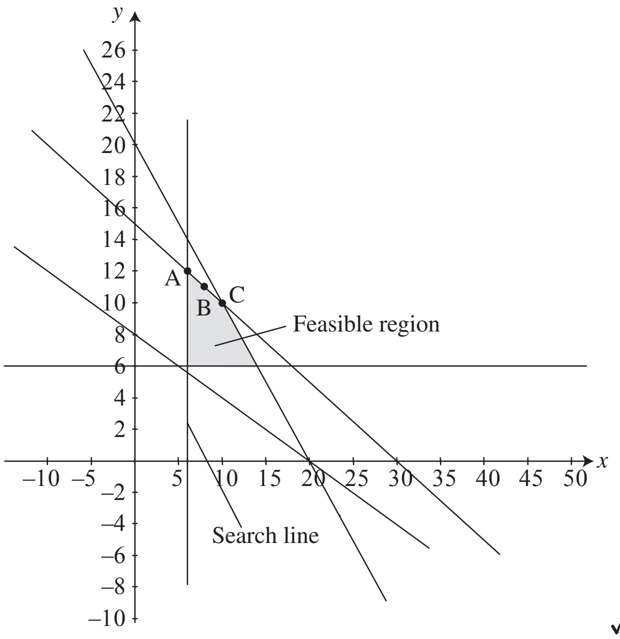
$x + y \leq 20$✓

$1\ 500x + 3\ 000y \leq 45\ 000$✓

$x \geq 6$✓

$y \geq 6$✓ (4)

9.2




✓✓✓✓✓ (5)

9.3 $P = 400x + 1\ 000y$✓ (1)

9.4 $-400x + P = 1\ 000y$

$-\frac{400}{1\ 000}x + \frac{P}{1\ 000} = y$

$-\frac{2}{5}x + \frac{P}{1\ 000} = y$✓

$\therefore$ Use search line of slope $-\frac{2}{5}$ … see sketch.✓

$\therefore$ Maximum profit for $x = 6$ and $y = 12$ at point A. ✓ (3)

9.5 New profit equation:

$P = 500x + 1\ 000y$

$-\frac{500}{1\ 000}x + \frac{P}{1\ 000} = y$

$-\frac{1}{5}x + \frac{P}{1\ 000} = y$✓

This line is parallel to one of the borders of the feasible region. Therefore maximum profit occurs at any whole-number point on this line. ✓

i.e. A(6; 12)

B(8; 11)

C(10; 10) ✓ (3)