



# education

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Department:  
Education  
**REPUBLIC OF SOUTH AFRICA**

**NASIONALE  
SENIOR SERTIFIKAAT**

**GRAAD 12**

**WISKUNDE V2**

**FEBRUARIE/MAART 2010**

**MEMORANDUM**

**PUNTE: 150**

**Hierdie memorandum bestaan uit 14 bladsye.**

**VRAAG 1**

|       |  |  |
|-------|--|--|
| 1.1   | Variasiewydte = $26 - 4 = 22$  | ✓ maksimum en minimumwaardes<br>✓ antwoord<br>SLEGS ANTWOORD:<br>Volpunte<br>(2) |
| 1.2   | Gemiddelde<br>$= \frac{4 + 5 + 8 + 13 + 19 + 22 + 25 + 26 + 23 + 17 + 14 + 7}{12}$ $= \frac{183}{12}$ $= 15,25$  | ✓ metode<br>✓ 183<br>✓ antwoord<br>(3)   |
| 1.3   | Standaardafwyking = 7,6 (7,59522.....)   | ✓✓ antwoord<br>(2)   |
| 1.4.1 | Verhoging in gemiddelde = $\frac{(3 \times 5) + (9 \times 1)}{12}$<br>$= 2^{\circ}\text{C}$ per maand.   | ✓✓ antwoord<br>(2)   |
| 1.4.2 | Die maksimumwaarde verhoog met $1^{\circ}\text{C}$ en die minimumwaarde verhoog met $5^{\circ}\text{C}$ . Dit impliseer dat die omvang van die data nou sal afneem. Dit sal veroorsaak dat die standaardafwyking kleiner word. (nuwe SD = 6,27.....) | ✓ afname in omvang<br>✓ afname in standaardafwyking<br>(2)<br><b>[11]</b>        |

**VRAAG 2**

|              |  |  |
|--------------|--|--|
| <p>2.1.1</p> | <p style="text-align: center;"><b>Studie van oefening en doele aangeteken</b></p>  | <p>✓✓✓ stip van punte</p> <p>Al 9 punte korrek – 3 punte<br/>5 of 7 punte korrek – 2 punte<br/>1 of 2 punte korrek – 1 punt<br/>0 punte korrek – 0 punte</p> <p style="text-align: right;">(3)</p> |
| <p>2.1.2</p> | <p>A(aangedui op die grafiek)</p>  | <p>✓ antwoord</p> <p style="text-align: right;">(1)</p>  |
| <p>2.1.3</p> | <p>8 doele</p>   | <p>✓✓ antwoord</p> <p style="text-align: right;">(2)</p>   |
| <p>2.2</p>   | <p>Laat die gemiddelde tyd vir al 560 leerdere <math>x</math> wees.<br/>Dan is die gemiddelde tyd vir leerdere wat in buurt C bly ook <math>x</math>.</p> $x = \frac{(135 \times 24) + (225 \times 32) + (200 \times x)}{560}$ $560x = 3240 + 7200 + 200x$ $360x = 10440$ $x = 29$ | <p>✓ gelyk aan gemiddelde tye</p> <p>✓ gemiddelde <math>\times</math> getal</p> <p>✓ vereenvoudiging</p> <p>✓ antwoord</p> <p style="text-align: right;">(4)<br/><b>[10]</b></p>                   |

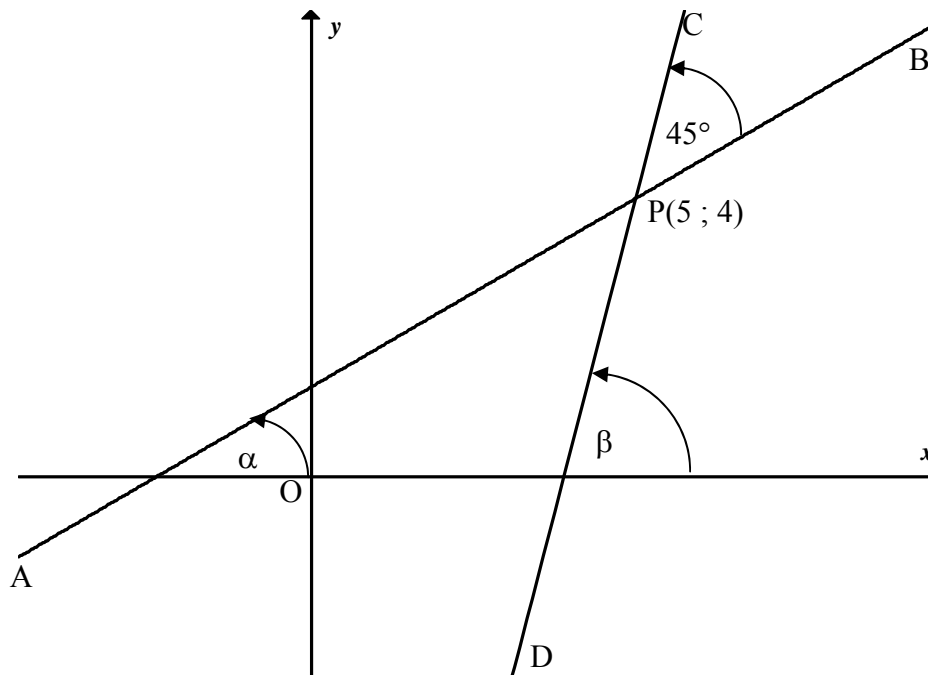
**VRAAG 3**

|                        |   |  |                  |                  |                  |                  |                  |            |   |   |    |    |   |                        |   |    |    |    |    |  |
|------------------------|---|--|------------------|------------------|------------------|------------------|------------------|------------|---|---|----|----|---|------------------------|---|----|----|----|----|--|
| <p>3.1</p>             | <table border="1"> <tr> <td>Tyd (in minute)</td> <td><math>11 \leq t &lt; 15</math></td> <td><math>15 \leq t &lt; 19</math></td> <td><math>19 \leq t &lt; 23</math></td> <td><math>23 \leq t &lt; 27</math></td> <td><math>27 \leq t &lt; 30</math></td> </tr> <tr> <td>Frekwensie</td> <td>6</td> <td>9</td> <td>13</td> <td>12</td> <td>8</td> </tr> <tr> <td>Kumulatiewe frekwensie</td> <td>6</td> <td>15</td> <td>28</td> <td>40</td> <td>48</td> </tr> </table> | Tyd (in minute)  | $11 \leq t < 15$ | $15 \leq t < 19$ | $19 \leq t < 23$ | $23 \leq t < 27$ | $27 \leq t < 30$ | Frekwensie | 6 | 9 | 13 | 12 | 8 | Kumulatiewe frekwensie | 6 | 15 | 28 | 40 | 48 | <p>✓ kumulatiewe frekwensie totale<br/>(1)</p> |
| Tyd (in minute)        | $11 \leq t < 15$  | $15 \leq t < 19$   | $19 \leq t < 23$ | $23 \leq t < 27$ | $27 \leq t < 30$ |                  |                  |            |   |   |    |    |   |                        |   |    |    |    |    |  |
| Frekwensie             | 6   | 9  | 13               | 12               | 8                |                  |                  |            |   |   |    |    |   |                        |   |    |    |    |    |  |
| Kumulatiewe frekwensie | 6   | 15   | 28               | 40               | 48               |                  |                  |            |   |   |    |    |   |                        |   |    |    |    |    |  |
| <p>3.2</p>             | <p style="text-align: center;"><b>Kumulatiewefrekwensie-kurwe wat die tyd om 'n taak te voltooi, aandui</b></p>   | <p>✓✓✓ stip punte by boonste limiete<br/>Al 6 punte korrek – 3 punte<br/>3 tot 5 punte korrek – 2 punte<br/>1 of 2 punte korrek – 1 punt<br/>Geen punte korrek – 0 punte<br/>✓ kurwe<br/>(4)</p> |                  |                  |                  |                  |                  |            |   |   |    |    |   |                        |   |    |    |    |    |  |
| <p>3.3</p>             | <p>Mediaanwaarde by posisie 24. Lees van die ogief af, gee Mediaan <math>\approx 22</math> minute<br/>LQ-waarde by posisie 12. Onderste kwartiel <math>\approx 18</math> minute (van ogief)<br/>UQ-waarde by posisie 36. Boonste kwartiel <math>\approx 25,5</math> minute (van ogief)<br/><br/>LET WEL: Laat speling toe om van die grafiek af te lees.</p>  | <p>✓ mediaan<br/>✓ onderste kwartiel<br/>✓ boonste kwartiel<br/>(3)</p>  |                  |                  |                  |                  |                  |            |   |   |    |    |   |                        |   |    |    |    |    |  |
| <p>3.4</p>             |   | <p>✓ mond<br/>✓ snor<br/>(2)</p>   |                  |                  |                  |                  |                  |            |   |   |    |    |   |                        |   |    |    |    |    |  |
| <p>3.5</p>             | <p>Die tye is na regs geskeef. 'n Paar mense het hierdie taak baie vinnig afgehandel, terwyl ander langer geneem het.</p>   | <p>✓ skeef na regs<br/>(1)<br/><b>[11]</b></p>   |                  |                  |                  |                  |                  |            |   |   |    |    |   |                        |   |    |    |    |    |  |

**VRAAG 4**

|     |  |  |
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| 4.1 | $m_{PQ} = \frac{2-0}{0-4} = -\frac{1}{2}$  | ✓ vervanging<br>(1)  |
| 4.2 | A: $\left(\frac{0+4}{2}; \frac{2+0}{2}\right)$<br>A (2 ; 1)  | ✓ x-koördinaat<br>✓ y-koördinaat<br>(2)  |
| 4.3 | $m_{AB} \cdot m_{PQ} = -1$<br>$m_{AB} \cdot (-1/2) = -1, \therefore m_{AB} = 2$<br>Vergelyking van AB is $y = 2x + c$<br>$\therefore 1 = 2(2) + c$<br>$c = -3$<br>Vergelyking van AB is $y = 2x - 3$ .<br><br><b>OF</b><br>$m_{AB} \cdot m_{PQ} = -1$<br>$m_{AB} \cdot (-1/2) = -1, \therefore m_{AB} = 2$<br>$y - 1 = 2(x - 2)$<br>$y - 1 = 2x - 4$<br>$y = 2x - 3$ | ✓ $m_{AB} \cdot m_{PQ} = -1$<br>✓ $m_{AB} = 2$<br>✓ vergelyking van AB<br>✓ $y = 2x - 3$<br>✓ $c = -3$<br>(5)<br><br>✓ $m_{AB} \cdot m_{PQ} = -1$<br>✓ $m_{AB} = 2$<br>✓ gradiënt van AB ✓<br>vervanging in formule<br>✓ vergelyking van AB<br>(5) |
| 4.4 | B is die punt (0 ; -3)<br>$BQ = \sqrt{(0-4)^2 + (-3-0)^2}$<br>$= 5$  | ✓ koördinate van B<br>✓ vervanging<br>✓ antwoord<br>(3)  |
| 4.5 | $BP = \sqrt{(0-0)^2 + (-3-2)^2}$<br>$= 5$<br>BP = BQ<br>$\therefore \Delta BPQ$ is gelykbenig.<br><br><b>OF</b><br>BP = 2 + 3<br>$= 5$<br>BP = BQ<br>$\therefore \Delta BPQ$ is gelykbenig   | ✓ BP = 5<br>✓ BP = BQ<br>(2)<br><br>✓ BP = 5<br>✓ BP = BQ<br>(2)   |
| 4.6 | As PBQR 'n ruit is, dan is A die middelpunt van BR.<br>Laat die koördinate van R (x ; y) wees.<br><br>$\frac{x+0}{2} = 2$ en $\frac{y-3}{2} = 1$<br>$x = 4$ $y = 5$<br>$\therefore R(4 ; 5)$<br><br><b>OF</b><br>RQ    PB dus $x_R = 4$<br>RQ = PB = 5, dus $y_R = 5$<br>$\therefore R(4 ; 5)$   | ✓ A is die middelpunt van BR<br>✓ x-koördinaat<br>✓ y-koördinaat<br>(3)<br><br>✓ RQ    PB<br>✓ x-koördinaat<br>✓ y-koördinaat<br>(3)<br><b>[16]</b>  |

**VRAAG 5**



|            |   |   |
|------------|---|---|
| <p>5.1</p> | <p>AB word gedefinieer as <math>5y - 3x - 5 = 0</math> wat geskryf kan word as</p> $y = \frac{3}{5}x + 1$ $m_{AB} = \frac{3}{5}$ <p>Laat <math>\alpha</math> die inklinasie van AB wees.</p> <p>Dan <math>\tan \alpha = \frac{3}{5}</math></p> $\alpha = 30,96^\circ .$ <p>Laat <math>\beta</math> die inklinasie van CD wees.</p> $\beta = 45^\circ + 30,96^\circ$ $= 75,96^\circ$ <p>Gradiënt van CD = <math>\tan 75,96^\circ = 4</math>.</p> <p><b>OF</b></p> $\tan \beta = \tan(\alpha + 45^\circ)$ $= \frac{\tan \alpha + \tan 45^\circ}{1 - \tan \alpha \cdot \tan 45^\circ}$ $= \frac{\frac{3}{5} + 1}{1 - \frac{3}{5} \times 1}$ $= 4$ $m_{CD} = \tan \beta$ $m_{CD} = 4$ | <p>✓ <math>m_{AB} = \frac{3}{5}</math></p> <p>✓ <math>\tan \alpha = \frac{3}{5}</math></p> <p>✓ <math>\alpha = 30,96^\circ</math></p> <p>✓ <math>\beta = 75,96^\circ</math></p> <p>✓ gradiënt van CD (5)</p> <p>✓ uitbreiding</p> <p>✓ <math>\tan 45^\circ = 1</math></p> <p>✓ <math>\tan \alpha = \frac{3}{5}</math></p> <p>✓ vervanging</p> <p>✓ antwoord (5)</p> |
|------------|---|---|

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| 5.2 | <p>Vergelyking van CD is <math>y = 4x + c</math><br/> <math>\therefore 4 = 4(5) + c</math><br/> <math>c = -16</math><br/> Vergelyking van CD is <math>y = 4x - 16</math>.</p> <p><b>OF</b></p> <p><math>y - 4 = 4(x - 5)</math><br/> <math>y - 4 = 4x - 20</math><br/> <math>y = 4x - 16</math></p> | <p>✓ y- afsnit<br/> ✓ vergelyking van CD<br/> (2)</p> <p>✓ vervanging<br/> ✓ vergelyking van CD<br/> (2)<br/> <b>[7]</b></p> |
|-----|---|--|

**VRAAG 6**

|     |   |   |
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| 6.1 | <p><math>x^2 + y^2 + 8x + 4y - 38 = 0</math><br/> <math>x^2 + 8x + 16 + y^2 + 4y + 4 = 16 + 4 + 38</math><br/> <math>(x + 4)^2 + (y + 2)^2 = 58</math><br/> Middelpunt is <math>(-4 ; -2)</math> en die radius is <math>\sqrt{58}</math></p>  | <p>✓ voltooi die kwadraat<br/> (een of beide)<br/> ✓ faktorvorm<br/> ✓ middelpunt<br/> ✓ radius<br/> (4)</p>                      |
| 6.2 | <p>Middelpunt van tweede sirkel is <math>(4 ; 6)</math><br/> Afstand tussen middelpunte is <math>\sqrt{(4 + 4)^2 + (6 + 2)^2} = \sqrt{128} = 11,31</math></p>   | <p>✓ middelpunt<br/> ✓ afstand<br/> (2)</p>   |
| 6.3 | <p>Som van radiusse = <math>\sqrt{58} + \sqrt{26} = 12,71</math><br/> Afstand tussen middelpunte is 11,31.<br/> Som van die radiusse &gt; afstand tussen die middelpunte<br/> <math>\therefore</math> die sirkels moet oorvleuel en dus moet die sirkels mekaar sny.</p>  | <p>✓✓ som van radiusse<br/> ✓ gevolgtrekking<br/> (3)</p>   |
| 6.4 | <p>Vergelyking van tweede sirkel:<br/> <math>(x - 4)^2 + (y - 6)^2 = 26</math><br/> <math>x^2 - 8x + 16 + y^2 - 12y + 36 = 26</math><br/> <math>x^2 - 8x + y^2 - 12y + 26 = 0</math></p> <p>Laat <math>(x ; y)</math> enige van die twee punte op die snypunte wees.<br/> Dan<br/> <math>x^2 + y^2 + 8x + 4y - 38 = 0</math><br/> en <math>x^2 + y^2 - 8x - 12y + 26 = 0</math></p> <hr/> <p>Trek af <math>16y + 16x - 64 = 0</math><br/> <math>y = -x + 4</math></p> <p>Beide snypunte lê op hierdie lyn.<br/> <math>\therefore y = -x + 4</math> is die vergelyking van die gemeenskaplike koord</p> <p><b>OF</b></p> | <p>✓ vergelyking van sirkel in vorm = 0</p> <p>✓ stelling – twee snypunte<br/> ✓ aftrekking</p> <p>✓ vereenvoudiging<br/> (4)</p> |

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|  | <p>Toets of die lyn <math>y = -x + 4</math> die twee sirkels op dieselfde punte sny:</p> $(x - 4)^2 + (-x - 2)^2 = 26$ $x^2 - 8x + 16 + x^2 + 4x + 4 = 26$ $2x^2 - 4x - 6 = 0$ $x^2 - 2x - 3 = 0$ $(x - 3)(x + 1) = 0$ $x = 3 \text{ of } x = -1$<br>$x^2 + y^2 + 8x + 4y - 38 = 0$ $x^2 + (4 - x)^2 + 8x + 4(4 - x) - 38 = 0$ $x^2 + 16 - 8x + x^2 + 8x + 16 - 4x - 38 = 0$ $2x^2 - 4x - 6 = 0$ $x^2 - 2x - 3 = 0$ $x = 3 \text{ of } x = -1$ | <p>✓ vervanging</p><br><p>✓ antwoord</p><br><p>✓ vervanging</p><br><p>✓ antwoord</p> <p style="text-align: right;">(4)</p> <p style="text-align: right;"><b>[13]</b></p> |
|--|--|--|

**VRAAG 7**

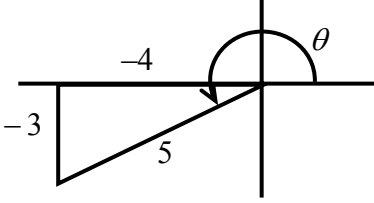
|       |   |   |
|-------|---|---|
| 7.1.1 | $P'(5; -2)$   | <p>✓ antwoord</p> <p style="text-align: right;">(1)</p>   |
| 7.1.2 | $P'(5; 2)$  | <p>✓ <math>x</math>-koördinaat</p> <p>✓ <math>y</math>-koördinaat</p> <p style="text-align: right;">(2)</p>   |
| 7.2.1 | <p><math>K \rightarrow K'' : (14; 4) \rightarrow (2; 2)</math><br/> <math>U \rightarrow U'' : (18; 6) \rightarrow (3; 9)</math><br/> <math>H \rightarrow H'' : (16; 8) \rightarrow (4; 8)</math><br/> <math>L \rightarrow L'' : (18; 10) \rightarrow (5; 9)</math><br/> <math>E \rightarrow E'' : (14; 12) \rightarrow (6; 7)</math></p> <p>Dus ‘halveer’ en ‘ruil koördinate in posisie’ of ‘ruil koördinate in posisie’ en ‘halveer’.</p> <p>Refleksie oor <math>y = x</math> gevolg deur kontraksie by <math>\frac{1}{2}</math></p> <p><b>OF</b></p> <p>Kontraksie by <math>\frac{1}{2}</math> gevolg deur refleksie oor <math>y = x</math>.</p> | <p>✓ gereflekteer</p> <p>✓ die lyn <math>y = x</math></p> <p>✓ vergroot</p> <p>✓ skaalfaktor van <math>\frac{1}{2}</math></p> <p style="text-align: right;">(4)</p> |
| 7.2.2 | <p><math>H' = \frac{1}{2}(16; 8) = (8; 4)</math></p> <p>OF <math>H'(8; 16)</math></p>   | <p>✓ (8 ; 4)</p> <p>✓ (8 ; 16)</p> <p style="text-align: right;">(2)</p>  |
| 7.2.3 | <p>Oppervlakte KUHLE : Oppervlakte <math>K''U''H''L''E'' = \left(\frac{2}{1}\right)^2 = 4 : 1</math></p>  | <p>✓ ✓ antwoord</p> <p style="text-align: right;">(2)</p> <p style="text-align: right;"><b>[11]</b></p>   |



**VRAAG 8**

|     |  |  |
|-----|--|--|
| 8.1 | <p>Vir antikloksgewyse rotasie:</p> $x' = x \cos \theta - y \sin \theta$ $= 3 \cos 120^\circ - 2 \sin 120^\circ$ $= 3(-\cos 60^\circ) - 2 \sin 60^\circ$ $= 3\left(-\frac{1}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{-3 - 2\sqrt{3}}{2}$ $y' = x \sin \theta + y \cos \theta$ $= 3 \sin 120^\circ + 2 \cos 120^\circ$ $= 3 \sin 60^\circ + 2(-\cos 60^\circ)$ $= 3\left(\frac{\sqrt{3}}{2}\right) + 2\left(-\frac{1}{2}\right)$ $= \frac{3\sqrt{3} - 2}{2}$ $P\left(\frac{-3 - 2\sqrt{3}}{2}; \frac{3\sqrt{3} - 2}{2}\right)$ | <p>✓ formule</p> <p>✓ vereenvoudiging<br/>✓ vervanging</p> <p>✓ antwoord</p> <p>✓ vereenvoudiging</p> <p>✓ antwoord</p> <p>(6)</p>   |
| 8.2 | $-2 = x\left(-\frac{1}{2}\right) - y\left(\frac{\sqrt{3}}{2}\right)$ $-4 = -x - \sqrt{3}y \quad \dots\dots \text{vergelyking 1}$ $0 = x\left(\frac{\sqrt{3}}{2}\right) + y\left(-\frac{1}{2}\right)$ $0 = \sqrt{3}x + y$ $y = -\sqrt{3}x \quad \dots\dots \text{vergelyking 2}$ <p>Vervang vergelyking 2 in vergelyking 1</p> $-4 = -x - \sqrt{3}(-\sqrt{3}x)$ $-4 = -x + 3x$ $-4 = 2x$ $x = -2$ $y = 2\sqrt{3}$ $Q(-2; 2\sqrt{3})$  | <p>✓ <math>-4 = -x - \sqrt{3}y</math></p> <p>✓ <math>y = -\sqrt{3}x</math></p> <p>✓ <math>x</math>-koördinaat<br/>✓ <math>y</math>-koördinaat</p> <p>(4)<br/><b>[10]</b></p> |

**VRAAG 9**

|              |   |  |
|--------------|---|--|
| <p>9.1.1</p> | $\sin \theta = -\frac{3}{5} \text{ en } \cos \theta = -\frac{4}{5}$ $\sin \theta + \cos \theta = -\frac{7}{5}$    | <p>✓ korrekte kwadrant en waardes.</p> <p>✓ <math>\sin \theta = -\frac{3}{5}</math></p> <p>✓ <math>\cos \theta = -\frac{4}{5}</math></p> <p>✓ antwoord</p> <p style="text-align: right;">(4)</p>   |
| <p>9.1.2</p> | $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta}$ $= \frac{2 \left(-\frac{3}{5}\right) \left(-\frac{4}{5}\right)}{\frac{16}{25} - \frac{9}{25}}$ $= \frac{24}{7}$ <p><b>OF</b></p> $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$ $= \frac{2 \left(\frac{3}{4}\right)}{1 - \left(\frac{3}{4}\right)^2}$ $= \frac{24}{7}$ | <p>✓ <math>\frac{\sin 2\theta}{\cos 2\theta}</math></p> <p>✓ <math>\sin 2\theta = 2 \sin \theta \cdot \cos \theta</math></p> <p>✓ <math>\cos 2\theta = \cos^2 \theta - \sin^2 \theta</math></p> <p>✓ vervanging</p> <p>✓ antwoord</p> <p style="text-align: right;">(5)</p> <p>✓✓ uitbreiding</p> <p>✓✓ vervanging</p> <p>✓ antwoord</p> <p style="text-align: right;">(5)</p> |
| <p>9.2.1</p> | $\frac{\cos(360^\circ - x) \cdot \tan^2 x}{\sin(x - 180^\circ) \cdot \cos(90^\circ + x)}$ $= \frac{(\cos x)(\tan^2 x)}{(-\sin x)(-\sin x)}$ $= (\cos x) \left(\frac{\sin^2 x}{\cos^2 x}\right) \left(\frac{1}{\sin^2 x}\right)$ $= \frac{1}{\cos x}$  | <p>✓ <math>\cos x</math></p> <p>✓ <math>-\sin x</math></p> <p>✓ <math>-\sin x</math></p> <p>✓ <math>\frac{\sin^2 x}{\cos^2 x}</math></p> <p>✓ antwoord</p> <p style="text-align: right;">(5)</p>   |
| <p>9.2.2</p> | $x = 30^\circ$ $\frac{1}{\cos 30^\circ} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$  | <p>✓ <math>x = 30^\circ</math></p> <p>✓ antwoord</p> <p style="text-align: right;">(2)</p> <p style="text-align: right;"><b>[16]</b></p>   |

**VRAAG 10**

|        |  |   |
|--------|--|---|
| 10.1.1 | $\sin 48^\circ = \sin(36^\circ + 12^\circ)$ $= \sin 36^\circ \cos 12^\circ + \cos 36^\circ \sin 12^\circ$ $= p + q$  | ✓ skryf $48^\circ$ in terme van $36^\circ$ en $12^\circ$<br>✓ uitbreiding<br>✓ antwoord<br>(3)  |
| 10.1.2 | $\sin 24^\circ = \sin(36^\circ - 12^\circ)$ $= \sin 36^\circ \cos 12^\circ - \cos 36^\circ \sin 12^\circ$ $= p - q$ <p><b>OF</b></p> $\sin 24^\circ = \sin(36^\circ - 12^\circ)$ $= \sin 36^\circ \cos 12^\circ - \cos 36^\circ \sin 12^\circ$ $= p - q$   | ✓ skryf $24^\circ$ in terme van $36^\circ$ en $12^\circ$<br>✓ uitbreiding<br>✓ $\sin 24^\circ = p - q$<br>(3)<br><br>✓ skryf $24^\circ$ in terme van $36^\circ$ en $12^\circ$<br>✓ uitbreiding<br>✓ $\sin 24^\circ = p - q$<br>(3)  |
| 10.1.3 | $\sin 48^\circ = 2 \sin 24^\circ \cos 24^\circ$ $\therefore p + q = 2(p - q) \cos 24^\circ$ $\therefore \cos 24^\circ = \frac{p + q}{2(p - q)}$ <p><b>OF</b></p> $\cos 48^\circ = 2 \cos^2 24^\circ - 1$ $\therefore \cos 24^\circ = \sqrt{\frac{1 + \cos 48^\circ}{2}} = \sqrt{\frac{1}{2} \left( 1 + \sqrt{1 - \sin^2 48^\circ} \right)}$ $= \sqrt{\frac{1}{2} \left( 1 + \sqrt{1 - (p + q)^2} \right)}$ <p><b>OF</b></p> $\cos^2 24^\circ = 1 - \sin^2 24^\circ$ $\cos^2 24^\circ = 1 - (p - q)^2$ $\cos 24^\circ = \sqrt{1 - (p - q)^2}$ | ✓ $\cos 48^\circ = 2 \cos^2 24^\circ - 1$<br>✓ $\sin 48^\circ = p + q$<br>✓ antwoord<br>(3)<br><br>✓ $\cos 48^\circ = 2 \cos^2 24^\circ - 1$<br>✓ $\sin 24^\circ = p - q$<br>✓ antwoord<br>(3)<br><br>✓ $\cos^2 24^\circ = 1 - \sin^2 24^\circ$<br>✓ $\sin 24^\circ = p - q$<br>✓ antwoord<br>(3) |

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| <p>10.2</p> | $\begin{aligned} & \sin^2 20^\circ + \sin^2 40^\circ + \sin^2 80^\circ \\ &= \sin^2 20^\circ + (\sin(60^\circ - 20^\circ))^2 + (\sin(60^\circ + 20^\circ))^2 \\ &= \sin^2 20^\circ + (\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ)^2 + (\sin 60^\circ \cos 20^\circ + \cos 60^\circ \sin 20^\circ)^2 \\ &= \sin^2 20^\circ + \left(\frac{\sqrt{3}}{2} \cos 20^\circ - \frac{1}{2} \sin 20^\circ\right)^2 + \left(\frac{\sqrt{3}}{2} \cos 20^\circ + \frac{1}{2} \sin 20^\circ\right)^2 \\ &= \sin^2 20^\circ + \frac{3}{4} \cos^2 20^\circ - \frac{\sqrt{3}}{2} \cos 20^\circ \sin 20^\circ + \frac{1}{4} \sin^2 20^\circ + \frac{3}{4} \cos^2 20^\circ \\ &\quad + \frac{\sqrt{3}}{2} \cos 20^\circ \sin 20^\circ + \frac{1}{4} \sin^2 20^\circ \\ &= \sin^2 20^\circ + \frac{3}{2} \cos^2 20^\circ + \frac{1}{2} \sin^2 20^\circ \\ &= \frac{3}{2} (\sin^2 20^\circ + \cos^2 20^\circ) \\ &= \frac{3}{2} \end{aligned}$ <p><b>OF</b></p> <p>Gebruik <math>\sin^2 \theta = \frac{1 - \cos 2\theta}{2}</math></p> <p><i>LHS</i></p> $\begin{aligned} &= \frac{3}{2} - \frac{1}{2} \{(\cos 40^\circ + \cos 80^\circ) + \cos 160^\circ\} \\ &= \frac{3}{2} - \frac{1}{2} \{(\cos 60^\circ \cdot \cos 40^\circ + \sin 60^\circ \sin 40^\circ + \cos 60^\circ \cdot \cos 40^\circ - \sin 60^\circ \sin 40^\circ) + \cos 160^\circ\} \\ &= \frac{3}{2} - \frac{1}{2} \{(2 \cos 60^\circ \cos 20^\circ) - \cos 20^\circ\} \\ &= \frac{3}{2} - \frac{1}{2} \left\{ \left( 2 \times \frac{1}{2} \cos 20^\circ \right) - \cos 20^\circ \right\} \\ &= \frac{3}{2} - 0 \\ &= \frac{3}{2} \end{aligned}$ | <p>✓ <math>40^\circ = 60^\circ - 20^\circ</math><br/>✓ <math>80^\circ = 60^\circ + 20^\circ</math></p> <p>✓ ✓ uitbreidings<br/>✓ vervanging</p> <p>✓ vereenvoudiging</p> <p>✓ faktorisering</p> <p>(7)</p> <p>✓ <math>40^\circ = 60^\circ - 20^\circ</math><br/>✓ <math>80^\circ = 60^\circ + 20^\circ</math></p> <p>✓ uitbreiding van <math>\cos 40^\circ</math><br/>✓ uitbreiding van <math>\cos 60^\circ</math><br/>✓ vereenvoudiging</p> <p>✓ vereenvoudiging</p> <p>✓ antwoord vir hake</p> <p>(7)</p> |
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| 10.3.1 | $\frac{\sin^4 x + \sin^2 x \cos^2 x}{1 + \cos x}$ $= \frac{\sin^2 x(\sin^2 x + \cos^2 x)}{1 + \cos x}$ $= \frac{\sin^2 x}{1 + \cos x}$ $= \frac{1 - \cos^2 x}{1 + \cos x}$ $= \frac{(1 - \cos x)(1 + \cos x)}{(1 + \cos x)}$ $= 1 - \cos x$ | <p>✓ faktorisering</p> <p>✓ <math>\sin^2 x + \cos^2 x = 1</math></p> <p>✓ identiteit</p> <p>✓ faktorisering</p> <p>(4)</p> |
| 10.3.2 | $1 + \cos x = 0$ $\cos x = -1$ $x = 180^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ <p>Ongedefinieer vir <math>x = 180^\circ + k \cdot 360^\circ; k \in \mathbb{Z}</math>.</p>   | <p>✓ <math>1 + \cos x = 0</math></p> <p>✓ <math>180^\circ + k \cdot 360^\circ</math></p> <p>(2)</p> <p>[22]</p>            |

**VRAAG 11**

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| 11.1 | $1 + \sin x = \cos 2x$ $1 + \sin x = 1 - 2 \sin^2 x$ $\sin x + 2 \sin^2 x = 0$ $\sin x(1 + 2 \sin x) = 0$ $\sin x = 0 \quad \text{of} \quad \sin x = -\frac{1}{2},$ $x = k \cdot 180 \quad \text{of} \quad \begin{aligned} x &= -30^\circ + k \cdot 360 \\ x &= 210^\circ + k \cdot 360 \end{aligned} \quad k \in \mathbb{Z}$ $x \in \{180^\circ; 210^\circ; 330^\circ; 360^\circ\}$ <p><b>OF</b></p> $1 + \sin x = \cos 2x$ $1 + \sin x = \cos^2 x - \sin^2 x$ $1 + \sin x = 1 - \sin^2 x - \sin^2 x$ $\sin x + 2 \sin^2 x = 0$ $\sin x(1 + 2 \sin x) = 0$ $\sin x = 0 \quad \text{of} \quad \sin x = -\frac{1}{2},$ $x = k \cdot 180 \quad \text{of} \quad \begin{aligned} x &= -30^\circ + k \cdot 360 \\ x &= 210^\circ + k \cdot 360 \end{aligned} \quad k \in \mathbb{Z}$ $x \in \{180^\circ; 210^\circ; 330^\circ; 360^\circ\}$ | <p>✓ uitbreiding</p> <p>✓ faktorisering</p> <p>✓ vergelykings</p> <p>✓ <math>x = k \cdot 180</math></p> <p>✓ oplossing vir <math>\sin x = -\frac{1}{2}</math></p> <p>✓✓ antwoorde</p> <p>(7)</p> <p>✓ uitbreiding</p> <p>✓ faktorisering</p> <p>✓ vergelykings</p> <p>✓ <math>x = k \cdot 180</math></p> <p>✓ oplossing vir <math>\sin x = -\frac{1}{2}</math></p> <p>✓✓ antwoorde</p> <p>(7)</p> |
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| 11.2 |  | <p> <math>1 + \sin x</math><br/>                     ✓ maks en min waardes<br/>                     ✓ vorm<br/> <math>\cos 2x</math><br/>                     ✓ amplitude<br/>                     ✓ snypunte                 </p> <p style="text-align: right;">(4)</p> |
| 11.3 | $180^\circ \leq x \leq 210^\circ$ or $330^\circ \leq x \leq 360^\circ$ | <p>✓✓✓ antwoord</p> <p style="text-align: right;">(3)<br/>[14]</p>   |

**VRAAG 12**

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| 12.1 | $\frac{b}{\sin[180^\circ - (\alpha + \beta)]} = \frac{BC}{\sin \alpha}$ $BC \sin(\alpha + \beta) = b \sin \alpha$ $BC = \frac{b \sin \alpha}{\sin(\alpha + \beta)}$ <p>maar <math>BC = DF</math></p> $\therefore DF = \frac{b \sin \alpha}{\sin(\alpha + \beta)}$ $\cos \theta = \frac{DF}{DE}$ $\therefore DE = \frac{DF}{\cos \theta}$ $\therefore DE = \frac{b \sin \alpha}{\sin(\alpha + \beta) \cos \theta}$ | <p>                     ✓ sinusreël<br/>                     ✓<br/> <math>\hat{A}BC = 180^\circ - (\alpha + \beta)</math><br/>                     ✓ <math>BC = \dots</math><br/>                     ✓ <math>BC = DF</math><br/>                     ✓ manipulering<br/>                     ✓ <math>DE = \dots</math> </p> <p style="text-align: right;">(6)</p> |
| 12.2 | $DE = \frac{2000 \sin 43^\circ}{\sin 79^\circ \cdot \cos 27^\circ}$ $= 1559,50 \text{ m}$   | <p>                     ✓ vervanging van noemer<br/>                     ✓ vervanging van deler<br/>                     ✓ antwoord                 </p> <p style="text-align: right;">(3)<br/>[9]</p>   |

**TOTAAL: 150**