



# education

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Department:  
Education  
**REPUBLIC OF SOUTH AFRICA**

**NATIONAL  
SENIOR CERTIFICATE**

**GRADE 12**

**MATHEMATICS P2**

**FEBRUARY/MARCH 2010**

**MEMORANDUM**

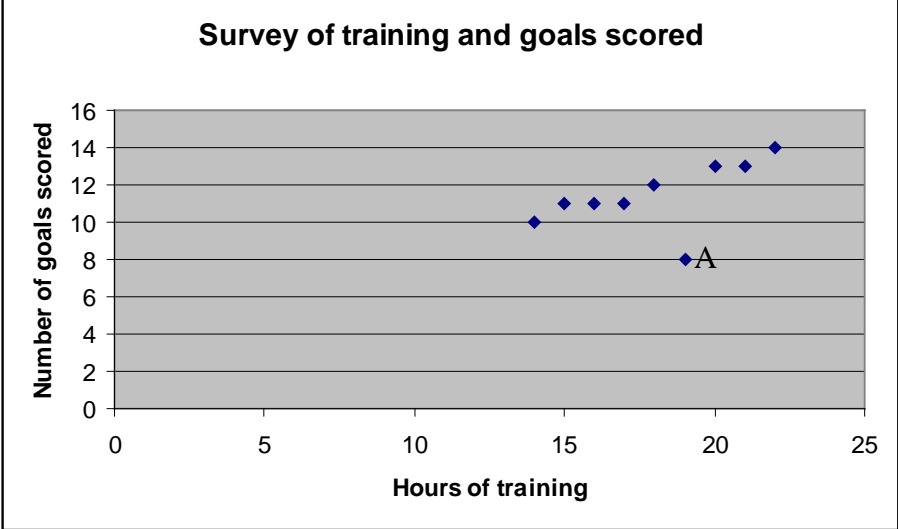
**MARKS: 150**

**This memorandum consists of 14 pages.**

**QUESTION 1**

1.1	Range = $26 - 4 = 22$	✓ maximum and minimum values ✓ answer ANSWER ONLY: Full Marks (2)
1.2	Mean $= \frac{4 + 5 + 8 + 13 + 19 + 22 + 25 + 26 + 23 + 17 + 14 + 7}{12}$ $= \frac{183}{12}$ $= 15,25$	✓ method ✓ 183 ✓ answer (3)
1.3	Standard deviation = 7,6 (7,59522.....)	✓✓ answer (2)
1.4.1	Increase in mean = $\frac{(3 \times 5) + (9 \times 1)}{12}$ $= 2^{\circ}\text{C per month.}$	✓✓ answer (2)
1.4.2	The maximum value increases by $1^{\circ}\text{C}$ and the minimum value increases by $5^{\circ}\text{C}$ . This implies that the range of the range of the data will now decrease. This will result in the standard deviation getting smaller. (new SD = 6,27.....)	✓ decrease in range ✓ decrease in standard deviation (2) <b>[11]</b>

**QUESTION 2**

<p>2.1.1</p>	<p style="text-align: center;"><b>Survey of training and goals scored</b></p> 	<p>✓✓✓ plotting the points</p> <p>All 9 point correct – 3 marks 5 or 7 points correct – 2 marks 1 or 2 points correct – 1 mark 0 points correct – 0 marks</p> <p style="text-align: right;">(3)</p>
<p>2.1.2</p>	<p>A(indicated on the graph)</p>	<p>✓ answer</p> <p style="text-align: right;">(1)</p>
<p>2.1.3</p>	<p>8 Goals</p>	<p>✓✓ answer</p> <p style="text-align: right;">(2)</p>
<p>2.2</p>	<p>Let the mean time for all 560 learners be <math>x</math>. Then the mean time for the learners living in neighbourhood C is also <math>x</math>.</p> $x = \frac{(135 \times 24) + (225 \times 32) + (200 \times x)}{560}$ $560x = 3240 + 7200 + 200x$ $360x = 10440$ $x = 29$	<p>✓ equal mean times</p> <p>✓ mean <math>\times</math> number</p> <p>✓ simplification</p> <p>✓ answer</p> <p style="text-align: right;">(4) <b>[10]</b></p>

**QUESTION 3**

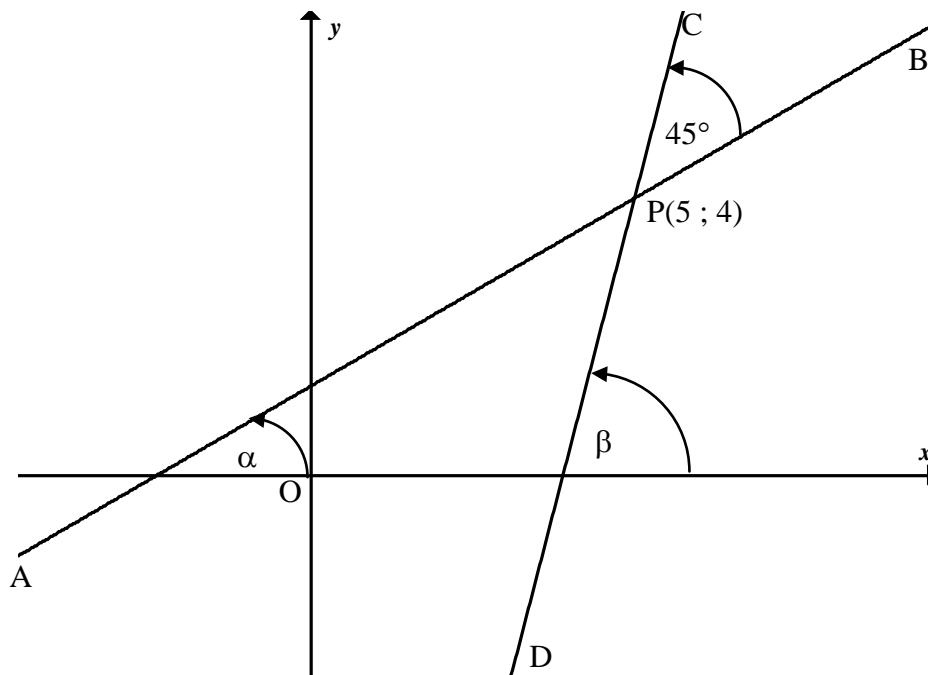
<p>3.1</p>	<table border="1"> <thead> <tr> <th>Time (in minutes)</th> <th><math>11 \leq t &lt; 15</math></th> <th><math>15 \leq t &lt; 19</math></th> <th><math>19 \leq t &lt; 23</math></th> <th><math>23 \leq t &lt; 27</math></th> <th><math>27 \leq t &lt; 30</math></th> </tr> </thead> <tbody> <tr> <td>Frequency</td> <td>6</td> <td>9</td> <td>13</td> <td>12</td> <td>8</td> </tr> <tr> <td>Cumulative Frequency</td> <td>6</td> <td>15</td> <td>28</td> <td>40</td> <td>48</td> </tr> </tbody> </table>	Time (in minutes)	$11 \leq t < 15$	$15 \leq t < 19$	$19 \leq t < 23$	$23 \leq t < 27$	$27 \leq t < 30$	Frequency	6	9	13	12	8	Cumulative Frequency	6	15	28	40	48	<p>✓ cumulative frequency totals (1)</p>
Time (in minutes)	$11 \leq t < 15$	$15 \leq t < 19$	$19 \leq t < 23$	$23 \leq t < 27$	$27 \leq t < 30$															
Frequency	6	9	13	12	8															
Cumulative Frequency	6	15	28	40	48															
<p>3.2</p>	<p style="text-align: center;"><b>Cumulative Frequency Curve showing the time taken to complete a task</b></p>	<p>✓✓✓ plotting points at upper limits 6 correct – 3 marks 3 to 5 correct – 2 marks 1 or 2 correct – 1 mark 0 correct – 0 marks  ✓ curve (4)</p>																		
<p>3.3</p>	<p>Median value at position 24. Reading off the ogive gives Median <math>\approx</math> 22 minutes LQ value at position 12. Lower quartile <math>\approx</math> 18 minutes (from ogive) UQ value at position 36. Upper quartile <math>\approx</math> 25,5 minutes (from ogive)</p> <p><b>NOTE:</b> Allow margin of error for reading off the graph.</p>	<p>✓ median ✓ lower quartile ✓ upper quartile (3)</p>																		
<p>3.4</p>		<p>✓ box ✓ whiskers (2)</p>																		
<p>3.5</p>	<p>The times are skewed to the right. A small number of people finished this task very quickly whilst others took more time.</p>	<p>✓ skewed to the right (1) <b>[11]</b></p>																		

**QUESTION 4**

4.1	$m_{PQ} = \frac{2-0}{0-4} = -\frac{1}{2}$	<ul style="list-style-type: none"> <li>✓ substitution</li> </ul> <p style="text-align: right;">(1)</p>
4.2	<p>A: <math>\left(\frac{0+4}{2}; \frac{2+0}{2}\right)</math></p> <p>A (2 ; 1)</p>	<ul style="list-style-type: none"> <li>✓ x-coordinate</li> <li>✓ y-coordinate</li> </ul> <p style="text-align: right;">(2)</p>
4.3	<p><math>m_{AB} \cdot m_{PQ} = -1</math></p> <p><math>m_{AB} \cdot (-1/2) = -1, \therefore m_{AB} = 2</math></p> <p>Equation of AB is <math>y = 2x + c</math></p> <p><math>\therefore 1 = 2(2) + c</math></p> <p><math>c = -3</math></p> <p>Equation of AB is <math>y = 2x - 3</math>.</p> <p><b>OR</b></p> <p><math>m_{AB} \cdot m_{PQ} = -1</math></p> <p><math>m_{AB} \cdot (-1/2) = -1, \therefore m_{AB} = 2</math></p> <p><math>y - 1 = 2(x - 2)</math></p> <p><math>y - 1 = 2x - 4</math></p> <p><math>y = 2x - 3</math></p>	<ul style="list-style-type: none"> <li>✓ <math>m_{AB} \cdot m_{PQ} = -1</math></li> <li>✓ <math>m_{AB} = 2</math></li> <li>✓ equation of AB</li> <li>✓ <math>y = 2x - 3</math></li> <li>✓ <math>c = -3</math></li> </ul> <p style="text-align: right;">(5)</p> <ul style="list-style-type: none"> <li>✓ <math>m_{AB} \cdot m_{PQ} = -1</math></li> <li>✓ <math>m_{AB} = 2</math></li> <li>✓ gradient of AB</li> <li>✓ substitution into formula</li> <li>✓ equation of AB</li> </ul> <p style="text-align: right;">(5)</p>
4.4	<p>B is the point (0 ; -3)</p> <p><math>BQ = \sqrt{(0-4)^2 + (-3-0)^2}</math></p> <p>= 5</p>	<ul style="list-style-type: none"> <li>✓ coordinates of B</li> <li>✓ substitution</li> <li>✓ answer</li> </ul> <p style="text-align: right;">(3)</p>
4.5	<p><math>BP = \sqrt{(0-0)^2 + (-3-2)^2}</math></p> <p>= 5</p> <p>BP = BQ</p> <p><math>\therefore \triangle BPQ</math> is isosceles.</p> <p><b>OR</b></p> <p>BP = 2 + 3</p> <p>= 5</p> <p>BP = BQ</p> <p><math>\therefore \triangle BPQ</math> is isosceles</p>	<ul style="list-style-type: none"> <li>✓ BP = 5</li> <li>✓ BP = BQ</li> </ul> <p style="text-align: right;">(2)</p> <ul style="list-style-type: none"> <li>✓ BP = 5</li> <li>✓ BP = BQ</li> </ul> <p style="text-align: right;">(2)</p>
4.6	<p>If PBQR is a rhombus then A is the midpoint of BR.</p> <p>Let the coordinates of R be (x ; y)</p> <p><math>\frac{x+0}{2} = 2</math>      and      <math>\frac{y-3}{2} = 1</math></p> <p><math>x = 4</math>                      <math>y = 5</math></p> <p><math>\therefore R(4 ; 5)</math></p> <p><b>OR</b></p> <p>RQ    PB so <math>x_R = 4</math></p> <p>RQ = PB = 5, so <math>y_R = 5</math></p> <p><math>\therefore R(4 ; 5)</math></p>	<ul style="list-style-type: none"> <li>✓ A is the midpoint of BR</li> </ul> <ul style="list-style-type: none"> <li>✓ x coordinate</li> <li>✓ y coordinate</li> </ul> <p style="text-align: right;">(3)</p> <ul style="list-style-type: none"> <li>✓ RQ    PB</li> <li>✓ x coordinate</li> <li>✓ y coordinate</li> </ul> <p style="text-align: right;">(3)</p>

**[16]**

**QUESTION 5**



<p>5.1</p> <p>AB is defined as <math>5y - 3x - 5 = 0</math> which can be written as <math>y = \frac{3}{5}x + 1</math></p> <p><math>m_{AB} = \frac{3}{5}</math></p> <p>Let <math>\alpha</math> be the inclination of AB.</p> <p><math>\tan \alpha = \frac{3}{5}</math></p> <p><math>\alpha = 30,96^\circ</math>.</p> <p>Let <math>\beta</math> be the inclination of CD</p> <p><math>\beta = 45^\circ + 30,96^\circ</math></p> <p><math>= 75,96^\circ</math></p> <p>Gradient of CD = <math>\tan 75,96^\circ = 4</math>.</p> <p><b>OR</b></p> <p><math>\tan \beta = \tan(\alpha + 45^\circ)</math></p> $= \frac{\tan \alpha + \tan 45^\circ}{1 - \tan \alpha \cdot \tan 45^\circ}$ $= \frac{\frac{3}{5} + 1}{1 - \frac{3}{5} \times 1}$ $= 4$ <p><math>m_{CD} = \tan \beta</math></p> <p><math>m_{CD} = 4</math></p>	<p><math>\checkmark m_{AB} = \frac{3}{5}</math></p> <p><math>\checkmark \tan \alpha = \frac{3}{5}</math></p> <p><math>\checkmark \alpha = 30,96^\circ</math></p> <p><math>\checkmark \beta = 75,96^\circ</math></p> <p><math>\checkmark</math> gradient of CD</p> <p style="text-align: right;">(5)</p> <p><math>\checkmark</math> expansion</p> <p><math>\checkmark \tan 45^\circ = 1</math></p> <p><math>\checkmark \tan \alpha = \frac{3}{5}</math></p> <p><math>\checkmark</math> substitution</p> <p><math>\checkmark</math> answer</p> <p style="text-align: right;">(5)</p>
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5.2	<p>Equation of CD is <math>y = 4x + c</math>  <math>\therefore 4 = 4(5) + c</math>  <math>c = -16</math>                  Equation of CD is <math>y = 4x - 16</math>.</p> <p><b>OR</b></p> <p><math>y - 4 = 4(x - 5)</math>  <math>y - 4 = 4x - 20</math>  <math>y = 4x - 16</math></p>	<p>✓ y- intercept                  ✓ equation of CD (2)</p> <p>✓ substitution                  ✓ equation of CD (2)</p> <p>[7]</p>
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**QUESTION 6**

6.1	<p><math>x^2 + y^2 + 8x + 4y - 38 = 0</math>  <math>x^2 + 8x + 16 + y^2 + 4y + 4 = 16 + 4 + 38</math>  <math>(x + 4)^2 + (y + 2)^2 = 58</math>                  Centre is <math>(-4 ; -2)</math> and the radius is <math>\sqrt{58}</math></p>	<p>✓ completing the square (both or one)                  ✓ factor form                  ✓ centre                  ✓ radius (4)</p>
6.2	<p>Centre of second circle is <math>(4 ; 6)</math>                  Distance between centres is <math>\sqrt{(4 + 4)^2 + (6 + 2)^2} = \sqrt{128} = 11,31</math></p>	<p>✓ centre                  ✓ distance (2)</p>
6.3	<p>Sum of radii = <math>\sqrt{58} + \sqrt{26} = 12,71</math>                  Distance between centres is 11,31.                  sum of the radii &gt; distance between the centres  <math>\therefore</math> the circles must overlap and hence the circles must intersect.</p>	<p>✓✓ sum of radii                  ✓ conclusion (3)</p>
6.4	<p>Equation of second circle:  <math>(x - 4)^2 + (y - 6)^2 = 26</math>  <math>x^2 - 8x + 16 + y^2 - 12y + 36 = 26</math>  <math>x^2 - 8x + y^2 - 12y + 26 = 0</math></p> <p>Let <math>(x ; y)</math> be either of the two points on intersection.                  Then  <math>x^2 + y^2 + 8x + 4y - 38 = 0</math>                  and <math>x^2 + y^2 - 8x - 12y + 26 = 0</math></p> <p>Subtract <math>\frac{16y + 16x - 64 = 0}{y = -x + 4}</math></p> <p>Both points of intersection lie on this line.  <math>\therefore y = -x + 4</math> is the equation of the common chord.</p> <p><b>OR</b></p>	<p>✓ equation of circle in form = 0                  ✓ statement – two points of intersection                  ✓ subtracting                  ✓ simplification (4)</p>

	<p>Check that the line <math>y = -x + 4</math> cuts the two circles at the same points:</p> $(x - 4)^2 + (-x - 2)^2 = 26$ $x^2 - 8x + 16 + x^2 + 4x + 4 = 26$ $2x^2 - 4x - 6 = 0$ $x^2 - 2x - 3 = 0$ $(x - 3)(x + 1) = 0$ $x = 3 \text{ or } x = -1$ $x^2 + y^2 + 8x + 4y - 38 = 0$ $x^2 + (4 - x)^2 + 8x + 4(4 - x) - 38 = 0$ $x^2 + 16 - 8x + x^2 + 8x + 16 - 4x - 38 = 0$ $2x^2 - 4x - 6 = 0$ $x^2 - 2x - 3 = 0$ $x = 3 \text{ or } x = -1$	<p>✓ substitution</p> <p>✓ answer</p> <p>✓ substitution</p> <p>✓ answer</p> <p style="text-align: right;">(4)</p> <p style="text-align: right;"><b>[13]</b></p>
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**QUESTION 7**

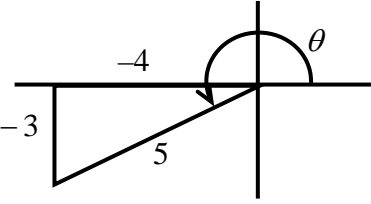
7.1.1	$P'(5; -2)$	<p>✓ answer</p> <p style="text-align: right;">(1)</p>
7.1.2	$P'(5; 2)$	<p>✓ x coordinate</p> <p>✓ y coordinate</p> <p style="text-align: right;">(2)</p>
7.2.1	$K \rightarrow K'' : (14; 4) \rightarrow (2; 2)$ $U \rightarrow U'' : (18; 6) \rightarrow (3; 9)$ $H \rightarrow H'' : (16; 8) \rightarrow (4; 8)$ $L \rightarrow L'' : (18; 10) \rightarrow (5; 9)$ $E \rightarrow E'' : (14; 12) \rightarrow (6; 7)$ So “halve” and ‘interchange” or ‘interchange” and “halve”.  Reflection across $y = x$ followed by contraction by $\frac{1}{2}$ OR Contraction by $\frac{1}{2}$ followed by reflection across $y = x$ .	<p>✓ reflected</p> <p>✓ the line <math>y = x</math></p> <p>✓ enlarged</p> <p>✓ scale factor of <math>\frac{1}{2}</math></p> <p style="text-align: right;">(4)</p>
7.2.2	$H' = \frac{1}{2}(16; 8) = (8; 4)$ OR $H'(8; 16)$	<p>✓ (8 ; 4)</p> <p>✓ (8 ; 16)</p> <p style="text-align: right;">(2)</p>
7.2.3	Area KUHLE : Area $K''U''H''L''E'' = \left(\frac{2}{1}\right)^2 = 4 : 1$	<p>✓ ✓ answer</p> <p style="text-align: right;">(2)</p> <p style="text-align: right;"><b>[11]</b></p>



**QUESTION 8**

<p>8.1</p>	<p>For anti-clockwise rotation:</p> $x' = x \cos \theta - y \sin \theta$ $= 3 \cos 120^\circ - 2 \sin 120^\circ$ $= 3(-\cos 60^\circ) - 2 \sin 60^\circ$ $= 3\left(-\frac{1}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{-3 - 2\sqrt{3}}{2}$ $y' = x \sin \theta + y \cos \theta$ $= 3 \sin 120^\circ + 2 \cos 120^\circ$ $= 3 \sin 60^\circ + 2(-\cos 60^\circ)$ $= 3\left(\frac{\sqrt{3}}{2}\right) + 2\left(-\frac{1}{2}\right)$ $= \frac{3\sqrt{3} - 2}{2}$ $P\left(\frac{-3 - 2\sqrt{3}}{2}; \frac{3\sqrt{3} - 2}{2}\right)$	<p>✓ formula</p> <p>✓ simplification ✓ substitution</p> <p>✓ answer</p> <p>✓ simplification</p> <p>✓ answer</p> <p>(6)</p>
<p>8.2</p>	$-2 = x\left(-\frac{1}{2}\right) - y\left(\frac{\sqrt{3}}{2}\right)$ $-4 = -x - \sqrt{3}y \quad \dots\dots \text{equation 1}$ $0 = x\left(\frac{\sqrt{3}}{2}\right) + y\left(-\frac{1}{2}\right)$ $0 = \sqrt{3}x + y$ $y = -\sqrt{3}x \quad \dots\dots \text{equation 2}$ <p>Substitute equation 2 into equation 1</p> $-4 = -x - \sqrt{3}(-\sqrt{3}x)$ $-4 = -x + 3x$ $-4 = 2x$ $x = -2$ $y = 2\sqrt{3}$ $Q(-2; 2\sqrt{3})$	<p>✓ <math>-4 = -x - \sqrt{3}y</math></p> <p>✓ <math>y = -\sqrt{3}x</math></p> <p>✓ x-coordinate ✓ y-coordinate</p> <p>(4) <b>[10]</b></p>

**QUESTION 9**

9.1.1	$\sin \theta = -\frac{3}{5} \text{ and } \cos \theta = -\frac{4}{5}$ $\sin \theta + \cos \theta = -\frac{7}{5}$ 	<ul style="list-style-type: none"> <li>✓ correct quadrant and values.</li> <li>✓ <math>\sin \theta = -\frac{3}{5}</math></li> <li>✓ <math>\cos \theta = -\frac{4}{5}</math></li> <li>✓ answer</li> </ul> <p style="text-align: right;">(4)</p>
9.1.2	$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta}$ $= \frac{2 \left(-\frac{3}{5}\right) \left(-\frac{4}{5}\right)}{\frac{16}{25} - \frac{9}{25}}$ $= \frac{24}{7}$ <p><b>OR</b></p> $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$ $= \frac{2 \left(\frac{3}{4}\right)}{1 - \left(\frac{3}{4}\right)^2}$ $= \frac{24}{7}$	<ul style="list-style-type: none"> <li>✓ <math>\frac{\sin 2\theta}{\cos 2\theta}</math></li> <li>✓ <math>\sin 2\theta = 2\sin\theta \cdot \cos\theta</math></li> <li>✓ <math>\cos 2\theta = \cos^2\theta - \sin^2\theta</math></li> <li>✓ substitution</li> <li>✓ answer</li> </ul> <p style="text-align: right;">(5)</p> <ul style="list-style-type: none"> <li>✓✓ expansion</li> <li>✓✓ substitution</li> <li>✓ answer</li> </ul> <p style="text-align: right;">(5)</p>
9.2.1	$\frac{\cos(360^\circ - x) \cdot \tan^2 x}{\sin(x - 180^\circ) \cdot \cos(90^\circ + x)}$ $= \frac{(\cos x)(\tan^2 x)}{(-\sin x)(-\sin x)}$ $= (\cos x) \left(\frac{\sin^2 x}{\cos^2 x}\right) \left(\frac{1}{\sin^2 x}\right)$ $= \frac{1}{\cos x}$	<ul style="list-style-type: none"> <li>✓ <math>\cos x</math></li> <li>✓ <math>-\sin x</math></li> <li>✓ <math>-\sin x</math></li> <li>✓ <math>\frac{\sin^2 x}{\cos^2 x}</math></li> <li>✓ answer</li> </ul> <p style="text-align: right;">(5)</p>
9.2.2	$x = 30^\circ$ $\frac{1}{\cos 30^\circ} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$	<ul style="list-style-type: none"> <li>✓ <math>x = 30^\circ</math></li> <li>✓ answer</li> </ul> <p style="text-align: right;">(2)</p>

**[16]**

**QUESTION 10**

<p>10.1.1</p>	$\sin 48^\circ = \sin(36^\circ + 12^\circ)$ $= \sin 36^\circ \cos 12^\circ + \cos 36^\circ \sin 12^\circ$ $= p + q$	<p>✓ writing <math>48^\circ</math> in terms of <math>36^\circ</math> and <math>12^\circ</math>                  ✓ expansion                  ✓ answer                  (3)</p>
<p>10.1.2</p>	$\sin 24^\circ = \sin(36^\circ - 12^\circ)$ $= \sin 36^\circ \cos 12^\circ - \cos 36^\circ \sin 12^\circ$ $= p - q$ <p><b>OR</b></p> $\sin 24^\circ = \sin(36^\circ - 12^\circ)$ $= \sin 36^\circ \cos 12^\circ - \cos 36^\circ \sin 12^\circ$ $= p - q$	<p>✓ writing <math>24^\circ</math> in terms of <math>36^\circ</math> and <math>12^\circ</math>                  ✓ expansion                  ✓ <math>\sin 24^\circ = p - q</math>                  (3)</p> <p>✓ writing <math>24^\circ</math> in terms of <math>36^\circ</math> and <math>12^\circ</math>                  ✓ expansion                  ✓ <math>\sin 24^\circ = p - q</math>                  (3)</p>
<p>10.1.3</p>	$\sin 48^\circ = 2 \sin 24^\circ \cos 24^\circ$ $\therefore p + q = 2(p - q) \cos 24^\circ$ $\therefore \cos 24^\circ = \frac{p + q}{2(p - q)}$ <p><b>OR</b></p> $\cos 48^\circ = 2 \cos^2 24^\circ - 1$ $\therefore \cos 24^\circ = \sqrt{\frac{1 + \cos 48^\circ}{2}} = \sqrt{\frac{1}{2} \left( 1 + \sqrt{1 - \sin^2 48^\circ} \right)}$ $= \sqrt{\frac{1}{2} \left( 1 + \sqrt{1 - (p + q)^2} \right)}$ <p><b>OR</b></p> $\cos^2 24^\circ = 1 - \sin^2 24^\circ$ $\cos^2 24^\circ = 1 - (p - q)^2$ $\cos 24^\circ = \sqrt{1 - (p - q)^2}$	<p>✓ <math>\cos 48^\circ = 2 \cos^2 24^\circ - 1</math>                  ✓ <math>\sin 48^\circ = p + q</math>                  ✓ answer                  (3)</p> <p>✓ <math>\cos 48^\circ = 2 \cos^2 24^\circ - 1</math>                  ✓ <math>\sin 24^\circ = p - q</math>                  ✓ answer                  (3)</p> <p>✓ <math>\cos^2 24^\circ = 1 - \sin^2 24^\circ</math>                  ✓ <math>\sin 24^\circ = p - q</math>                  ✓ answer                  (3)</p>

<p>10.2</p> $\begin{aligned} & \sin^2 20^\circ + \sin^2 40^\circ + \sin^2 80^\circ \\ &= \sin^2 20^\circ + (\sin(60^\circ - 20^\circ))^2 + (\sin(60^\circ + 20^\circ))^2 \\ &= \sin^2 20^\circ + (\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ)^2 + (\sin 60^\circ \cos 20^\circ + \cos 60^\circ \sin 20^\circ)^2 \\ &= \sin^2 20^\circ + \left(\frac{\sqrt{3}}{2} \cos 20^\circ - \frac{1}{2} \sin 20^\circ\right)^2 + \left(\frac{\sqrt{3}}{2} \cos 20^\circ + \frac{1}{2} \sin 20^\circ\right)^2 \\ &= \sin^2 20^\circ + \frac{3}{4} \cos^2 20^\circ - \frac{\sqrt{3}}{2} \cos 20^\circ \sin 20^\circ + \frac{1}{4} \sin^2 20^\circ + \frac{3}{4} \cos^2 20^\circ \\ &\quad + \frac{\sqrt{3}}{2} \cos 20^\circ \sin 20^\circ + \frac{1}{4} \sin^2 20^\circ \\ &= \sin^2 20^\circ + \frac{3}{2} \cos^2 20^\circ + \frac{1}{2} \sin^2 20^\circ \\ &= \frac{3}{2} (\sin^2 20^\circ + \cos^2 20^\circ) \\ &= \frac{3}{2} \end{aligned}$ <p><b>OR</b></p> <p>Use <math>\sin^2 \theta = \frac{1 - \cos 2\theta}{2}</math></p> <p><i>LHS</i></p> $\begin{aligned} &= \frac{3}{2} - \frac{1}{2} \{(\cos 40^\circ + \cos 80^\circ) + \cos 160^\circ\} \\ &= \frac{3}{2} - \frac{1}{2} \{(\cos 60^\circ \cdot \cos 40^\circ + \sin 60^\circ \sin 40^\circ + \cos 60^\circ \cdot \cos 40^\circ - \sin 60^\circ \sin 40^\circ) + \cos 160^\circ\} \\ &= \frac{3}{2} - \frac{1}{2} \{(2 \cos 60^\circ \cos 20^\circ) - \cos 20^\circ\} \\ &= \frac{3}{2} - \frac{1}{2} \left\{ \left( 2 \times \frac{1}{2} \cos 20^\circ \right) - \cos 20^\circ \right\} \\ &= \frac{3}{2} - 0 \\ &= \frac{3}{2} \end{aligned}$	<p>✓40°= 60° - 20° ✓80°= 60° + 20°</p> <p>✓ ✓expansions ✓ substitution</p> <p>✓ simplification ✓ factorisation</p> <p>(7)</p> <p>✓40°= 60° - 20° ✓80°= 60° + 20°</p> <p>✓ expansion of cos 40° ✓ expansion of cos 60° ✓ simplification ✓ simplification ✓ answer for bracket</p> <p>(7)</p>
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<p>10.3.1</p>	$\frac{\sin^4 x + \sin^2 x \cos^2 x}{1 + \cos x}$ $= \frac{\sin^2 x(\sin^2 x + \cos^2 x)}{1 + \cos x}$ $= \frac{\sin^2 x}{1 + \cos x}$ $= \frac{1 - \cos^2 x}{1 + \cos x}$ $= \frac{(1 - \cos x)(1 + \cos x)}{(1 + \cos x)}$ $= 1 - \cos x$	<p>✓ factorisation</p> <p>✓ <math>\sin^2 x + \cos^2 x = 1</math></p> <p>✓ identity</p> <p>✓ factorisation</p> <p>(4)</p>
<p>10.3.2</p>	<p><math>1 + \cos x = 0</math></p> <p><math>\cos x = -1</math></p> <p><math>x = 180^\circ + k.360^\circ; k \in Z</math></p> <p>Undefined for <math>x = 180^\circ + k.360^\circ; k \in Z</math>.</p>	<p>✓ <math>1 + \cos x = 0</math></p> <p>✓ <math>180^\circ + k.360^\circ</math></p> <p>(2)</p> <p><b>[22]</b></p>

**QUESTION 11**

<p>11.1</p>	<p><math>1 + \sin x = \cos 2x</math></p> <p><math>1 + \sin x = 1 - 2\sin^2 x</math></p> <p><math>\sin x + 2\sin^2 x = 0</math></p> <p><math>\sin x(1 + 2\sin x) = 0</math></p> <p><math>\sin x = 0</math>      or      <math>\sin x = -\frac{1}{2}</math>,</p> <p><math>x = k.180</math>      or      <math>x = -30^\circ + k.360</math>      <math>k \in Z</math></p> <p style="padding-left: 150px;"><math>x = 210^\circ + k.360</math></p> <p><math>x \in \{180^\circ; 210; 330^\circ; 360^\circ\}</math></p> <p><b>OR</b></p> <p><math>1 + \sin x = \cos 2x</math></p> <p><math>1 + \sin x = \cos^2 x - \sin^2 x</math></p> <p><math>1 + \sin x = 1 - \sin^2 x - \sin^2 x</math></p> <p><math>\sin x + 2\sin^2 x = 0</math></p> <p><math>\sin x(1 + 2\sin x) = 0</math></p> <p><math>\sin x = 0</math>      or      <math>\sin x = -\frac{1}{2}</math>,</p> <p><math>x = k.180</math>      or      <math>x = -30^\circ + k.360</math>      <math>k \in Z</math></p> <p style="padding-left: 150px;"><math>x = 210^\circ + k.360</math></p> <p><math>x \in \{180^\circ; 210; 330^\circ; 360^\circ\}</math></p>	<p>✓ expansion</p> <p>✓ factorisation</p> <p>✓ equations</p> <p>✓ <math>x = k.180</math></p> <p>✓ solution for <math>\sin x = -\frac{1}{2}</math></p> <p>✓✓ answers</p> <p>(7)</p> <p>✓ expansion</p> <p>✓ factorisation</p> <p>✓ equations</p> <p>✓ <math>x = k.180</math></p> <p>✓ solution for <math>\sin x = -\frac{1}{2}</math></p> <p>✓✓ answers</p> <p>(7)</p>
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11.2		<p>1+sinx ✓ max and min values ✓ shape</p> <p>cos2x ✓ amplitude ✓ intercepts</p> <p>(4)</p>
11.3	$180^\circ \leq x \leq 210^\circ$ or $330^\circ \leq x \leq 360^\circ$	<p>✓✓✓ answer</p> <p>(3)</p> <p><b>[14]</b></p>

**QUESTION 12**

12.1	$\frac{b}{\sin[180^\circ - (\alpha + \beta)]} = \frac{BC}{\sin \alpha}$ $BC \sin(\alpha + \beta) = b \sin \alpha$ $BC = \frac{b \sin \alpha}{\sin(\alpha + \beta)}$ <p>but <math>BC = DF</math></p> $\therefore DF = \frac{b \sin \alpha}{\sin(\alpha + \beta)}$ $\cos \theta = \frac{DF}{DE}$ $\therefore DE = \frac{DF}{\cos \theta}$ $\therefore DE = \frac{b \sin \alpha}{\sin(\alpha + \beta) \cos \theta}$	<p>✓ sine rule ✓ <math>\hat{A}BC = 180^\circ - (\alpha + \beta)</math></p> <p>✓ <math>BC = \dots</math></p> <p>✓ <math>BC = DF</math></p> <p>✓ manipulation</p> <p>✓ <math>DE = \dots</math></p> <p>(6)</p>
12.2	$DE = \frac{2000 \sin 43^\circ}{\sin 79^\circ \cdot \cos 27^\circ}$ $= 1559,50 \text{ m}$	<p>✓ substitution numerator ✓ substitution denominator ✓ answer</p> <p>(3)</p> <p><b>[9]</b></p>

**TOTAL: 150**