



education

Department:
Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

**MATHEMATICS P1
FEBRUARY/MARCH 2009
MEMORANDUM**

MARKS: 150

This memorandum consists of 15 pages.

QUESTION 1

1.1.1	$3x + \frac{1}{x} = 4$ $3x^2 - 4x + 1 = 0$ $(3x - 1)(x - 1) = 0$ $x = \frac{1}{3} \text{ or } x = 1$	✓ standard form ✓ factors ✓✓ answers (4)
1.1.2	$5x(x - 3) = 2$ $5x^2 - 15x - 2 = 0$ $x = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(5)(-2)}}{2(5)}$ $x = \frac{15 \pm \sqrt{265}}{10}$ $x = 3,13 \text{ or } x = -0,13$	✓ standard form ✓ substitution into formula ✓ 265 ✓✓ answers (5)
1.1.3	$x^2 - 2x > 3$ $x^2 - 2x - 3 > 0$ $(x - 3)(x + 1) > 0$ $\begin{array}{cccccc} + & 0 & - & 0 & + & \\ & -1 & & 3 & & \end{array} \quad \text{OR} \quad \begin{array}{c} \text{Graph of } y = x^2 - 2x - 3 \\ \text{with x-intercepts at } -1 \text{ and } 3 \end{array}$ $x < -1 \text{ or } x > 3$	✓ standard form ✓ critical values ✓✓ answers (4)
1.2	$x - 3y = 1$ $x = 1 + 3y$ $(1 + 3y)^2 - 2(1 + 3y)y + 9y^2 = 17$ $1 + 6y + 9y^2 - 2y - 6y^2 + 9y^2 - 17 = 0$ $12y^2 + 4y - 16 = 0$ $3y^2 + y - 4 = 0$ $(3y + 4)(y - 1) = 0$ $y = -\frac{4}{3} \text{ or } y = 1$ $x = -3 \text{ or } x = 4$	✓ $x = 1 + 3y$ ✓ substitution ✓ simplification ✓ factors ✓ answers ✓✓ answers (7)

	<p>OR</p> $x - 3y = 1$ $y = \frac{x-1}{3}$ $x^2 - 2x\left(\frac{x-1}{3}\right) + 9\left(\frac{x-1}{3}\right)^2 = 17$ $x^2 - \frac{2x^2 - 2x}{3} + 9\left(\frac{x^2 - 2x + 1}{9}\right) = 17$ $x^2 - \frac{2x^2 - 2x}{3} + x^2 - 2x + 1 = 17$ $3x^2 - 2x^2 + 2x + 3x^2 - 6x + 3 = 51$ $4x^2 - 4x - 48 = 0$ $x^2 - x - 12 = 0$ $(x+3)(x-4) = 0$ $x = -3 \text{ or } x = 4$ $y = -\frac{4}{3} \text{ or } y = 1$	
1.3	<p>let $1234567893 = n$ Then $1234567893 \times 1234567894 - 1234567895 \times 1234567892$ $= n(n+1) - (n+2)(n-1)$ $= n^2 + n - n^2 - n + 2$ $= 2$</p> <p>OR</p> <p>Full marks for $(3 \times 4) - (5 \times 2) = 2$</p>	<p>✓ method ✓ simplification</p> <p>✓ answer</p> <p>(3)</p> <p>[23]</p>

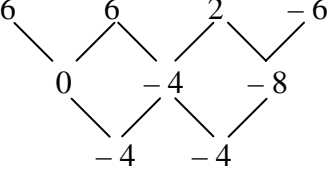
QUESTION 2

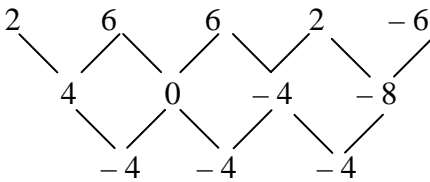
2.1.1	$S_2 = \frac{1}{1 \times 2} + \frac{1}{2 \times 3}$ $= \frac{1}{2} + \frac{1}{6}$ $= \frac{2}{3}$	✓ answer (1)
2.1.2	$S_3 = \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4}$ $= \frac{2}{3} + \frac{1}{12}$ $= \frac{9}{12}$ $= \frac{3}{4}$	✓ answer (1)
2.1.3	$S_4 = \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5}$ $= \frac{3}{4} + \frac{1}{20}$ $= \frac{16}{20}$ $= \frac{4}{5}$	✓ answer (1)
2.2	<p>If n represents the number of terms then the sum will be n divided by $n + 1$.</p> <p>OR</p> $S_n = \frac{n}{n+1}$	✓ ✓ answer (2)
2.3	$S_n = \frac{2008}{2009}$	✓ answer (1) [6]

QUESTION 3

3.1	$(2p - 3) - (1 - p) = (p + 5) - (2p - 3)$ $2p - 3 - 1 + p = p + 5 - 2p + 3$ $3p - 4 = -p + 8$ $4p = 12$ $p = 3$	✓ equating differences ✓ simplification ✓ answer (3)
3.2.1	First term = $1 - p = 1 - 3 = -2$	✓ -2 (1)
3.2.2	$-2 ; 3 ; 8$ Common difference = 5 OR $3p - 4 = 3(3) - 4 = 5$	✓ answer (1)
3.3	After the first term -2, all the other terms end in either a 3 or an 8. Perfect squares never end in a 3 or an 8.	✓✓ explanation (2) [7]

QUESTION 4

4.1	$6 ; 6 ; 2 ; -6 ; -18 ; \dots$ First difference: $0, -4, -8, -12, \dots$ Therefore the next term is: $-18 - 16 = -34$	✓ -34 (1)
4.2	 $2a = -4$ $a = -2$ $T_n = -2n^2 + bn + c$ $6 = -2 + b + c$ $8 = b + c \quad \dots(i)$ $6 = -8 + 2b + c$ $14 = 2b + c \quad \dots(ii)$ $(ii) - (i): \quad 6 = b$ $\therefore c = 2$ $T_n = -2n^2 + 6n + 2$ <p>OR</p>	✓ $a = -2$ ✓ solving simultaneously ✓ $b = 6$ ✓ $c = 2$ ✓ general term (5)

	 <p> $2a = -4$ $a = -2$ $T_0 = c = 2$ $T_1 = -2 + b + 2 = 6$ $b = 6$ $T_n = -2n^2 + 6n + 2$ </p> <p>OR</p> <p> $T_2 - T_1 = 0$ $T_3 - T_2 = -4$ $T_4 - T_3 = -8$ \vdots $T_n - T_{n-1} = -4(n-2) = 8 - 4n$ </p> <p>Adding both sides</p> <p> $T_n - 6 = -4[1 + 2 + 3 + \dots + (n-2)]$ $T_n = -4\left[\frac{(n-2)(n-1)}{2}\right] + 6$ $T_n = -2n^2 + 6n + 2$ </p>	<p> $\checkmark a = -2$ $\checkmark c = 2$ \checkmark substitution $\checkmark b = 6$ \checkmark general term </p> <p>(5)</p> <p> \checkmark obtaining differences </p> <p> \checkmark adding both sides \checkmark substituting $T_1 = 6$ \checkmark expression for sum of terms </p> <p> \checkmark simplification </p> <p>(5)</p>
<p>4.3</p>	<p> $-6838 = -2n^2 + 6n + 2$ $-2n^2 + 6n + 6840 = 0$ $n^2 - 3n - 3420 = 0$ </p> <p> $n = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-3420)}}{2}$ </p> <p> $n = \frac{3 \pm 117}{2}$ </p> <p> $n = 60$ or $n = -57$ not possible </p> <p> $\therefore T_{60} = -6838$ </p>	<p> \checkmark standard form \checkmark substitution \checkmark values of n \checkmark answer </p> <p>(4) [10]</p>

QUESTION 5

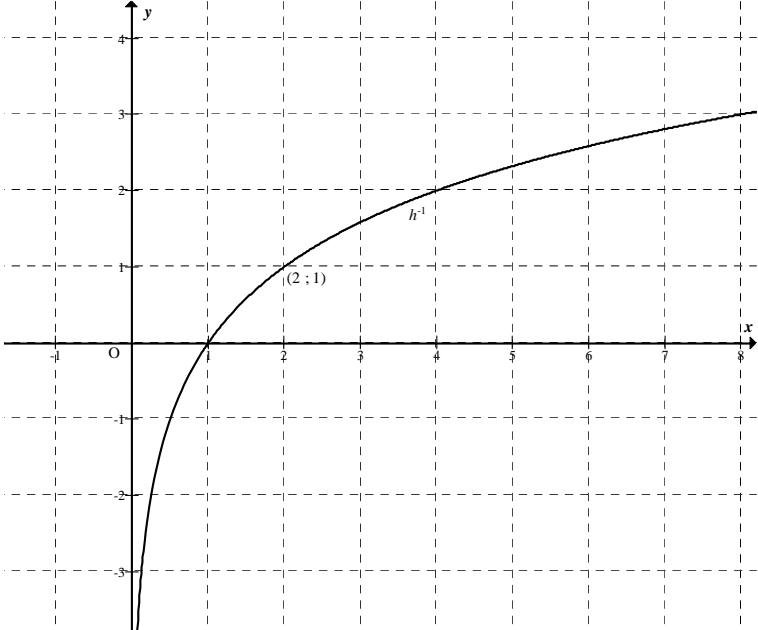
5.1	Area of unshaded square = $1 - \frac{1}{16}$ $= \frac{15}{16}$	✓ 1 ✓ $\frac{1}{16}$ ✓ answer (2)
5.2	<p>Sum of the unshaded areas of the first seven squares</p> $= (1-1) + \left(1 - \frac{1}{4}\right) + \left(1 - \frac{1}{4^2}\right) + \dots + \left(1 - \frac{1}{4^6}\right)$ $= 7 - \left(1 + \frac{1}{4} + \frac{1}{4^2} + \dots + \frac{1}{4^6}\right)$ $= 7 - \left(\frac{1\left(1 - \left(\frac{1}{4}\right)^7\right)}{1 - \frac{1}{4}}\right)$ $= 7 - 1,333251953\dots$ $= 5,666748047\dots$ $= 5,67$ <p>OR</p> <p>Sum of the unshaded areas of the first seven squares</p> $= (1-1) + \left(1 - \frac{1}{4}\right) + \left(1 - \frac{1}{4^2}\right) + \dots + \left(1 - \frac{1}{4^6}\right)$ $= 6 - \left(\frac{1}{4} + \frac{1}{4^2} + \dots + \frac{1}{4^6}\right)$ $= 6 - \frac{\frac{1}{4}\left(1 - \left(\frac{1}{4}\right)^6\right)}{1 - \frac{1}{4}}$ $= 6 - 0,332519531\dots$ $= 5,666748047\dots$ $= 5,67$	✓ expression for sum of unshaded areas ✓ simplification ✓ answer (5)

[7]

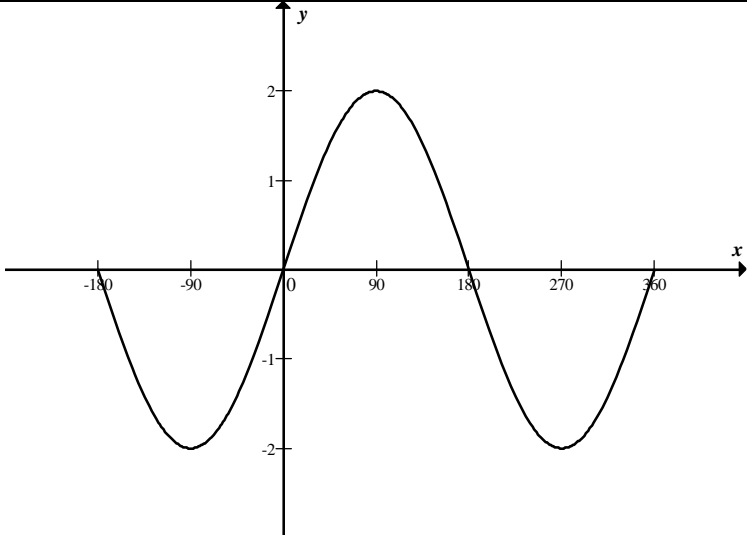
QUESTION 6

6.1	$y = \frac{a}{x-1} + 2$ $0 = \frac{a}{0-1} + 2$ $0 = -a + 2$ $a = 2$ $y = \frac{2}{x-1} + 2$ <p>OR</p> $(x-1)(y-2) = k$ <p>But g passes through the origin</p> $\therefore (-1)(-2) = k = 2$ $\therefore (x-1)(y-2) = 2$	$\checkmark b = 1$ $\checkmark c = 2$ \checkmark substitution of $(0 ; 0)$ $\checkmark a = 2$ \checkmark equation \checkmark through origin \checkmark substitution \checkmark equation (4)
6.2	$g(x) = (x-1)^2 + q$ $0 = (2,5-1)^2 + q$ $q = -\frac{9}{4}$ <p>Turning point $\left(1 ; -\frac{9}{4}\right)$</p>	$\checkmark p = 1$ \checkmark substitution of $(2,5 ; 0)$ $\checkmark q = -\frac{9}{4}$ \checkmark answer (4)
6.3	$y = 2$ $x = 2$	\checkmark answer \checkmark answer (2)
6.4	$h(x) = -(x-1)^2 + \frac{9}{4}$	\checkmark answer (1) [11]

QUESTION 7

7.1	<p>Any base raised to the power 0 is 1 which means the y-intercept of the graph $h(x) = a^x$ will be (0 ; 1) therefore Q(0 ; 1)</p> <p>OR $h(0) = a^0 = 1$ $\therefore Q(0;1)$</p>	<p>✓ y-intercept ✓ any base raised to power 0 is 1 (2)</p>
7.2	$a^{-1} = \frac{1}{2}$ $\frac{1}{a} = \frac{1}{2}$ $a = 2$	<p>✓ substitution ✓ answer (2)</p>
7.3	$2^y = x$ $y = \log_2 x$	<p>✓ interchanging x and y ✓ answer (2)</p>
7.4		<p>✓ point (0,5 ; - 1) or any other valid point ✓ point (1 ; 0) ✓ shape (3)</p>
7.5	$x > 0,5$	<p>✓ reading off from graph ✓ answer (2)</p>
7.6	$\therefore 2 + x \log 3 = x \log 2$ $\therefore x = \frac{2}{\log \frac{2}{3}} = -11.36$ <p>OR $\left(\frac{2}{3}\right)^x = 100$ $\therefore x \log \left(\frac{2}{3}\right) = 2$ $\therefore x = \frac{2}{\log \frac{2}{3}} = -11.36$</p>	<p>✓ equating ✓ logs both sides ✓ answer (3) [14]</p>

QUESTION 8

8.1		✓ shape ✓ amplitude (2)
8.2	$y \in [-4 ; 4]$	✓✓ answer (2)
8.3	720°	✓✓ answer (2)
8.4	Shift f 90° to the left to get $2\cos x$. $\therefore \theta = 90^\circ + n.360^\circ$	✓✓ answer Accept 90° (2) [8]

QUESTION 9

9.1	$2000\left(1 + \frac{i}{12}\right)^{18} = 2860$ $\left(1 + \frac{i}{12}\right)^{18} = 1,43$ $1 + \frac{i}{12} = 1,020069541\dots$ $\frac{i}{12} = 0,020069541\dots$ $i = 0,24083\dots$ $i = 24,08\%$	✓ substitution $\checkmark \left(1 + \frac{i}{12}\right)^{18} = 1,43$ $\checkmark i = 0,24083\dots$ ✓ answer (4)
9.2	$F_v = \frac{100 \left[\left(1 + \frac{0,08}{12}\right)^{12} - 1 \right]}{\frac{0,08}{12}}$ $= R1\,244,99$ <p>The accumulated amount is less than R1 300 required to buy the bike. Farouk will not be able to buy the bike on 1 January 2009.</p>	✓ formula ✓ substitution $\checkmark\checkmark$ answer ✓ conclusion (5) [9]

QUESTION 10

10.1	$\text{Loan} = 125000 - \frac{15}{100} \times 125000$ $\text{Loan} = R\,106\,250$ <p>OR</p> $\text{Loan} = 0,85 \times 125\,000$ $\text{Loan} = R\,106\,250$	✓ answer (1)
10.2	$106250 = x \left[\frac{1 - \left(1 + \frac{0,125}{12}\right)^{-6 \times 12}}{\frac{0,125}{12}} \right]$ $1106,770833 = x \left(1 - \left(1 + \frac{0,125}{12}\right)^{-6 \times 12} \right)$ $x = R\,2104,94$	✓ substitution $\checkmark - 72$ $\checkmark \left(\frac{0,125}{12}\right)$ ✓ simplification \checkmark answer (5) [6]

QUESTION 11

11.1	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{(x+h)^2 - 2(x+h) - x^2 + 2x}{h}$ $= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 2x - 2h - x^2 + 2x}{h}$ $= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 2h}{h}$ $= \lim_{h \rightarrow 0} \frac{h(2x + h - 2)}{h}$ $= \lim_{h \rightarrow 0} (2x - 2 + h)$ $= 2x - 2$	✓ method ✓ substitution ✓ simplification ✓ factorising ✓ answer (5)
11.2.1	$D_x[(x^3 - 3)^2]$ $= D_x[x^6 - 6x^3 + 9]$ $= 6x^5 - 18x^2$	✓ simplification ✓✓ answer (3)
11.2.2	$y = \frac{4}{\sqrt{x}} - \frac{x^3}{9}$ $y = 4x^{-\frac{1}{2}} - \frac{1}{9}x^3$ $\frac{dy}{dx} = -2x^{-\frac{3}{2}} - \frac{x^2}{3}$	✓ power form ✓✓ answer (3) [11]

QUESTION 12

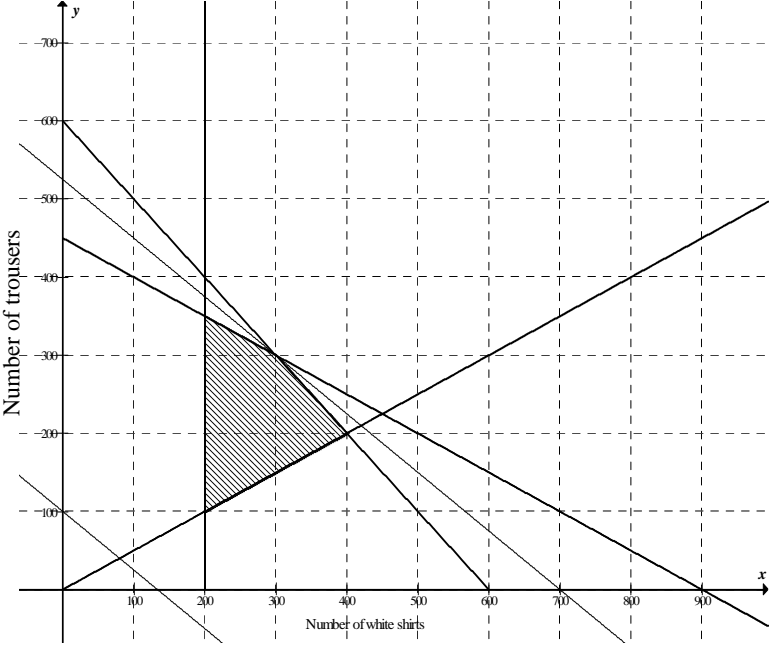
12.1	$h'(x) = -3x^2 + 2ax + b$ $h'(-1) = -3(-1)^2 + 2a(-1) + b$ $0 = -3 - 2a + b$ $2a - b = -3 \quad \dots (i)$ $h'(2) = -3(2)^2 + 2a(2) + b$ $0 = -12 + 4a + b$ $4a + b = 12 \quad \dots (ii)$ $(ii) + (i): \quad 6a = 9$ $a = \frac{3}{2}$ $\therefore 2\left(\frac{3}{2}\right) - b = -3$ $b = 6$ <p>OR</p> $h(-1) = -(-1)^3 + a(-1)^2 + b(-1) = \frac{-7}{2}$ $\therefore a - b = \frac{-9}{2}$ $2a - 2b = -9 \quad \dots (i)$ $h(2) = -(2)^3 + a(2)^2 + b(2) = 10$ $4a + 2b = 18 \quad \dots (ii)$ $(i) + (ii): \quad 6a = 9$ $a = \frac{3}{2}$ $\left(\frac{3}{2}\right) - b = \frac{-9}{2}$ $b = 6$	$\checkmark h'(x)$ \checkmark substitution of $x = -1$ $\checkmark h'(x) = 0$ \checkmark simplification \checkmark substitution \checkmark solving simultaneously (6)
12.2	Average Gradient $= \frac{10 - (-3,5)}{2 - (-1)}$ $= \frac{13,5}{3}$ $= \frac{9}{2}$	\checkmark substitution \checkmark answer (2)

12.3	$h'(x) = -3x^2 + 3x + 6$ $h'(-2) = -3(-2)^2 + 3(-2) + 6$ $h'(-2) = -12$ Point of contact $(-2 ; 2)$ $y - 2 = -12(x + 2)$ $y = -12x - 22$	✓ $h'(x)$ ✓ substitution ✓ gradient ✓ point ✓ answer (5)
12.4	$h'(x) = -3x^2 + 3x + 6$ $h''(x) = -6x + 3$ $-6x + 3 = 0$ $x = \frac{1}{2}$ OR $x = \frac{-1 + 2}{2}$ $x = \frac{1}{2}$	✓ second derivative ✓ = 0 ✓ answer (3)
12.5	$p > 3,5 \text{ or } p < -10$	✓✓ answer (2) [19]

QUESTION 13

13.1	$AB^2 = (a^2 - 0)^2 + (a - 3)^2$ $AB^2 = a^4 + a^2 - 6a + 9$	✓ substitution ✓ simplification (2)
13.2	$\frac{d}{da} AB^2 = 4a^3 + 2a - 6$ $0 = 4a^3 + 2a - 6$ $0 = 2a^3 + a - 3$ $0 = (a - 1)(2a^2 + 2a + 3)$ $a = 1$ There is no real solution for $2a^2 + 2a + 3 = 0$	✓ $\frac{d}{da} AB^2$ ✓ $\frac{d}{da} AB^2 = 0$ ✓ simplification ✓ factorisation ✓ answer (5) [7]

QUESTION 14

14.1	$x \geq 200$ $x + y \leq 600$ $50x + 100y \leq 45000$	✓ answer ✓ answer ✓ answer (3)
14.2 and 14.3		✓ 600 ✓ 450 ✓ 200 ✓ 600 ✓ 900 ✓ feasible region (5) (1)
14.4	$P = 30x + 40y$	✓✓ answer (2)
14.5	Maximum at (300 ; 300)	✓ search line ✓ answer (2) [13]

TOTAL: 150