CHAPTER 11
Financial mathematics

In this chapter you will:

- Calculate interest using the simple interest formula ($S\ I = P \times i \times n$)
- Use the simple interest formula to calculate the principal (P)
- Use the simple interest formula to calculate the interest rate (i)
- Use the simple interest formula to calculate the investment period (n)
- Compare offers from different banks
- Calculate interest using the compound interest formula ($A = P(1 + i)^n$)
- Use the compound interest formula to calculate interest
- Calculate different time periods and their corresponding interest rates
- Calculate compound interest over different time periods
- Compare interest rates
- Use the compound interest formula to calculate the principal (P)
- Use the compound interest formula to calculate the interest rate (i)
- Use the compound decrease formula to calculate final amounts
- Use the compound decrease formula to calculate population decreases
- Calculate repayments on hire purchase agreements
- Calculate percentage profit and percentage loss

This chapter covers material from Topic 5: Financial Mathematics

SUBJECT OUTCOME 5.2:

Use simple and compound interest to explain and define a variety of situations

Learning Outcome 1:	Differentiate between simple and compound interest and extrapolate the advantages and disadvantages of each in specific situations
Learning Outcome 2:	Calculate simple and compound interest over different periods at specific rates
Learning Outcome 3:	Do calculations using computational tools efficiently and correctly and verify solutions in terms of the context
Learning Outcome 4:	Use solutions to calculations effectively to define the changes that occur over a period

§ 11.1 THE SIMPLE INTEREST FORMULA

- We calculate the amount of *simple interest* earned by using the formula $\boldsymbol{SI = P \times i \times n}$ where: ***SI*** = simple interest, ***P*** = the principal (the money invested), ***i*** = the interest rate p.a., written as a decimal, and ***n*** = the number of time periods
- If we want to calculate the *final amount received at the end of the time period*, we use the formula $\boldsymbol{A = P + SI}$ where: ***A*** = the final amount received at the end of the time period, ***P*** = the principal or the amount invested and ***SI*** = simple interest.
- Note that p.a. means *per annum* or *per year*.
- Note also that we always round amounts of money off to 2 decimal places.

EXAMPLES	SOLUTIONS
1) Write each of the following as a decimal: a) 9%	$9\% = \frac{9}{100} = 0{,}09$
b) 14,5%	$14{,}5\% = \frac{14{,}5}{100} = 0{,}145$
2) a) Calculate SI, the simple interest earned when R5 430 is invested for 5 years at an interest rate of 9,5% p.a.	P = R5 430 n = 5 year $i = 9{,}5\% = i = \frac{9{,}5}{100} = 0{,}095$ p.a. $\boldsymbol{SI = P \times i \times n}$ = R5 430 × 0,095 × 5 = R2 579,25
b) Calculate the final amount received at the end of 5 years.	$\boldsymbol{A = P + SI}$ = R5 430 + R2 579,25 = R8 009,25

Exercise 11.1

1) Write the following percentages as decimals:

a) 11,25% ……

b) 10,4% ……..

c) 8,9% ……

2) A principal of R7 395 is invested for 6 years at an interest rate of 10,25% p.a.

a) Calculate SI, the simple interest earned

P = …… n = …… i = …….

$\boldsymbol{SI = P \times i \times n}$

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…….

b) Calculate A, the amount received at the end of 6 years.

$\boldsymbol{A = P + SI}$

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§ 11.2 USING THE SIMPLE INTEREST FORMULA TO CALCULATE THE PRINCIPAL (P)

EXAMPLE	SOLUTION
A sum of money is invested at a simple interest rate of 10,7% p.a. for 5 years.	i = 10,7 % p.a. = $\frac{10,7}{100}$ p.a.= 0,107 p.a. n = 5 years
1) How much needs to be invested in order to make R550 interest? $P = ?$	SI = R550 P = ? $SI = P \times i \times n$ $550 = P \times 0,107 \times 5$ $\frac{550}{0,107\times5} = \frac{P\times0,107\times5}{0,107\times5}$ R1 028,04 = P So R1 028,04 needs to be invested.
2) What amount is received at the end of 5 years? $A = ?$	$A = P + SI$ = 1 028,04 + 550 = 1 578,04 R1 578,04 is received at the end of 5 years.

Exercise 11.2

1) A sum of money is invested at an interest rate of 11,6 % p.a. simple interest for 6 years.
   a) How much needs to be invested in order to make R1055 interest?

   i = …… n = …… SI = ……

   $SI = P \times i \times n$

   ……

   ……

   ……

   So ……. needs to be invested.

   b) What amount will be received at the end of 6 years?

   $A = P + SI$

   ……

   ……

2) A sum of money is invested at an interest rate of 10,75% p.a. for 4 years.
   a) How much must be invested in order to make R950 *S I* on the investment?

   i = …… n = …… SI = ……

   $SI = P \times i \times n$

   ……

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   So ……. needs to be invested.

   b) How much is saved in total at the end of 4 years?

   $A = P + SI$

   ……

   ……

§ 11.3 USING THE SIMPLE INTEREST FORMULA TO CALCULATE THE INTEREST RATE (*i*)

EXAMPLE	SOLUTION
R12 457 is invested for 6 years. At the end of this time, an amount of R18 278 is paid out.	***P*** = R12 457 ***A*** = R18 278 ***n*** = 6 years
1) Calculate the simple interest, S I, received ***S I = ?***	***A = P + SI*** 18 278 = 12 457 + *S I* *S I* = 18 278 – 12 457 *S I* = R5 821
2) At what % interest rate p.a. was the money invested? Answer correct to 2 decimal places. ***i = ?***	***S I = P × i × n*** 5 821 = 12 457 × *i* × 6 $\frac{5\ 821}{12\ 457\times 6} = \frac{12\ 457\times i\times 6}{12\ 457\times 6}$ *i* = 0,02436… *i* = 0,07788…× 100 % *i* = 7,79 % p.a.

Exercise 11.3

1) Convert the following decimals to a percentage (correct to 2 decimal places);
   a) 0,08 …….
   b) 0,12 …….
   c) 0,325 …….
   d) 0,615 …….
   e) 0,022 5 …….
   f) 0,0257788… …….

2) R15 901 is invested for 4 years. At the end of this time, an amount of R19 780 is paid out.
   a) Calculate the simple interest earned.

      A = ……... ***P =*** ……... ***n =*** ……. ***S I =*** ……..

      A = P + SI

      …….

      …….

      …….

   b) Calculate the interest rate and write your answer as a percentage correct to 2 decimal places.

      S I = P × i × n

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§ 11.4 USING THE SIMPLE INTEREST FORMULA TO CALCULATE THE INVESTMENT PERIOD (n)

EXAMPLE	SOLUTION
R6 058 is invested at a simple interest rate of 9,25% p.a. How many years will it take in order to receive an amount of R10 540,92 at the end of the investment period?	i = 9,25% p.a. = $\frac{9,25}{100}$ p.a. = 0,0925 p.a. $\boldsymbol{A}$ = R10 540,92 $\boldsymbol{P}$ = R6 058 $\boldsymbol{A = P + SI}$ 10 540,92 = 6 058 + SI 10 540,92 – 6 058 = SI – 6 058 SI = R4 482,92 $\boldsymbol{SI = P \times i \times n}$ 4 482,92 = 6 058 × 0,0925 × n $\frac{4482,92}{6\,058\times 0,0925} = \frac{6\,058\times 0,0925\times n}{6\,058\times 0,0925}$ n = 8 The money must be invested for 8 years.

Exercise 11.4

1) R10 000 is invested at a simple interest rate of 9,35% p.a. How many years will it take in order to receive an amount of R14 675 at the end of the investment period?

***A* =** ……... ***P* =** …….. ***i* =** ……..

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2) R8 990 is invested at a simple interest rate of 10,25% p.a. How many years will it take in order to receive an amount of R12 675,90 at the end of the investment period?

***A* =** ……... ***P* =** …….. ***n* =** ……. ***SI* =** ……..

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§ 11.5 COMPARING OFFERS FROM DIFFERENT BANKS

EXAMPLE

Mpho has R5 980 to invest for 6 years.

- Bank A offers him a fixed simple interest rate of 9,5% p.a. PLUS a bonus of 5% of the principal.
- Bank B offers him a fixed simple interest rate of 10,5% p.a.

Which bank offers the better investment?

SOLUTION

BANK A	**BANK B**
P = R5 980 n = 6 years i = 9,5 % p.a. = $\frac{9,5}{100}$ p.a. = 0,095 p.a. $SI = P \times i \times n$ = 5 980 × 0,095 × 6 = R3 408,60 Bonus = 5% of R5 980 = 0,05 × 5 980 = R299 At the end of 6 years Mpho would receive R3 408,60 + R299 = ***R3 707,60 from Bank A***.	P = R5 980 n = 6 years i = 10,5 % p.a. = $\frac{10,5}{100}$ p.a. = 0,105 p.a. $SI = P \times i \times n$ = 5 980 × 0,105 × 6 = R3 767,40 At the end of 6 years Mpho would receive ***R3 767,40 from Bank B***, which is more than the R3 707,60 he would receive from Bank A So Bank B offers the better investment.

Exercise 11.5

Patricia has R6 400 to invest for 4 years.

- Bank A offers her a fixed simple interest rate of 9,75 % p.a. plus a bonus of 2% of the principal.
- Bank B offers her a fixed simple interest rate of 10% p.a.

In which bank should she invest her money?

BANK A

P **=** ……... ***n*** **=** …….. ***i*** **=** ……..

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BANK B

P **=** ……... ***n*** **=** …….. ***i*** **=** ……..

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§ 11.6 MIXED EXERCISE

✧ Remember that $SI = P \times i \times n$ and $A = P + SI$

1) Jean invests R10 450 for 5 years at 9,5% p.a. *S I*.

a) How much interest will she receive at the end of this period?

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b) How much will she have saved at the end of this period?

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***A* =** …….
***P* =** .....
***i* =** ……
***n* =** ……...
***S I* =** ……

2) How much must Sipho invest at 9,5% *S I* p.a. for 6 years in order to earn interest of R2 500?

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***A* =** …….
***P* =** .....
***i* =** ……
***n* =** ……...
***S I* =** ……

3)

a) Calculate the number of whole years needed for an investment of R5 990 to grow to R7 876,85 if the interest rate is 9% *S I* p.a.

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b) At what interest rate (correct to 2 decimal places) must it be invested for R5 990 to grow to R8 500 after 5 years?

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***A* =** …….
***P* =** .....
***i* =** ……
***n* =** ……...
***S I* =** ……

4) For how long (whole years) must John invest R11 545 at 9,5% p.a. *S I* to save R16 000 in total?

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***A* =** …….
***P* =** .....
***i* =** ……
***n* =** ……...
***S I* =** ……

§ 11.7 COMPOUND INTEREST

- When a person is paid ***interest*** at regular intervals on a sum of money invested, without the interest being added to the sum invested, the investment is called ***simple interest***.
  The formula for calculating simple interest is $SI = P \times i \times n$.
  We use the formula $A = P + SI$ to calculate the final amount (A).
- When the interest is added to the sum invested, and ***interest is calculated on this new amount***, the investment is called ***compound interest***.
  The formula used for calculating compound interest is $A = P(1+i)^n$, where
  A is the final amount obtained after the investment period
  P is the principal or the amount invested
  i is the interest rate (written as a decimal)
  n is the number of time periods of the investment

EXAMPLE

James invests R4 000 at 8% p.a. for 5 years. Should he choose:
1) Simple interest or
2) Compound interest?

SOLUTION

SIMPLE INTEREST	COMPOUND INTEREST
P = 4 000 i = 8% p.a. = 0,08 p.a. n = 5 years $SI = P \times i \times n$ $= 4\,000 \times 0{,}08 \times 5$ = R1 600 $A = P + SI$ = 4 000 + 1 600 = R5 600	P = 4 000 i = 8% p.a. = 0,08 p.a. n = 5 years $A = P(1+i)^n$ $= 4\,000\,(1 + 0{,}08)^5$ $= 4\,000\,(1{,}08)^5$ = R5 877,31

- James should choose compound interest as he would end up with a larger final amount.

Exercise 11.7

Paula invests R5 500 at 10% p.a. for 4 years. Should she choose ***simple interest*** or ***compound interest***?

SIMPLE INTEREST

P = ……

……..... i = …….

……..... n = …….

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…….....

COMPUND INTEREST

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CONCLUSION: …….....

§ 11.8 USING THE COMPOUND INTEREST FORMULA TO CALCULATE INTEREST (*i*)

- We use the formula $A = P(1+i)^n$ to calculate the final amount received at the end of the investment period.
- We use the formula $CI = A - P$ to calculate the interest obtained at the end of the investment period.

EXAMPLE	SOLUTION
1) Use the *CI* formula to calculate the amount received at the end of 8 years if R7 000 is invested at 5% p.a.	P = R7 000 n = 8 i = 5 % = 0,05 p.a. $A = P(1+i)^n$ $= 7\,000(1 + 0{,}05)^8$ $= 7\,000(1{,}05)^8$ = R10 342,19
2) How much interest is received on this investment?	$CI = A - P$ = R10 342,19 – R7 000 = R3 342,19

Exercise 11.8

1) Use the *CI* formula to calculate the amount paid out at the end of an investment of R6 955 at an interest rate of 10,4% p.a. compounded annually for 7 years.

P = …… n = …… i = ……

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2) How much of this amount is interest?

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§ 11.9 TIME PERIODS AND INTEREST RATES

- Interest on an investment is calculated at the end of a given time period. This time period is not necessarily one year. The interest can be compounded (calculated) ***annually***, ***half-yearly***, ***quarterly***, ***monthly*** and sometimes ***daily***.
- To find the ***interest rate*** when the interest is compounded more than once per year, ***divide*** the interest rate p.a. by the number of time periods in one year
- To find the number of ***time periods***, ***multiply*** the number of years by the required number of time periods in one year

EXAMPLE

Suppose the investment period is **6 years** at an **interest rate of 9% p.a.**
Suppose the interest is compounded more than once per year,

- we divide the interest rate up into the different time periods
- we adjust the time periods accordingly.

SOLUTION

TIME PERIOD	INTEREST RATE PER TIME PERIOD WRITTEN AS A DECIMAL (i)	NUMBER OF TIME PERIODS (n)
annually	9% p.a. = 0,09 per year	$n = 6 \times 1 = 6$ years
half-yearly	9% p.a. = $\frac{0{,}090}{2} = 0{,}045$ per half year	$n = 6 \times 2 = 12$ half-years
quarterly	9% p.a. = $\frac{0{,}0900}{4} = 0{,}0225$ per quarter	$n = 6 \times 4 = 24$ quarters
monthly	9% p.a. = $\frac{0{,}090000000}{12} = 0{,}008\dot{3}$ per month	$n = 6 \times 12 = 72$ months

Exercise 11.9

1) Complete the table for an investment period of 7 years and an interest rate of 4% p.a.

Time period	Interest rate per time period as a decimal (i)	Number of time periods (n)
annually		
half-yearly		
quarterly		
monthly		

2) Complete the table for an investment period of 8 years and an interest rate of 12,5% p.a.

Time period	Interest rate per time period as a decimal (i)	Number of time periods (n)
annually		
half-yearly		
quarterly		
monthly		

§ 11.10 CALCULATING COMPOUND INTEREST OVER DIFFERENT TIME PERIODS

EXAMPLE	SOLUTION
R8 550 is invested at a financial institution for 6 years at an interest rate of 7,2 % p.a. compounded **quarterly**.	P = R8 550 i = 7,2% p.a. = 0,072 p.a. $= \frac{0,072}{4}$ per quarter = 0,018 per quarter $n = 6 \times 4 = 24$ quarters
1) What amount is due after 6 years?	$A = P(1+i)^n$ $= 8\,550(1 + 0,018)^{24} = 8\,550(1,018)^{24}$ = R13 119,36
2) How much of this is interest?	$CI = A - P$ = R13 119,36 – R8 550 = R4 569,36

Exercise 11.10

1) An investment of R8 550 is made at a financial institution. The interest rate is 7,2 % p.a. compounded annually for 6 years.
   a) What amount is due after 6 years?

      P = …… i = …… n = ……..

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      ……

   b) How much is interest?

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      ……..

   c) Compare your answers with those in the example above. Do you earn more interest by compounding annually or compounding quarterly?

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2) Use the numbers in question 1 to compound the investment half-yearly.
   a) What amount is due after 6 years?

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   b) How much is the interest?

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      ……..

3) Is compounding an investment more frequently an advantage? ……

§ 11.11 COMPARING INTEREST RATES

EXAMPLE

Joe wants to invest R13 450 for 6 years.

- Bank A offers him 9,6% p.a. interest compounded monthly.
- Bank B offers 10% p.a. interest compounded quarterly.

1) Calculate the amount received from each bank.
2) How much interest does each option yield?
3) Which offer should Joe take?

SOLUTION

BANK A	**BANK B**
P = R13 450 i = 9,6% p.a. = 0,096 p.a. $= \frac{0,096}{12}$ per month = 0,008 per month $n = 6 \times 12 = 72$ months 1) $A = P(1+i)^n$ $= 13\,450(1 + 0,008)^{72}$ $= 13\,450(1,008)^{72}$ $=$ R23 871,55 2) $CI = A - P$ $=$ R23 871,55 – R13 450 $=$ R10 421,55	P = R13 450 i = 10% p.a. = 0,1 p.a. $= \frac{0,1}{4}$ per quarter = 0,025 per quarter $n = 6 \times 4 = 24$ quarters 1) $A = P(1+i)^n$ $= 13\,450(1 + 0,025)^{24}$ $= 13\,450(1,025)^{24}$ $=$ R24 327,36 2) $CI = A - P$ $=$ R24 327,36 – R13 450 $=$ R10 877,36
3) Joe should take the offer by Bank B, since he receives more interest there.	

Exercise 11.11

Vusi wants to invest R9 560 for 5 years.
Bank A offers him 10,2% p.a. interest compounded monthly.
Bank B offers 10,8 % p.a. interest compounded quarterly.

	BANK A	**BANK B**
1) Calculate the amount received from each bank.	***P*** = ……..…… ***i*** = ……...…. ……..…….. ***n*** = ……..…… ……..…… …… …… ……..	***P*** = ……..…… ***i*** = ……...…. ……..…….. ***n*** = ……..…… ……..…… …… …… ……..
2) How much interest will each option yield?	…….. ……... ……...	…….. ……... ……...
3) Which offer should Vusi take?	……	

§ 11.12 USING THE COMPOUND INTEREST FORMULA TO CALCULATE THE PRINCIPAL (P)

EXAMPLE

How much money must be invested in order to receive an amount of R14 600 in 5 years
1) at an interest rate of 10,4 % p.a. compounded annually?
1) at an interest rate of 10,4 % p.a. compounded quarterly?

SOLUTION

1) A = R14 600

i = 10,4 % $= \frac{10,4}{100}$ = 0,104 p.a.

$n = 5$

$$A = P(1+i)^n$$

$$14\,600 = P(1 + 0{,}104)^5$$

$$14\,600 = P(1{,}104)^5$$

$$\frac{14\,600}{(1{,}104)^5} = \frac{P(1{,}104)^5}{1{,}104^5}$$

8 902,41 = P

R8 902,41 must be invested.

Key Sequence						
14 600	÷	1.104	$x^{\blacksquare}$	5	=	

2) A = R14 600

i = 0,104 p.a. $= \frac{0,103}{4}$ = 0,026 per quarter

$n = 5 \times 4 = 20$ quarters

$$A = P(1+i)^n$$

$$14\,600 = P(1 + 0{,}026)^{20}$$

$$14\,600 = P(1{,}026)^{20}$$

$$\frac{14\,600}{1.026^{20}} = \frac{P(1{,}026)^{20}}{1{,}026^{20}}$$

8 737,87 = P

R8 737,87 must be invested.

Key Sequence						
14 600	÷	1.026	$x^{\blacksquare}$	20	=	

Exercise 11.12

An investment amounted to R24 987,50 at the end of 5 years.

1) How much was the initial investment at

a) 9,5% p.a. compounded annually?

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b) 9,5% p.a. compounded half-yearly ?

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2) For which one would you have to invest less?

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§ 11.13 CALCULATING THE INTEREST RATE (i)

EXAMPLE

An investment of R17 000 grows to R25 000 in 5 years.

1) What was the percentage interest rate p.a. compounded annually (correct to 2 decimal places)?
2) What was the percentage interest rate p.a. compounded half-yearly (correct to 2 decimal places)?

SOLUTION

1) P = R17 000
A = R25 000
n = 5 years
i = ? % p.a.

$$A = P(1+i)^n$$
$$25\,000 = 17\,000(1+i)^5$$
$$\frac{25\,000}{17\,000} = \frac{17\,000(1+i)^5}{17\,000}$$
$$1{,}470\,588\ldots = (1+i)^5$$
$$\sqrt[5]{1{,}47\ldots} = \sqrt[5]{(1+i)^5}$$
$$1{,}08018\ldots = 1+i$$
$$1{,}080\,18\ldots - 1 = 1+i-1$$
$$0{,}080\,18\ldots = i$$
$$i = 0{,}080\,18\ldots \times 100\%$$
$$= 8{,}02\ \%$$

Key Sequence

$\sqrt{\square}$ 5 ► 25 000 ÷ 17 500 =

2) P = R17 000
A = R25 000
n = 5 × 2 = 10 half-years
i = ? % p.a.

$$A = P(1+i)^n$$
$$25\,000 = 17\,000(1+i)^{10}$$
$$\frac{25\,000}{17\,000} = \frac{17\,000(1+i)^{10}}{17\,000}$$
$$1{,}470\,588\ldots = (1+i)^{10}$$
$$\sqrt[10]{1{,}470588\ldots} = \sqrt[10]{(1+i)^{10}}$$
$$1{,}039\,319\ldots = 1+i$$
$$1{,}039\,319\ldots - 1 = 1+i-1$$
$$i = 0{,}039\,319\ldots \text{per half year}$$
$$i = 0{,}039\,319\ldots \times 100\%$$
$$= 3{,}9319\ \ldots\% \text{ per half year}$$
$$= 7{,}86\ \% \text{ p.a.}$$

Key Sequence

$\sqrt{\square}$ 10 ► 25 000 ÷ 17 500 =

Exercise 11.13

An investment of R5 000 increases to R7 500 in 6 years
Calculate the interest rate as a percentage per annum, correct to 1 decimal place

1) when interest is compounded annually

2) when interest is compounded half-yearly

§ 11.14 A MIXED EXERCISE

1) Calvin's overseas trip in five years time will probably cost R23 000. How much must he invest now at 12,25% p.a. compounded annually to fund the trip?

2) Doris has R10 450 to invest and requires R20 000 at the end of 6 years. What interest rate compounded half-yearly will she need? Answer correct to 1 decimal place.

3) Sophie invested R15 500 and received R13 546,70 in interest at the end of 5 years.
   a) How much has she saved in total after 5 years?
   b) At what interest rate did she invest her money if it was compounded monthly for 5 years?

§ 11.15 THE COMPOUND DECREASE FORMULA

- An investment which **grows or increases** in value, over time, is said to **appreciate** in value.
- An investment which **decreases** in value, over time, is said to **depreciate** in value. Office equipment and motor cars are investments that depreciate with time and become worth less than what was paid for them. The purchasing power of your money decreases owing to **inflation**. The price of most goods and services show a steady, compounded increase, from year to year.
- For depreciation we use the formula $A = P(1-i)^n$

  where A = the amount received at the end of the investment time
  P = the principal (the money invested)
  i = the interest rate p.a. written as a decimal
  n = the number of time periods of the investment

EXAMPLE	SOLUTION
A new car cost R98 500 in 2004. The car depreciates by 11% of its value each year	P = R98 500 i = 11 % p.a. = 0,11
1) How much will the car be worth in 2008? Give the answer correct to the nearest rand.	n = 2008 – 2004 = 4 years $A = P(1-i)^n$ $= 98\,500(1 - 0{,}11)^4$ $= 98\,500(0{,}89)^4$ = 61 801 The car is worth R61 801 in 2008.
2) By how much will the car have depreciated?	The car has depreciated by R98 500 – R61 801 = R36 699 in four years.

Exercise 11.15

A motorcar cost R71 000 in July 2002. Its value depreciates by 10% p.a. compounded annually.

1) How much will the motorcar be worth in July 2008?

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2) By how much will the car have depreciated?

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§ 11.16 POPULATION DECREASE

✧ The compound decrease formula may also be used for calculating population decrease.

EXAMPLE	SOLUTION
A town has a population of 78 895 in January 2008. An annual average rate of compound decrease of 6% p.a. is expected in the population after this date.	P = 78 895 i = 6% p.a. = 0,06 p.a.
1) What will be the expected population in January 2012?	n = 2012 – 2008 = 4 years $A = P(1-i)^n$ A = 78 895$(1 – 0{,}06)^4$ = 78 895$(0{,}94)^4$ = 61 597 The expected population in January 2012 is 61 597.
2) Calculate the population decrease.	Population decrease = 78 895 – 61 597 = 17 297

Exercise 11.16

1) A town has a population of 54 678 in January 2008. An annual average rate of compound decrease of 8% p.a. is expected in the population after this date.
   a) What will be the expected population in January 2010?

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   b) Find the population decrease.

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2) The population of a city in July 2008 is 98 979 people. The city has experienced an annual average rate of compound decrease of 7% p.a..
   a) What was the population in July 2004?

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   b) Calculate the population decrease.

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§ 11.17 MIXED EXERCISE

1) Equipment depreciates at 11% p.a. In June 2008, the equipment was worth R230 000.
   a) What will the equipment be worth in June 2011?

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   b) By how much will it have depreciated?

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2) If the inflation rate is steady at 6% p.a. for 5 years compounded annually, what is the purchasing power of R100 after 5 years?

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3) Machinery worth R256 300 depreciates at a rate of 12% p.a. How much is it worth after 6 years if the rate of depreciation is compounded annually?

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§ 11.18 HIRE PURCHASE (H.P.) AGREEMENTS

- Goods bought on HP are taken from the store **before the full purchase price is paid**.
- A deposit may be paid at the time of purchase and interest is charged on the balance. The equal repayments made may be weekly or monthly.
- These agreements often charge *S I* (***S I = P × i × n***) on the full loan for the full period, not just on the amount still to be paid.

EXAMPLE

Musa buys goods for R34 600 on H.P. He puts down a 15% deposit and repays the balance at 16% p.a. *S I* in equal monthly instalments over 5 years.

1) Calculate the deposit
2) Calculate the balance owing after interest has been included
3) How much will he pay per month?

SOLUTION

1) Deposit
= 15% of R34 600
= 0,15 × R34 600
= R5 190

2) Balance owing after paying deposit
= R34 600 – R5 190
= R29 410

P = R29 410 n = 5 years
i = 16% p.a. = 0,16 p.a.

$SI = P \times i \times n$
= 29 410 × 0,16 × 5
= R23 528

Total amount owing,
$A = P + SI$
= R23 528 + R29 410
= R52 938

3) n = 5 years
= 5 × 12
= 60 months

Monthly repayments
$= \frac{52\,938}{60}$
= R882,30 per month

Exercise 11.18

Neo purchases a motorcar for R71 650. She pays a deposit of 15% and pays the rest off in equal monthly instalments at an interest rate of 12,5% p.a. *SI* over 5 years.

1) How much is her deposit?

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2) How much does of the original price does she still owe?

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3) How much does she owe in total once interest has been added?

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4) What is her monthly repayment?

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§ 11.19 PROFIT AND LOSS

- The profit (P) or the loss (L) is the difference between the selling price (SP) and the cost price (CP). The profit will be a positive amount and the loss will be negative.

$$P = SP - CP \qquad L = CP - SP$$

- The profit or loss is usually calculated as a percentage of the cost price.

$$\textit{Percentage Profit} = \frac{P}{CP} \times 100\% \qquad \textit{Percentage Loss} = \frac{L}{CP} \times 100\%$$

EXAMPLE	SOLUTION
1) A baker sells biscuits for R12,55 per bag. He calculates that each bag of biscuits costs him R6,50 to produce. a) What is his profit on each bag? b) What is his percentage profit (correct to 2 decimal places) on each bag?	a) $P = SP - CP$ $= R12,55 - R6,50 = R6,05$ b) Percentage Profit $= \frac{6,05}{6,50} \times 100\% = 93,08\%$
2) The baker sells his old stock at half-price, at R6,28 per bag. a) What is his loss per bag? b) What is his percentage loss (correct to 2 decimal places) on each bag?	a) $L = CP - SP$ $= R6,50 - R6,28 = R0,22$ His loss per bag is 22c b) Percentage Loss $= \frac{0,22}{6,50} \times 100\% = 3,38\%$

Exercise 11.19

1) A bookshop sells a book marked R149,50. The cost of the book to the shop is R75,50.

   a) Calculate the profit made on each book sold.

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   b) Find the percentage profit (correct to 1 decimal place) made by the shop on each book sold.

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2) Owing to poor sales, the bookshop decides to sell the book in 1) for R109,20.

   a) What discount are they giving on the original selling price?

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   b) Calculate the profit made on each book sold at the new selling price

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   c) Calculate the new percentage profit, correct to the nearest whole number.

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