Basic Skills for Mathematical Literacy

This guide has been developed for the Department of Education by Brombacher and Associates.

Brombacher and Associates cc

94 Myburgh Road	(021) 715-6161 (tel)
Diepriver, 7800	(021) 715-4529 (fax)
aarnout@brombacher.co.za	073-244-6488 (cell)

www.brombacher.co.za

Department of Education

Sol Plaatje House
123 Schoeman Street
Private Bag X895
Pretoria 0001
South Africa
Tel: +27 12 312-5911
Fax: +27 12 321-6770

120 Plein Street
Private Bag X9023
Cape Town 8000
South Africa
Tel: +27 21 465-1701
Fax: +27 21 461 8110

http://education.pwv.gov.za

Basic Skills for Mathematical Literacy

CONTENTS

TOPIC 1

NUMBERS

INDEX

1.1 BASIC OPERATIONS

1.1.1 Number Sentences

Consider the following scenario:

Jemima, Xolani and Luanda visit a supermarket and buy 6 samoosas at R3,20 each and 3 cold drinks at R5,50 each.

Jemima calculates the amount that they will have to pay in the following way:

Samoosas = 6 × R3,20 = R19,20

Cold Drinks = 3 × R5,50 = R16,50

Total = R19,20 + R16,50 = R35,70

Xolani approaches the problem in a slightly different way.

- First he constructs a *number sentence* to describe the situation:

  Cost = 6 × R3,20 + 3 × R5,50

A *number sentence* is a method of using numbers and mathematical operators to describe a situation. Using a number sentence often provides us with a convenient way of summarising the information and calculations involved in the situation, and for helping us to see what calculations we have to do.

- Then he completes the different calculations in the number sentence:

  = R19,20 + R16,50

  = R35,70

Luanda also constructs a number sentence, but uses a different order to Xolani:

Cost = 3 × R5,50 + 6 × R3,20

= R16,50 + R19,20

= R35,70

Some important things to note:

1. Jemima broke the problem down into compartments and then calculated the cost of the samoosas, cost of the cool drinks and total cost individually and separately.
   Xolani and Luanda, on the other hand, constructed *number sentences* to represent the situations and then calculated the cost of the food together in one sum. The answers, though, are identical. The point is that, in this scenario, whether the calculations are preformed individually or all together the result is the same.
2. Number sentences are a useful way for identifying the mathematical calculations and operations involved in a situation.
3. Xolani calculated the cost of the samoosas first and the cost of the cool drinks second; while Luanda calculated the cost of the cool drinks first and the cost of the samoosas second. Their answers, though, are identical. So, in this scenario, the order in which the calculations are performed does not matter.

Practice Exercise: Number Sentences

1. Mandy buys 3 bars of soap at R5,99 each, 1 tube of toothpaste at R6,20 and 2 chocolates at R4,30 each.
a. Write a number sentence to represent the total cost of Mandy's shopping.

____

____

b. Use two different ways to show how much Mandy paid for her shopping?

____

____

____

____

____

1. b. … ______________________________

____

____

____

____

2. Khosi buys a loaf of bread at R7,50 per loaf, 2 packets of rice at R12,99 per packet and 2 packets of maize meal at R28,30 per packet. She pays for the groceries with a R100,00 note.
a. Write a number sentence to represent the total cost of Khosi's shopping.

____

____

2. b. Use two different methods to calculate how much *change* Khosi received?

3. The entry fee into a game reserve is R20,00 per car and R12,00 per person.

a. Write a number sentence to represent the cost of a family of 3 people entering the reserve in one car.

b. If the family pays for the entrance fee with a R100,00 note, how much change will they receive?

4. Three friends live in the same house. They go shopping and buy 1 packet of washing powder at R18,99 per packet, 2 bottles of milk at R15,20 each and 6 bread rolls at R0,85 per roll.

a. If they share the cost of the groceries equally amongst the three of them, write a number sentence to describe how much money each person will have to pay towards the groceries.

b. Calculate how much each person will have to pay towards the groceries.

1.1.2 The Importance of Brackets and "BODMAS"

Consider the following scenario:

Sipho buys 1 loaf of bread at R7,20 per loaf and 3 bottles of milk at R5,45 each. He pays with a R50,00 note.

Sipho constructs the following number sentence to represent this situation:

Change = R50,00 − R7,20 + 3 × R5,45

He then uses a basic calculator to calculate how much change he will receive. He presses the buttons on the calculator in the following order $\boxed{50}$ $\boxed{-}$ $\boxed{7.2}$ $\boxed{+}$ $\boxed{3}$ $\boxed{\times}$ $\boxed{5.45}$ and gets the answer 249.61 — at which point he jumps for joy since the change he is supposed to receive is more than he paid for the food!

Clearly this is not correct. But why? If we follow the order in which Sipho pressed the buttons on the calculator, then the calculator has performed the calculations in the following way:

1. 50 − 7.2 = 42.8
2. 42.8 + 3 = 45.8
3. 45.8 × 5.45 = 249.61

The reason why using the calculator in this way gives the wrong answer is that the calculator is performing the calculations based on the order in which the numbers and operations appear in the number sentence rather than on the order in which the events actually happened in the given scenario. If we return to the context of the supermarket, then the first thing that Sipho needed to do was to multiply 5,45 by 3; then he needed to add 7,2 to this answer; and finally he needed to subtract this result from 50. Instead, by pressing the buttons in the order in which the numbers appeared in the number sentence, Sipho first subtracted 7,2 from 50; then he added 3 to this result; and finally he multiplied this result by 5,45.

The point is that performing calculations according to the order in which they appear in a number sentence does not guarantee a correct answer. Rather the calculations must be performed in the order in which they occurred in the given scenario.

There are two ways that we can use to help us to identify the order in which calculations in a number sentence must happen:

A. Brackets

To avoid confusion about the order in which different operations in a number sentence must happen, we can make use of ***brackets***. Brackets provide us with a way to group together certain numbers and operations in the order in which they happened in the situation.

For example, using brackets in Sipho's number sentence would give:

$$\text{Cost} = \text{R}50{,}00 - [\text{R}7{,}20 + (3 \times \text{R}5{,}45)]$$

Inserting the brackets in appropriate places in this number sentence now makes it very clear that the R5,45 must first be multiplied by 3; then R7,20 must be added to this result; and finally this value must be subtracted from R50,00.

B. "BODMAS"

In order to help us to remember in which order we must perform operations, we make use of the concept of "**B O D M A S**". This stands for:

"Brackets of, Division, Multiplication, Addition and Subtraction"

In other words, in any calculation, if there are brackets involved then the operation in the brackets must be performed first; multiplication or division (in any order) must be performed second; and addition or subtraction (in any order) must be performed last.

For example, consider the number sentence that Xolani constructed to represent the cost of the goods that he bought from the supermarket:

$$\text{Cost} = 6 \times \text{R}3{,}20 + 3 \times \text{R}5{,}50$$

Applying the BODMAS convention means that any multiplication must happen first, followed by addition. This gives:

$$\begin{aligned} \text{Cost} &= 6 \times \text{R}3{,}20 + 3 \times \text{R}5{,}50 \\ & \qquad \text{Multiplication} \\ &= \text{R}19{,}20 + \text{R}16{,}50 \\ & \qquad \text{Addition} \\ &= \text{R}35{,}70 \end{aligned}$$

Notice that we could have organised the operations in the number sentence according to the order in which they happened in the scenario by using brackets.

i.e. $\text{Cost} = (6 \times R3{,}20) + (3 \times R5{,}50)$

$= R19{,}20 + R16{,}50$

$= R35{,}70$

But also note that if there are no brackets in the number sentence and we use the BODMAS convention, then it replaces the need to insert brackets.

Practice Exercise: Brackets and BODMAS

1. Thuleleni buys 6 bananas at R0,55 each and 2 pineapples at R4,80 each. She pays for the fruit with a R20,00 note.

a. Write a number sentence to represent the change that Thuleleni will receive from her shopping. Make sure to put brackets in the appropriate place(s).

b. Calculate how much change Thuleleni will receive.

2. Place brackets in the appropriate places in the following number sentences:

a. 3 × 7 + 4 − 5 × 2

b. 11 + 5 − 9 ÷ 3 + 2 × 10

c. 12 ÷ 4 × 5 + 2 − 6 ÷ 2

3. Determine the value of the number sentences in 2.

a.

b.

c.

4. Determine the value of the following:

a. (6 − 2) + 3 × (5 + 2)

b. [4 + (2 × 3) − 5] ÷ 5

4. c. $10 - [(5 \times 2) + 9 \div 3)] + 8$

____

____

____

5. Fill in the missing numbers in each of the questions below:

a. $3 \times$ ____ $- 4 = 2$

b. $6 + (4 \times$ ____ $) = 90$

c. $25 - (16 \div$ ____ $) = 21$

6. The following equation is used to determine the monthly repayment on a particular loan:

$$Repayment = (loan \div 1000) \times 23{,}05$$

a. Calculate the repayment on a R250 000,00 loan.

____

____

____

6. b. Calculate the repayment on a R1 000 000,00 loan.

____

____

____

7. The following formula is used to determine the amount of money in a particular investment after 2 years.

$$Amount = R4\ 000 \times [(1 + \frac{3}{100})^2]$$

Calculate how much money there will be in the investment after 2 years.

____

____

____

____

____

Test Your Knowledge: Basic Operations

1. A group of 5 friends are going away for the weekend. The total cost for the weekend comes to R852,00. How much does each person have to pay?

2. A mother is taking her four children to the uShaka Sea World in Durban. How much will it cost her if the tariffs are:

- Adults → R98,00
- Children → R66,00

3. Layla gives the shopkeeper a R100,00 note to pay for her purchases of R73,58. How much change will she receive?

4. Faisel buys 2 cokes for R5,20 each and 3 samoosas for R3,50 each. How much must he pay?

5. How much will it cost Zikhona if she buys 3 packets of chips for R3,75 per packet and 3 chocolates for R4,50 per chocolate? Show 2 ways of doing this sum.

6. There are 35 sweets in one packet and 46 of the same type of sweet in another packet. Divide these sweets equally amongst three friends.

7. In a particular town, electricity users pay a fixed monthly service fee of R85,00 and a consumption fee of R0,40 per kWh of electricity used.

a. Write a number sentence to represent the cost of electricity in this town. Be sure to include brackets in appropriate places in the number sentence.

b. Use the number sentence to determine the cost of using 367 kWh of electricity during the month.

8. Calculate:

a. $6 + 7 \times 2$

b. $8 - 3 \times 2$

c. $19 - 4 \times 3$

8. d. $3 \times 6 - 9$

e. $15 - 4 + 7 \times 2$

f. $11 \times 3 + 2$

g. $16 \times 4 - 3$

h. $6 + 7 \times 2 - 20 \div 4$

i. $18 \times 2 - (4 + 7)$

j. $16 - 5 \times 2 + 3$

9. Decide whether each of the statements below is true or false:

a. $6 \times 7 - 2 = 40$

b. $8 \times (6 - 2) + 3 = 56$

c. $35 - 7 \times 2 = 56$

d. $3 + 7 \times 3 = 30$

e. $18 - (4 + 7) = 21$

f. $43 - 3 + 2 = 42$

g. $18 \div 2 + 6 = 10$

h. $64 - 10 + 2 = 52$

i. $(4 + 2) + 7 = 4 + (2 + 7)$

j. $(8 - 2) - 1 = 8 - (2 - 1)$

k. $(8 \div 4) \div 2 = 8 \div (4 \div 2)$

10. Calculate:

a. $8{,}2 \div 0{,}2 - 0{,}1$

b. $3{,}6 \times 0{,}2 - 0{,}1$

10. c. $8,2 \times (6 - 5,4)$

d. $2,2 - 0,7 \times 0,2$

11. Fill in the missing numbers in each of the questions below:

a. $0,8 + \_\_\_ \times 0,6 = 3,2$

b. $\_\_\_ \times 0,5 + 6 \times 0,4 = 3,9$

c. $0,9 + 4,8 \div \_\_\_ = 6,9$

11. d. $2,7 \div \_\_\_ - 1,4 = 1,6$

1.2 ROUNDING

1.2.1 Definition

Rounding is the process of reducing the number of *significant digits* in a number.

Significant digits are the digits of a number that are known with certainty.

The first significant digit in a number is the first non-zero digit (reading from left to right). The remaining non-zero digits (or a zero considered to be the exact value) to the right of this number are all significant figures.

In both of the numbers below, the digit "3" is the first significant digit. The number on the left has 6 significant digits — 3, 4, 5, 6, 7 & 8 — while the number on the right has only got 3 significant digits — 3, 5 & 6.

3456,78 0,00356

First significant figure

The result of rounding is a "shorter" number having fewer non-zero digits yet similar in size to the original number. When rounding, the value of the number that results from rounding is less precise; but the number that we get from rounding is easier to use.

Because of this, it is important that we understand the impact of rounding on the calculations that we perform. Since the rounded number is less precise than the original number, if we do a lot of calculations with the rounded number then the result of those calculations may be quite different from the result that we would have produced had we not rounded the number.

It is possible to round a number with different degrees of precision. Rounding can happen to a **given number of significant digits** — to the nearest **whole number, nearest ten** or **nearest hundred;** or to a **given number of decimal places.**

There are three methods of rounding that we need to consider: **rounding off; rounding down; and rounding up**.

1.2.2 Rounding Off, Rounding Down and Rounding Up

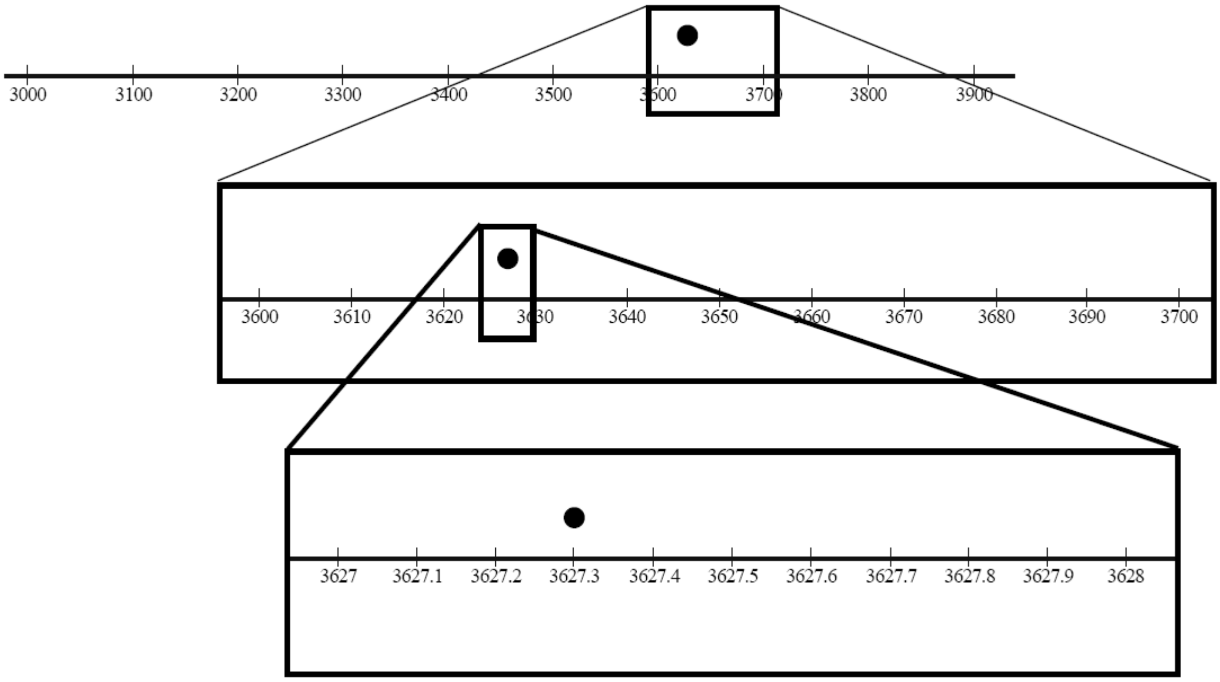
Rounding involves reducing the number of significant digits in a number. When we speak of rounding to the nearest 10, or 100 or to two decimal places we are simply indicating that we intend to stop working with the digits of the number that are less significant than the one indicated. In other words, when rounding to the nearest 100 we intend to ignore the tens and units digits from here on in the calculation or reporting of the number.

A. Rounding Off

Rounding off refers to rounding a given number to the nearest number with a given property.
As such, rounding off involves identifying the number with a given property that is closest to the number that we are working with. To do this we rely on a mental number line.

Example 1:

Consider the number lines and the bullets showing the number 3 627,3 below.



It should be clear from the diagram that:

- The number 3 627,3 is closer to 3600 than it is to 3 700 – if we had to round this number off to the nearest 100 then the number becomes 3 600.
- The number 3 627,3 is closer to 3630 than it is to 3620 – if we had to round this number off to the nearest 10 then the number becomes 3 630.
- The number 3 627,3 is closer to 3627 than it is to 3628 – if we had to round this number off to the nearest whole number then the number becomes 3 627.

Example 2:

Consider the number 3,1279.

- To round this number off to *<u>three decimal places</u>* we need to consider the value of the number in the 4th digit after the decimal − in this case "9". If this value is bigger than "5" then the digit in the 3rd position after the decimal must be rounded up to the nearest whole number; if this value is smaller than "5" then the digit in the 3rd position after the decimal must be rounded down to the nearest whole number.
  So, rounding 3,1279 to three decimal places gives: 3,128
- Using the same method, rounding the number off to two decimal places gives: 3,13
- Rounding off to one decimal place will give: 3,1
  In this case the "1" stays the same since the value in the 2nd digit after the decimal is less than 5.

Practice Exercise: Rounding Off

1. Round off 3 467 to the nearest:

a. Ten

b. Hundred

c. Thousand

a. ______________________________

b. ______________________________

c. ______________________________

2. Round off 3 428,629 to:

a. Two decimal places

____

b. One decimal place

____

c. The nearest whole number (i.e. 0 decimal places)

____

d. The nearest ten

____

e. The nearest hundred

____

f. The nearest thousand.

____

2. The bank calculates that they must pay R2,3157 in interest to one of their clients. If the bank rounds this value off to two decimal places before giving the interest to the client, how much money will the client receive?

____

3. Bob calculates that he needs 30,157 m^3 of concrete for the foundations of a house. For ease of use he rounds off this value to one decimal place.

What volume of concrete does Bob need?

____

4. Benni calculates that he needs 6,8 m of wood to fix his fence. If the shop only sells wood in whole meter lengths, how many metres of wood will he need to ask for at the shop?

____

5. The cost of pre-paid electricity in the Msunduzi Municipality is R0,47516 per kWh of electricity used.

a. Calculate the cost of using 300 kWh of electricity to two decimal places.

b. Calculate the cost of using 428,2 kWh of electricity to two decimal places.

c. If Cindy pays R385,00 for electricity, how many kWh of electricity has she used? Give your answer to one decimal place.

6. A cell phone company charges R0,0427 per second to make calls on its network.

a. How much will it cost (in Rand and cents) to make a 45 second call on this network?

b. How much will it cost (in Rand and cents) to make a 6 minute 47 second call on this network?

B. Rounding Down

Rounding down refers to the process of rounding a given number down to the nearest number with a given property. In the case of the number 3 627,3 above, when we rounded the number to the nearest 100 to get 3 600 we actually **rounded down**.

Example:

How many egg boxes can be filled if we have 155 eggs and each egg box can hold 6 eggs?

Since each egg box holds 6 eggs we need to divide 155 by 6 to determine the number of boxes that can be filled:

No. of boxes = 155 eggs ÷ 6 eggs per box

= 25,833 boxes

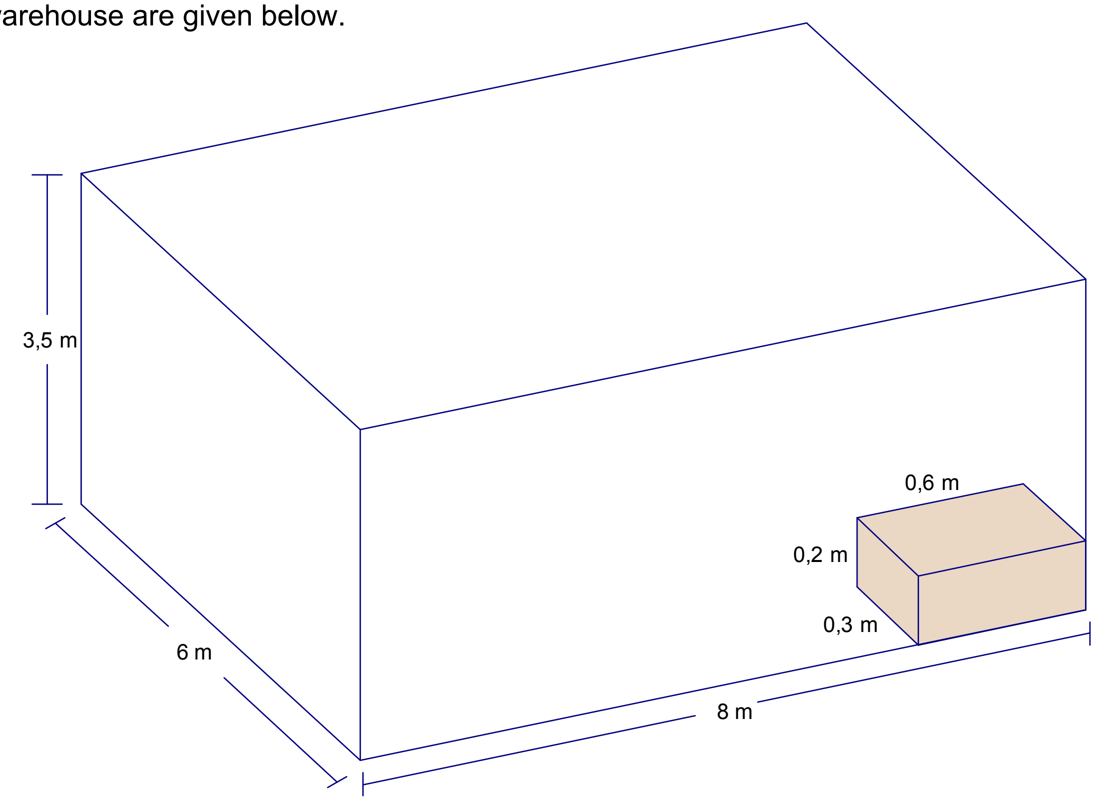
Since we are interested in whole egg boxes only, we want to round 25,833 to a whole number. Clearly 26 is the closest whole number to 25,833, but to give 26 as the answer would be to suggest that the 26^{th} egg box can be filled. This is not the case. We are 26 x 6 –155 = 1 egg short.
In this problem it is more appropriate to **round down** than to **round off.**
As such, the answer is: 25 egg boxes can be filled.

Practice Exercise: Rounding Down

1. Xolani is packing oranges into boxes. Each box can hold 30 oranges. If Xolani has 400 oranges to pack into the boxes, how many *full* boxes of oranges will he have?

2. Vusi buys a 6 m long pole to make a small fence. If the fence will be 0,7 m high, how many supports for the fence will Vusi be able to cut from the 6 m long pole?

3. Zanele needs to package boxes in a warehouse. A picture of the dimensions of each box and the warehouse are given below.



a. How many boxes will Zanele be able to fit along the length of the warehouse?

3. b. How many boxes will Zanele be able to fit along the width of the warehouse?

____

____

____

d. Now calculate the total number of boxes that Zanele will be able to store in the warehouse.

____

____

____

____

3. c. How many boxes high will Zanele be able to stack the boxes?

____

____

____

C. Rounding Up

Rounding up refers to the process of rounding a given number up to the nearest number with a given property. In the case of the number 3 627,3 above, when we rounded the number to the nearest 10 to get 3 630 we actually **rounded up**.

Example:

How many mini busses are needed to transport 122 adults (passengers) if each mini bus can hold exactly 15 passengers?

Since each mini bus holds 15 passengers we need to divide 122 by 15 to determine the number of mini busses that are needed:

$$\text{No. of mini busses} = 122 \text{ passengers} \div 15 \text{ passengers per mini bus}$$
$$= 8{,}133 \text{ mini busses}$$

Since we can only work with whole mini busses, we want to round 8,133 to a whole number. Clearly 8 is the closest whole number to 8,133. However, the answer of 8,133 tells us that we need *more than 8* taxis and to give 8 as the answer would be to leave 2 passengers stranded: i.e. 8 taxis x 15 people per taxi = 120 people and not 122.
Since we want all of the passengers to be transported it is more appropriate to **round up** than to **round off.**
As such, the answer is: 9 mini busses are needed.

The decision about whether to round off, round down or round up is determined by the context or situation in which the rounding happens, and there are many contexts in which we consciously choose to round up or round down rather than round off.

Practice Exercise: Rounding Up

1. 33 tourists are planning a sightseeing trip around Cape Town. If the company who will take them on the trip uses 14-seater mini-busses, how many mini-busses will be used?

2. Mandy is organising a dinner function for 74 people. The people are going to be seated at tables that can hold 8 people per table. How many tables will Mandy need?

3. The table below shows the coverage ratios for two different types of paint.

Paint Type	Coverage
Acrylic	9 m^2 per litre
Enamel	7,5 m^2 per litre

a. Which paint type is thicker? Explain.

3. b. How many litres of Acrylic paint will a painter need to *buy* to paint a wall with an area of 75 m^2?

c. How many litres of Enamel paint will a painter need to *buy* to paint a wall with an area of 104,2 m^2?

d. A painter buys 10 litres of Acrylic paint. What is the maximum size wall that he will be able to paint with this tin of paint?

4. The table below shows the number of bags of cement needed to plaster a wall.

WALL AREA (m^2)	No. BAGS CEMENT
60	5
120	10

(Adapted from: PPC Cement, Pamphlet – *The Sure Way to Estimate Quantities*, www.ppccement.co.za)

a. How many bags of cement will a builder need to *buy* to plaster a 30 m^2 wall?

____

____

____

b. How many bags of cement will a builder need to *buy* to plaster a 103 m^2 wall?

____

____

____

____

____

5. Hamilton is planning a trip from Durban to Johannesburg. The distance is 565 km.

a. If Hamilton's car has an average petrol consumption rate of 8 litres per 100 km, calculate how many litres of petrol he will need to complete the journey.

____

____

____

____

5. b. If the current petrol price is R10,30 per litre, show that Hamilton will need to put R465,56 worth of petrol in his car.

____

____

____

____

____

____

c. Explain why if Hamilton puts exactly R465,56 worth of petrol into his car then there is a possibility that he could run out of petrol during the trip.

____

____

____

d. What Rand value of petrol would you suggest Hamilton put in his car? Explain.

____

____

____

1.2.3 The Impact of Rounding

Telkom charges R0,00284 per second for a local call made during Callmore time. Explore the impact of rounding on the costs of a telephone call.

Consider a call that lasts for 8 minutes and 39 seconds (08:39) – without any rounding the cost of the call would be determined as follows:

→ Time spent on the call in seconds = (8 min × 60 min per sec) + 39 sec

= 480 sec + 39 sec

= 519 sec

∴ Cost of the call = 519 sec × R0,00284 per second

= R1,47396

Of course it is only possible to pay up to one-hundredth of a Rand (cents) and so the third, fourth and fifth decimal places make no sense in real life. As it is Telkom records the cost of the call to three decimal places on your telephone account. Although you might have rounded the amount off to R1,47 because this makes sense, Telkom records the amount as R1,474 — 4-tenths of a cent more than you would have had to pay if the amount had been rounded to R1,47. In itself, this 4-tenth of a cent does not seem to be that significant, but over many phone calls and many subscribers, you should be able to imagine how recording the third decimal place will have a significant impact on Telkom's income.

Consider the impact that rounding too early in a calculation can make.

Case 1: Rather than dealing with minutes and seconds – round off to whole minutes and then calculate. Remember we are using 60 as our rounding value:

→ Length of the call = 8 minutes and 39 seconds ≈ 9 minutes

∴ Cost of the call = (9 min × 60 sec per min) × R0,00284 per second

= R1,5336

Comparing this cost to the cost of the call if the time is not rounded off gives:

$$\text{Percentage difference} = \frac{\text{R}1.5336 - \text{R}1.47396}{\text{R}1.47396} \times 100\%$$

$$\approx 4\% \quad \text{(rounded off to one decimal place)}$$

Case 2: Rather than using the per second charge correct to 5 decimal places, round off to 3 decimal places and calculate:

→ Per second charge = R0,00284 ≈ R0,003

∴ Cost of the call = (8 min × 60 sec per min + 39 sec) × R0,003 per second

= R1,557

Comparing this cost to the cost of the call if the per second charge is not rounded off gives:

$$\text{Percentage difference} = \frac{R1.557 - R1.47396}{R1.47396} \times 100\%$$

$$\approx 6\%$$

Case 3: Rather than dealing with minutes and seconds – round off to whole minutes and rather than using the per second charge correct to 5 decimal places round off to 3 decimal places and calculate:

→ Length of the call = 8 minutes and 39 seconds ≈ 9 minutes

→ Per second charge = R0,00284 ≈ R0,003.

∴ Cost of the call = (9 min x 60 sec per min) x R0,003 per second = R1,62

Comparing this cost to the cost of the call if the time and the per second charge is not rounded off gives:

$$\text{Percentage difference} = \frac{R1.62 - R1.47396}{R1.47396} \approx 10\%$$

Although the amounts in these illustrations do not seem significant in Rand and cents terms, the percentage differences are quite substantial.

The point is that we need to be aware of the impact of rounding in general and in particular of the impact of rounding too early in any calculation.

Practice Exercise: The Impact of Rounding

1. A map has a scale of 1 : 100 000.

a. Bongani measures the distance between two towns on the map to be 11,8 cm.
According to Bongani's measurements, what is the actual distance between the two towns (in km)?

____

____

____

____

____

____

b. Songi measures the distance between the same two towns to be 11,7 cm.
According to Songi's measurements, what is the actual distance between the two towns (in km)?

____

____

____

____

____

____

c. Donnie measures the distance between the two towns to be 11,8 cm. If she rounds this value off to 12 cm and then uses 12 cm to determine the actual distance between the two towns, what effect will this have on the accuracy of her answer? You must show all working.

____

____

____

____

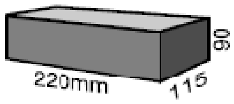
____

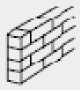
____

____

____

2. The picture below shows the number of bricks, bags of cement and m^3 of sand needed to build a wall. (Source: Adapted from: PPC Cement, Pamphlet – The Sure Way to Estimate Quantities, www.ppccement.co.za)



MATERIAL REQUIRED FOR 1000 BRICKS			
WALL SIZE	m2 WALL AREA	No. BAGS SUREBUILD	m3 SAND
 Single brick wall	20	4	1,0

Mpho calculates that the wall he is planning to build will have an area of 105,4 m^2.

a. Calculate how many bags of cement Mpho will need to build this wall.

b. If Mpho rounds this value off to 105 m^2 and then calculates the number of bags of cement that he will need, what will his answer be?

c. Explain what the implications of rounding off the area of the wall value will be for Mpho.

Test Your Knowledge: Rounding

1. a. Round off 2 973 to the nearest:

i. Ten

ii. Hundred

iii. Thousand

i. ______________________________

ii. ______________________________

iii. ______________________________

b. Round off R134,78 to the nearest Rand.

____

c. Round R12 456 987,00 to the nearest million Rand.

____

d.

i. Round off 3,18 to one decimal place.

____

ii. Round off 5,52 to one decimal place.

____

iii. Round off 24,148 to two decimal places.

____

d. iv. Round off 3,5 to the nearest whole number.

____

v. Round off 24,145 to one decimal place.

____

2. Nomalunge is packing apples into packets to sell at the local market. She has 250 apples and is putting 7 apples into a packet. How many packets containing seven apples will she have?

____

____

____

____

3. John is making bookcases and is using a plank of wood that is 2,6 m long to make the shelves. If each shelf has a length of 70 cm, how many shelves can he cut from one plank of wood?

____

____

____

____

____

4. Your college needs to transport 743 learners.

6. b. Repeat the above for 1 decimal place. Give

The bus company says that their buses can take a maximum of 60 learners. How many buses does your school need?

5. Your college is expecting about 345 people to attend a fashion show. How many rows of chairs are needed if each row takes 18 chairs?

6. Consider the statement: $\frac{1}{11} \times \frac{3}{11} \times \frac{5}{11} \times \frac{7}{11}$

a. Using your calculator, convert each fraction to its decimal equivalent correct to 3 decimal places and then determine the product.

your final answer to three decimal places.

c. Compare the answers that the calculations above produce and make comment on the differences.

1.3 PERCENTAGE

1.3.1 Definition

A percentage is a portion of a whole, where the whole is one hundred. Every percentage is then a fraction out of 100 (the whole). It is for this reason that we write a percentage as a fraction with a denominator of 100.

E.g. 40% is shorthand for: $\frac{40}{100}$ or 0,40.

Percentage has been adopted quite comfortably into day-to-day language because:

- People find it easier to visualise/comprehend percentages than actual amounts. For example one would have a better sense of how popular a candidate was if you heard that *"Karen got 70% of the votes"* as compared with: *"Karen got 4 389 of the 6 270 votes cast'.*
- It makes comparisons easier. For example, people find it easier to make sense of the statement: *"37,5% of the population got ill this year in comparison with 44,4% last year"* than they would the statement: " $\frac{3}{8}$ *of the population got ill this year in comparison with* $\frac{4}{9}$ *last year".*
- Percentages are a whole lot easier to write and type into text such as newspaper articles than fractions of the form $\frac{a}{b}$ are.

While percentage makes visualisation and comparison easier in day-to-day discussion, one should be aware that in using a percentage the actual values are lost (or hidden). For example:

- There may well be more water in a dam that is 25% full than in a dam that is 75% full – provided that the first dam has a greater capacity than the second.
- When we are told that 85% of the respondents to a survey favoured one product over another – we do not know either how many respondents there were and/or how representative that sample was of all users.

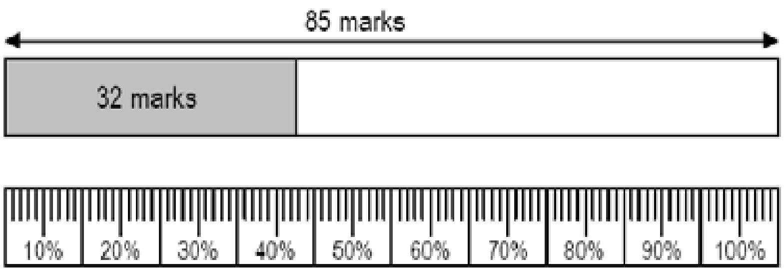
1.3.2 Typical calculations involving percentage

We will consider 6 different, yet related, calculations that involve percentage.

A. Expressing a Part of a Whole as a Percentage

E.g. *What percentage did Fabian get for his test if he scored 32 marks out of a possible 85?*

In this problem 85 marks represent the whole or 100%. 32 marks represent a part of that whole.



The following thought processes would help to solve the problem:

Step 1 (estimate):

32 *is less than one-half or 50% of the whole since one-half of 85 is a little more than 40.*

Step 2 (relate the marks in the whole to percentages):

Since 85 marks represents 100%, it follows that 85 marks ÷ 100% = 0,85 marks per %.

Step 3 (convert the actual marks to a percentage of the whole):

32 marks represents 32 marks ÷ 0,85 marks per % ≈ 37,65%.

Calculator work:

The same calculation can be achieved using a basic calculator as follows:

$\boxed{3}\,\boxed{2}\,\boxed{\div}\,\boxed{8}\,\boxed{5}\,\boxed{\%}$

Practice Questions: Expressing a Part of a Whole as a %

1. Xolani gets 18 out of 30 for a test. What percentage did he get for the test?

____

____

____

____

2. A town has a total population of 2 450 people. During an election 1 666 people in the town vote. What percentage of the total population of the town voted in the election?

____

____

____

____

3. The table below shows the number of teachers in each province in South Africa in 2005.

Province	No. of Teachers
Eastern Cape	67 230
Free State	23 400
Gauteng	60 121
KwaZulu-Natal	80 979
Limpopo	56 160
Mpumalanga	27 701
North West	27 454
Northern Cape	6 641
Western Cape	32 447

(National Department of Education. 2006. Education Statistics in South Africa at a Glance in 2005. p.4)

3. a. How many teachers were there in South Africa in 2005?

____

____

b. What percentage of the total number of teachers in South Africa teach in:

i. Mpumalanga?

ii. Western Cape?

iii. Gauteng AND KwaZulu-Natal?

(Give your answers to one decimal place)

b. i. ______________________________

____

____

ii. ______________________________

____

____

iii. ______________________________

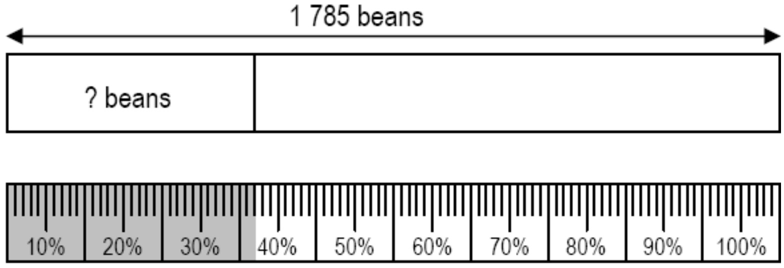
____

____

B. Determining a Percentage of an Amount

E.g. *How many beans are there in 32% of 1 785 beans?*

In this problem 1 785 beans represents the whole or 100%. 32% represents the fraction of beans in the whole that we want to identify.



The following thought processes would help to solve the problem:

<u>Step 1 (estimate):</u>

32% is approximately one-third (33%) and one-third of 1 785 is approximately 600 since 1 800 ÷ 3 = 600.

<u>Step 2 (relate the beans in the whole to percentages):</u>

Since 1 785 beans represents 100%, it follows that 1 785 beans ÷ 100% = 17,85 beans per %

<u>Step 3 (determine the number of beans represented by the desired percentage):</u>

32% represents 32% × 17,85 beans per % ≈ 571 beans

<u>Calculator work:</u>

The same calculation can be achieved using a basic calculator as follows:

$$\boxed{1}\,\boxed{7}\,\boxed{8}\,\boxed{5}\,\boxed{\times}\,\boxed{3}\,\boxed{2}\,\boxed{\%}$$

Practice Questions: Determining a % of an Amount

1. Trudy is given a 5% discount on a shirt that costs R125,00. How much *discount* does Trudy receive?

____

____

____

____

2. Sindiwe earns R4 200,00 per month and receives an 8% increase in salary. How much *increase* does Sindiwe receive?

____

____

____

____

3. The table below shows the percentage of learners in each province in South Africa in 2005.

Province	Percentage
KwaZulu Natal	20,9%
Gauteng	17,2%
Eastern Cape	17,0%
Limpopo	14,9%
Western Cape	8,5%
Mpumulanga	7,0%
North West	6,9%
Free State	5,9%
Northern Cape	1,6%

(National Department of Education. 2006. Education Statistics in South Africa at a Glance in 2005. p.4)

3 …

If there was a total 13 936 737 learners in South Africa in 2005, calculate how many learners there were in:

a. Eastern Cape

b. Western Cape

c. North West

d. Northern Cape

3. a. ______________________________

____

____

____

b. ______________________________

____

____

____

c. ______________________________

____

____

____

d. ______________________________

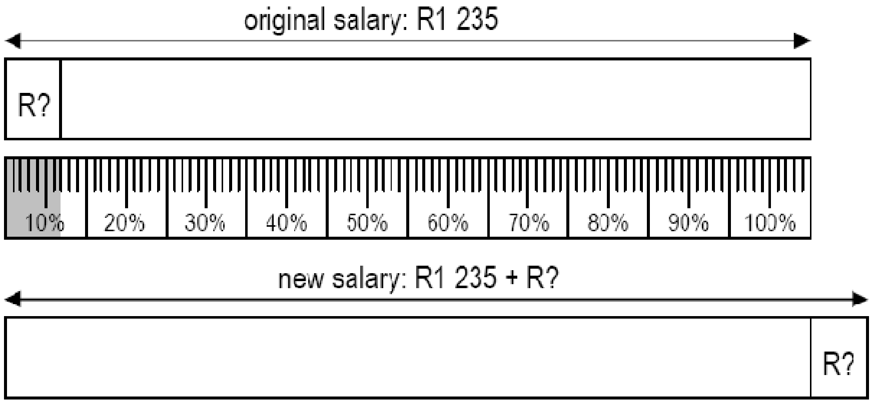
____

____

C. Adding a Percentage of an Amount to an Amount

E.g. *What will Victor's new salary be if he currently earns R1 235,00 per week and he gets a 7% increase?*

In this problem R1 235,00 represents the whole or 100% of Victor's original salary. 7% represents the fraction/part of his salary that we want to identify and add to his salary.



The following thought processes would help to solve the problem:

<u>*Step 1 (estimate):*</u>

10% of R1 235,00 ≈ R124,00. Since he only gets a 7% increase in salary, his new salary will be less than R1 235,00 + R124,00 ≈ R1 360,00.

<u>*Step 2 (relate the original salary — i.e. the whole — to percentages):*</u>

Since R1 235,00 represents 100%, it follows that R1 235,00 ÷ 100% = R12,35 per %.

<u>*Step 3 (determine the number of Rand represented by the desired percentage):*</u>

7% represents 7% × R12,35 per % ≈ R86,45.

<u>*Step 4 (add the amount to the whole)*</u>

New salary = R1 235,00 + R86,45 = R1 321,45.

<u>*Calculator work:*</u>

The same calculation can be achieved using a basic calculator in one of two different ways:

Method 1: $\boxed{1}\boxed{2}\boxed{3}\boxed{5}\boxed{\times}\boxed{7}\boxed{\%}\boxed{+}\boxed{1}\boxed{2}\boxed{3}\boxed{5}\boxed{=}$

Method 2: $\boxed{1}\boxed{2}\boxed{3}\boxed{5}\boxed{+}\boxed{7}\ \boxed{\%}$

Practice Questions: Adding a %

1 The price of a can of cool drink that costs R5,50 increases by 5%. What will the new price of the can of cool drink be?

2. Sindiwe earns R4 200,00 per month and receives an 8% increase in salary. What will Sindiwe's new salary be

3. Mandy makes and sells bracelets. It costs her R9,50 to make each bracelet and she sells the bracelet with 110% mark up. Determine how much she sells the bracelets for.

4. A supermarket owner is looking to increase the prices of certain goods in his shop. The table below shows the current price of the goods and the percentage by which the owner wants to increase the prices.

Goods	Current Price	% Increase
Maize-Meal	R55,45	17%
Chicken	R32,99	9%

Calculate the new price of each of the goods.

Maize-Meal: ______

Chicken: ______

5. Bob is mixing concrete in order to build a wall. He decides to buy slightly more cement, sand and stone than he needs to account for wastage.

Goods	Accurate Quantity Needed	Extra Needed for Wastage
Cement	58 bags	10%
Sand	87 wheelbarrows	15%
Stone	90 wheelbarrows	15%

Determine how many bags of cement and wheelbarrows of sand and stone Bob will need to *buy*.

Cement: ______________________________

____

____

____

____

Sand: ______________________________

____

____

____

____

____

Stone: ______________________________

____

____

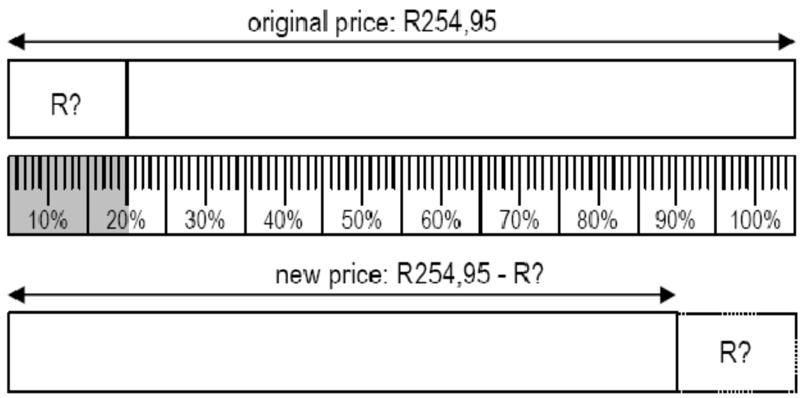
____

____

D. Subtracting a Percentage of an Amount from an Amount

E.g. *How much will you pay for a pair of jeans if they cost R254,95 and you are offered a 15% discount?*

In this problem R254,95 represents the whole or 100% of the original price for the pair of jeans. 15% represents the fraction/part of this price that we want to identify and subtract.



The following thought processes would help to solve the problem:

Step 1 (estimate):

10% of R254,95 ≈ R25,00. 15% is one-and-a-half times 10%, so 15% discount will be ≈ R37,00 and the discount price will be ≈ R255,00 – R37,00 ≈ R220,00.

Step 2 (relate the original price — i.e. the whole to percentages):

Since R254,95 represents 100%, it follows that R254,95 ÷ 100% = R2,5495 per %

Step 3 (determine the number of Rand represented by the desired percentage)

15% represents 15% x R2,5495 per % ≈ R38,24

Step 4 (subtract the amount from the whole)

Amount to pay = R254,95 – R38,24 = R216,71

Calculator work:

The same calculation can be achieved using a basic calculator in one of two different

Method 1: 2 5 4 . 9 5 × 1 5 % M+ ; 2 5 4 . 9 5 – MRC =

Method 2: 2 5 4 . 9 5 – 1 5 %

Alternatively, when you deduct 15% from the price of a garment you are left with 85% of the price of the garment. The discounted price can therefore also be regarded as 85% of the original amount and calculated as in B above.

Practice Questions: **Subtracting a %**

1. Trudy is given a 5% discount on a shirt that costs R125,00. How much will she have to pay for the shirt?

2. The average rainfall in Mphophomeni decreased by 13% from 2006 to 2007. If the average rainfall in 2006 was 28,3 mm, determine the average rainfall in 2007.

(Give your answer to one decimal place)

3. Sindi buys a car that costs R75 000,00. The value of her car decreases by 15% per year.

a. How much will the car be worth after 1 year?

b. How much will the car be worth after 2 years?

3. a.

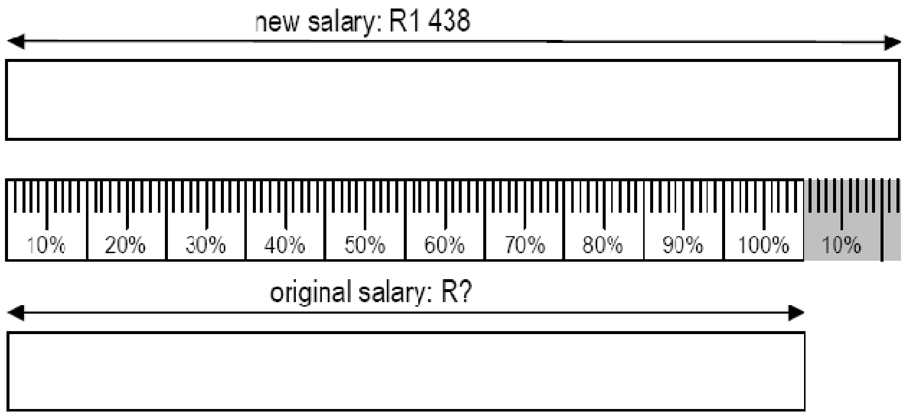
b.

4. Would it be possible to decrease the price of a radio that costs R390,00 by 105%? Explain your answer.

E. Calculating the Original Amount after a Percentage has been Added or Subtracted

E.g. *How much did Paul originally earn if his new salary after a 12% increase is R1 438,00 per week?*

In this problem R1 438,00 represents the original salary increased by 12%. We want to know what the original salary was — the key to solving this problem lies in understanding that *the increased salary is 112% of the original salary*.



The following thought processes will then help to solve the problem:

Step 1 (estimate):

R1 438 represents 112 parts:

- *if each part was R11 (R10 + R1) then that would give:*
  R1 120 + R112 = R1 232;
- *if each part was R12 (R10 + R2) then that would give:*
  R1 120 + R112 × 2 = R1 232 + R112 = R1 344.

So each part is ≈ R12 and the original salary would have been ≈ R1 200,00.

Step 2 (relate the final salary — i.e. original 100% plus the increase to percentages): Since R1 438 represents 112%, it follows that R1 438 ÷ 112% ≈ R12,8393 per %.

Step 3 (determine the original amount represented by 100%):

The original salary represents 100%, hence 100% × R12,8393 per % ≈ R1 283,93.

Calculator work:

The same calculation can be achieved using a basic calculator as follows:

$\boxed{1}\boxed{4}\boxed{3}\boxed{8}\boxed{\div}\boxed{1}\boxed{1}\boxed{2}\ \boxed{\%}$

Practice Questions: Calculating the Original Amount

1. Donny is given a 7% increase in salary so that she now earns R6 210,00 per month. How much did she earn before the increase?

2. A bicycle costs R755,00 *including* VAT (Value Added Tax). If VAT is 14%, how much VAT is included in the price of the bicycle?

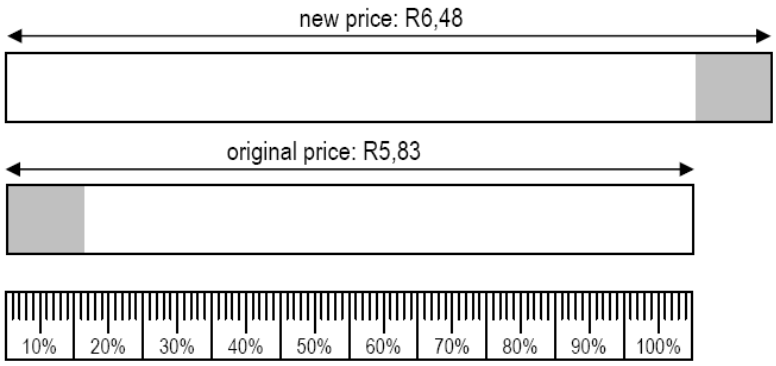
3. Jemima sells necklaces with a 40% mark up on what it costs her to make the necklaces. If she sells the necklaces for R55,00, how much does it cost her to make the necklaces?

3.

4. House prices in KwaZulu-Natal increased on average, by 18% from 2006 to 2007. If a house cost R680 000,00 in 2007, how much would that same house have cost in 2006?

F. Calculating a Percentage Change

E.g. *What was the percentage change in the price of petrol if it changed from R5,83 per litre to R6,48 per litre?*



In this problem R6,48 represents the original amount of R5,83 increased by some percentage. We want to express the actual increase (R6,48 – R5,83 = R0,65) as a percentage of the original petrol price.

The following thought processes will help to solve the problem:

Step 1 (estimate):

10% of R5,83 ≈ R0,60 and R5,83 + R0,60 ≈ R6,40 so the increase seems to be ≈ 10%.

Step 2 (determine the change in the price of petrol):

R6,48 – R5,83 = R0,65

Step 3 (express the change in price as a percentage of the original amount as in problem 1):

Since R5,83 ÷ 100% = R0,0583 per %, it follows that R0,65 = R0,65 ÷ R0,0583 per % = 11,15%. So the percentage increase is 11,15%.

The same calculation can be achieved using the formula:

$$\%\ \text{increase} = \frac{\text{final value} - \text{original value}}{\text{original value}}$$

Practice Questions: **% Change**

1. The price of bread increased from R7,20 to R7,80. Calculate the percentage increase in price to one decimal place.

2. In 2007, 12 003 people entered the Comrades Marathon and in 2008 11 191 people entered. Calculate the percentage decrease in the number of entrants from 2007 to 2008.

3. The table below shows the number of teachers in South Africa over the period 2001 – 2004.

Year	No. of Teachers
2001	354 201
2002	360 155
2003	362 598
2004	362 042

Calculate the percentage increase per year in the number of teachers in South Africa to one decimal place.

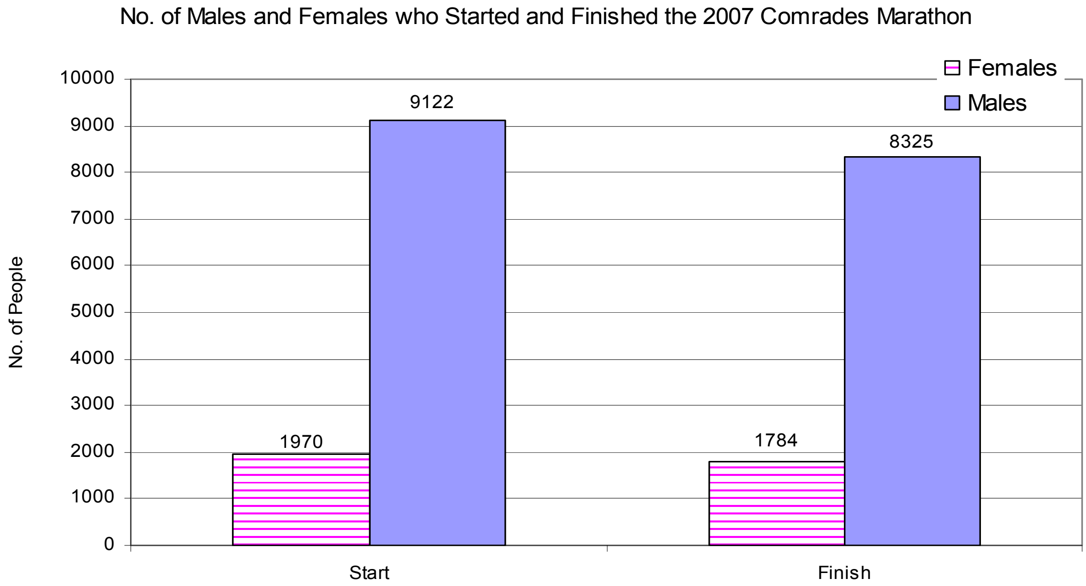
3 …

2001 – 2002:

2002 – 2003:

2003 – 2004:

4. The graph below shows the number of females and males who started and finished the 2007 Comrades Marathon.



4. a. What percentage of the females who started the race finished (to one decimal place)?

b. What percentage of the males who started the race finished (to one decimal place)?

c. Did the females or males perform better in the 2007 Comrades Marathon? Explain.

Test Your Knowledge: Percentages

1. a.

i. You get $\frac{27}{60}$ for your first Mathematical Literacy test. Express your result as a percentage.

____

____

____

____

ii. If you get $\frac{17}{40}$ for your second Mathematical Literacy test, in which test did you do better?

____

____

____

____

1. b. 26 590 people watched Bafana Bafana play against Ghana. If the stadium can accommodate 30 000 people, what percentage of the stadium was full?

____

____

____

____

2. a. How much will a waitron receive as a tip if she gets a 10% tip on a bill of R349,56?

____

____

____

b. 75% of the money raised at your school's market day was given to charity. How much money went to charity if your school raised R15 486,00?

____

____

____

2. c. 18% of the 11 046 athletes in the 2007 Comrades Marathon were females. How many females were there in the race?

____

____

____

3. a. Jimmy earns R18,00 an hour. How much will Jimmy earn per hour if he gets a 6,5% increase?

____

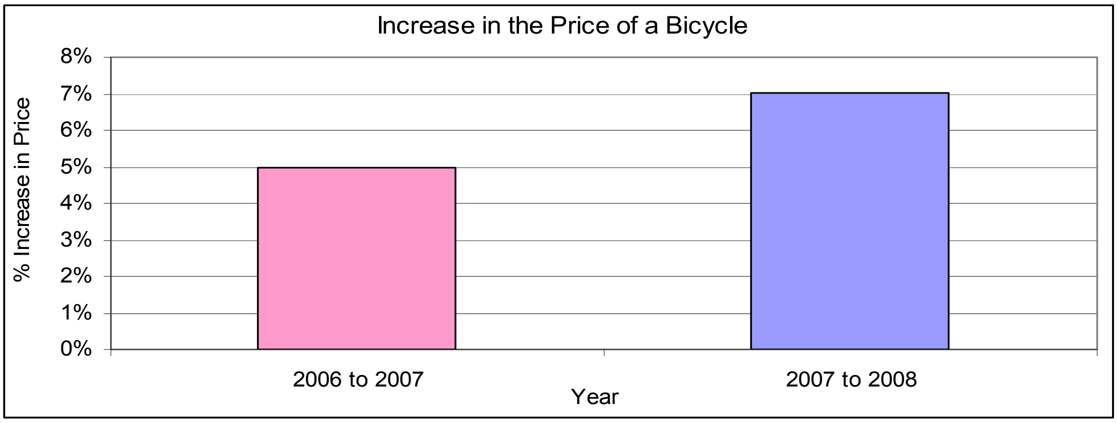
____

____

____

3. b. A new car will cost R179 500,00 without VAT. What will it cost you with 14% VAT included? (VAT = Value added tax)

3. c. The graph below shows how the price of a bicycle increased from 2006 to 2007 and from 2007 to 2008.



If the price of a bicycle in 2006 was R3 200,00, calculate how much that same bicycle would have cost in 2008.

4. a. A shop advertises a 33% discount on all goods in the shop. How much would you pay for a pair of pants that was selling for R200,00?

b. 15% of the people who had bought tickets to a concert did not arrive. If the organisers had sold 5 880 tickets, how many people were at the concert?

4. c. Bongiwe earns R5 460,00 each month. She decides that she wants to save 5% of her salary each month. How much money does she have left after she has banked her savings?

5. a. The price of milk increased from R6,50 to R7,80 per litre. What was the percentage increase?

5. b. A supermarket advertises that they are selling bottles of cooking oil that cost R13,99 at a discounted price of R10,99. Calculate the percentage discount (to one decimal place).

5. c. In 2001 the population of Cape Town was approximately 2 900 000 and in 2007 the population was approximately 3 500 000. What was the percentage increase in the population from 2001 to 2007 (to one decimal place)?

6. The VAT inclusive price of a washing machine is R1 580,00. Calculate how much the machine costs without VAT. Take VAT to be 14%.

1.4 RATIO

1.4.1 Definition

A ratio is a comparison of two (or more) numbers called the *terms* in the ratio.

Ratios have no units since the quantities being compared are of the same kind or type.

A ratio can be expressed in different ways:

- In words → *a to b*
- With a colon → *a:b*
- As a fraction → $\frac{a}{b}$

A number of ratios that we come across in daily life include:

- Mixing ratios
    - o cold drink which is made by mixing concentrate and water in some ratio − e.g. 1 : 4
    - o concrete which is made by mixing gravel, sand and cement in a ratio − e.g. 6 : 3 : 1
    - o paint colours which are made by mixing component colours in a given ratio

- Scales
    - o comparing lengths on maps and plans to actual distances − e.g. 1 : 50 000

There are two important ideas that we need to understand if we are to work successfully with ratios. The first is that the ratio does not give us the units and the second is that ratios can be written in equivalent forms.

1.4.2 Ratios and Units

A ratio gives a comparison of the size of one (or more) values in relation to another value of the same type or kind. For this reason, we do not include units in a ratio since the relationship in size between the values is the same irrespective of the units being used.

For example:

- If the mixing ratio for mixing concrete is given as gravel : sand : cement = 6 : 3 :1 then this means that for every 6 units of gravel you will need 3 units of sand and 1 unit of cement.

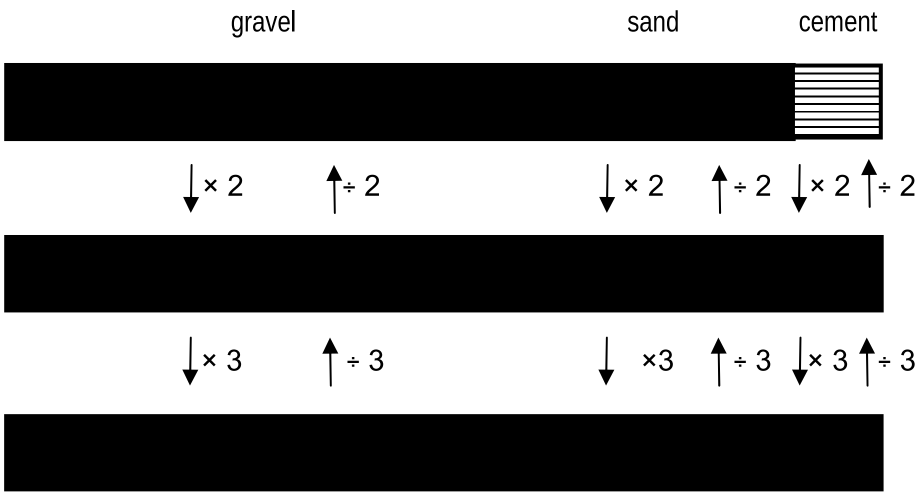
  In other words, you can mix 6 **buckets** of gravel with 3 **buckets** of sand and 1 **bucket** of cement; or you can mix 6 **wheelbarrows** of gravel with 3 **wheelbarrows** of sand and 1 **wheelbarrow** of cement — and both mixtures will have the same strength.

- If the scale for a plan is 1 : 50 then it does not matter what units you use to measure on the plan as long as you use the same units on the ground. i.e. 1 **mm** on the plan corresponds to 50 **mm** in actual size and 1 **cm** in the plan corresponds to 50 **cm** in actual size.

1.4.3 Equivalent ratios

It is very important to understand that ratios can be written in equivalent forms and to be able to convert between these forms.

The ratio of gravel to sand to cement is given as 6 : 3 : 1 — this can also be written as 12 : 6 : 2 and 36 : 18 : 6. These ratios are *equivalent* because they have the same meaning. i.e. There is six times as much gravel as cement in each mixture and three times as much sand as cement in each mixture. We can write: 6 : 3 : 1 = 12 : 6 : 2 = 36 : 18 : 6. A diagram can help to develop this understanding.



It is useful to be able to move flexibly between equivalent forms of the ratio and this can be done in several ways. For example, two approaches to determine the other terms in the ratio of a mixture that contains 12 bags of cement are shown below.

Method 1:

6 : 3 : 1

× 3   × 12

72 : 36 : 12

× 6

Method 2:

6 : 3 : 1

× 12   × 12   × 12

72 : 36 : 18

Or the equivalent ratios:

6 : 3 : 1 = 12 : 6 : 2 = 36 : 18 : 6 = 72 : 36 : 12

6 : 3 : 1 is called the ratio in its ***simplest form*** because the numbers in the ratio have no common factor; i.e. there is no common number that can be divided equally into all three terms.

Expressing ratios in simplest form can help us to compare ratios.

For example, in one college there are 1 152 students and 36 lecturers while in another college there are 1 568 students and 48 lecturers. We can write the ratio of students to lecturers for the two colleges as: College 1 → 1 152 : 36   College 2 → 1 568 : 48

These ratios are hard to compare because the values in the ratios are so different. By converting these ratios to equivalent and/or simplest forms it may be easier to see in which college the student to lecturer ratio is better.

College 1:

1 152 : 36

÷ 12   ÷ 12

96 : 3

College 2:

1 568 : 48

÷ 16   ÷ 16

98 : 3

Although these ratios are not in simplest form and it is possible to see that the second college has slightly more students per lecturer than the first college, the numbers in reality will seldom work out as easily as these did. For example if the student : lecturer ratios had been 1 358 : 49 and 793 : 36 it would have been much more difficult to simplify these ratios to equivalent fractions since there is no common factor for 36 and 49. And although we could have converted the ratios to something like 582 : 21 and 462 : 21, this would have meant working out an appropriate decimal value to divide each of the terms by in order to still end up with whole values to represent the students and lecturers.

It is important to note that, as with percentage, when we simplify ratios we create ratios that are easier to make sense of but the actual values are lost (or hidden). In the examples of the colleges above, we get a better sense of the relationship of the number of students to each lecturer, but we lose any sense of how many of either there are in the college.

1.4.4 Unit ratios (1 : n or n : 1)

Unit ratios are ratios in which the smallest of the numbers in the ratio is a 1 (a single unit). In the case of the colleges above it is easier to convert the student : lecturer ratios to unit ratios. Unit ratios also make comparison easier because they give us a sense of how many students have been allocated to *each* lecturer.

In order to convert a ratio to a unit ratio you divide one of the values in the ratio (usually the smaller value) by itself. This will reduce that value to a single unit (i.e. 1). You then divide the other value in the ratio by that same value.

Applying this method to the student : lecturer ratios in the previous scenario gives:

College 1:	1 358 : 49	College 2:	793: 36
	÷ 49 ÷ 49		÷36 ÷ 36
	27,71 : 1		22,03 : 1

It is now very clear that College 2 has a lower student : lecturer ratio than College 1, with approximately 22 students to every lecturer compared with 28 students to every lecturer in College 1. Notice how the values have been *rounded up*. This is because the scenario deals with people and although is it fine to use decimal values (27,71 students and 22,03 students) for purposes of making a comparison, it is also important to remember that it is not possible to have a decimal portion of a person.

1.4.5 Typical calculations involving ratio

There are at least three different calculations involving ratios:

A. Converting Between Different Forms of a Ratio

1. *Write 36 : 42 in simplest form:*

   In converting this ratio to simplest form we need to find the largest number (factor) that will divide into both terms of the ratio.

Option 1:	Option 2:	Option 3:
36 : 42	36 : 42	36 : 42
÷ 2 ÷ 2	÷ 3 ÷3	÷ 6 ÷ 6
18 : 21	12 : 14	6 : 7

   Clearly 18 : 21 ; 12 : 14 and 6: 7 are all equivalent forms of the ratio 36 : 42, but 6 : 7 is considered the *simplest form* of the ratio since 6 and 7 do not have a common factor that divides equally into both numbers.

2. *Write 14 : 72 in the form 1 : n*:
   To do this we divide both 14 and 72 by 14.

   14 : 72
   ÷ 14 ÷14
   1 : 5,143

   It is generally a good idea to round off any values in the ratio to three decimal places in case you need to compare the values to other decimal values in other ratios.

Practice Questions: Using Ratios

1. Write the following ratios in simplest form:

a. 20 : 32

b. 72 : 56

c. 27 : 81

a. ______________________________

____

____

b. ______________________________

____

____

c. ______________________________

____

____

2. Write the following ratios in unit form (i.e. in the form 1 : n or n : 1):

a. 25 : 75

b. 728 : 91

c. 12 : 59

d. 107 : 11

a. ______________________________

____

____

2. b. ______________________________

____

____

c. ______________________________

____

____

d. ______________________________

____

____

3. The *pupil : teacher* ratios in two schools are given below.

- School 1 → 782 : 32
- School 2: → 1 328 : 57

Show by calculation which school has the better pupil : teacher ratio.

____

____

____

____

____

____

____

4. The scale of a map is 1 : 20 000.

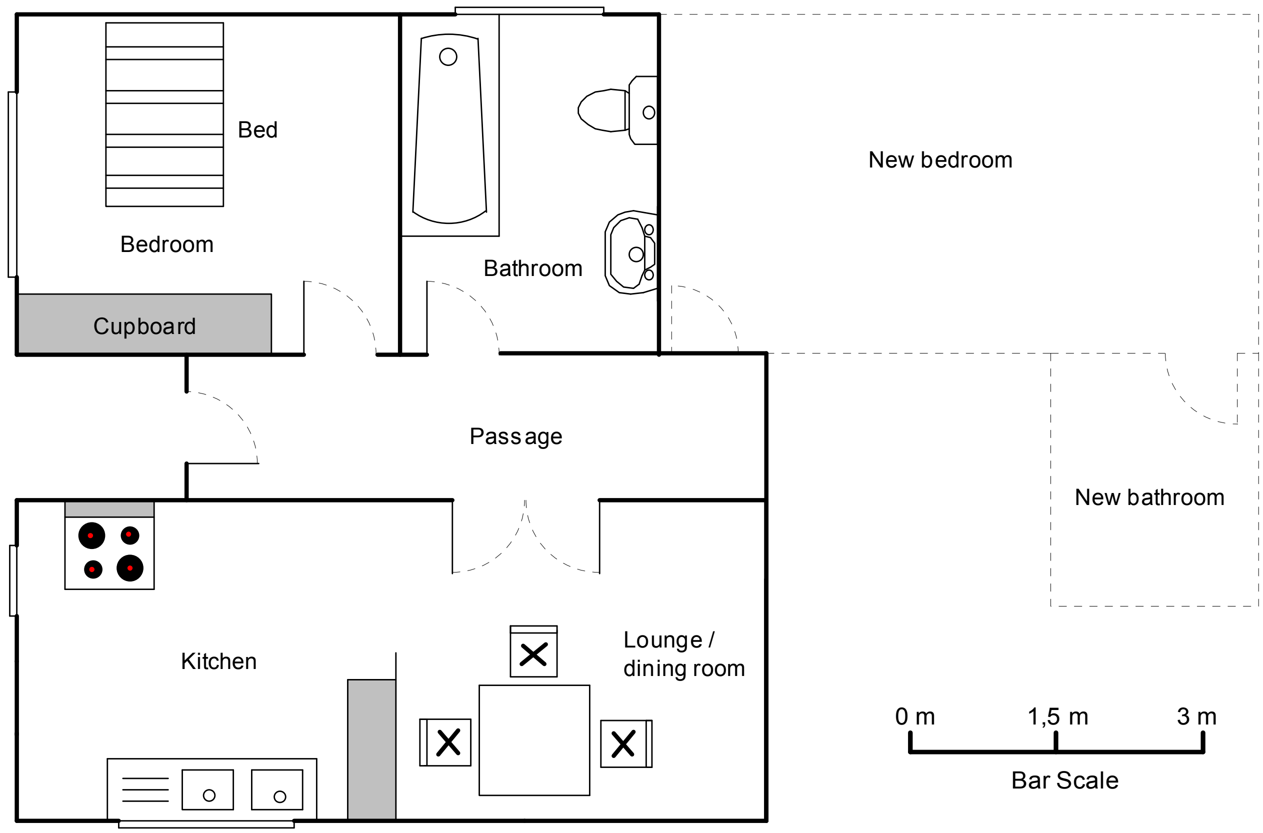
If the distance measured on the map is 24 cm, how far will this be in actual distance (in km)?

5. The scale of a plan is 1 : 20.

a. If the length of an item on the plan is 185 mm, determine the actual length of this item in metres.

5. b. If the actual length of an item is 5 m, determine how long this item will have been drawn on the plan (in cm).

6. The picture below shows a 2-dimensional top-view picture of the layout of a house. The owners of the house are planning on building a new bedroom and bathroom on to the house.

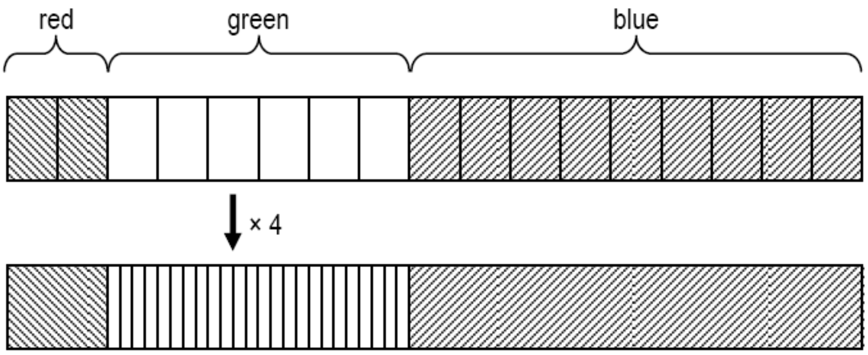


Use the bar scale to determine the dimensions (length and width) of the new bedroom and new bathroom. If necessary, give your answers to one decimal place.

B. Determining Missing Numbers in a Ratio

E.g. *If paint is mixed in the ratio red : green : blue = 2 : 6 : 9, determine the number of units of red and blue that are needed if 24 units of green are to be used.*

The following picture should assist in making sense of this problem.



The mixing ratio is 2 : 6 : 9 as shown in the top line. In the second line we can see that we have 24 units of green — to get from 6 units to 24 units we needed to divide each unit into 4 equal parts — that is, we had to multiply by 4, it follows that we need to multiply the number of units of red in the mixing ratio and the number of units of blue in the mixing ratio by 4 as well. It follows that we will need 8 units of red and 36 units of green to complete the mixture.

$$2 : 6 : 9 \xrightarrow{\times 4} 8 : 24 : 36$$

Practice Questions: Determining Missing Numbers in a Ratio

1. If paint is mixed in the ratio

red : green : blue = 2 : 6 : 9

a. Determine the number of units of green and blue that are needed if 10 units of red are to be used.

b. Determine the number of units of red and blue that are needed if 18 units of green are to be used.

c. Determine the number of units of red and green that are needed if 25 units of blue are to be used.

2. Energade concentrate energy drink recommends that 1 unit of concentrate be mixed with 5 units of water.

a. How many ml of water must be added to 50 ml of concentrate?

b. How many litres of water must be added to 300 ml of concentrate?

c. How much juice (water & concentrate) will you make with 100 ml of concentrate?

2. d. Simphiwe mixes together 400 ml of water with 50 ml of concentrate. Will the juice be too sweet, not sweet enough or just right?

____

____

____

____

____

3. For making low strength concrete, the ratio of cement : sand : stone is 1 : 4 : 4.

a. How many wheelbarrows of sand and stone will you need if you use 8 wheelbarrows of cement?

____

____

____

b. How many spades of cement will you need to mix with 36 spades of sand?

____

____

____

____

3. c. How many bags of cement will you need to buy if you use 37 bags of stone?

____

____

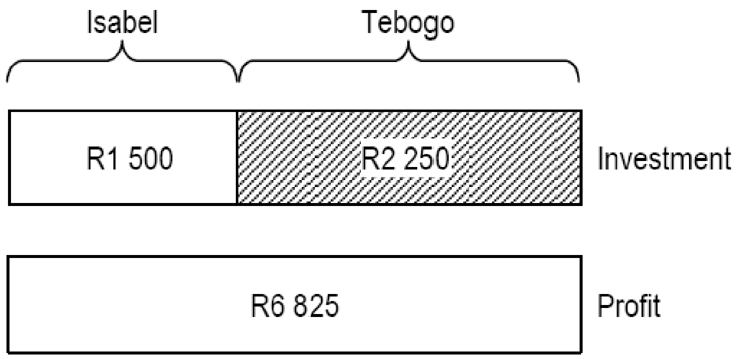
____

____

C. Dividing or Sharing an Amount in a Given Ratio

If Isabel and Tebogo invested R1 500,00 and R2 250,00 respectively in a business, how much will they each get if the company makes a profit of R6 825,00 and they have agreed to share the profits in the ratio of their investments?

A picture should assist in making sense of this problem.



Isabel and Tebogo have each invested into the business and have done so in the ratio 1 500 : 2 250 — this makes a total of 1 500 + 2 250 = 3 750 units or parts (rand). The business has realised a profit of R6 825,00 which must be divided evenly among the 3 750 investment units.

It follows that each unit will get: R6 825 ÷ 3 750 units = R1,82 per unit.

Since Isabel invested 1 500 units she will get: 1 500 units × R1,82 per unit = R2 730,00.

Since Tebogo invested 2 250 units she will get: 2 250 units × R1,82 per unit = R4 095,00.

Practice Questions: Dividing an Amount in a Given Ratio

1. Sean and Zinhle invest R3 000,00 and R4 200,00 into an investment. After 3 years their combined money has grown to R9 352,00. If they divide the money in the same ratio in which they invested, how much money will each person receive?

2. Mpho and Sello worked together on a project and received R450,00 for their completed work. Mpho worked for 3 days and Sello worked for 4 days, and they agree to divide the money between them in the ratio 3 : 4. How much should each person receive?

3. A hairdresser needs to make up a 40 ml mixture of tint and hydrogen peroxide. The ratio of tint : peroxide is 1 : 2.

How many milliliters of tint and how many milliliters of peroxide will the hairdresser need to use to make the 40 ml mixture.

4. Energade concentrate energy drink recommends that 1 unit of concentrate be mixed with 5 units of water.

a. How many ml of concentrate and ml of water must be mixed to make 500 ml of juice?

4. b. How many ml of concentrate and ml of water must be mixed to make 3 litres of juice?

5. Three brothers combine their money and then invest the money. The table below shows the amount that each brother invests:

	Amount Invested
Brother 1	R8 000,00
Brother 2	R13 000,00
Brother 3	R20 000,00

a. After 5 years the money has grown by an effective 48% from its original value. Determine how much money there will be in the investment after 5 years.

5. b. If after 5 years the brothers decide to withdraw and divide the money in the ratio of their initial investments, how much will each brother receive?

Test Your Knowledge: Ratios

1. The instructions on the label of an energy drink say that you must dilute the concentrate with water in the ratio of 1 : 4.

a. Explain what this means.

b. If I have 2 litres of the energy drink concentrate mentioned in question 1 (a), how many litres of water do I need to add to make up the mixture?

2. High Strength Concrete is made up of gravel, sand and cement. The mixing ratio is 4 : 2 : 1.

a. If I have 2 wheelbarrows of gravel, how many wheel barrows of sand and cement do I need to make up a batch of concrete?

2. b. If I have 3 bags of cement, how many bags of gravel and sand of the same size do I need to make up a batch of concrete?

c. If I have 3 wheelbarrows full of sand, how many wheelbarrows of gravel and cement do I need to make up a batch of concrete?

3. A new green colour of paint is made by mixing blue paint and yellow paint in the ratio 4 : 3.

If I have 12 litres of blue paint, how many litres of yellow paint do I need to make up the new green colour?

____

____

____

____

4. Grace and Nikiswa received a total of R640,00 for the work that they did. Grace worked for 14 hours and Nikiswa worked for 18 hours.

a. Write the hours that they worked as a simplified ratio.

____

____

____

____

4. b. Calculate how much each of the girls should be paid.

____

____

____

____

____

5. The instructions on the label of an energy drink say that you must dilute the concentrate energy drink with water in the ratio of 1 : 4.

If I want to make 6 litres of diluted energy drink, how much concentrate must I use and how much water?

____

____

____

____

____

____

6. The following recipe caters for 6 people.

- $1\frac{1}{2}$ cups cooked rice
- 650 g chicken
- 375 ml chicken stock
- $\frac{1}{2}$ teaspoon salt
- 2 tablespoons flour

Calculate how much of each ingredient you would need to cater for 15 people.

Rice: ________________________________

____

____

____

____

Chicken: ______________________________

____

____

____

Stock: ________________________________

____

____

____

Salt: _________________________________

____

____

____

Flour: ________________________________

____

____

____

7. A map is drawn with a scale of 1 : 50 000.

For each of the following distances on the map, calculate the actual distance on the ground (give your answers in kilometres):

a. 2 cm on the map:

____

____

____

____

____

b. 9 cm on the map:

____

____

____

____

____

c. 30 cm on the map:

____

____

____

____

____

8. A map has a scale of 1 : 200 000. The distance between two towns is 60 km.

How far apart are the towns on the map (in cm)?

____

____

____

____

____

9. On a map, a distance of 5 cm represents an actual distance of 15 km. Determine the scale of the map and write the scale in the form 1 : n.

1.5 PROPORTION

1.5.1 Definition

When two ratios are equal, for example a : b = c : d , the four quantities, a, b, c and d are said to form a proportion. In other words, proportions are no more than the comparison of ratios — when ratios are equal we say that they are in proportion to each other.

We will work with two types of proportions:

Direct Proportion: → Two quantities are said to be in **direct proportion** if as the one quantity increases (or decreases) the other quantity increases (or decreases) by the same ratio. When two quantities are directly proportional then the ratios of any two pairs of quantities will be equal.

Inverse Proportion → Two quantities are said to be in **inverse proportion** if as the one quantity increases (or decreases) the other quantity decreases (or increases) by the same ratio. When two quantities are inversely proportional then the product of any pair of quantities is always constant.

The value of proportions lies in recognising that a situation is either a direct proportion or an inverse proportion situation and then using the properties of such situations to solve the problem.

1.5.2 Direct Proportion

Consider the following situations.

- ***Taxi A*** → charges a start-up or flag-fall fee of R18,00 per trip and after that charges R6,00 per kilometre travelled.
- ***Taxi B*** → only charges a fee of R7,00 per kilometre travelled.

We can develop a table of values for a number of different distances.

Distance travelled		5km	10km	15km	20km	25km
Cost	Taxi A	R48,00	R78,00	R108,00	R138,00	R168,00
	Taxi B	R35,00	R70,00	R105,00	R140,00	R175,00

Using this table, when we consider the *distance : cost* ratio for ***Taxi A*** we get:

5 : 48 10 : 78 15 : 108 and so on …

It should be immediately obvious that as the distance increases so the cost of the journey also increases. However, the ratio of any one pair of values is not equal to the ratio of any other pair — this can be made more obvious by converting all of the ratios to unit ratios:

5 : 48 = 1 : 9,6

10 : 78 = 1 : 7,8

15 : 108 = 1 : 7,2

By contrast, when we consider the *distance : cost* ratio for ***Taxi B*** we get:

5 : 35 10 : 70 15 : 105 and so on …

Once again it should be obvious that as the distance increases so the cost of the journey also increases. In this situation, however, the ratio of any one pair of values is equal to the ratio of any other pair — this can be made more obvious by converting all of the ratios to unit ratios:

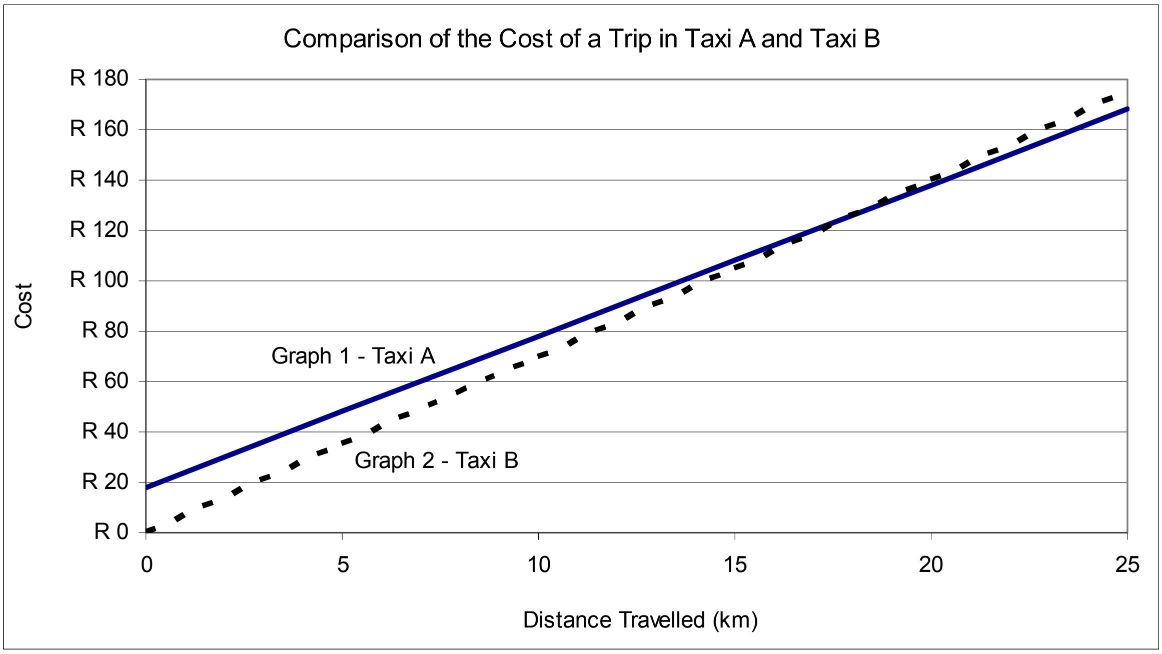
5 : 35 = 1 : 7

10 : 70 = 1 : 7

15 : 105 = 1 : 7

In summary, there is a direct proportion between the cost of a trip and the distance travelled for Taxi B but not for Taxi A.

If we plot the relationship between the cost of a taxi trip and the distance traveled for both Taxi A and Taxi B we get the following graph.



Notice that both graphs are linear (straight lines), even though the relationship represented by *Graph 1 − Taxi A* is not a direct proportion. In other words, it is possible for a graph to be a straight line even though there is not a direct proportion between the variables represented in the graph.

The difference between the direct proportion relationship of *Graph 2 – Taxi B* and the linear (but not direct proportion) relationship of *Graph 1 − Taxi A* is that the graph for *Taxi B* goes through the origin whereas the graph for *Taxi A* does not.

Because the relationship for *Taxi B* is a direct proportion relationship we can use equivalent ratios to solve problems related to this taxi. For example, to determine the cost of a 45 km trip in *Taxi B,* we can apply ratios as shown in the following way:

1 km : R7,00

× 45 × 45

45 km : ?

It follows that the cost of a 45 km trip is R7,00 × 45 = R315,00.

It is important to note that the same method ***cannot*** be applied for *Taxi A*. This is because there is not a direct proportion between the cost of the trip and the distance travelled.

Practice Questions: Direct Proportion

1. Determine whether or not the following ratios are in proportion:

a. 4 : 10 and 16 : 40

b. 20 : 220 and 37 : 407

c. 5 : 17 and 20 : 63

d. 6 : 7 and 30 : 35 and 102 : 119

1. a. ______________________________

____

____

b. ______________________________

____

____

c. ______________________________

____

____

d. ______________________________

____

____

____

2. The tables below show the cost of talking on various different cell phone options.

Determine by calculation whether or not the values given in the table are in direct proportion.

2. a.

Time	0 min	5 min	10 min	15 min
Cost	R0,00	R12,50	R25,00	R37,50

____

____

____

____

____

b.

Time	10 min	20 min	30 min	40 min
Cost	R105	R125	R145	R165

____

____

____

____

____

c.

Time (min)	60	120	240	360
Cost	R168	R336	R504	R672

____

____

____

____

____

3. The table below shows the monthly cost of electricity for a user in the Msunduzi Municipality.

Electricity used (kWh)	10	20	30
Monthly Cost	R5,42	R10,84	R16,26

a. Explain why there is a *direct proportion* between the electricity used per month and the cost of that electricity.

____

____

____

____

____

____

____

b. Use the fact that there is a direct proportion to calculate the monthly cost of using the following kWh of electricity during the month:

i. 50 kWh

ii. 100 kWh

iii. 372 kWh

iv. 512,7 kWh

3. b. i. ______________________________

____

____

ii. ______________________________

____

____

____

iii. ______________________________

____

____

____

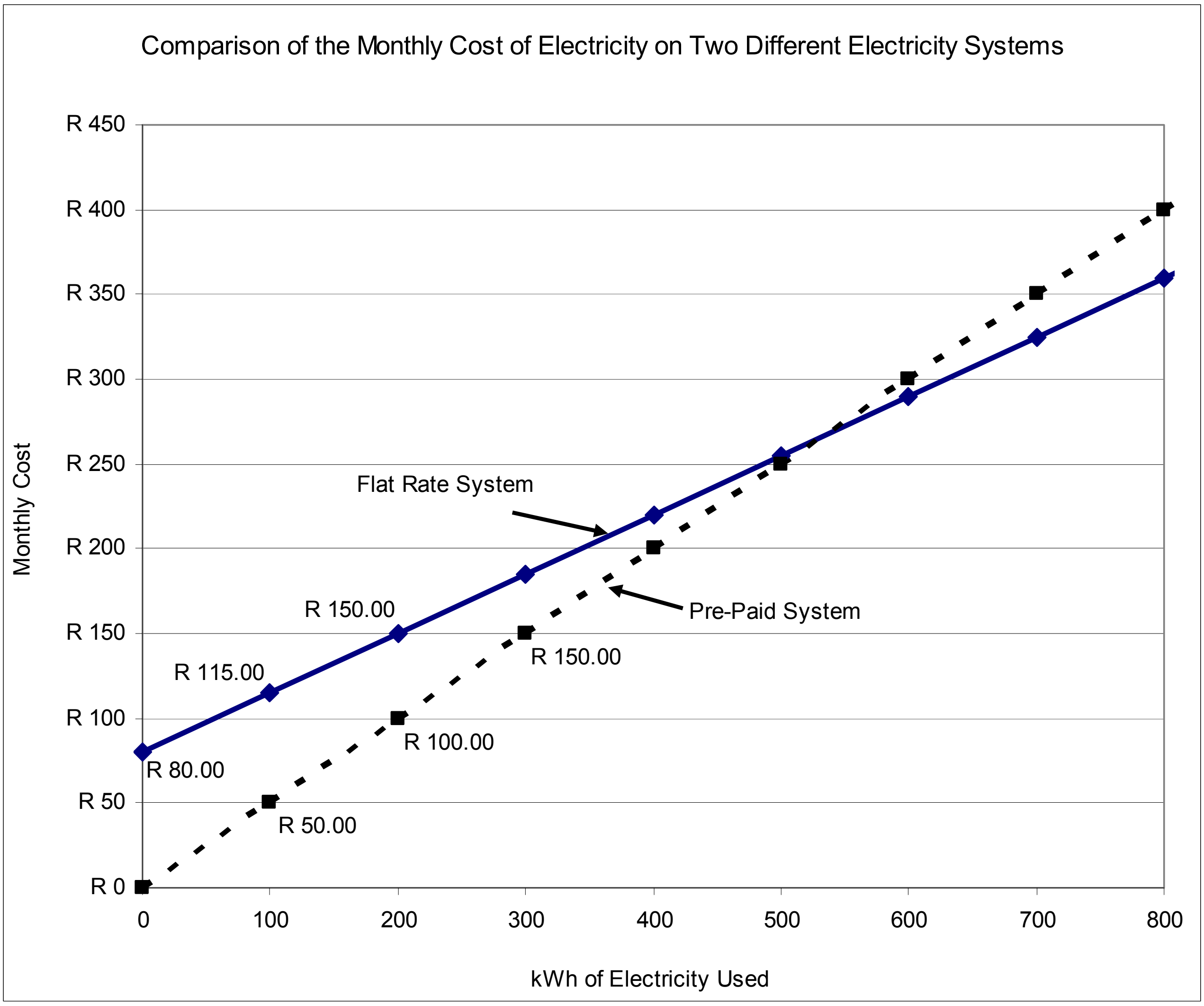
iv. ______________________________

____

____

____

4. The graphs below show the cost of electricity of two different systems in a municipality.



a. Does the Pre-Paid system or the Flat-Rate represent a direct proportion? Explain.

____

____

____

____

____

____

b. On the Flat-Rate system there is a fixed monthly service fee as well as a charge per unit (kWh) of electricity used during the month. How much is the fixed monthly service fee?

____

____

4. c. Calculate the per unit fee (i.e. the cost of using 1 kWh of electricity) for electricity on the:

i. Pre-paid system

ii. Flat rate system

i. ______________________________

____

____

____

____

ii. ______________________________

____

____

____

____

4. d. How much would it cost to use 1 000 kWh of electricity on the Pre-Paid system?

____

____

____

____

4. e. How much would it cost to use 1 000 kWh of electricity on the Flat Rate system?

____

____

____

____

1.5.3 Inverse Proportion

Scenario 1: Train Ticket

A monthly train ticket costs R240,00. If you use the train ticket once you will effectively pay R240,00 for the trip. If you use the train ticket 10 times you will effectively be paying R24,00 per trip.

We can illustrate this situation in the following table.

Number of trips	1	10	20	30	40
Effective cost per trip	R240,00	R24,00	R12,00	R8,00	R6,00

Notice how as the value of one variable increases the value of the other variable decreases.

We can describe the relationship between the number of trips and the effective cost per trip as:

No. of trips × Effective cost per trip = R240,00

R240,00 is referred to as the **Constant Product.**

OR

No. of trips = R240,00 ÷ Effective cost per trip

OR

Effective cost per trip = R240,00 ÷ No. of trips

In the equations above, the value R240,00 is referred to as the **Constant Product** − this is the constant value that results when the two variables are multiplied together.

This train ticket price situation is an example of an ***inverse proportion*** situation. This is because one variable is directly proportional to the multiplicative inverse of the other variable:

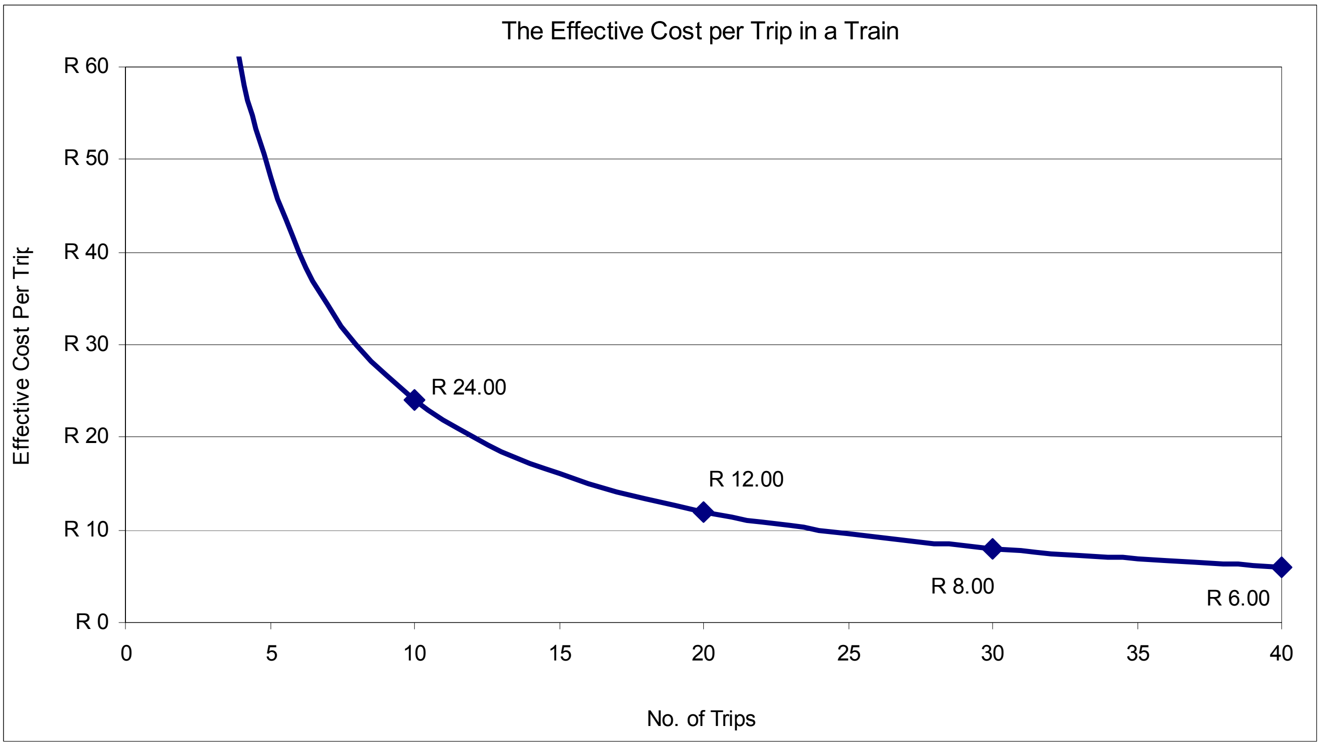
$$1 : \frac{1}{240} = 240 : 1$$

$$10 : \frac{1}{24} = 240 : 1$$

$$20 : \frac{1}{12} = 240 : 1$$

Or, put another way, if two variables are inversely proportional to each other then the product of those variables will always give the same value − i.e. the constant product.

We can represent this inversely proportional relationship between the number of trips and the effective cost per trip on the following graph:



Scenario 2: Train Seats

Now consider the following situation:

There are 60 seats in the train coach. If 10 are occupied then 50 are unoccupied; if 20 seats are occupied then 40 are unoccupied.

We can illustrate this situation in the following table.

Occupied seats	10	20	30	40	50
Unoccupied seats	50	40	30	20	10

As with the Train Ticket situation, in this situation as the value of one variable increases (occupied seats) so the value of the other variable decreases (unoccupied seats).

What is not the same, however, is that in this Train Seats situation one variable is ***not*** directly proportional to the multiplicative inverse of the other variable. i.e.:

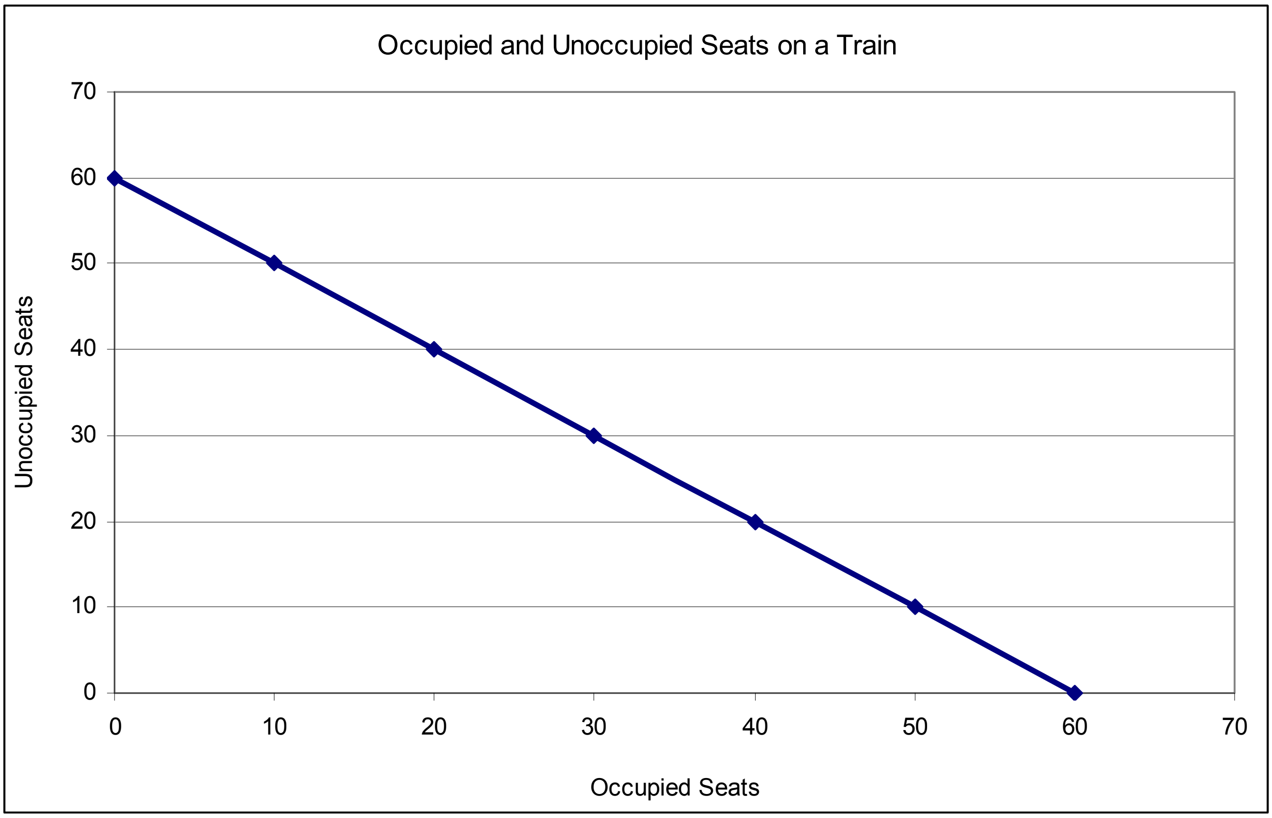
$$10 : \frac{1}{50} = 500 : 1$$

$$20 : \frac{1}{40} = 800 : 1$$

$$30 : \frac{1}{30} = 900 : 1$$

And so, unlike with the Train Tickets scenario, in this scenario there is no *constant product*.

If we draw a graph to represent the relationship between the number of occupied and unoccupied seats on the train, the result is the following:



Unlike the *curved graph* representing the Train Ticket situation, this graph is a straight line. This is because there is a constant relationship between the number of occupied and unoccupied seats – for every 1 seat that is occupied the number of unoccupied seats decreases by 1.

Practice Questions: Inverse Proportion

1. The table below shows the number of days that it takes to build a wall as dependent on the numbers of workers building the wall.

No. of Workers	1	2	3
Days to build the wall	24	12	8

a. Explain why the relationship between the number of workers and the number of days needed to build the wall is an inverse proportion relationship.

____

____

____

____

____

____

____

b. What is the constant product?

____

____

____

____

c. Use the constant product to determine how many days it would take to build the wall if there were 6 workers.

____

____

____

____

d. Write down an equation to represent the relationship between the number of workers and the number of days needed to build the wall.

____

____

e. If a graph were to be drawn to represent this situation, would the graph be a straight line or a curved graph? Explain.

____

____

____

____

____

2. Zinzi uses her car to drive to work. If she drives alone, then she has to pay all of the petrol costs. If she finds people to travel with her then they all share the travel costs.

The table below shows Zinzi's petrol costs as dependent on the number of people who travel in the car with her.

No. of People in the Car	1	2	3
Zinzi's Petrol Costs	R380,00	R190,00	R126,67

a. Explain why the relationship between the number of people in Zinzi's car and Zinzi's petrol costs is an inverse proportion relationship? Explain.

____

____

____

____

____

____

____

b. What is the constant product?

____

____

____

____

c. Use the constant product to determine what Zinzi's petrol costs will be if she travels to work with 5 people in the car every month.

____

____

____

____

____

d. Write down an equation to represent Zinzi's petrol costs.

____

____

____

3. The table below shows the cost of travelling in a Yellow Cab taxi.

Distance (km)	10	20	100
Cost	R125	R250	R1 250

a. Is there an inverse proportion relationship between the distance travelled in the taxi and the cost of the trip? Explain.

____

____

____

3. b. Calculate how much the taxi charges per kilometer travelled.

c. Calculate the cost of travelling 147 km in this taxi.

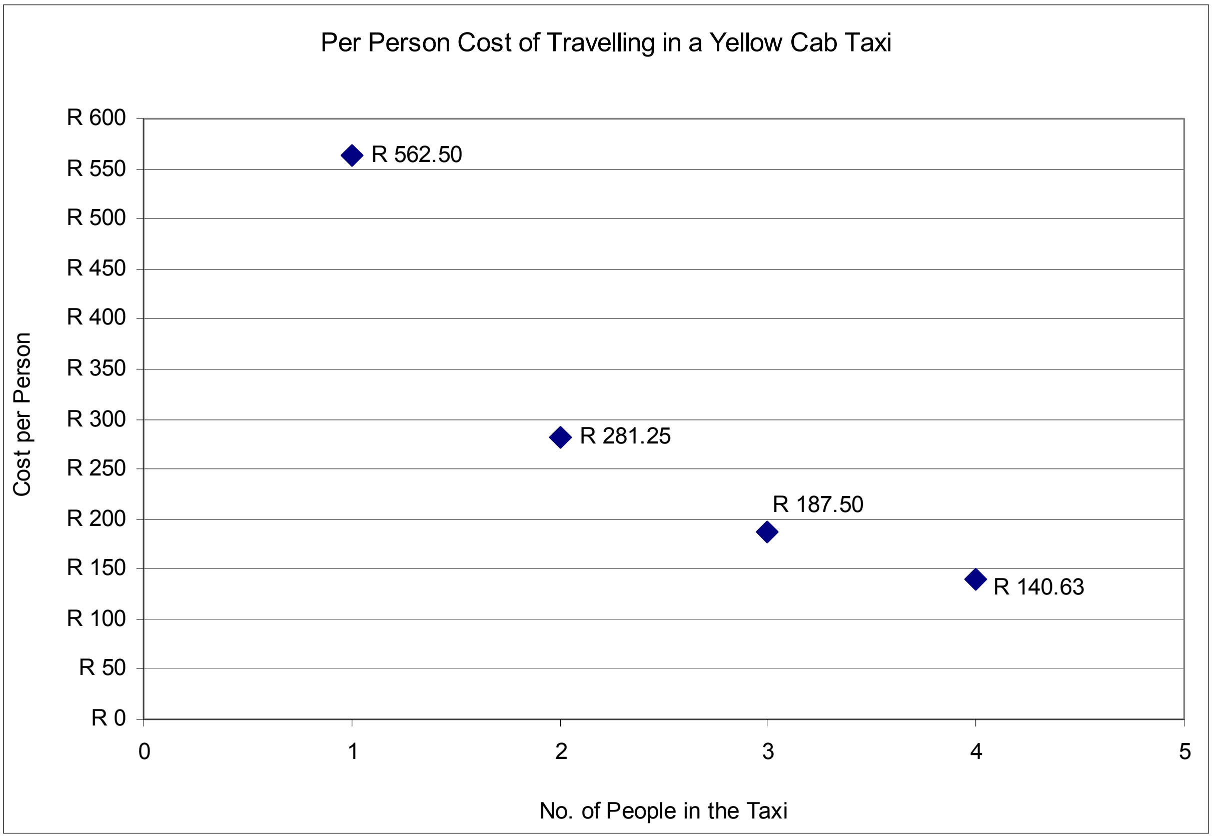
3. d. Write down an equation to represent the cost of a trip in the Yellow Cab Taxi.

e. If a graph were drawn to represent the cost of a trip in the taxi, what would the graph look like?

i.e. → would the graph be straight or curved;

→ in which direction would the graph go?

4. Ryan wants to catch a Yellow Cab Taxi from university to the bus station. If he catches the taxi alone then he will pay R562,50. If he shares the taxi with one friend, each of them will pay R281,25. The graph below illustrates this scenario.



a. Is there an inverse proportion relationship between the number of people in the taxi and the amount that each person has to pay for the trip? Explain.

____

____

____

b. Why have the points on the graph not been joined?

____

____

____

c. How much will each person have to pay for the taxi trip if there are 5 people in the taxi?

____

____

____

____

____

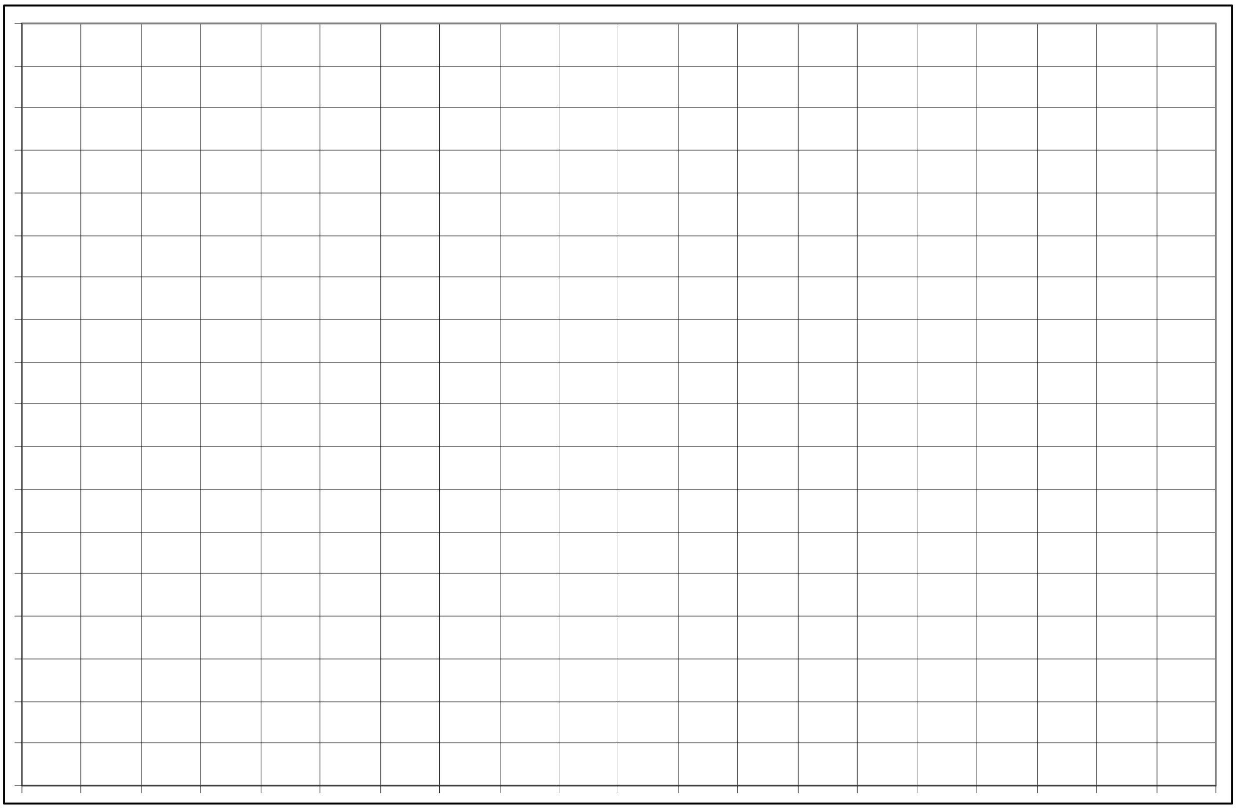
Test Your Knowledge: Proportion

1. A chocolate bar costs R5,45.

a. Complete the following table:

Number of chocolate bars	1	2	5	10	20	30
Cost						

b. On the set of axes below, draw a graph to represent the above situation



c. What type of proportional relationship is represented in this situation? Explain.

2. A teacher has 36 learners in her class. She buys enough sweets to give each child 5 sweets.

a. What type of proportional relationship between the number of children in the class and the number of sweets that each child receives? Explain.

b. How many sweets will each learner get if only 30 learners come to school?

3. A scout troop wants to go on an expedition. The bus company quotes them R650,00 for a 30-seater bus. The price of hiring the bus stays the same even if not all of the 30 seats are taken.

a. What type of proportional relationship is there between the number of scouts on the bus and the amount that each parent has to pay? Explain.

b. How much will it cost the parents of each scout for transport if only 17 scouts go on the expedition?

4. Riyaad gets paid R15,50 per hour for his holiday job.

a. What type of proportional relationship is there between the amount that Riyaad gets paid and the number of hours that he works? Explain.

b. How much will Riyaad earn in 12 hours?

1.6 RATE

1.6.1 Definition

A rate is a special kind of ratio in which the two (or more) quantities being compared have different units. Examples of rates include:

- comparing the distance travelled by a car to the time taken to travel that distance − this gives the speed of the car in kilometers per hour (km/h);
- comparing the time spent on a telephone call to the total cost of the call − in Rand per minute (R/min);
- comparing the value of the Rand currency to the US Dollar currency − this gives the exchange rate of the Rand to the Dollar in Rand per Dollar (R/$).

Since rate is a special ratio, working with rate is no different to working with ratio and proportion in terms of the mathematical processes. The only real difference is in the use of units. Working with units provides the advantage that they provide guidance to us as we perform calculations.

Three important concepts to understand when working with rates are ***constant rates***, ***average rates*** and ***unit rates***.

1.6.2 Constant Rates

When the quantities being compared by the rate are in **direct proportion** we say that we have a **constant rate.** Phrased differently, the rate between the two quantities is independent of the size of the quantities being compared.

Constant rates do not change or vary. Consider the following examples of constant rates:

- conversion rates used to convert from metres to centimetres and from grams to kilograms − e.g. 10 mm = 1 cm and 1 000 g = 1 kg;
- rates at which telephone calls are charged − e.g. R2,80 per minute;
- the interest rate used by the bank to determine the fee to be paid when repaying a loan − e.g. 9% per year.

Example:

At a particular petrol station, petrol costs R10,30 per litre. How much would it cost to fill a car with 35 litres of petrol?

In this situation the petrol cost of R10,30 per litre is a constant or fixed rate.

We can use this constant rate to determine the cost of 35 litres of petrol in the following way:

$$\begin{array}{ccc} \text{R}10{,}30 & : & 1\,\ell \\ \downarrow \times 35 & & \downarrow \times 35 \\ ? & : & 35\,\ell \end{array}$$

→ Cost of petrol = R10,30 per litre × 35 litres

= R360,50

Practice Exercise: Constant Rates

1. Petrol costs R10,50 per litre. How much would it cost to put 40 ℓ of petrol into a car?

2. Mince is selling for R42,99 per kilogram. How much would it cost to buy 3 kilograms of mince?

3. The cost of a telephone call on a Telkom landline during peak time is R2,80 per minute. Calculate the cost of making a 17 minute call.

4. The cost of a call on a particular cell phone contract is R0,04 per second. How much would it cost to make a call that lasts
6 min 23 seconds?

5. Cheese is selling for R58,49 per kilogram. How much would it cost to buy a 400 g block of cheese?

6. A particular type of paint has a coverage of 9 m^2 per litre. Calculate how many litres of paint will be needed to paint a wall that has a surface area of 23 m^2?

1.6.3 Unit Rates

A useful method for performing calculations involving rates is to use **unit rates**. As with unit ratios, unit rates are rates in which one of the quantities in the rate is compared with a single unit of the second quantity in the rate.

Example:

Consider a store selling two different bottles of the same cooking oil:

- <u>Bottle 1:</u> 2 litres; cost R42,99
- <u>Bottle 2:</u> 750 mℓ; cost R17,59

The question is: *Which of these bottles offer better value for money?*

Given that the values in the rates are so different (R42,99 for 2 ℓ and R17,59 for 750 mℓ), it is very hard to know from these values which option is the better value for money.

Converting to unit rates gives us a useful way for comparing these rates.

There are two options for converting to unit rates

- Convert the rates to *volume : R1,00* → i.e. a comparison of the amount of oil that you get for every one Rand that you pay;

OR

- Convert the rates to *Rand : unit volume* (litres or millilitres) → i.e. a comparison of how much you are paying per unit of oil.

More typically we tend to determine the *cost : unit volume* rate.

Let's consider the ***2/ bottle*** first: We know that R42,99 = 2 ℓ

We can convert the litre quantity in this rate to a unit value (i.e. 1 ℓ) by diving by 2. This gives:

R21,50 = 1 ℓ

We can illustrate the process used in converting the rate supplied to a unit rate in the following way:

$$\begin{array}{ccc} R42{,}99 & : & 2\,\ell \\ \downarrow \div 2 & & \downarrow \div 2 \\ ? & : & 1\,\ell \end{array}$$

Now let's consider the ***750 mℓ*** bottle:

In this case the conversion involves *2 steps* as the volume is in different units and not in litres.

- First we need to convert the *mℓ* value to a litre equivalent;

  To do this we use the constant rate 1 000 mℓ : 1 ℓ in the following way:

$$\begin{array}{ccc} 1\,000\text{ m}\ell & : & 1\,\ell \\ \downarrow \div 1\,000 & & \downarrow \div 1\,000 \\ 1\text{ m}\ell & : & 0{,}001\,\ell \\ \downarrow \times 750 & & \downarrow \times 750 \\ 750\text{ m}\ell & : & 0{,}750\,\ell \end{array}$$

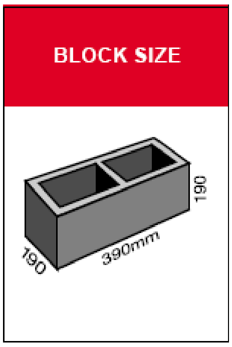
- Now that we have the rate in Rands and litres we can convert to an equivalent unit rate:

$$\begin{array}{ccc} R17{,}59 & : & 0{,}750\,\ell \\ \downarrow \div 0{,}750 & & \downarrow \div 0{,}750 \\ R23{,}45 & : & 1\,\ell \end{array}$$

It would seem as if the 2 litre bottle at R42,99 is better value for money (R21,50 per litre) than the 750ml bottle at R17,59 (R23,45 per litre).

Practice Exercise: Unit Rates

1. The table below shows the number of blocks and bags of cement needed to build a wall.



FOR 1000 BLOCKS		
WALL SIZE m^2	No. BAGS SUREBUILD	m^3 SAND
80	9	2,00

(PPC Cement, Pamphlet – *The Sure Way to Estimate Quantities*, www.ppccement.co.za)

a.

i. How many bags of cement are needed to make a 160 m^2 wall?

ii. How many bags of cement are needed to make a 40 m^2 wall?

iii. How many bags of cement are needed to make a 150 m^2 wall?

1. b.

i. How many blocks are needed to make a 200 m^2 wall?

ii. How many m^3 of sand is needed to make a 150 m^2 wall?

c.

i. A builder buys 15 bags of cement to make a wall. How big is the wall?

c. ii. A builder buys 250 blocks to make a wall. How many bags of cement will he need to buy?

2. Which is the better value for money:

a. 300 g box of chocolates that costs R13,05

OR

1 kg box costs R44,99?

2. b. 350 ml bottle of juice that costs R6,25

OR

1 litre bottle of juice that costs R12,80?

c. 200 g packet of biscuits that costs R7,25

OR

1,2 kg box of biscuits that costs R44,50?

3. Two cars leave Durban at the same time. Car A travels 535 km in 5 hours and Car B travels 980 km in $8\frac{1}{2}$ hours. Which car is travelling the fastest? Explain.

1.6.4 Average Rates

In situations where the rate varies over time we use **average rate** to refer to the **effective rate.** The effective rate is what the rate would have been had it been constant over the time period under consideration.

We use average rates frequently on a day to day basis; we talk about average petrol consumption in litres per 100 km; or the average speed at which we completed a journey in kilometres per hour; average run rates for cricketers expressed in runs per innings; and average rainfall rates expressed in mm per day or month or year. What all of these examples have in common is that although we talk about the average speed for a journey or the average petrol consumption per 100 km or the average number of runs scored in an innings: the actual speed of the car, the actual petrol consumption per km and the actual runs per innings vary enormously from journey to journey and match to match. The value of average rates is that they allow us to predict or **estimate**.

Example:

Estimate the petrol cost of travelling 800 km in a car with an average consumption rate of 11,5 litres per 100 km if petrol costs R8,48 per litre.

This question makes use of both an *average rate* — the average consumption rate — and a *constant rate* — the cost of the petrol.

The question is answered in two steps:

- First we estimate the amount of petrol that will be needed to complete the journey:

$$\begin{array}{ccc} 11{,}5\ \ell & : & 100\text{ km} \\ \downarrow \times 8 & & \downarrow \times 8 \\ ? & : & 800\text{ km} \end{array}$$

→ Petrol needed = 11,5 ℓ x 8 km
= 92 ℓ

- Next we calculate the cost of that petrol using proportional thinking:

$$\begin{array}{c} R8{,}48 : 1\,\ell \\ \downarrow \times 92 \quad \downarrow \times 92 \\ ? : 92\,\ell \end{array}$$

→ Cost = 92 ℓ × R8,48 per ℓ
= R780,16

Although the answer that we have developed is correct to two decimal places, in reality we should expect the cost to vary a little because of the anticipated variation in the petrol consumption rate. For this reason it is probably best to anticipate that the petrol cost will be approximately R800,00.

Practice Exercise: Average Rates

1. a. A car has an average petrol consumption rate of 8 litres per 100 km.

a. How much petrol will the car use to travel 370 km?

____

____

____

____

____

b. If the current price of petrol is R10,30 per litre, how much will it cost to travel 370 km?

____

____

____

____

____

c. If the owner of the car puts R550,00 worth of petrol in the car, how far will she be able to travel until the petrol runs out?

____

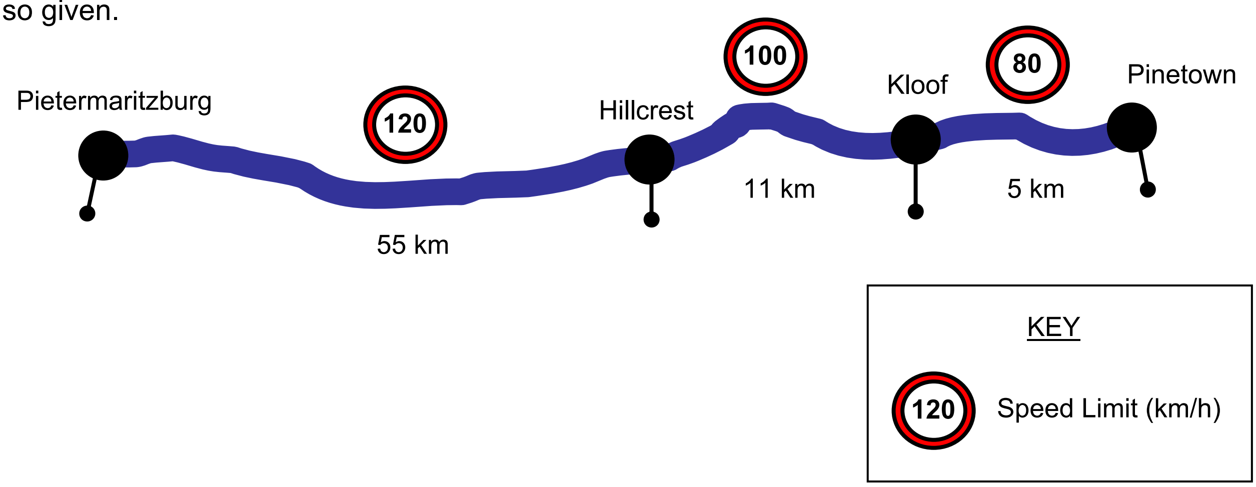
____

____

____

____

2. The picture below shows a distance chart with the distances between different towns on route from Pietermaritzburg to Pinetown. The speed limits between the various towns on the route are also given.



a. Mpumi is driving from Pietermaritzburg to Pinetown. If she drives the whole way at the speed limit, calculate how long it will take for her to travel from:

i. Pietermaritzburg to Hillcrest (in minutes and seconds)

ii. Hillcrest to Kloof (in minutes and seconds)

iii. Kloof to Pinetown (in minutes and seconds)

i. ____

____

____

____

____

ii. ____

____

____

____

____

iii. ____

____

____

____

____

2. b. Mpumi's car has an average petrol consumption rate of 9 litres per 100 km. If the current petrol price if R10,30 per litre, calculate how much it will cost her in petrol costs to travel from Pietermaritzburg to Pinetown.

3. The table below shows the running times of the winner of the 2008 Comrades Marathon at different places on the route.

Place on the Route	Total Running Time (h : min : sec)	Total Distance Run (km)
Cowies Hill	01:04:50	16,7
Drummond	02:42:44	42,6
Cato Ridge	03:37:43	57,1
Camperdown	04:03:54	63,8
Polly Shorts	04:57:13	79,1
Finish	05:24:46	86,8

a.

i. Calculate how long it took for the athlete to run from the Start to Cowies Hill in *minutes*. Round off your answer to 3 decimal places.

3. a. ii. Determine the average speed (in minutes and seconds per km) at which the athlete ran from the Start to Cowies Hill.

b. Determine the average speed (in minutes and seconds per km) at which the athlete ran from Polly Shorts to the Finish.

c. Determine the average running speed (in minutes and seconds per km) of the athlete over the whole race.

____

____

____

____

____

____

____

____

d. Why do we use the word “average” when referring to the running speed of the athlete?

____

____

____

1.6.5 Constructing Rates to Solve Problems

In some situations it will be necessary to first identify and construct rates before being able to use the rates to solve problems.

Example:

Mike is packing apples into boxes. He must pack 500 boxes and thinks that he can do so in 4 hours. After 2 hours he has packed 260 boxes.

a. Is he ahead of or behind schedule at this stage?

b. If he stops packing and talks on the phone for half an hour how many boxes will he have to pack per minute for the remainder of the time to still complete the task in 4 hours?

In this problem we are expected to develop our own rates and then to use them to solve a problem.

a. Is he ahead of or behind schedule at this stage?

What we know is that Mike expects to pack 500 boxes of apples in 4 hours.

The rate then is: 500 boxes per 4 hours.

We start out by converting that to a unit rate:

500 box : 4 hrs

÷ 4 ÷ 4

? : 1hr

→ 500 boxes per 4 hours = 125 boxes per hour

After working for two hours at a rate of 125 boxes per hour we would expect Mike to have packed:

125 boxes : 1 hr

× 2 × 2

? : 2 hrs

→ 2 hrs x 125 boxes per hour = 250 boxes

Since Mike has already packed 260 boxes, he has packed more than we would have expected and we say that he is ahead of schedule.

b. If he stops packing and talks on the phone for half an hour how many boxes will he have to pack per minute for the remainder of the time to still complete the task in 4 hours?

He spends half an hour on the phone and as such has only $1\frac{1}{2}$ hours left to pack the remaining 500 — 260 = 240 boxes.

Once again we determine a unit rate:

$$\begin{array}{c} \text{240 boxes : 1,5 hrs} \\ \downarrow \div 1{,}5 \quad \downarrow \div 1{,}5 \\ \text{160 boxes : 1 hr} \end{array}$$

→ 240 boxes in $1\frac{1}{2}$ hours ÷ 1,5 hours= 160 boxes per hour

And, since there are 60 minutes in an hour, it follows that:

160 boxes per hour = (160 ÷ 60) boxes per minute

≈ 2,7 boxes per minute.

Practice Exercise: Constructing Rates

1. In a cricket match between South Africa and England, South Africa scored 235 runs off 50 overs. After 28 overs, England had managed to score 125 runs.

a. Determine South Africa's run rate in runs per over (to one decimal place).

____

____

____

____

b. Determine England's run rate in runs per over (to one decimal place).

____

____

____

____

c. Based on your answers in a. and b., who do you think might win the match?

____

____

d. At what run rate (in runs per over) must England score runs from now until the end of the game in order to win the match?

____

____

____

____

____

____

2. Trudy is driving from Pietermaritzburg to Durban airport, a distance of 120 km.

After 45 minutes she has travelled 72 km.

a. Determine the average speed (in km/h) at which she has travelled for this part of the journey.

____

____

____

____

____

____

b. Trudy left home at 9:00 am and she needs to be at the airport by 10:30 am. If she continues to drive at this speed, will she arrive in time?

____

____

____

____

____

____

____

____

3. In 2008 Leonid Shvetsov broke the record for the Comrades Marathon. The table below shows the running time of this athlete at various places along the route.

Place on the Route	Total Running Time (h : min : sec)	Total Distance Run (km)
Cowies Hill	01:04:50	16,7
Drummond	02:42:44	42,6
Cato Ridge	03:37:43	57,1
Camperdown	04:03:54	63,8
Polly Shorts	04:57:13	79,1
Finish	---	86,8

In order to break the record Leonid Shvetsov had to finish in a time faster than 5 hours 25 min and 35 seconds.

Calculate how fast (in minutes and seconds per km) Leonid Shvetsov had to run from Polly Shorts to the Finish in order to break the record.

Test Your Knowledge: Rates

1. a. If I bought a packet of apples for R12,99 and there were 9 apples in the packet, what is the cost per apple?

b. If 1,3 kg of mince costs R42,84, what is the price per kilogram?

c. If I used 22 kℓ of water in June and it cost me R144,98, what is the price of water per kilolitre?

2. a. If petrol costs R8,24 per litre, how much would it cost to fill a 50 ℓ tank.

2. b. If you earn R650,00 per week for working for 5 days in the week, what is your daily rate of pay?

c. Boerewors costs R32,45 / kg.

How much would $3\frac{1}{2}$ kg of boerewors cost me?

3. The Tariffs for *uShaka Sea World* are as follows:

Adults:	R98 per person
Senior citizens (aged 60 +):	R85 per person
Children:	R66 per person

Calculate the cost for a family to visit *uShaka* if the family consists of 2 adults, 1 Grandpa and 3 children.

4. Which of the following items give better value for money?

a. 2,5kg of sugar at R15,69

OR

5kg of sugar at R29,75?

b. 100 Trinco teabags at R7,89

OR

80 Freshpak teabags at R6,80?

5. a. If I travel at a constant speed of 80 km/h, how long will it take me to complete a journey of 65 km? Round off your answer to the nearest minute.

b. If my car has a petrol consumption rate of 6 ℓ per 100 km and the cost of petrol is R10,44 per litre, calculate how much it would cost to travel the 65 km journey.

TOPIC 2

PATTERNS AND RELATIONSHIPS

INDEX

2.1 MOVING BETWEEN TABLES, EQUATIONS AND GRAPHS

Consider the following scenario:

Sipho currently has a pre-paid cell phone where the cost of a call is R2,50 per minute. He is considering changing to a contract where there is a monthly subscription fee of R100,00 and the cost of a call is R2,00 per minute.

We can use three different methods to help us to represent and make sense of this situation:

Method 1: Constructing a Table

A table is a useful way for summarizing information.

Using the information for Sipho's cell phone dilemma, we can construct the following table:

Time (min)	0	1	2	3	4	5
Monthly Cost → Pre-Paid	R0,00	R2,50	R5,00	R7,50	R10,00	R12,50
Monthly Cost → Contract	R100,00	R102,00	R104,00	R106,00	R108,00	R110,00

There are several important points that are evident from the table:

- On the *contract option*, even if Sipho talks for no minutes during the month he will still have to pay R100,00. This is because of the fixed monthly subscription fee on the contract.
- For every 1 minute that Sipho talks on the *pre-paid option*, the monthly cost increases by the constant amount of R2,50. This is because the cost of a call on the pre-paid option is R2,50 per minute.
- For every 1 minute that Sipho talks on the *contract option*, the monthly cost increases by the constant amount of R2,00. This is because the cost of a call on the pre-paid option is R2,00 per minute.

Although a table provides us with a useful way for summarising information, one of the disadvantages of a table is that it only provides a very limited view of a situation or of all of the possible values in a situation.

i.e. By comparing the pre-paid and contract information in the table, it would appear that the pre-paid option is significantly better than the contract option. The problem with this assumption, though, it that the table only provides information on the cost of both options for up to and including 5 minutes. But what happens after 5 minutes? Does the situation change around?

So, while a table is a useful way of summarising information, it is also very limiting in the amount of information that can be displayed. For this reason, together with a table we often make use of a second representation to help us to make sense of a situation like Sipho's cell phone dilemma − namely, an *equation*.

Method 2: Constructing an Equation

From the information provided on the different cell phone options, we know the following:

- On the pre-paid option, the monthly cost is dependent entirely on how long Sipho talks for on his phone, and this cost is based on the fee of R2,50 per minute.
- On the contract option, there is a fixed fee of R100,00 that Sipho will have to pay even if he makes no calls. Over and above this fee he then has to pay R2,00 for every minute that he spends making calls.

Using this information we can construct the following equations to represent the monthly cost of making calls on the pre-paid and contract options:

→ Pre-paid: *Monthly cost = R2,50 × time (min)*

→ Contract: *Monthly cost = R100,00 + (R2,00 × time (min))*

The advantage of an equation is that we can use the equation to determine the cost of a call for <u>*any*</u> *number of minutes* − something that was not possible in the limited space available in a table.

For example, if Sipho talks for 220 minutes during the month then:

→ Monthly cost on the pre-paid option = R2,50 × 220 = R550,00

→ Monthly cost on the contract option = R100,00 + (R2,00 × 220) = R540,00

So, while the information in the table seemed to say that the pre-paid option is better than the contract option, using the equations shows us that for 220 minutes the contract option would be better. So, somewhere between 5 minutes and 220 minutes the monthly cost of making calls on the pre-paid option changes from being cheaper than the contract option, to becoming more expensive.

The disadvantage of an equation is that it can only provide a picture of what is happening in the scenario for one value at a time. So, while the equation is useful in that it allows us the flexibility of being able to perform calculations for any value in the scenario, it is also limiting in how much information it allows us to see at one time.

And so, to help us to see more clearly what is happening in the situation for many values we make use of a third representation − *drawing graphs* to represent the situation.

Method 3: Drawing a Graph

To draw a graph to show the monthly cost of the pre-paid and contract options for a large number of time intervals, we are going to use the equations that we constructed above to help us to extend the table:

Pre-Paid

If time = 50 minutes then:

Cost = R2,50 × 50

= R125,00

If time = 100 minutes then:

Cost = R2,50 × 100

= R250,00

and so on …

Contract

If time = 50 minutes then:

Cost = R100,00 + (R2,00 × 50)

= R200,00

If time = 100 minutes then:

Cost = R100,00 + (R2,00 × 100)

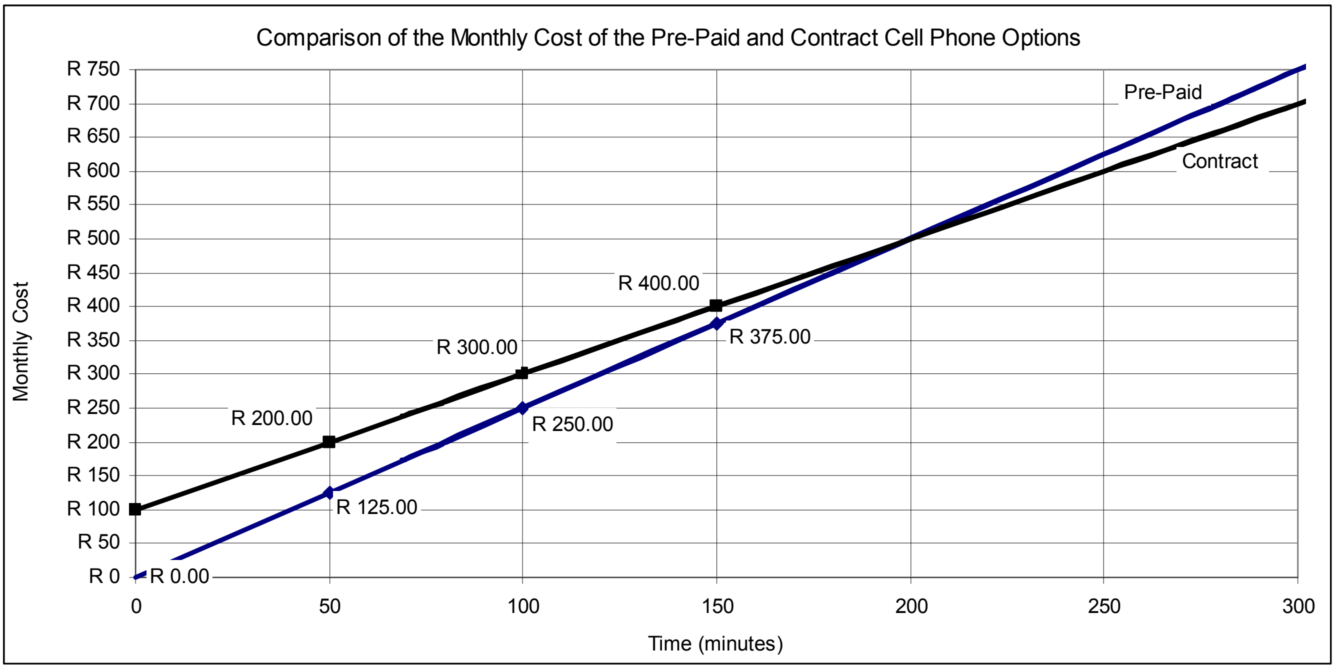
= R300,00

and so on …

This gives the following extended table:

Time (min)	0	50	100	150
Monthly Cost − Pre-Paid	R0,00	R125,00	R250,00	R375,00
Monthly Cost − Contract	R100,00	R200,00	R300,00	R400,00

We can now plot these values on a set of axes to give two graphs − one to represent the monthly cost of the pre-paid option and one to represent the monthly cost of the contract option.



Using the graph we can now see clearly that although the pre-paid option starts out much cheaper, if Sipho talks for more than 200 minutes per month then it will be more expensive to be on the pre-paid option than on the contract option.

The purpose and advantage of a graph is that it creates a visual picture of the situation being dealt with. This visual picture often makes it possible to see things that are not evident in either the table or the equation.

The disadvantage of a graph is that it also only represents a portion of all of the possible values that could exist in a scenario. For example, this graph only represents the cost of speaking for up to and including 300 minutes per month. But what about the person who speaks for 500 minutes per month? So, as with the table, a graph also only provides a limited impression of a situation or scenario.

Summary:

We can often describe a situation using three different representations — table, equation and graph. Each representation serves a different purpose and each representation provides a slightly different impression of what is happening in a scenario. The important thing to remember, though, is that each representation is linked to every other representation and in many ways each representation is just a different version of every other representation. i.e. Sometimes we use a table of values to help us to determine an equation to represent a situation; other times we use an equation to construct a table of values; and a graph then provides a picture version of the table of values and the relationship described in the equation.

And so, a table, equation and graph are simply different representations of the same relationship.

Practice Exercise: Tables, Equations and Graphs

1. Sipho is planning a birthday party and is looking for a venue to hold the party. A local sports club charge R500,00 per evening for the venue and R50,00 per person.

a. Complete the following table:

Number of guests	10	20	30	40	50	60	70	80	90	100
Cost of the party										

b. How much will it cost if 120 people attend the party?

____

____

c. Write down an equation to describe the cost of the party.

____

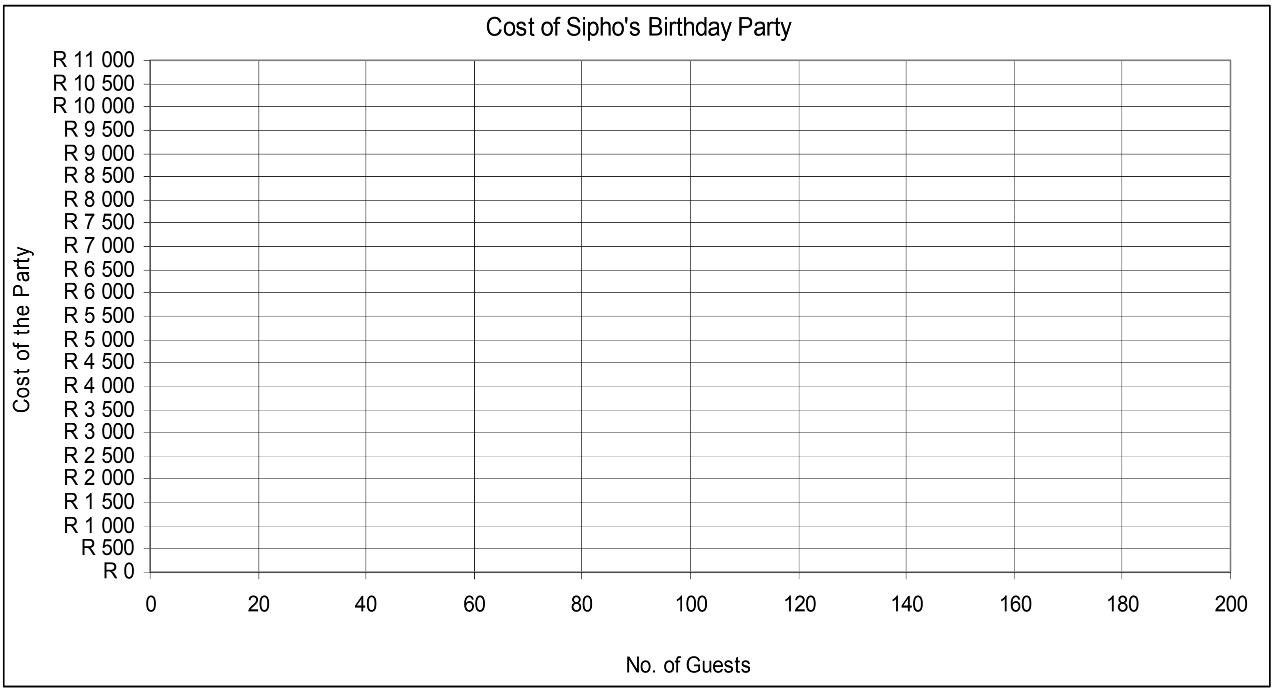
____

d. Use the equation to determine the cost of the party if 167 people attend.

____

____

e. On the set of axes below, draw a graph to show the cost of the party for up to an including 200 people.



f. Use the graph to answer the following

f. iii. If Sipho has to pay R9 000,00 for the party, questions:

i. How much will it cost if 180 people attend the party?

____

____

ii. How much will it cost of 130 people attend the party?

____

____

how many people attended.

____

____

2. A metered taxi has the following rates:

- R3,00 flat-rate
- R8,50 per km travelled.

a. Complete the following table. The first two blocks in the table have been completed for you.

Distance Travelled (km)	1	2	3	4	10	20	30	40
Cost of the ride	R11,50	R20,00						

b. Construct an equation to represent the cost of a trip in this taxi.

____

____

c. Use the equation to determine how much it would cost to travel

i. 120 km

____

____

____

ii. 157 km

____

____

____

d. How many km did you travel if the ride cost you R215,50?

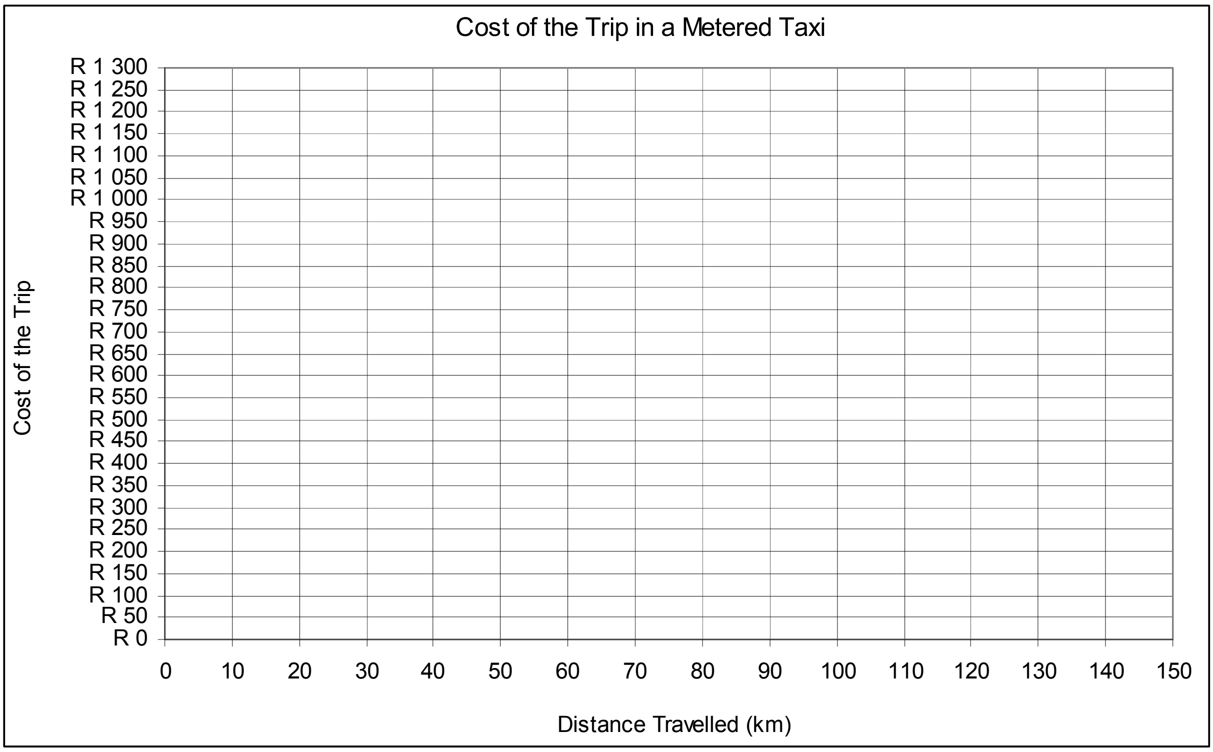
____

____

____

____

e. On the set of axes below, draw a graph to show the relationship between the cost of a trip in the taxi and the distance travelled by the taxi for up to and including 150 km.



f. Use the graph to answer the following questions:

i. Approximately how much will it cost to travel 130 km in the taxi?

____

____

ii. Approximately how much will it cost to travel 143 km in the taxi?

____

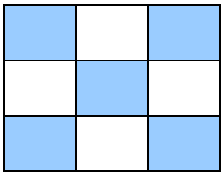
____

iii. If the cost of a trip in a taxi is R1 023,00, apprximately how far did the taxi travel?

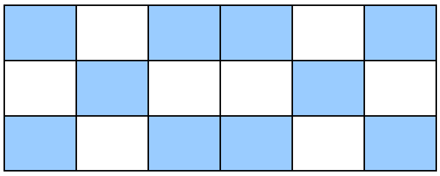
____

____

3. Moira is tiling the floor in her kitchen. The picture below shows the pattern that she is going to use:



1 repeat



2 repeats

a. Complete the following table:

Repeat of the pattern	1	2	3	4	5	10	20	50
No. of blue tiles	5							

b. Draw a picture to show how many blue tiles there will be in 3 repeats of the pattern.

c. How many blue tiles will Moira need if she repeats the pattern 17 times?

____

____

d. If Moira were to use 75 blue tiles, how many repeats of the pattern would there be?

____

____

e. Write down an equation to represent the relationship between the number of repeats of the pattern and the number of blue tiles in the pattern.

____

f. Use this equation to determine how many blue tiles Moira will need if she repeats the pattern 23 times.

____

____

____

g. If a graph were drawn to represent this pattern, the graph would be a straight line. Explain why this is the case?

____

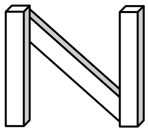
____

h. How many *white* tiles will Moira need if she repeats the pattern 13 times?

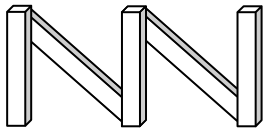
____

____

4. Jessi is building a fence around his farm. The picture below shows the design of the fence.



1 extension



2 extensions

a. Complete the following table:

No. of extensions of the fence	1	2	3	4	10	20
No. of pieces of wood						

b. Draw a picture to show how many pieces of wood there will be in 3 extensions of the fence.

c. How many pieces of wood will there be in 7 extensions of the fence?

____

____

d. Write down an equation to represent the relationship between the number of extensions in the fence and the number of pieces of wood needed.

____

____

e. Use the equation to determine how many pieces of wood Jessi will need to build a fence that has 18 extensions of the pattern.

____

____

____

f. If a graph were to be drawn to represent the relationship between the number of extensions in the fence and the number of pieces of wood needed to make the fence, what would this graph look like and why?

____

____

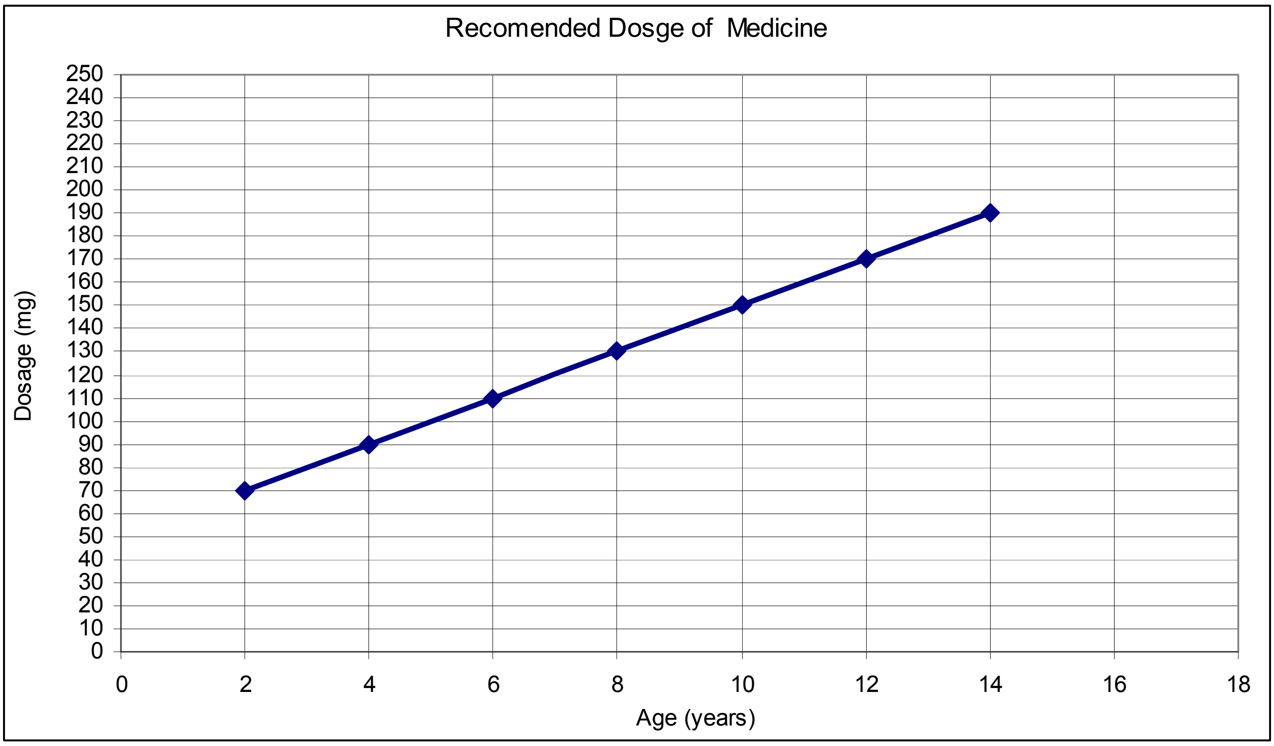
____

____

____

____

5. The graph below lists the dosage (in *mg)* of a particular drug that should be administered to children according to their age.



a. How many mg of the drug should be administered to a child who is 6 years old

____

b. How old is a child if a doctor prescribes a dosage of 150mg?

____

c. Estimate the dosage that should be given to a child who is 9 years old.

____

d. How old do you estimate a child to be if the doctor has prescribed a dosage of 80mg?

____

e. Extend the graph to determine the reccomended dosage of medicine for a person who is 17 years old.

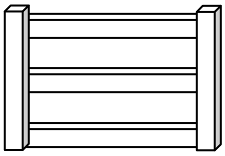
____

f. Write down an equation to represent the recommended dosage of medicine as dependent on the age of the child.

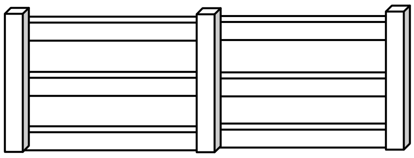
____

____

6. Muchacha is building a fence around his house. The picture below shows the design of the fence:



Extension 1



Extension 2

a. Use any method to determine how many pieces of wood Muchacha will need to build a fence with 37 extensions.

b. If Muchacha were to use 41 pieces of wood, how many extensions of the fence would there be?

7. The table below shows the cost of pre-paid electricity and flat-rate electricity in the Mtuntili Municipality:

System	Fixed Fee	Charge per kWh
Pre-Paid	None	R0,50
Flat-Rate	R80,00	R0,30

a. Use the table below to show the difference in cost between electricity on the pre-paid system and the flat-rate system for up to and including *50 kWh* of electricity. Use an interval of 10 in the table.

b. Use the table to help you to draw two separate graphs on the same set of axes to represent the cost of electricity on the pre-paid and flat-rate systems for up to and including *500 kWh* of electricity. You need to construct your own set of axes.

c. If a person uses an average of 320 kWh of electricity per month, should they be on the pre-paid system or the flat-rate system?

____

d. If a person uses an average of 450 kWh of electricity per month, should they be on the pre-paid system or the flat-rate system?

____

e. How many kWh must a person be using every month for the cost of being on the pre-paid system to be the same as being on the flat-rate system?

____

f.

i. Write down separate equations to represent the monthly cost of electricity on the pre-paid and flat-rate systems.

____

____

____

____

ii. Use the equations to determine how much it would cost on both systems to use 257,3 kWh of electricity in a month.

____

____

____

____

____

____

2.2 SUBSTITUTION AND SOLVING EQUATIONS

2.2.1 What is an equation?

An equation is a mathematical expression showing a relationship between two or more variables and/or numbers.

- Every equation contains *variables* − a variable is a symbol or letter used to describe the relationship being represented by the equation. Variables do not have a fixed value and their value can vary or change.
- Every equation has an *equal sign* showing precisely how the variables and/or numbers are related to each other.

2.2.2 Dependent and Independent Variables

When working with equations, it is always important to establish which variable in the equation is the *dependent variable* and which variable(s) is the *independent variable*.

- The dependent variable is a variable whose value is determined by the value of one or more other variables. In other words, the value of the dependent variable is dependent on the value of other variables.
- The independent variable(s) is a variable whose value is not dependent on the value of any other variable.

Example:

In a particular municipality, the cost of electricity during the month for a person living in a small house is dependent on:

- a fixed monthly service fee of R80,00;
- a charge per kWh of electricity used of R0,62 per kWh.

Using this information, the following formula can be used to represent the cost of electricity in this municipality:

Monthly Cost (Rand) = R80,00 + (R0,62 × kWh of electricity used)

- In this formula, the "*Monthly Cost"* is determined and depends entirely on how much electricity a person uses during the month. So, *Monthly Cost* is the *dependent variable*.
- The "*kWh of Electricity Used"* is the *independent variable* as the value of this variable could be anything and is affected by many factors like how cold it has been, or how many electrical appliances the person has used during the month, and so on. The value of this variable though, is not determined by the *Monthly Cost* and is entirely independent of the monthly cost.

When drawing a graph to represent an equation and, hence, the relationship between the variables in the equation, it is convention to place the *independent variable* on the horizontal axis and the *dependent variable* on the vertical axis. So, for the above equation the "*kWh of Electricity Used*" values would appear on the horizontal axis and the "*Monthly Cost values*" on the vertical axis.

2.2.3 Substitution

To substitute a value into an equation is to replace the independent variable(s) with a specific value in order to determine the value of the dependent variable.

Example:

Continuing with the equation representing the cost of electricity consumption:

Monthly Cost (Rand) = R80,00 + (R0,62 × kWh of electricity used)

How much would a person who uses 312 kWh of electricity during the month have to pay in electricity costs?

To answer this question we need to replace the independent variable "kWh of Electricity Used" with 312 kWh and then calculate the value of the dependent variable − "Monthly Cost".

→ Monthly Cost = R80,00 + (R0,62 × 312)

= R80,00 + R193,44

= R273,44

So, to determine the monthly cost of using 312 kWh of electricity during a month, we *substituted* the fixed value of 312 kWh into the independent variable and then performed the necessary calculations as outlined in the equation to determine the value of the dependent variable.

Practice Exercise: Substitution

1. If $p = 5$ and $q = 4$, determine the value of each of the following:

a. $p + 2 \times q$

b. $3 \times (p + q) + p \times q$

c. $q \div 3 + 1$

2. The equation below represents the cost of pre-paid electricity in a particular municipality:

Monthly Cost = R0,72 × kWh of electricity used

a. How much will it cost to use 200 kWh of electricity?

2. b. How much will it cost to use 418,7 kWh of electricity?

3. The equation below represents the transaction fee charged for withdrawing money from a bank account over the counter at a branch.

Fee = R20,00 + (0,95% × amount withdrawn)

a. How much will it cost in transaction fees to withdraw R100,00 from the bank account at the branch?

b. How much will it cost in transaction fees to withdraw R1 550,00 from the bank account at the branch?

3. c. Sindi withdraws R620,00 from her bank account at the branch and is charged R32,00 in transaction fees. Has she been charged the correct fee?

4. The formula below is used to determine the Body Mass Index (BMI) of an adult.

$$BMI\ (kg/m^2) = \frac{weight\ (kg)}{[height\ (m)]^2}$$

a. Determine the BMI of an adult who weighs 62 kg and is 1,65 m tall.

b. Determine the BMI of an adult who weighs 92 kg and is 1,73 m tall.

4. c. An adult who weighs 75 kg and is 2,1 m tall works out that their BMI is 17 kg/m^2. Are they correct?

d. This BMI of a person is used to determine the weight status of the adult according to the following categories.

BMI	Weight Status
<18.5	Underweight
>= 18.5 and < 25	Normal
>= 25 and < 30	Overweight
> 30	Obese

Determine the weight status of the adults with the following weights and heights:

i. Weight – 73 kg; height − 1,68 m

ii. Weight – 105 kg; height − 1,7 m

iii. Weight – 41 kg; height − 1,55 m

5. To calculate the monthly repayment on a bank loan the following formula can be used:

Repayment = (loan amount ÷ 1 000) × factor

The "factor" is a value that is determined by the length of the loan and the current interest rate − various factors are given in the table below:

Factor Table					
Length	**13.5%**	**14%**	**15%**	**15.5%**	**16%**
15	12.98	13.32	14	13.34	14.69
20	12.07	12.44	13.17	13.54	13.91
25	11.66	12.04	12.81	13.20	13.59

a. Calculate the monthly repayment on a R200 000,00 loan if the length of the loan is 20 years and the interest rate is 15%.

5. b. Calculate the monthly repayment on a R725 500,00 loan if the length of the loan is 25 years and the interest rate is 16%.

c. Calculate the monthly repayment on a R2 150 000,00 loan if the length of the loan is 20 years and the interest rate is 14%.

d. Calculate the monthly repayment on a R1,25 million loan if the length of the loan is 25 years and the interest rate is 15,5%.

5. e. Based on the information presented in the table and on your answers above:

i. What effect does a longer loan length have on the monthly repayments of a loan?

____

____

____

ii. What effect do changes in the interest rate have on the monthly repayments of a loan?

____

____

____

2.2.4 Solving Equations

To solve an equation means to replace the dependent variable with a specific value and then find the value of the independent variable.

Example:

Continuing with the equation representing the cost of electricity consumption:

Monthly Cost (Rand) = R80,00 + (R0,62 × kWh of electricity used)

So far we have only substituted values into the independent variable − kWh of electricity used − in order to determine the monthly cost of electricity.

But what if a person receives a bill of R524,00 and wants to check if the electricity consumption value listed on the bill is correct? To answer this question we need to perform the following calculations:

- Step 1: Substitute the given dependent variable value into the equation.

  → R524,00 = R80,00 + (R0,62 × kWh of electricity used)

- Step 2: Manipulate the equation to get the independent variable for which you are solving on its own on one side of the equal sign and all the other terms on the other side of the equal sign. You do this by performing the ***opposite operations*** to those that appear in the original equation and in the ***reverse order***.

In the case of monthly electricity cost, to solve for "kWh of Electricity Used" we need to do the opposite operations to what appears in the original equation − this means that we must *subtract* R80,00 from the both sides of the equation and then *divide* both sides of the equation by R0,62.

i.e. R524,00 = R80,00 + (R0,62 × kWh of electricity used)

R524,00 − R80,00 = R80,00 − R80,00 + (R0,62 × kWh of electricity used)

R444,00 = R0,62 × kWh of electricity used

R80,00 − R80,00 = 0

$$\frac{R444{,}00}{R0{,}62} = \frac{R0{,}62}{R0{,}62} \times \text{kWh of electricity used}$$

R0,62 ÷ R0,62 = 1

∴ kWh of electricity used = 716,13 kWh

Practice Exercise: Solving Equations

1. Determine the value of p in each of the following equations:

a. $p + 7 = 15$

b. $3 \times p - 8 = 28$

c. $2 \times (p - 2) = 14$

2. a. The equation below represents the cost of pre-paid electricity in a particular municipality:

Monthly Cost = R0,72 × kWh of electricity used

i. If a person spends R250,00 on electricity, how many kWh of electricity have they used?

ii. If a person spends R317,50 on electricity, how many kWh of electricity have they used?

2. b. The equation below represents the cost of flat-rate electricity in the same municipality:

Monthly Cost = R92,00 + (R0,55 × kWh)

i. What is the fixed monthly service fee on the flat-rate system?

ii. What is the per kWh charge for electricity on the flat-rate system?

iii. If a person receives an electricity bill for R300,00, how many kWh of electricity have they used during the month?

iv. If a person receives an electricity bill for R412,27, how many kWh of electricity have they used during the month?

2. c. A person uses an average of 420 kWh of electricity per month. Should they be on the pre-paid system or the flat-rate system? Explain.

3. The equation below represents the transaction fee charged for withdrawing money from a bank account over the counter at a branch.

Fee = R20,00 + (0,95% × amount withdrawn)

a. Write 0,95% as a decimal value.

b. If a person pays R24,75 in transaction fees, how much have they withdrawn from the bank?

3. c. If a person pays R110,25 in transaction fees, how much have they withdrawn from the bank?

4. The formula below is used to determine the Body Mass Index (BMI) of an adult.

$$BMI\ (kg/m^2) = \frac{weight\ (kg)}{[height\ (m)]^2}$$

a. If a person is 1,68 m tall and has a BMI of 21,05 kg/m^2, how much do they weigh?

b. If a person is 1,77 m tall and has a BMI of 25,86 kg/m^2, how much do they weigh?

c. If a person weighs 75 kg and has a BMI of 25,95 kg/m^2, how tall are they?

d. If a person weighs 61 kg and has a BMI of 28,23 kg/m^2, how tall are they?

5. To calculate the monthly repayment on a bank loan the following formula can be used:

Repayment = (loan amount ÷ 1 000) × factor

The "factor" is a value that is determined by the length of the loan and the current interest rate − various factors are given in the table below:

Factor Table					
Length	**13.5%**	**14%**	**15%**	**15.5%**	**16%**
15	12.98	13.32	14	13.34	14.69
20	12.07	12.44	13.17	13.54	13.91
25	11.66	12.04	12.81	13.20	13.59

5. a. On a R800 000,00 loan at an interest rate of 15,5%, a person pays R10 832,00 in monthly repayments. What is the length of the loan?

b. On a R1 250 000,00 loan at an interest rate of 14%, a person pays R15 050,00 in monthly repayments. What is the length of the loan?

c. On a R320 000,00 loan with a length of 15 years, a person pays R4 480,00 in monthly repayments. What is the interest rate on the loan?

5. d. On a R2 500 000 loan with a length of 20 years, a person pays R34 775,00 in monthly repayments. What is the interest rate on the loan?

e. A person pays R5 045,26 in monthly repayments on a loan. If the interest rate on the loan is 13,5% and the length of the loan is 20 years, calculate the size of the loan.

6. The table below shows the transfer fees that have to be paid when buying a house.

Property Value	Transfer Fee
≤ R500 000	0%
R500 001 to R1 Million	R25 000,00
Above R1 000 001	R25 000,00 + 8% on the value above R1 Million

6. a. Calculate the transfer fee on a R1 250 000,00 house.

b. Calculate the transfer fee on a R2 425 500,00 house.

c. If the transfer duty on a house amounts to R65 000,00, what is the price of the house?

d. If the transfer duty on a house amounts to R201 000,00, what is the price of the house?

Test Your Knowledge: Tables, Equations and Graphs

The table below shows the electricity tariffs for pre-paid electricity and flat-rate electricity in a municipality.

Electricity System	Fixed Monthly Service Fee	Charge per kWh
Pre-Paid	Nil	R0,75
Flat-Rate	R75,00	R0,50

1. Calculate the cost of using 317,2 kWh of electricity on the:

a. Pre-paid system

b. Flat-rate system

2. Write down separate equations to represent the monthly cost of pre-paid electricity and the monthly cost of flat-rate electricity.

3. a. Use the appropriate equation to determine how many kWh of electricity a person has used during the month if they spend R325,00 on pre-paid electricity.

b. Use the appropriate equation to determine how many kWh of electricity a person has used during the month if they spend R516,29 on flat-rate electricity.

4. Use the equations to construct a table of values showing the cost of pre-paid and flat-rate electricity. Use the table below to help you.

kWh of Electricity Used	Pre-Paid	Flat-Rate
	Monthly Cost	Monthly Cost
0		
50		
100		
200		
300		
400		
500		

5. Use the table to draw two separate graphs on the same set of axes to represent the cost of pre-paid electricity and flat-rate electricity. Use the blank set of axes given below.

Use the *graph* to answer the following questions:

6. a. If a person uses 250 kWh of electricity, approximately how much will they pay in electricity costs on the:

i. Pre-paid system?

____

____

ii. Flat-rate system?

____

____

6. b. If a person spends on average R260,00 on electricity every month, how many kWh of electricity would they be using on the:

i. Pre-paid system?

____

____

ii. Flat-rate system?

____

____

c. If a person uses an average of 450 kWh of electricity per month, should they be on the pre-paid or the flat-rate system? Explain.

____

____

____

d. Approximately how many kWh of electricity must a person be using every month in order for it to be more expensive to be on the pre-paid system rather than the flat-rate system?

____

____

e. Approximately how much money must a person be spending on electricity every month in order for it to be more expensive to be on the flat-rate system rather than the pre-paid system?

____

____

TOPIC 3

SPACE, SHAPE & ORIENTATION

INDEX

3.1 CONVERTING UNITS OF MEASUREMENT

When performing calculations involving space and shape it is often necessary to convert from one unit of measurement to another − for example from cm to m, or ml to litres, or m^3 to litres.

The important thing to remember about converting from one unit of measurement to another is that you are essentially working with ratios (see Topic 1 – Numbers). As such, we will use the same method of unit ratios and equivalent ratios that you learned about in *Topic 1 – Numbers* to convert units of measurement.

Examples:

The table below shows the conversion ratios for length, volume and weight:

Length	Volume	Weight
1 km = 1 000 m	1 litre = 1 000 ml	1 kg = 1 000 g
1 m = 100 cm	1 m^3 = 1 000 litres	1 g = 1 000 mg
1 cm = 10 mm	1 ml = 1 cm^3	1 tonne = 1 000 kg

1. Pule measures the height of the door to be 220 cm. To work out the height of the door in metres we can use the following method:

$$100 \text{ cm} = 1 \text{ m}$$

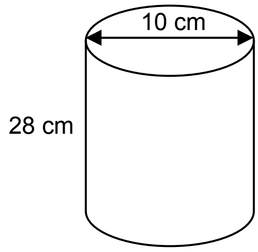
$$\downarrow \div 100 \qquad \downarrow \div 100$$

$$1 \text{ cm} = 0{,}01 \text{ m}$$

$$\downarrow \times 220 \qquad \downarrow \times 220$$

$$220 \text{ cm} = 2{,}2 \text{ m}$$

2. Benni is making containers for holding water. He measures the dimensions of the containers in cm and then works out that the volume of each container is 2 200 cm^3.



To work out how much water each container will hold, he needs to convert this volume value from cm^3 to ml or litres.

$\rightarrow$ 1 cm^3 = 1 ml

× 2 200 × 2 200

2 200 cm^3 = 2 200 ml

And, since: 1 000 ml = 1 litre

÷ 1 000 × 2 200 ÷ 1 000 × 2 200

2 200 ml = 2,2 litres

Practice Exercise: Converting Units of Measurement

1. Use the table below to convert the given values to the given unit of measurement.

Length	**Volume**	**Weight**
1 km = 1 000 m	1 litre = 1 000 ml	1 kg = 1 000 g
1 m = 100 cm	1 m^3 = 1 000 litres	1 g = 1 000 mg
1 cm = 10 mm	1 ml = 1 cm^3	1 tonne = 1 000 kg

a. 1 500 m = _________ km

b. 15,325 km = _________ m

c. 165 mm = _________ cm = _________ m

d. 1,25 m = _________ cm = _________ mm

e. 1,275 litres = _________ ml

f. 723 ml = _________ litres

g. 450 g = _________ kg

h. 312 kg = ________ tones = _______ grams

i. 575 cm^3 = ________________ m^3

2. The table below shows the conversion ratios for converting from metric to imperial measurements.

Length	Capacity	Weight
1 mile = 1,609 km	1 gallon = 4,5461 litres	1 pound = 0,4536 kg
1 foot = 0,3048 m		
1 inch = 25,4 mm		

a. 3 miles = _________ km

b. 8,5 miles = _________ km

c. 5 feet = _________ m

d. 143 pounds = _________ kg

e. 1 km = _________ miles

f. 1 m = _________ feet

g. 1 litre = _________ gallons

h. 125 mm = _________ inches

i. 72 kg = _________ pounds

j. 3 feet = _________ cm

k. 1 572 ml = _________ gallons

l. 3 500 m = _________ miles

3. The table below shows the conversion ratios for converting from ml to grams and grams to ml for different cooking ingredients.

Ingredients	5 ml	12,5 ml	25 ml	100 ml
Flour	3 g	8 g	15 g	60 g
Margarine	5 g	12,5 g	25 g	100 g
Mealie Meal	3 g	6 g	12 g	50 g
Rice	4 g	10 g	20 g	80 g
Brown & White Sugar	4 g	10 g	20 g	80 g

1 cup = 250 ml 1 tablespoon = 15 ml 1 teaspoon = 5 ml

a. How many ml of flour is equal to 6 g of flour?

____

____

b. How many ml of sugar is equal to 40 g of sugar?

____

____

c. How many grams of margarine is equal to 100 ml of margarine?

____

d. How many ml of mealie meal is equal to 112 g of mealie meal?

____

____

____

3. e. How many ml of sugar is equal to 130 g of sugar?

____

____

____

f. How many ml of rice is equal to 450 g of rice?

____

____

____

g. How many ml of mealie meal is equal to 280 grams of mealie meal?

____

____

____

h. How many grams of flour is equal to 290 ml of flour?

____

____

____

3. i. How many grams of rice is equal to 2 cups of rice?

____

____

____

j. How many grams of sugar is equal to 3 tablespoons of sugar?

____

____

____

____

k. How many cups of flour is equal to 450 g of flour?

____

____

____

____

L. How many tablespoons of sugar is equal to 60 g of sugar?

____

____

____

____

3.2 WORKING WITH 2- AND 3-DIMENSIONS

Situations involving space and shape will often involve both 2-dimensional (2-D) and 3-dimensional (3-D) situations.

For example, consider the process involved in designing a house: Architects start by drawing a picture of the house so that the person whose house is being built knows what the house will look like. The architect then draws 2-D diagrams of the front, both sides, top, and insides of the house so that the builder can see the dimensions of the house from all sides. The builder then has to interpret, make sense of and use these 2-D diagrams in order to build the 3-D house.

3.2.1 Moving From 2-D Diagrams to 3-D Objects

To help you to develop the ability to move comfortably from 2-D picture to 3-D objects you are going to fold 2-D *nets* into 3-D object.

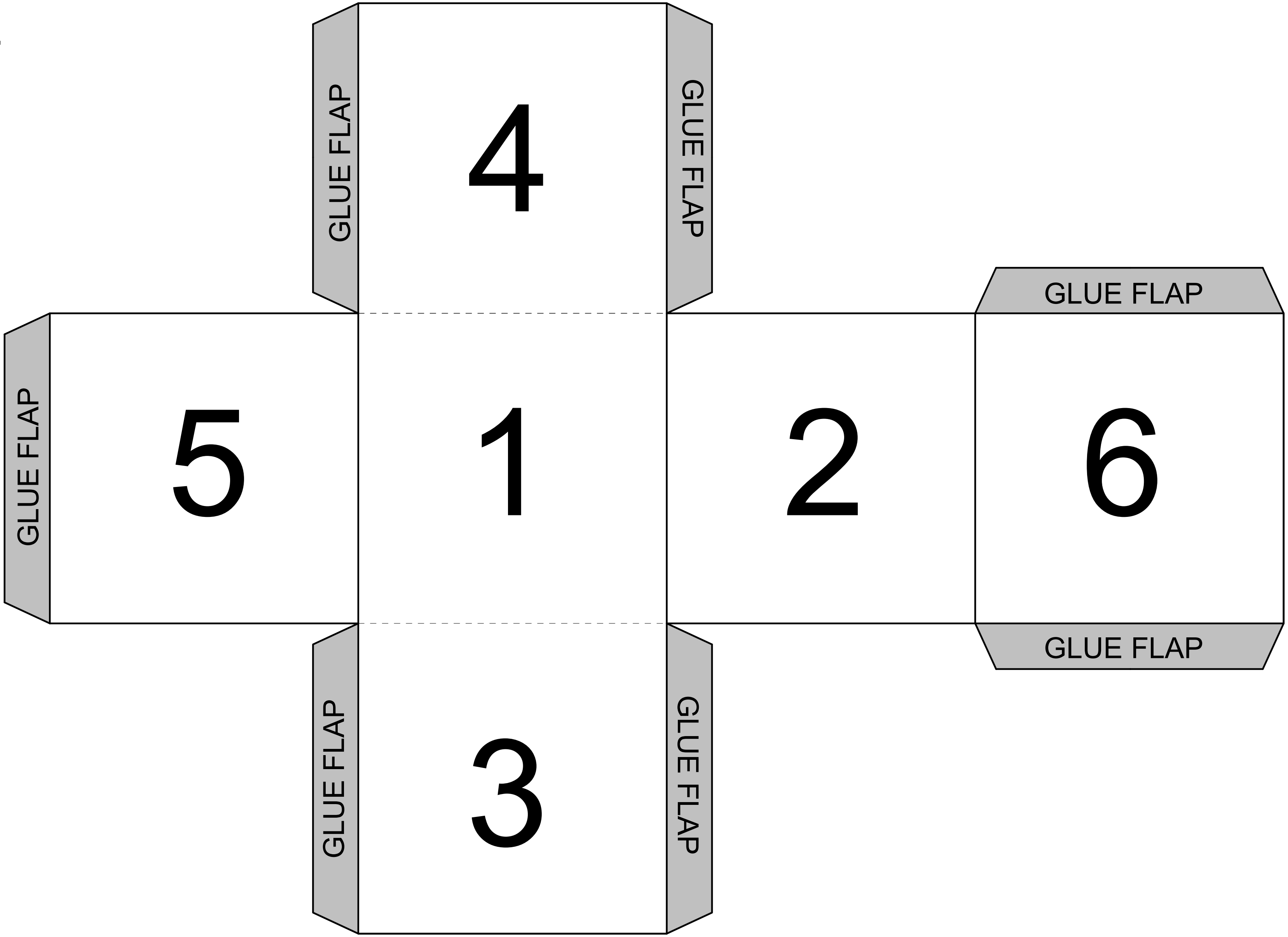
A *net* is a 2-D plan of a 3-D object.

Activity: **Building 3-D objectss from 2-D nets**

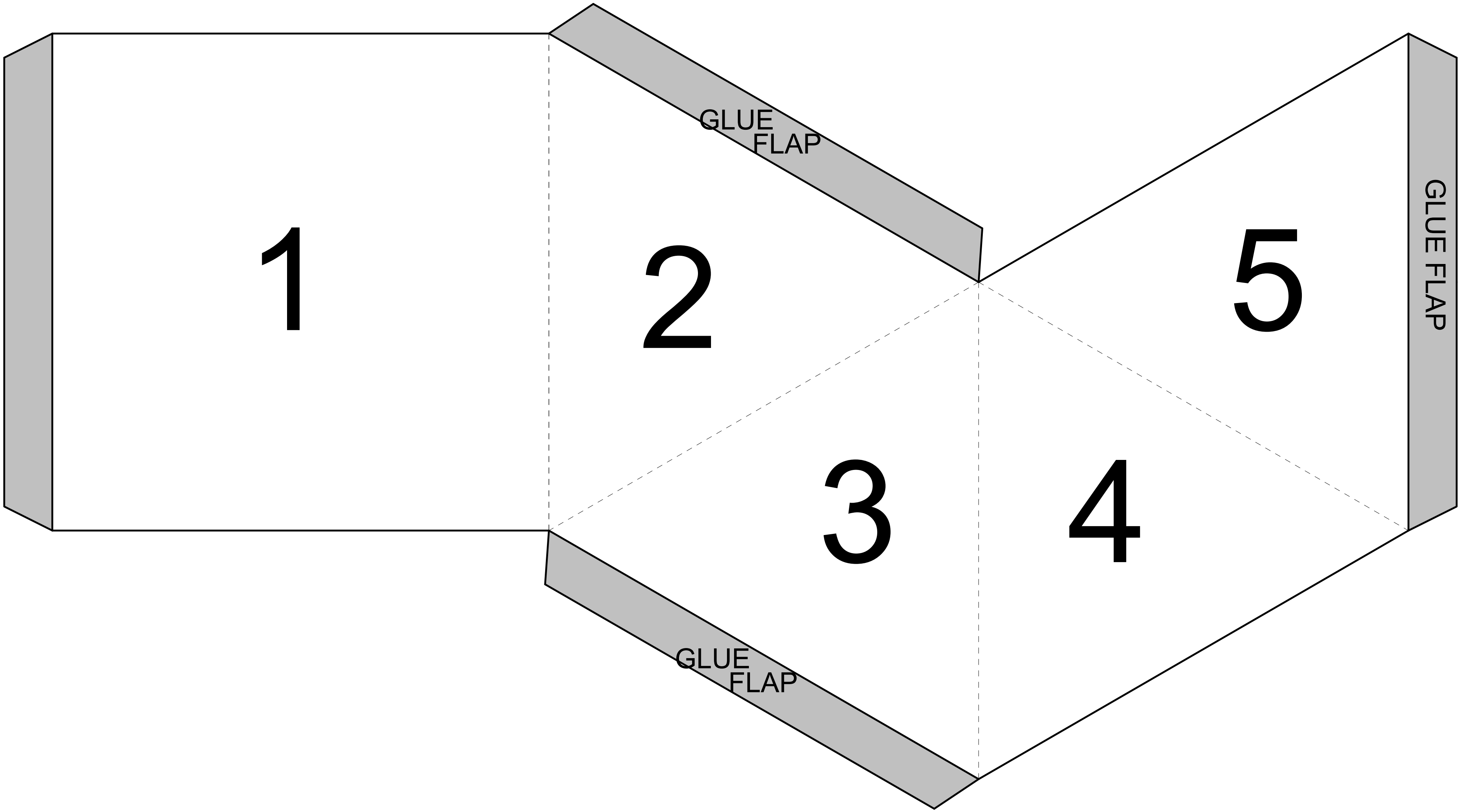
Instructions:

1. On the pages below you are given the nets for two shapes − a cube and a pyramid.
2. Cut out the nets for each of the shapes.
3. Fold the shape along the dotted lines.
4. Place glue on the flaps and then glue the sides of the shape together in the order in which the sides are numbered.

CUBE


GLUE FLAP
4
GLUE FLAP
GLUE FLAP
GLUE FLAP
5
1
2
6
GLUE FLAP
GLUE FLAP
3
GLUE FLAP

PYRAMID



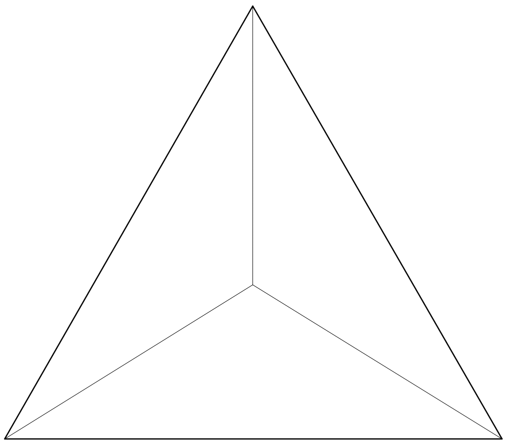
3.2.2 Moving from 3-D Objects to 2-D Pictures

Now that you have had some practice at building 3-D objects from 2-D pictures, let’s try to do things the other way around − i.e. working from 3-D objects to 2-D pictures.

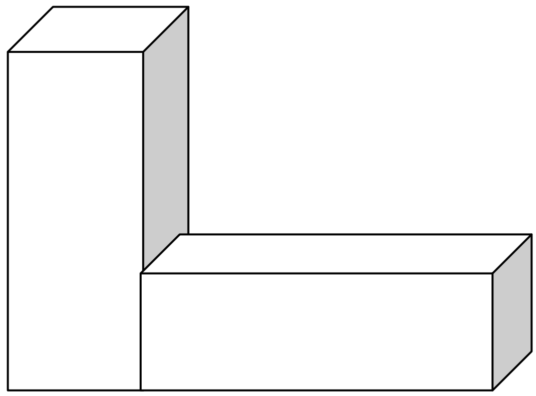
Activity 1: Constructing nets

Construct 2-D nets to represent the 3-D shapes below. Make sure to indicate on the nets where the glue flaps and fold lines will be:

1.



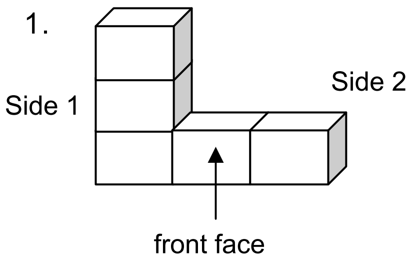
2.



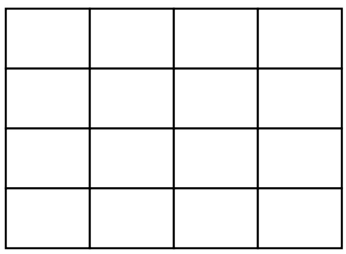
Activity 2: Drawing different perspectives

3-D pictures of objects are given below. You need to use the given grids to draw appropriate 2-D pictures to show what the objects will look like from the front, side, back and top. The position of the front face of the shape has been labeled.

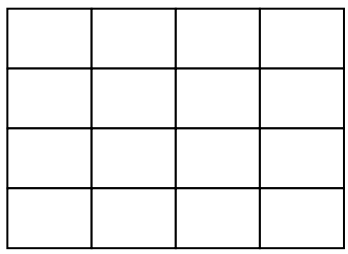
1.



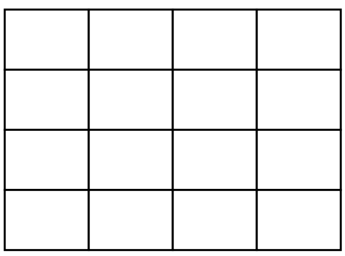
Front



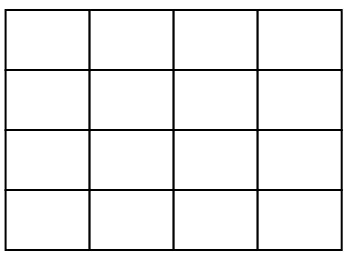
Back



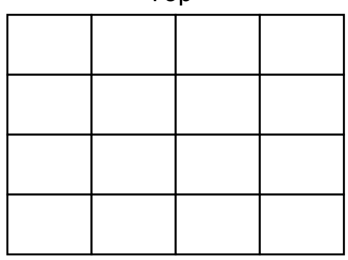
Side 1



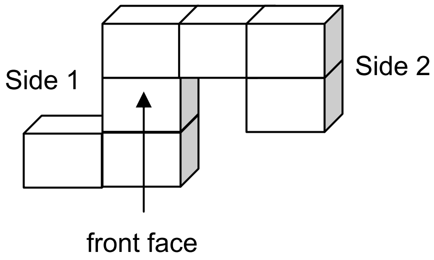
Side 2



Top



2.



Front

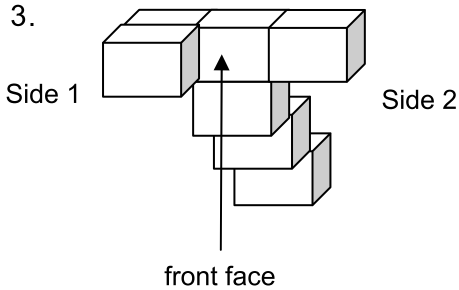
Back

Side 1

Side 2

Top

3.



Front

Back

Side 1

Side 2

Top

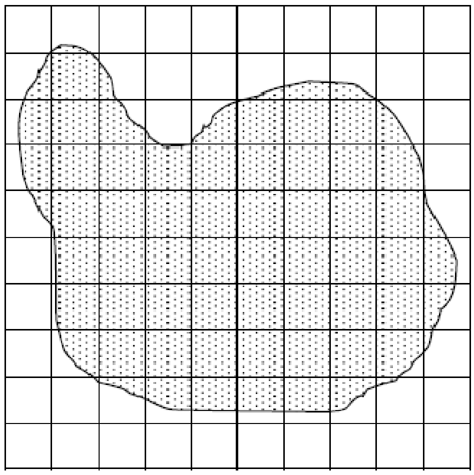
3.3 AREA

3.3.1 Definition

The area of an object is the number of square units of a certain size needed to cover the surface of a figure. Put another way, the area of an object is the amount of 2-dimensional space that an object takes up.

Working out the area of an object involves working with *two dimensions* of the object and working out the number of *square units* needed to cover the surface of the object. For this reason, the area of an object is always expressed in units^2 i.e. mm^2, cm^2, m^2, and so on.

The easiest way of thinking about **area** is to imagine having a piece of grid paper, placing this over a shape and counting the number of squares needed to “cover the surface".



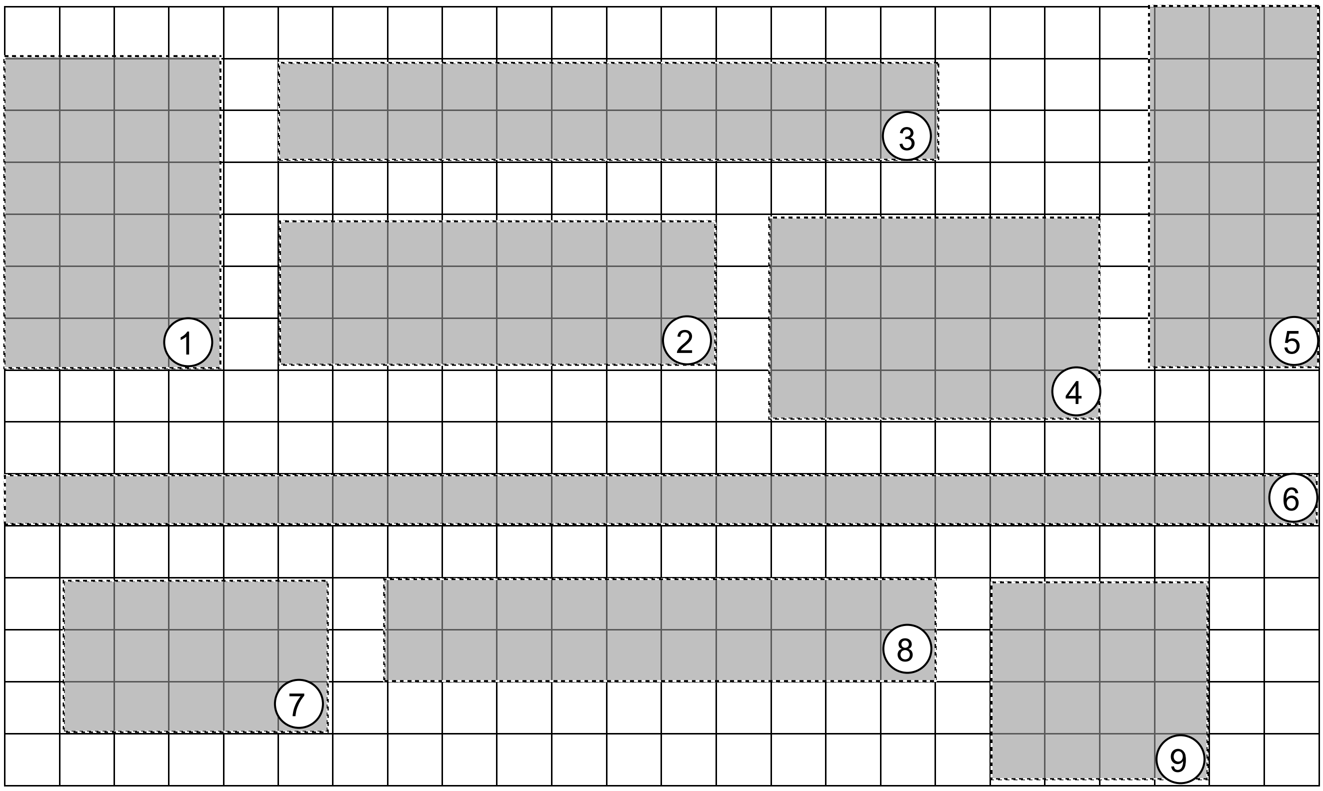
Of course, this approach raises questions about what to do with the squares that are not fully covered. Furthermore it may not always be practical to cover the shape whose area we want to determine with a piece of grid paper. To make life easier we have developed formulae for determining the area of a few common shapes — in particular rectangles, triangles, and squares.

3.3.2 Discovering Area Formulae

A. Area of a Rectangle / Square

Activity:

1. To help you to discover the formula for calculating the area of a square or rectangle, complete the table of values given below for each of the shapes in the following picture:



Shape number	1	2	3	4	5	6	7	8	9
Length									
Breadth									
Number of squares									

2. Based on your answers in the table, can you think of an equation that could be used to describe the relationship between the length and breadth of a rectangular figure and the area of that figure? Write your answer below.

Developing a formula:

Hopefully you came up with the following pattern from the values in the table:

Area (rectangle) = length × breadth

In relation to the picture, *length* represents the number of square units in each row and *breadth* represents the number of square units in each column. It follows that *length* × *breadth is* simply the total number of square units in the rectangle.

Example:

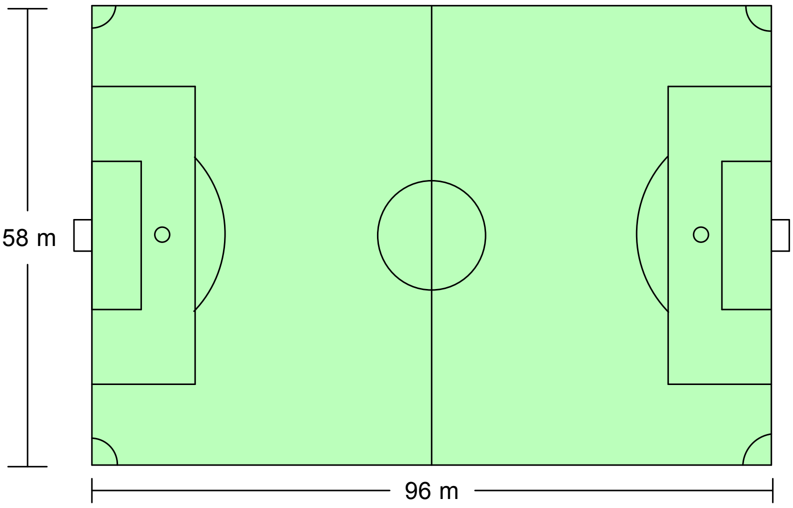
The playing field of a standard soccer pitch is 96 m long and 58 m wide.

If the caretaker wants to replace the grass on the pitch he will need to first work out the area of the pitch.

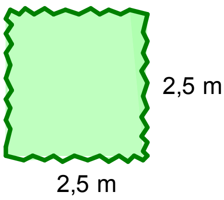
Area of the pitch = length × breadth

= 96 m × 58 m

= 5 568 m^2



The caretaker is going to replace the grass on the pitch with grass squares that are 1,2 m long and 1,2 m wide. It is tempting to think that we can solve the problem as follows.



To determine how many of these square patches of grass he will need we need to compare the *area of the grass patch* to the *area of the whole field*.

Area of a grass patch = 1,2 m × 1,2 m = 1,44 m^2

No. of grass patches needed for the soccer field = 5 568 m^2 ÷ 1,44 m^2

= 3 866,7 (rounded off to one decimal place)

→ The caretaker needs more than 3 866 patches, so we round this *answer up*.

The correct answer is 3 867 patches of grass.

However, the solution is not that realistic as it assumes that the grass patches (or tiles in the case bathroom tiles etc.) can be cut up and redistributed. This is not true. A more realistic approach would be:

Divide the length of the pitch by the length of the grass patch and round up the answer.

Divide the width of the pitch by the width of the grass patch and round up the answer. Thes two answers are then multiplied together. Our example would be:

Number of grass patches in the length = 96 m ÷ 1,2 m = 80 grass patches

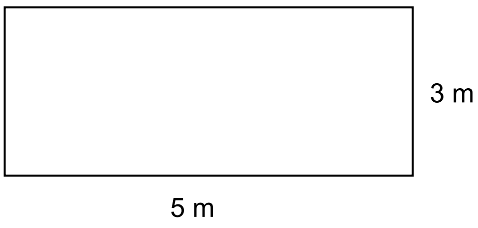
Number of grass patches in the width = 58 m ÷ 1,2 m = 48,3 grass patches rounded up to 49

Total number of grass patches needed = 80 × 49 = 3 920 grass patches.

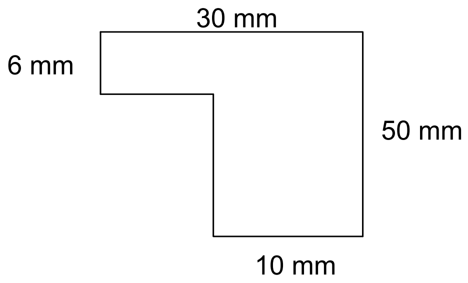
Practice Exercise: Area of Rectangles

1. Calculate the areas of the following shapes:

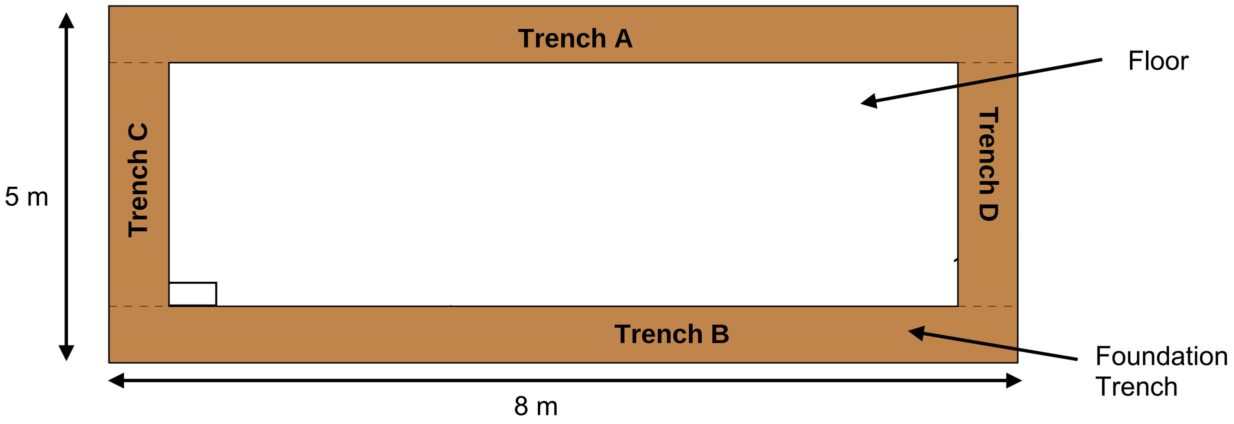
a.



b.



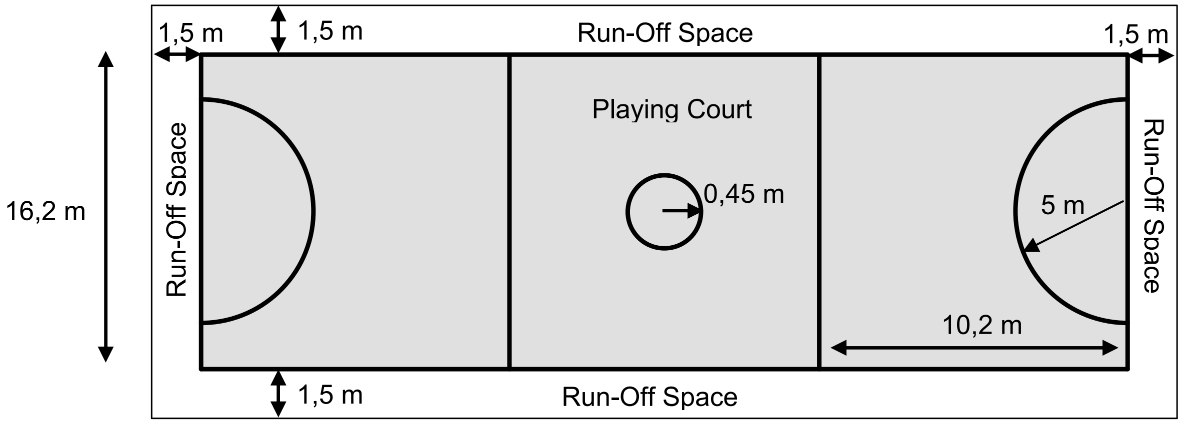
2. Zipho is building a house. The picture below shows the dimensions of the floor and foundation trench of the house.



a. Determine the area of the floor.

b. Once the floor has been built, Zipho plans to tile the floor with square tiles that are 0,8 m long and 0,8 m wide. Use both methods shown above to determine how many tiles Zipho need for the floor?

3. The picture below shows the dimensions of a netball court. The court is surrounded by a "runoff space". This is extra space around the side of the playing court so that the players have space to run if they leave the court.



a. The caretaker wants to repaint the *playing court*.

i. Calculate the surface area of the playing court.

b. The caretaker also wants to repaint the lines on the playing court. Calculate how many metres of lines he needs to repaint.

(You may need to use the following formula: *Perimeter of a circle = 2 × π × radius of circle; let π = 3,142*)

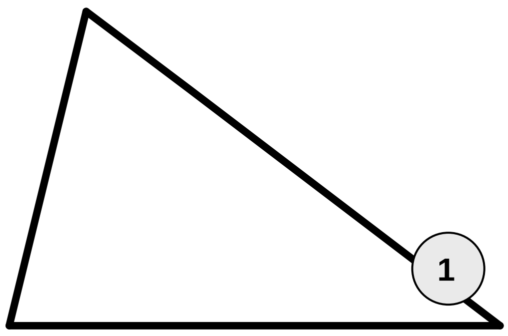
ii. If the paint that the caretaker will use has a coverage of 4 m^2 per litre, calculate how many litres of paint the caretaker will need to buy.

B. Area of a Triangle

Now that we have determined a formula for the area of a rectangle we can use this to determine the area of a triangle.

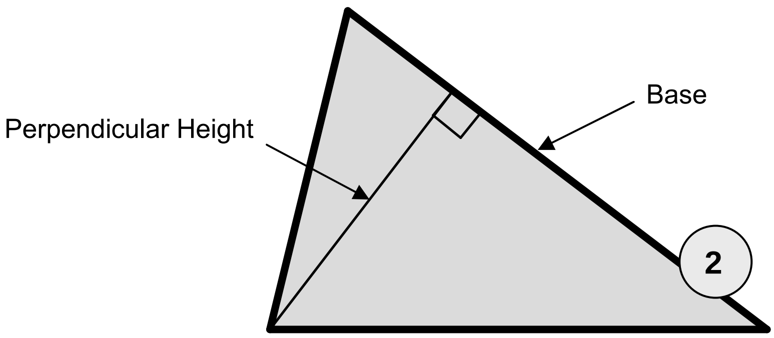
Activity:

Consider the following triangle:

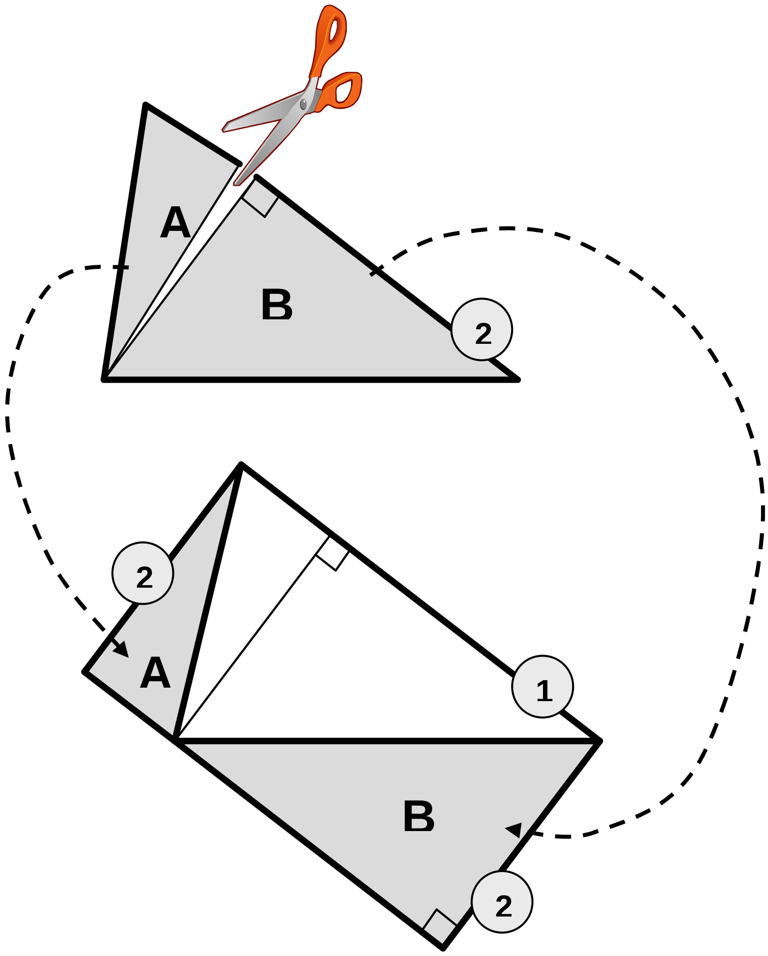


To determine the area of this triangle using the rectangle formula, do the following:

1. Construct a second triangle (Triangle 2) that is identical to Triangle 1.
2. On Triangle 2 construct a perpendicular line from one of the three corners (vertices) to the opposite side — we call this line a <u>*perpendicular height*</u> of the triangle. We call the side of the triangle to which the perpendicular height has been drawn the <u>*base*</u> of the triangle. *NOTE: base does not mean "the bottom"; it means the side to which the perpendicular height has been drawn.*



3. Now cut the second triangle along the perpendicular height and place the two pieces as shown below.



Developing a formula:

We have now created a rectangle that is made up of two identical triangles. As such, *the area of the rectangle is twice the area of the original triangle*. From this observation it follows that:

Area (rectangle) = length × breadth

However, in the rectangle above, *the* *<u>length</u>* *of the rectangle is the same as the base of the original triangle* and *the* *<u>width</u>* *of the triangle is the same as the perpendicular height of the original triangle*.

As such: Area (rectangle) = base x perpendicular height

BUT: Area (rectangle) = 2 × Area (triangle)

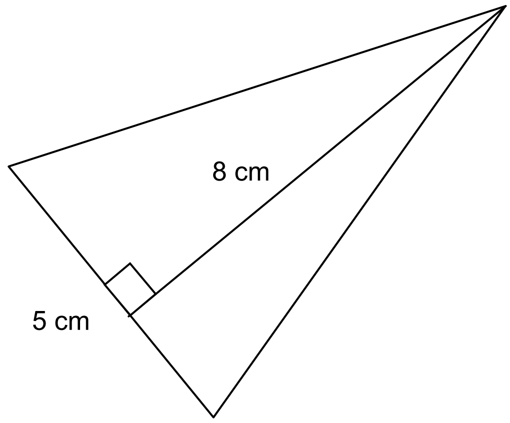
→ 2 × Area (triangle) = base × perpendicular height

$$\therefore \textbf{\textit{Area (triangle)}} = \frac{\textbf{\textit{base × perpendicular height}}}{\textbf{\textit{2}}}$$

This formula is often summarised as: $\textbf{\textit{Area (triangle)}} = \frac{1}{2} \times \textbf{\textit{base}} \times \textbf{\textit{perpendicular height}}$

Example 1:

The triangle alongside has a perpendicular height of 8 cm and a base of 5 cm.



Area (triangle) = ½ × base × perpendicular height

= ½ × 5 cm × 8 cm

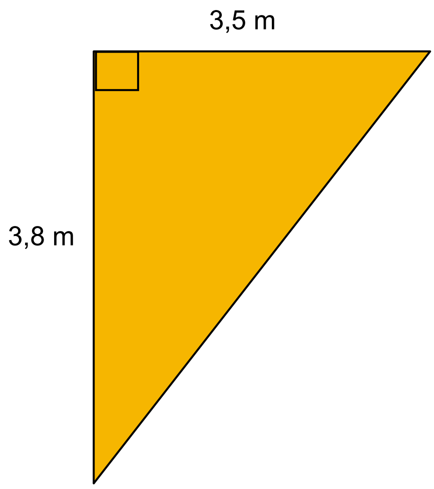
= ½ × 40 cm^2

= 20 cm^2

Example 2:

In his garden, Dennis has a triangular piece of land that he wants to cover with gravel.

In this scenario, the triangular portion of land is right-angled. This means that two of the sides of the triangle are perpendicular to each other.



As such, one of the perpendicular sides of this triangle will be the “base” and one will be the “height”.

→ Area (triangle) = ½ × base × perpendicular height

= ½ × 3,5 m × 3,8 m

= ½ × 13,3 m^2

= 6,65 m^2

Notice that because the triangle in this situation is right-angled, we could have used 3,8 m as the “base” and 3,5 m as the “height” with the same result:

→ Area (triangle) = ½ × base × perpendicular height

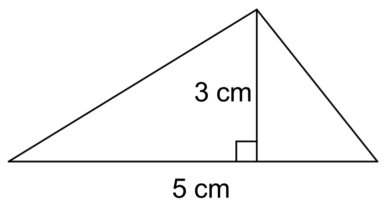
= ½ × 3,8 m × 3,5 m

= 6,65 m^2

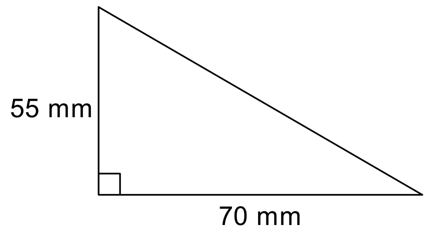
Practice Exercise: Area of Triangles and Rectangles

1. Calculate the areas of the following triangles:

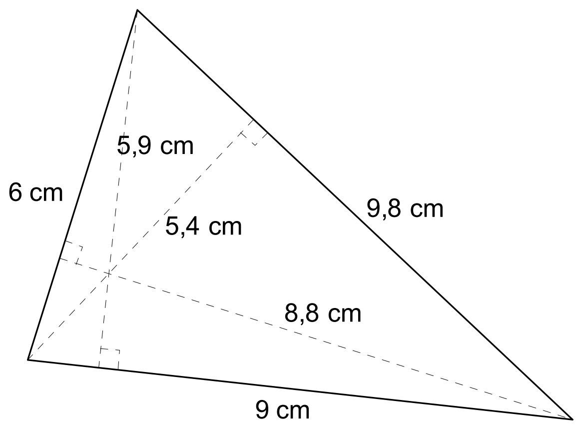
a.



b.



2.



a. Determine the area of the triangle using:

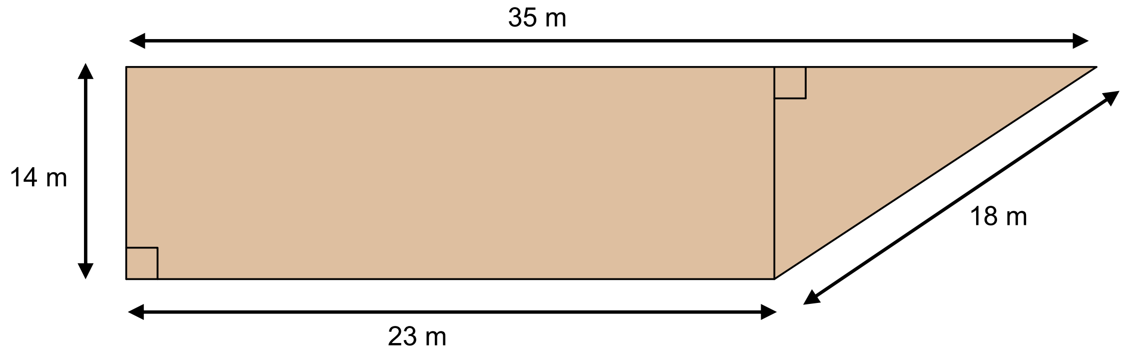
i. Height 5,9 cm and base 9 cm.

ii. Height 8,8 cm and base 6 cm.

iii. Height 5,4 cm and base 9,8 cm.

b. Compare the areas that you calculated in (a). What do you notice?

3. Imraan owns the piece of land pictured below.



a. Imraan needs to work out the area of the land so that he knows how much land he has to buy fertiliser for.

i. Calculate the area of the rectangular portion of the piece of land.

____

____

ii. Calculate the area of the triangular portion of the piece of land.

____

____

____

____

iii. Calculate the total area of the piece of land.

____

____

iv. The fertilizer that Imraan intends to use has a coverage of 1,5 m^2 per bag. How many bags of fertilizer will Imraan need to fertilise the whole plot of land?

____

____

____

____

b. Imraan wants to erect a fence around the outside of the piece of land. The fence will be supported by wooden poles that will be spaced 2 m apart from each other.
How many wooden poles will Imraan for the whole fence?

____

____

____

____

____

____

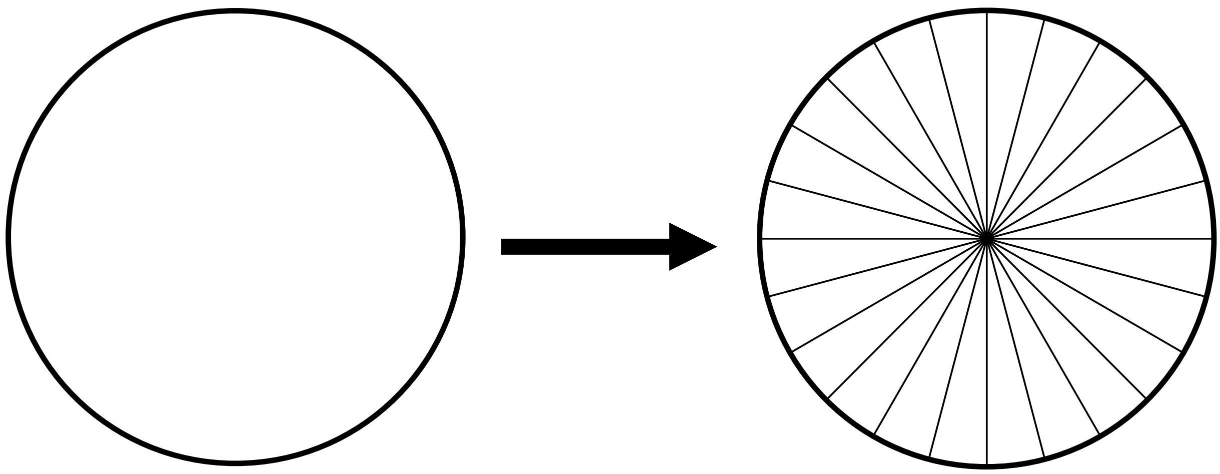
C. Area of a Circle

The formula for the area of a circle can also be derived using the formula for the area of a rectangle.

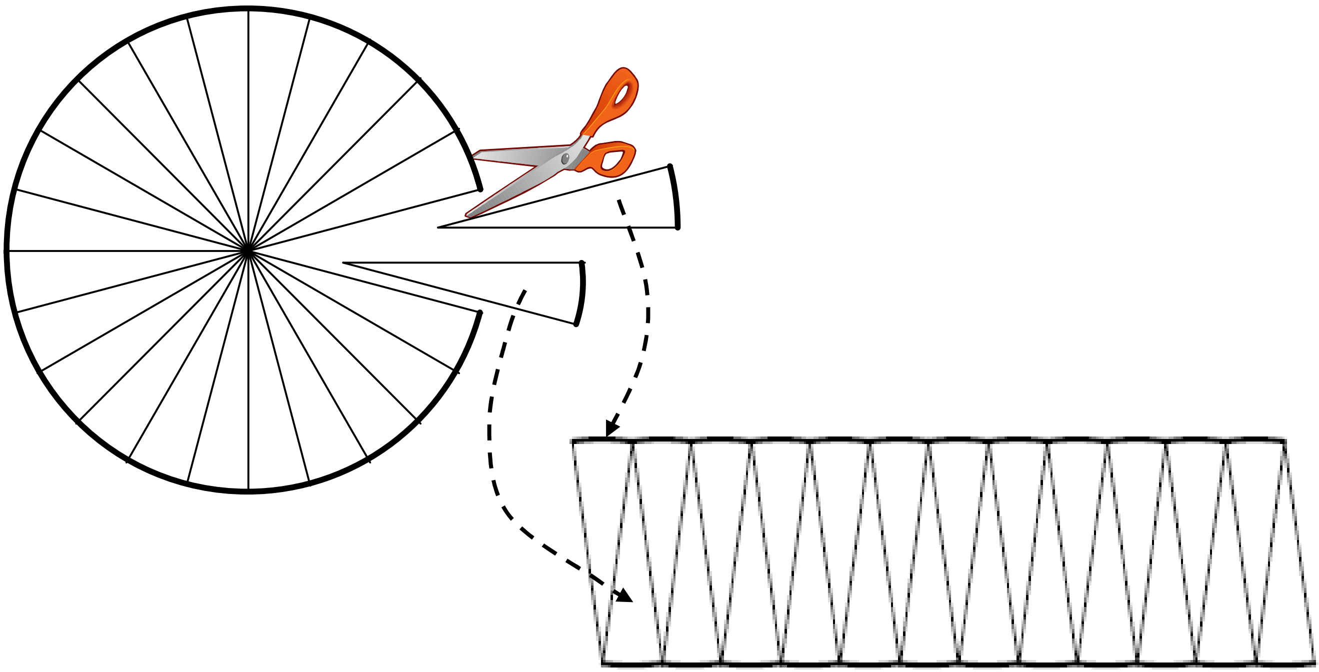
Activity:

To determine the area of a circle using the rectangle formula, imagine doing the following.

1. Take a circle and divide it into a large number of segments.



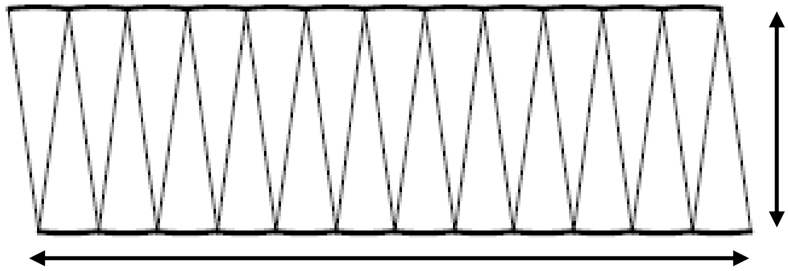
2. Cut out the segments and rearrange them as illustrated below.



Developing a formula:

We have now created a very good approximation to a rectangle. The larger the number of segments we use the better the rearranged segments approximate a rectangle.

If we look closely at this “rectangle” there are a couple of things to notice:



Width = radius of the circle

Length = ½ of the circumference of the circle

- The length of the “rectangle” is the same as ½ of the circumference of the original circle.
- The width of the circle is the same as the radius of the original circle.

From this observation it follows that:

Area (rectangle) = length × breadth

Area (rectangle) = ($\frac{1}{2}$ × circumference of the circle) × (radius of the circle)

BUT:

Area (rectangle) = Area (circle)

$\therefore$ Area (circle) = $\frac{1}{2}$ × circumference x radius

AND:

Since circumference = π × diameter it follows that:

$\rightarrow$ Area (circle) = $\frac{1}{2}$ × (π × diameter) × radius

$\boxed{\text{Circumference} = \pi \times \text{diameter}}$

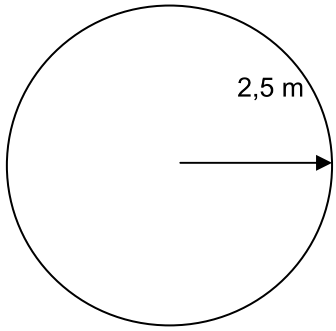
= $\frac{1}{2}$ × π × (2 × radius) × radius

$\boxed{\text{Diameter} = 2 \times \text{radius}}$

= π × (radius)^2

Worked Example:

Vusi is painting a circular area on a wall. The picture alongside shows the dimension of the radius of this circular area.



To determine how much paint he will need, Vusi must first work out the area of this circle and then work out how much paint is needed. (*let* $\pi = 3,142$)

Area (circle) = $\pi \times (\text{radius})^2$

$= \pi \times (2,5 \text{ m})^2$

$= \pi \times 2,5 \text{ m} \times 2,5 \text{ m}$

$= 19,638 \text{ m}^2$ (rounded off to three decimal places)

The paint that Vusi is using has a coverage of 5 m^2 per litre.

→ Paint needed: 5 m^2 : 1 litre

19,638 m^2 : 1 litre ÷ 5 × 19,638

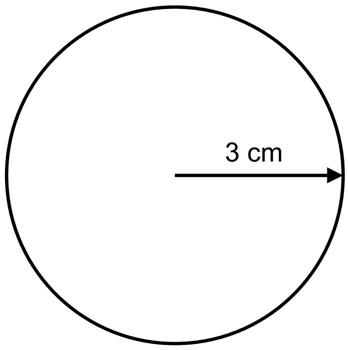
≈ 3,9 litres (rounded off to one decimal place)

So, Vusi will need to buy at least 4 litres of paint.

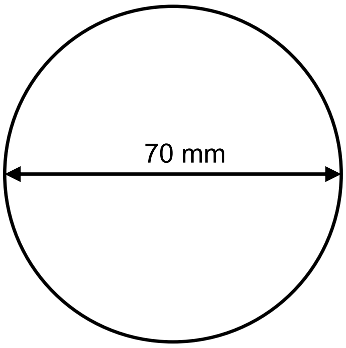
Practice Exercise: Area of Circles (+ Rectangles)

1. Calculate the areas of the following circles:

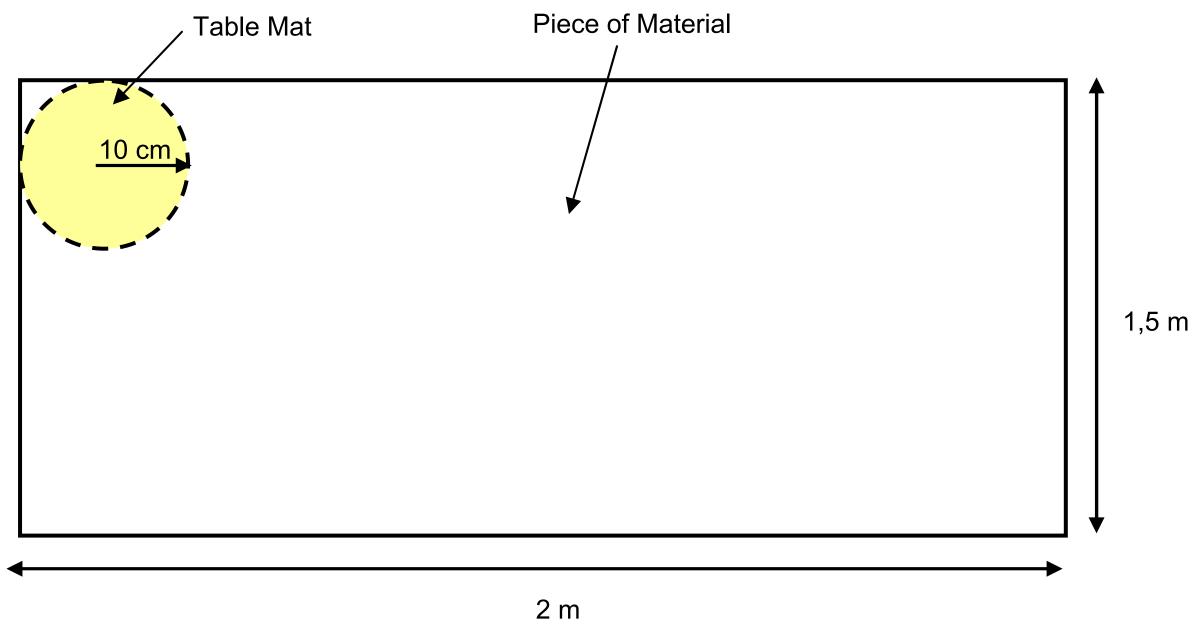
a.



b.



2. Luanda makes circular table mats. She cuts the mats out from a rectangular piece of material. The picture below shows the dimensions of each table mat and the dimensions of the rectangular piece of material out of which she cuts the circular mats.



a.

i. Calculate the area of each circular table mat.

____

____

____

ii. Calculate the area of the rectangular piece of material.

____

____

____

iii. Use both methods discussed in Section 3.3.2 calculate how many table mats Luanda will be able to cut from the rectangular material.

____

____

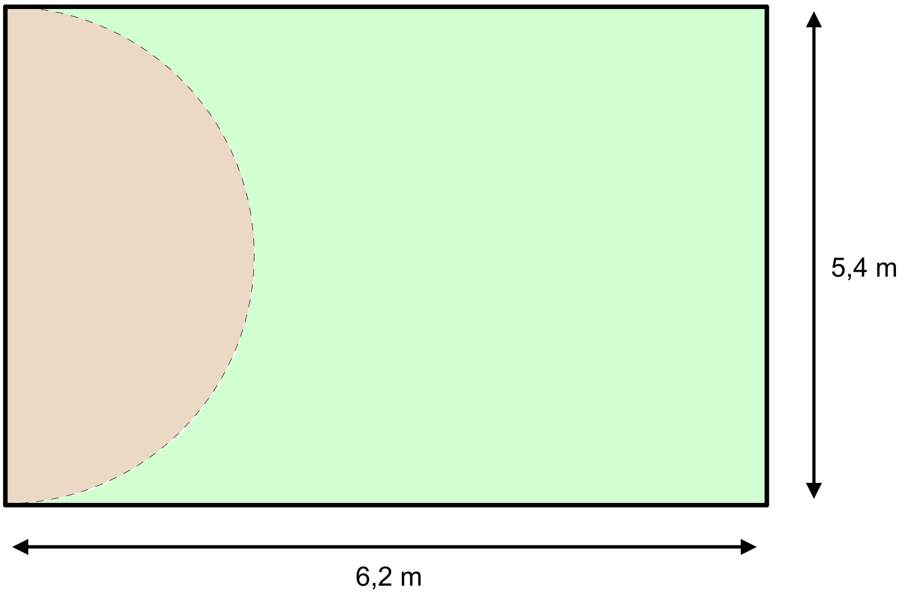
____

____

____

____

3. Bulelwa is landscaping a garden. She wants to create a semi-circular flower bed at one end of the garden and then plant grass for the rest of the garden.



a. Determine how much top-soil Bulelwa will need for the flower bed.

____

____

____

____

____

____

b. Determine how much grass Bulelwa will need for the rest of the garden.

____

____

____

____

____

____

3.4 VOLUME

3.4.1 Definition

The volume of an object can be described as the number of cubic units of a certain size needed to fill the inside of an object. Or put another way:

- for a hollow object, volume is the amount that the object can hold; and
- for a solid object, volume is the amount of 3-D space that the object takes up.

To work out the volume of an object involves working out how much 3-D space the object takes up. This requires working with *three dimensions* of the object. For this reason, the volume of an object is always expressed in units^3 i.e. mm^3, cm^3, m^3, and so on.

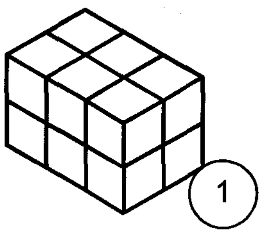
3.4.2 Discovering Volume Formulas

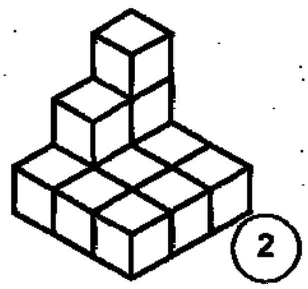
One of the easiest ways of thinking about volume is to imagine having a collection of unit blocks and counting the number that are needed to fill a particular shape.

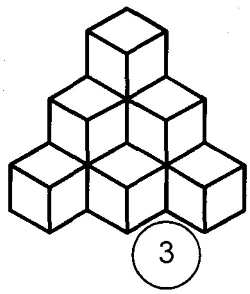
A. Volume of a Rectangular Box

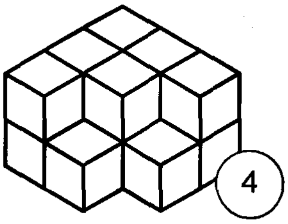
Activity:

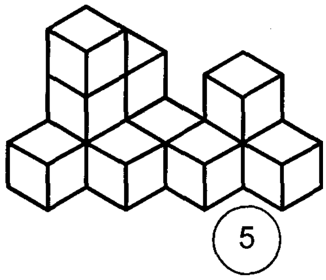
1. Determine the volume of the following 5 objects by counting the number of unit blocks in each object.



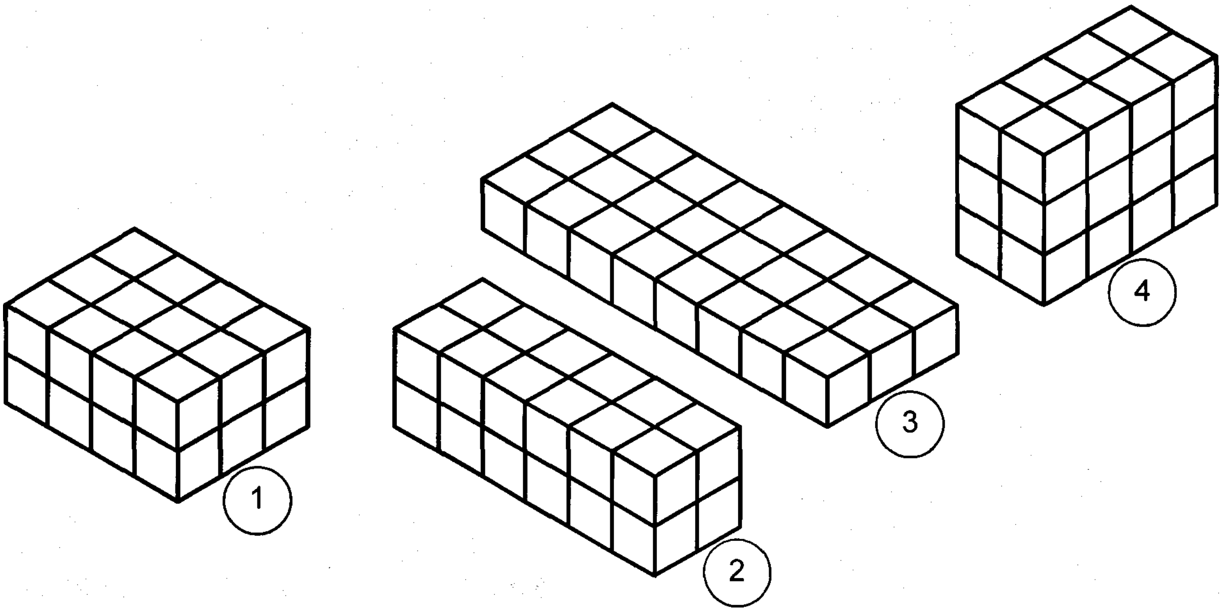








2. For each of the following objects, complete the table of values given below:



Shape number	1	2	3	4
Length				
Breadth				
Height				
Number of cubes				

3. Based on your answers in the table, can you think of an equation that could be used to describe the relationship between the length, breadth and height of a rectangular object and the volume of that object? Write your answer below.

Developing a formula:

Hopefully you came up with the following pattern from the values in the table:

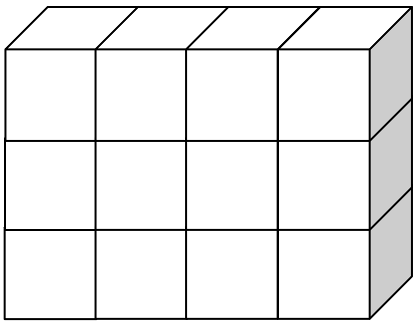
Volume (rectangular prism) = length × breadth × height

In relation to the pictures, *length* × *breadth* represents the number of unit cubes in the bottom layer of the prism and *height* represents the number of layers of cubes. It follows that *length* × *breadth* × *height is* simply the total number of cubic units in the rectangular prism.

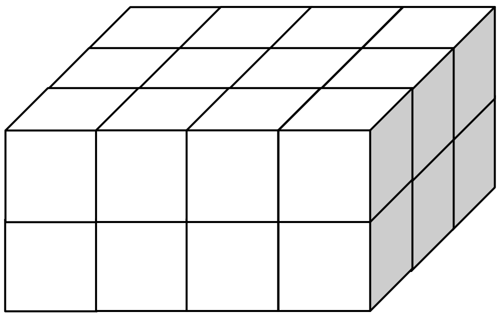
Practice Exercise: Volume of Rectangular Boxes

1. The boxes below are made from unit cubes. Calculate the volumes of the boxes.

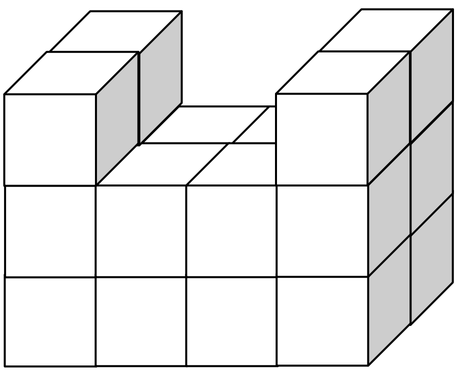
a.



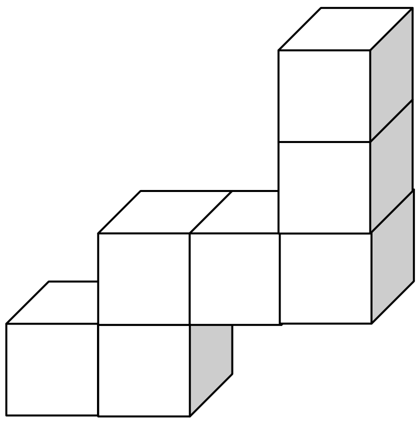
b.



c.

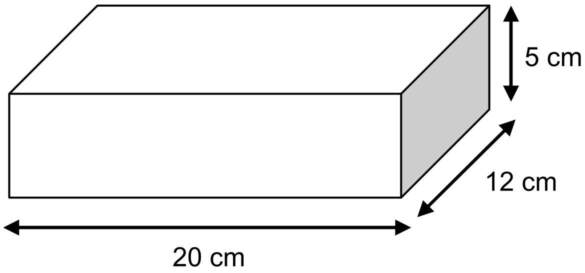


d.

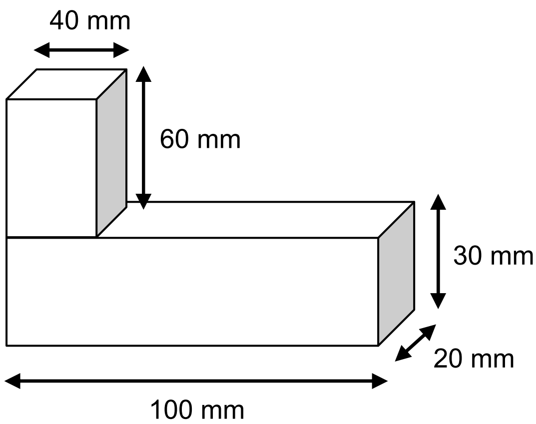


2. Calculate the volumes of the following boxes:

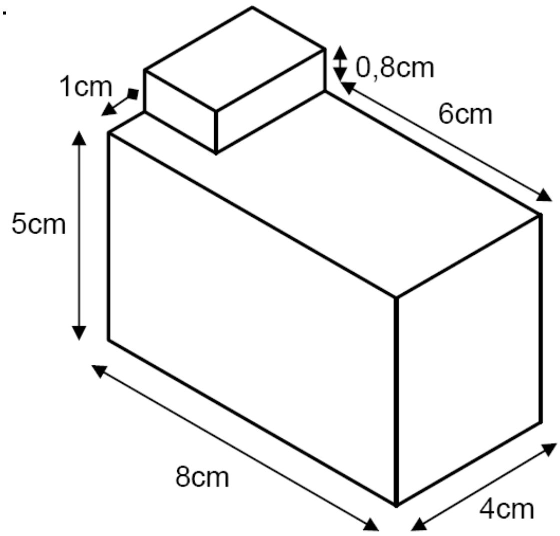
a.



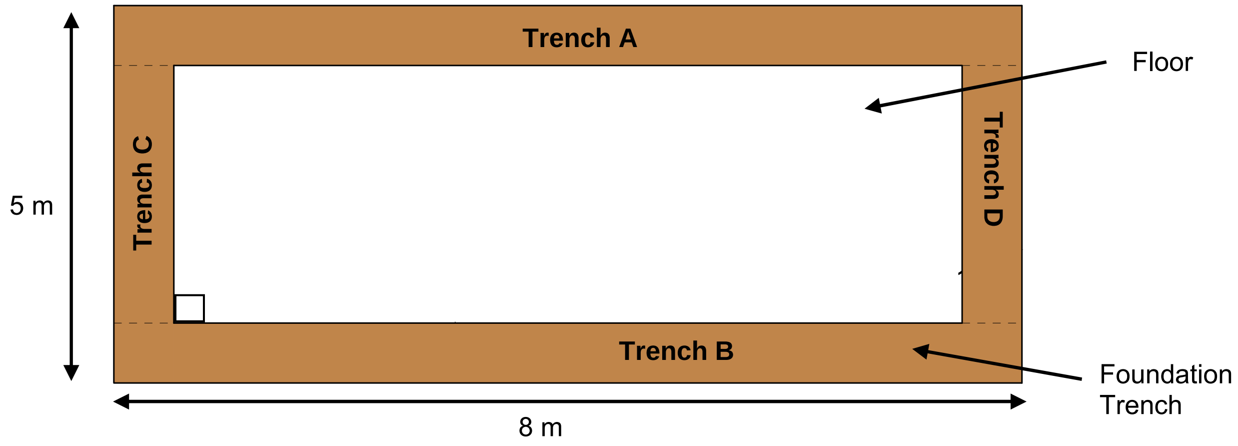
b.



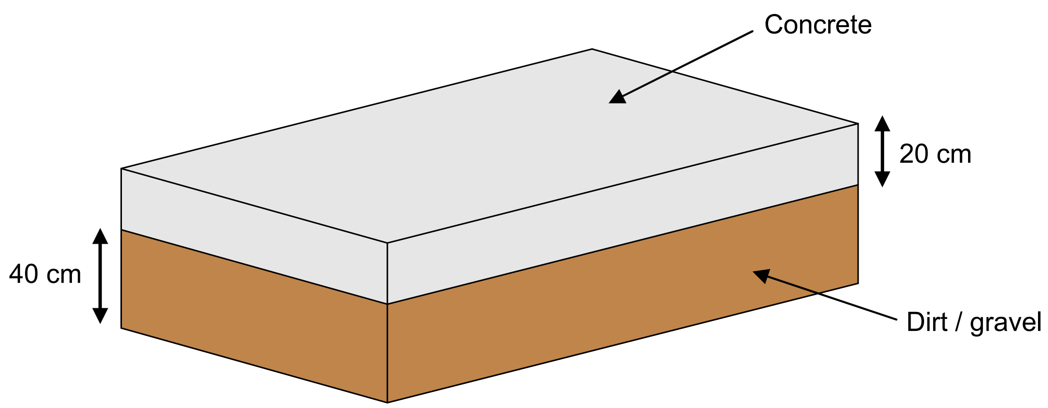
c.



3. Zipho is building a house. The picture below shows the dimensions of the floor and foundation trench of the house.



a. The picture below shows a 3-D picture of the *floor*.



Determine the volume of concrete needed for the floor.

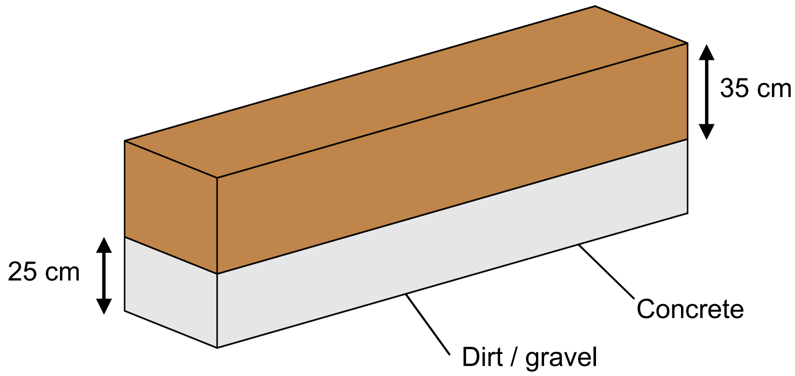
____

____

____

____

b. The picture below shows a 3-D picture of a segment of the *foundation trench*.



Determine the volume of concrete needed for the foundation trench.

c. PPC Cement provides the following guideline for the number of bags of cement, m^3 of sand and m^3 of stone needed to make a particular quantity of concrete.

(PPC Cement, Pamphlet – The Sure Way to Estimate Quantities, www.ppccement.co.za)

LOW STRENGTH CONCRETE

Foundations, Post Holes, Paths & Patios

Strength Guide	*10 - 15 Mpa*
Mix Proportions	*1:4:4 (OPC use 1:5:5)*
Water Guide	*80ℓ per 2 bags of cement*

MIX QUANTITY m^3	No.BAGS SUREBUILD	SAND m^3	STONE m^3
0,2	1	0,1	0,1
0,4	2	0,3	0,3
0,6	3	0,4	0,4
0,7	4	0,5	0,5
0,9	5	0,6	0,7
1,1	6	0,8	0,8
1,8	10	1,3	1,3
3,7	20	2,6	2,7
7,5	40	5,2	5,4
15,0	80	10,4	10,7

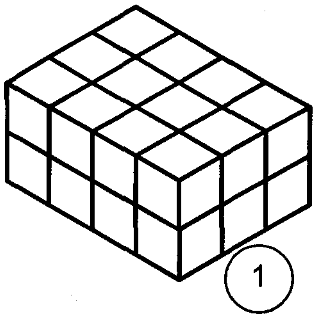
i. Use the guideline to determine how many bags of cement Zipho will need to buy to make enough concrete for the <u>*foundations*</u> of the house.

ii. If the ratio of *cement : sand : stone* is *1 : 4 : 4* and if *1 wheelbarrow of cement = 2 bags of cement*, determine how many wheelbarrows of sand and stone Zipho will need for the concrete for the <u>*foundations*</u> of the house.

B. Developing a *General Formula* for Volume:

In the section above we used arrangements of blocks similar to the one shown alongside to determine that the volume of the shape is given by:

Volume (rectangular prism) = length × breadth × height



In terms of the picture, *length* × *breadth* gives the total number of unit cubes in the bottom layer of the prism and *height* represents the number of layers of cubes. It follows that *length* × *breadth* × *height is* simply the total number of cubic units in the rectangular prism.

Thinking about the volume in this way gives rise to the following:

Volume (rectangular prism) = (length × breadth) × height

→ Volume (rectangular prism) = (area of the bottom layer) × height

∴ **Volume (rectangular prism) = area of the "base" x height**

This gives us a *general formula* for working out the volume or the volume formula for any rectangular prism.

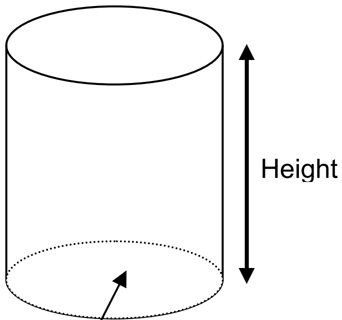
C. Volume of a Cylinder

We can use the *general formula* for the volume of a rectangular prism to determine a formula for the volume of a cylinder in the following way:

Volume (cylinder) = area of the "base" × height

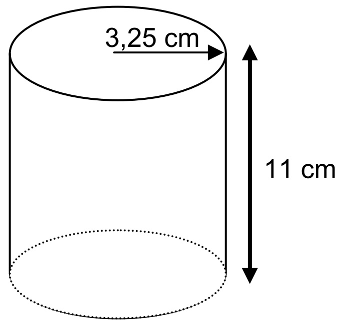
→ Volume (cylinder) = area of a circle × height

∴ ***Volume (cylinder) =*** $\boldsymbol{\pi \times (radius)^2 \times height}$



Example:

A particular cool drink can is 11 cm high and has a radius of 3,25 cm.



The volume of this can (in cm^3) will be:

Volume (cylinder) = $\pi \times (\text{radius})^2 \times \text{height}$

$= \pi \times (3{,}25 \text{ cm})^2 \times 11 \text{ cm}$ (use $\pi \approx 3{,}142$)

$= 365{,}1 \text{ cm}^3$ (one decimal place)

To work out how much cool drink or liquid this can will be able to hold, we will use the fact that $1 \text{ cm}^3 = 1$ ml (see 3.1 – Converting Units of Measurement):

$1 \text{ cm}^3 = 1$ ml

$\rightarrow 365{,}1 \text{ cm}^3 = 365{,}1$ ml

$\therefore$ The can will be able to hold ≈ 365 ml of cool drink.

Where is the "base"?

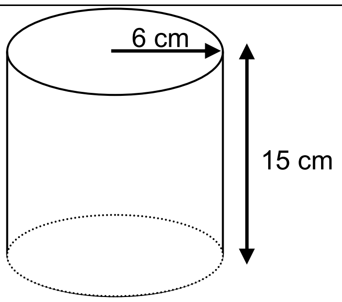
The "base" of a 3-dimensional object is usually the lid or the bottom of the object. However, the "base" does not always have to be positioned at the "bottom" of the object. For example, if a cylinder is lying on its side, then the "base" − which is represented by a circle – will be positioned on the side of the object.

The base of the 3-dimensional object, then, will either be a rectangle, square, circle, or triangle, but will not necessarily be positioned at the "bottom" of the object.

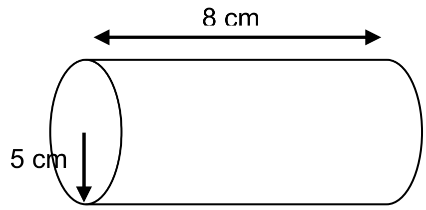
Practice Exercise: Volume of Cylinders

1. Calculate the volumes of the following shapes:

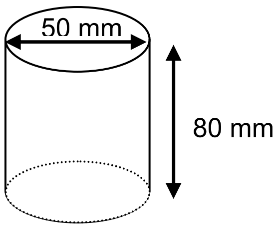
a.



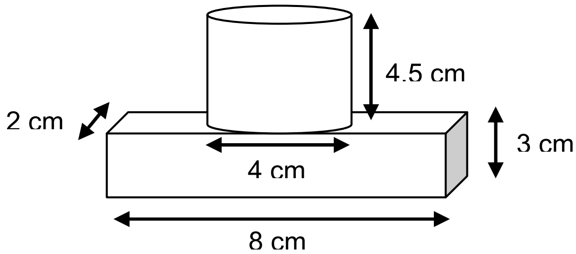
c.



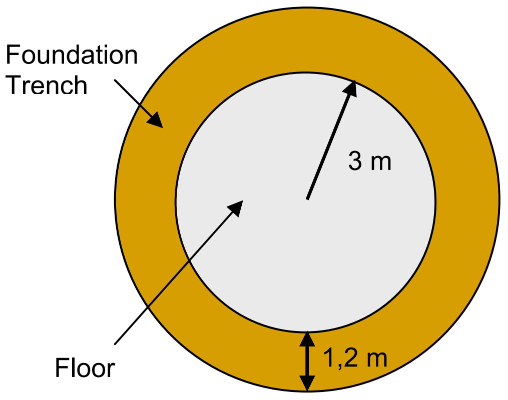
b.



d.



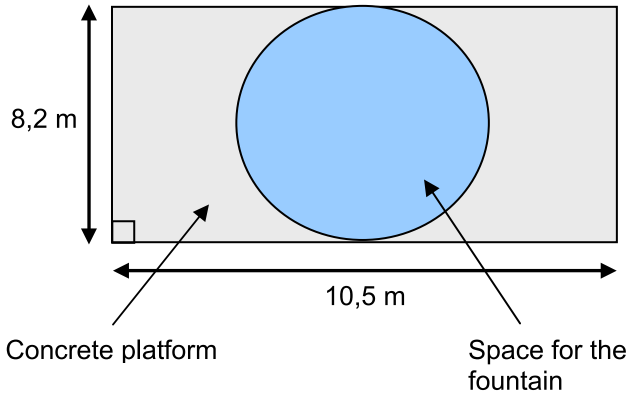
2. The picture below shows the radius of the floor and the width of the foundation trench for a circular house (rondavel).



a. If the floor is going to be *25 cm thick*, calculate the volume of concrete that the builder will need to make for the floor.

b. The foundation trench will be filled with concrete that is *30 cm thick*. Calculate the volume of concrete that the builder will need to make for the foundations of the house.

3. Vilikazi is landscaping a garden and decides to build a circular fountain in the middle of a concrete platform.

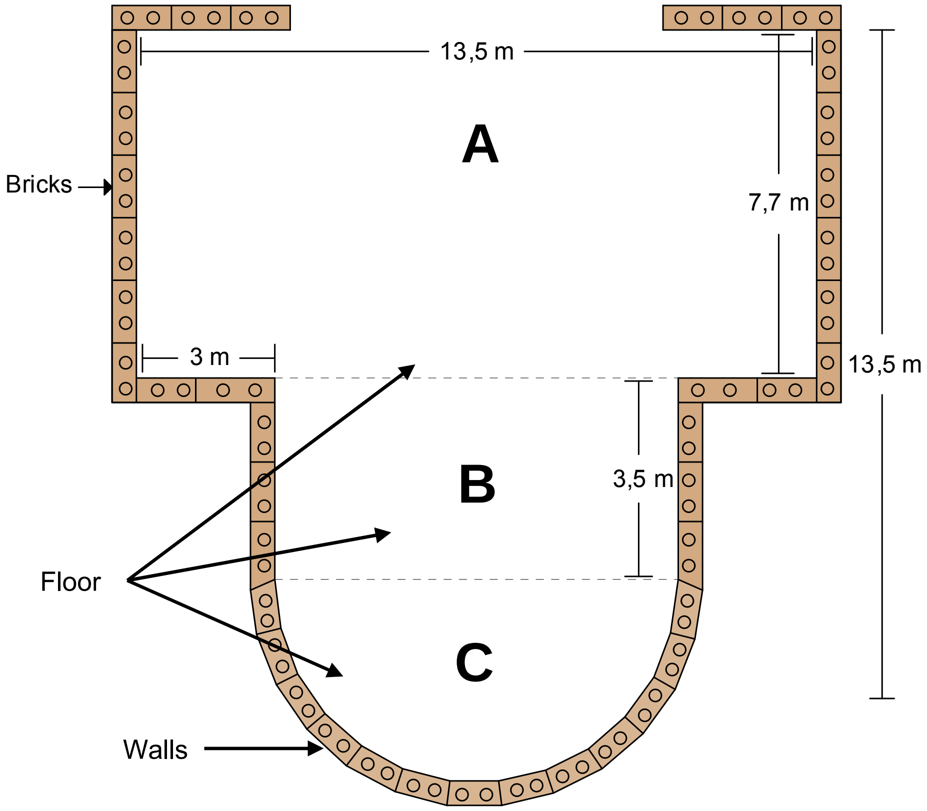


a. If the circular fountain is going to be 20 cm deep, calculate what volume of dirt Vilikazi will remove from the ground to make space for the fountain.

b. If the concrete platform will be 20 cm deep, calculate the volume of concrete that Vilikazi will need to make the platform.

Test Your Knowledge: 2-D & 3-D Pictures, Area and Volume

The picture below shows the outline of a building.



1. a. Calculate the area of Part A of the floor.

____

____

____

____

____

b. Calculate the area of Part B of the floor.

____

____

____

____

____

1. c. Calculate the area of Part C of the floor.

____

____

____

____

____

d. The builder plans to carpet the floor. If the cost of carpeting is R85,00 per m^2, calculate how much it will cost to carpet this building.

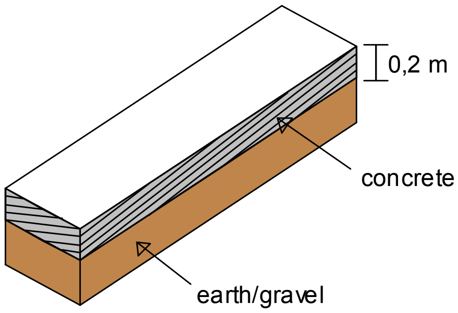
____

____

____

____

2. The picture below shows a 3-D cross-section of the floor.



a. Calculate the volume of concrete needed for Part A of the floor.

____

____

____

____

____

b. Calculate the volume of concrete needed for Part B of the floor.

____

____

____

____

____

2. c. Calculate the volume of concrete needed for Part C of the floor.

____

____

____

____

____

d. The table below shows the number of bags of cement needed for making different volumes of concrete.

Concrete (m^3)	Bags of Cement
0,1	1
0,3	2
0,6	4
1,5	10
3	20
15	100

Use the table to determine how many bags of cement the builder will need to make the floor.

____

____

____

____

____

SOLUTIONS

TOPIC 1

NUMBERS

1.1 BASIC OPERATIONS

1.1.1 Number Sentences

Practice Exercise: Number Sentences

1. Mandy buys 3 bars of soap at R5,99 each, 1 tube of toothpaste at R6,20 and 2 chocolates at R4,30 each.
a. Write a number sentence to represent the cost of Mandy's shopping.

Cost = (3 × R5,99) + R6,20 + (2 × R4,30)

b. Use two different ways to show how much Mandy paid for her shopping?

(1) Cost = (3 × R5,99) + R6,20 + (2 × R4,30)

= R17,97 + R6,20 + R8,60

= R32,77

(2) Soap: 3 × R5,99 = R17,97

Toothpaste: 1 × R6,20

Chocolates: 2 × R4,30 = R8,60

Total = R17,97 + R6,20 + R8,60 = R32,77

2. a. Khosi buys a loaf of bread at R7,50 per loaf, 2 packets of rice at R12,99 per packet and 2 packets of maize meal at R28,30 per packet. She pays for the groceries with a R100,00 note.
Write a number sentence to represent the cost of Khosi's shopping.

Cost = R7,50 + (2 × R12,99) + (2 × R28,30)

b. Use two different methods to calculate how much change Khosi received?

(1) Change = R100,00 − [R7,50 + (2 × R12,99) + (2 × R28,30)]

= R100 − (R7,50 + R25,98 + R56,60)

= R100 − R90,08

= R9,92

(2) Bread: R7,50

Rice: 2 × R12,99 = R25,98

Maize Meal: 2 × R28,30 = R56,60

→ Total cost = R7,50 + R25,98 + R56,60

= R90,08

→ Change = R100,00 − R90,08 = R9,92

3. The entry fee into a game reserve is R20,00 per car and R12,00 per person.

a. Write a number sentence to represent the cost of a family of 3 people entering the reserve in one car.

Cost = R20,00 + (R12,00 × 3)

b. If the family pays for the entrance fee with a R100,00 note, how much change will they receive?

Entry fee = R20,00 + (R12,00 × 3)

= R20,00 + R36,00

= R56,00

∴ **Change = R100,00 − R56,00 = R44,00**

4. Three friends live in the same house. They go shopping and buy 1 packet of washing powder at R18,99 per packet, 2 bottles of milk at R15,20 each and 6 rolls at R0,85 per roll.

a. If they share the cost of the groceries equally amongst the three of them, write a number sentence to describe how much money each person will have to pay towards the groceries.

Amount each person must pay

= [R18,99 + (2 × R15,20) + (6 × R0,85)] ÷ 3

b. Calculate how much each person will have to pay towards the groceries.

Amount each person must pay

= [R18,99 + (2 × R15,20) + (6 × R0,85)] ÷ 3

= [R18,99 + R30,40 + R5,10] ÷ 3

= R54,49 ÷ 3

= R18,16

1.1.2 The Importance of Brackets and "BODMAS"

Practice Exercise: Brackets and BODMAS

1. Thuleleni buys 6 bananas at R0,55 each and 2 pineapples at R4,80 each. She pays for the fruit with a R20,00 note.

a. Write a number sentence to represent the change that Thuleleni will receive from her shopping. Make sure to put brackets in the appropriate place(s).

Cost = R20,00 − [(6 × R0,55) + (2 × R4,80)]

b. Calculate how much change Thuleleni will receive.

Cost = R20,00 − [(6 × R0,55) + (2 × R4,80)]

= R20,00 − [R3,30 + R9,60]

= R20,00 − [R3,30 + R9,60]

= R20,00 − R12,90

= R7,10

2. Place brackets in the appropriate places in the following number sentences:

a. 3 × 7 + 4 − 5 × 2

(3 × 7) + 4 − (5 × 2)

b. 11 + 5 − 9 ÷ 3 + 2 × 10

11 + 5 − (9 ÷ 3) + (2 × 10)

c. 12 ÷ 4 × 5 + 2 − 6 ÷ 2

(12 ÷ 4 × 5) + 2 − (6 ÷ 2)

3. Determine the value of the number sentences in 2.

a. **(3 × 7) + 4 − (5 × 2) = 21 + 4 − 10**

= 15

b. **11 + 5 − (9 ÷ 3) + (2 × 10)**

= 11 + 5 − 3 + 20

= 33

c. **(12 ÷ 4 × 5) + 2 − (6 ÷ 2)**

= (3 × 5) + 2 − 3

= 15 + 2 − 3

= 14

4. Determine the value of the following:

a. (6 − 2) + 3 × (5 + 2)

= 4 + [3 × 7]

= 4 + 21

= 25

b. [4 + (2 × 3) − 5] ÷ 5

(4 + 6 − 5) ÷ 5

= 5 ÷ 5

= 1

4. c. 10 − [(5 × 2) + 9 ÷ 3)] + 8

10 − [10 + 3] + 8

= 10 − 13 + 8

= 5

5. Fill in the missing numbers in each of the questions below:

a. 3 × **2** − 4 = 2

b. 6 + (4 × **21**) = 90

c. 25 − (16 ÷ **4**) = 21

6. The following equation is used to determine the monthly repayment on a particular loan:

Repayment = (loan ÷ 1000) × 23,05

a. Calculate the repayment on a R250 000,00 loan.

Repayment = (R250 000,00 ÷ 1000) × 23,05

= R250 × 23,05

= R5 762,50

6. b. Calculate the repayment on a R1 000 000,00 loan.

Repayment = (R1 000 000 ÷ 1000) × 23,05

= R1 000 × 23,05

= R23 050,00

7. The following formula is used to determine the amount of money in a particular investment after 2 years.

$$Amount = R4\ 000 \times [(1 + \frac{3}{100})^2]$$

Calculate how much money there will be in the investment after 2 years.

Amount = R4 000 × [(1 + $\frac{3}{100}$**)^2]**

= R4 000,00 × [(1 + 0,03)^2]

= R4 000,00 × [1,03^2]

= R4 000,00 × 1,0609

= R4 243,60

Test Your Knowledge: Basic Operations

1. A group of 5 friends are going away for the weekend. The total cost for the weekend comes to R852,00. How much does each person have to pay?

Cost per person = R852,00 ÷ 5 = R170,40

2. A mother is taking her four children to the uShaka Sea World in Durban. How much will it cost her if the tariffs are:

- Adults → R98,00
- Children → R66,00

No. of adults = 1

No. of children = 4

∴ **Total cost = R98,00 + (4 × R66,00)**

= R98,00 + R264,00

= R362,00

3. Layla gives the shopkeeper a R100,00 note to pay for her purchases of R73,58. How much change will she receive?

Change = R100,00 − R73,58 = R26,42

4. Faisel buys 2 cokes for R5,20 each and 3 samoosas for R3,50 each. How much must he pay?

Cost = (2 × R5,20) + (3 × R3,50)

= R10,40 + R10,50

= R20,90

5. How much will it cost Zikhona if she buys 3 packets of chips for R3,75 per packet and 3 chocolates for R4,50 per chocolate? Show 2 ways of doing this sum.

(1) Cost = (3 × R3,75) + (3 × R4,50)

= R11,25 + R13,50

= R24,75

(2) Chips: 3 × R3,75 = R11,25

Chocolates: 3 × R4,50 = R13,50

Total cost = R11,25 + R13,50 = R24,75

6. There are 35 sweets in one packet and 46 of the same type of sweets in another packet. Divide these sweets equally amongst three friends.

Total sweets = 35 + 46 = 81

No. of sweets per person = 81 ÷ 3 = 27

7. In a particular town, electricity users pay a fixed monthly service fee of R85,00 and a consumption fee of R0,40 per kWh of electricity used.

a. Write a number sentence to represent the cost of electricity in this town. Be sure to include brackets in appropriate places in the number sentence.

Cost = R85,00 + (R0,40 × kWh used)

b. Use the number sentence to determine the cost of using 367 kWh of electricity during the month.

Cost = R85,00 + (R0,40 × 367)

= R85,00 + R146,80

= R231,80

8. Calculate:

a. 6 + 7 × 2

= 6 + 14 = 20

b. 8 – 3 × 2

= 8 − 6 = 2

c. 19 – 4 × 3

= 19 − 12 = 7

d. 3 × 6 – 9

= 18 − 9 = 9

e. 15 – 4 + 7 × 2

= 15 − 4 + 14 = 25

f. 11 × 3 + 2

= 33 + 2 = 35

g. 16 × 4 – 3

= 64 − 3 = 61

8. h. 6 + 7 × 2 – 20 ÷ 4

= 6 + 14 − 5 = 15

i. 18 × 2 – (4 + 7)

= 36 − 11 = 25

j. 16 – 5 × 2 + 3

= 16 − 10 + 3 = 9

9. Decide whether each of the statements below is true or false:

a. 6 × 7 – 2 = 40

6 × 7 − 2 = 42 − 2

= 40

∴ **True!**

b. 8 × (6 – 2) + 3 = 56

8 × (6 − 2) + 3 = 8 × 4 + 3

= 32 + 3

= 35

∴ **False!**

c. 35 – 7 × 2 = 56

35 − 7 × 2 = 35 − 14

= 21

∴ **False!**

9. d. $3 + 7 \times 3 = 30$

$3 + 7 \times 3 = 3 + 21$

$= 24$

∴ False!

e. $18 - (4 + 7) = 21$

$18 - (4 + 7) = 18 - 11$

$= 7$

∴ False!

f. $43 - 3 + 2 = 42$

$43 - 3 + 2 = 40 + 2$

$= 42$

∴ True!

g. $18 \div 2 + 6 = 10$

$18 \div 2 + 6 = 9 + 6$

$= 15$

∴ False!

h. $64 - 10 + 2 = 52$

$64 - 10 + 2 = 54 + 2$

$= 56$

∴ False!

9. i. $(4 + 2) + 7 = 4 + (2 + 7)$

$\rightarrow (4 + 2) + 7 = 6 + 7$

$= 13$

$\rightarrow 4 + (2 + 7) = 4 + 9$

$= 13$

∴ True!

j. $(8 - 2) - 1 = 8 - (2 - 1)$

$\rightarrow (8 - 2) - 1 = 6 - 1$

$= 5$

$\rightarrow 8 - (2 - 1) = 8 - 1$

$= 7$

∴ False!

k. $(8 \div 4) \div 2 = 8 \div (4 \div 2)$

$\rightarrow (8 \div 4) \div 2 = 2 \div 2$

$= 1$

$\rightarrow 8 \div (4 \div 2) = 8 \div 2$

$= 4$

∴ False

10. Calculate:

a. 8,2 ÷ 0,2 – 0,1

= 41 − 0,1

= 40,9

b. 3,6 × 0,2 – 0,1

= 0,72 − 0,1

= 0,62

c. 8,2 × (6 – 5,4)

= 8,2 × 0,6

= 4,92

d. 2,2 – 0,7 × 0,2

= 2,2 − 0,14

= 2,06

11. Fill in the missing numbers in each of the questions below:

a. 0,8 + **4** × 0,6 = 3,2

i.e. (3,2 − 0,8) ÷ 0,6 = 4

b. **3** × 0,5 + 6 × 0,4 = 3,9

i.e. [3,9 − (6 × 0,4)] ÷ 0,5 = 3

11. c. 0,9 + 4,8 ÷ **0,8** = 6,9

i.e. 6,9 − 0,9 = 6

Then: 4,8 ÷ ? = 6

This means that 6 × ? = 4,8

∴ ? = 4,8 ÷ 6

= 0,8

d. 2,7 ÷ **0,9** – 1,4 = 1,6

i.e. 1,6 + 1,4 = 3

Then: 2,7 ÷ ? = 3

This means that 3 × ? = 2,7

∴ ? = 2,7 ÷ 3

= 0,9

1.2 ROUNDING

1.2.2 Rounding Off, Rounding Down and Rounding Up

A. Rounding Off

Practice Exercise: Rounding Off

1. Round off 3 467 to the nearest:

a. Ten

b. Hundred

c. Thousand

a. **3 470**

b. **3 500**

c. **3 000**

2. Round off 3 428,629 to:

a. Two decimal places − **3 428,63**

b. One decimal place − **3 428,6**

c. The nearest whole number (i.e. 0 decimal places) − **3 429**

d. The nearest ten − **3 430**

e. The nearest hundred − **3 400**

f. The nearest thousand − **3 000**

2. The bank calculates that they must pay R2,3157 in interest to one of their clients. If the bank rounds this value off to two decimal places before giving the interest to the client, how much money will the client receive?

R2,32

3. Bob calculates that he needs 30,157 m^3 of concrete for the foundations of a house. For ease of use he rounds off this value to one decimal place.

What volume of concrete does Bob need?

30,2 m^3

4. Benni calculates that he needs 6,8 m of wood to fix his fence. If the shop only sells wood in whole meter lengths, how many metres of wood will he need to ask for at the shop?

7 m

5. The cost of pre-paid electricity in the Msunduzi Municipality is R0,47516 per kWh of electricity used.

a. Calculate the cost of using 300 kWh of electricity to two decimal places.

Cost = R0,47516 × 300 kWh

= R142,548

= R142,55 (to two decimal places)

b. Calculate the cost of using 428,2 kWh of electricity to two decimal places.

Cost = R0,47516 × 428,2 kWh

= R203,46 (to two decimal places)

c. If Cindy pays R385,00 for electricity, how many kWh of electricity has she used? Give your answer to one decimal place.

R385,00 = R0,47516 × kWh

R385,00 ÷ R0,47516 = kWh

∴ **kWh = 810,3 (to one decimal place)**

6. A cell phone company charged R0,0427 per second to make calls on its network.

a. How much will it cost (in Rand and cents) to make a 45 second call on this network?

Cost = R0,0427 × 45

= R1,92

(to two decimal places − i.e. Rand and cents)

b. How much will it cost (in Rand and cents) to make a 6 minute 47 second call on this network?

6 min 47 sec = (6 × 60) sec + 47 sec

= 407 sec

→ **Cost = R0,0427 × 407**

= R17,38

(to two decimal places − i.e. Rand and cents)

B. Rounding Down

Practice Exercise: Rounding Down

1. Xolani is packing oranges into boxes. Each box can hold 30 oranges. If Xolani has 400 oranges to pack into the boxes, how many *full* boxes of oranges will he have?

No. of boxes = 400 oranges ÷ 30 per box

= 13,3 boxes

∴ No. of full boxes = 13

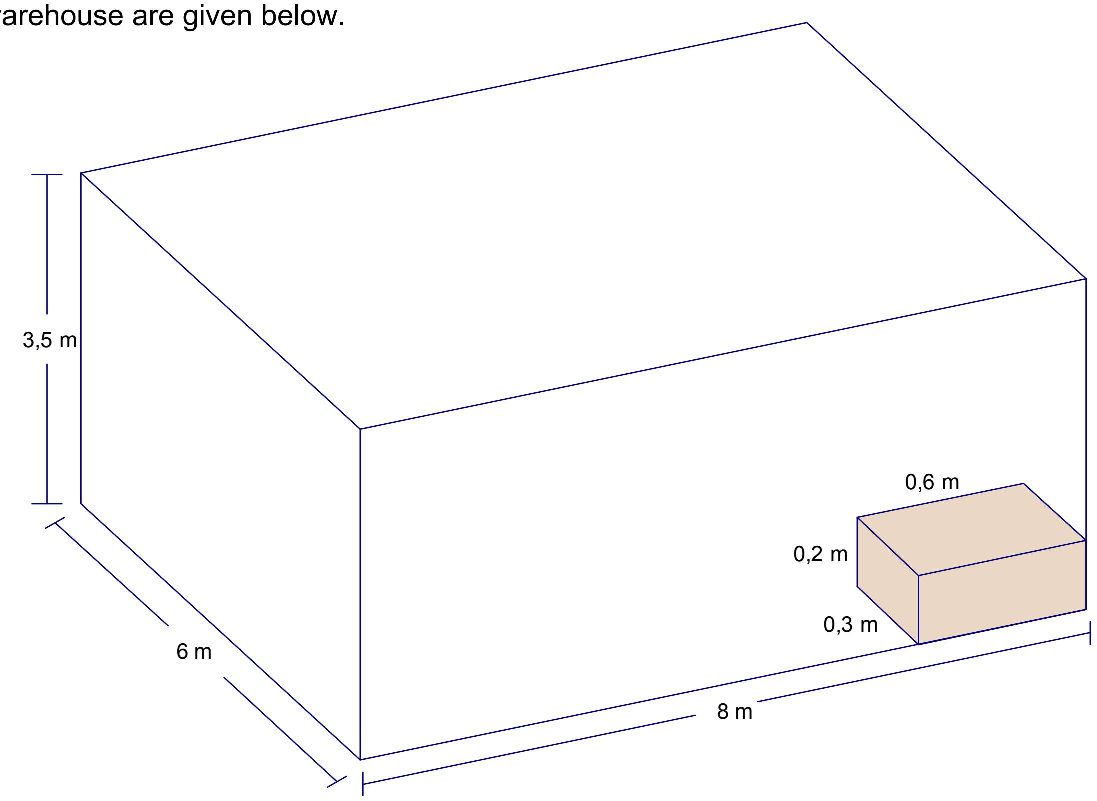
2. Vusi buys a 6 m long pole to make a small fence. If the fence will be 0,7 m high, how many supports for the fence will Vusi be able to cut from the 6 m long pole?

No. of supports = 6 m pole ÷ 0,7 m/support

= 8,6 supports

∴ No. of full length supports = 8

3. Zanele needs to package boxes in a warehouse. A picture of the dimensions of each box and the warehouse are given below.



a. How many boxes will Zanele be able to fit along the length of the warehouse?

No. of boxes along the length = 8 m ÷ 0,6 m

= 13,3 boxes

= 13 full boxes

3. b. How many boxes will Zanele be able to fit along the width of the warehouse?

No. of boxes along width = 6 m ÷ 0,3 m

= 20 boxes

3. c. How many boxes high will Zanele be able to stack the boxes?

No. of boxes high = 3,5 m ÷ 0,2 m

= 17,5 boxes

= 17 full boxes

d. Now calculate the total number of boxes that Zanele will be able to store in the warehouse.

Total boxes in the warehouse

= 13 long × 20 wide × 17 high

= 4 420

C. Rounding Up

Practice Exercise: Rounding Up

1. 33 tourists are planning a sightseeing trip around Cape Town. If the company who will take them on the trip uses 14-seater mini-busses, how many mini-busses will be used?

No. of busses = 33 people ÷ 14 people/bus

= 2,357 busses

∴ 3 busses are needed (but only 2 might be completely full).

2. Mandy is organising a dinner function for 74 people. The people are going to be seated at tables that can hold 8 people per table. How many tables will Mandy need?

No. of tables = 74 people ÷ 8 people/table

= 9,25 tables

∴ 10 tables are needed (but only 9 might be full).

3. The table below shows the coverage ratios for two different types of paint.

Paint Type	Coverage
Acrylic	9 m^2 per litre
Enamel	7,5 m^2 per litre

a. Which paint type is thicker? Explain.

Enamel – for 1 litre of paint you can cover less wall space than with Acrylic.

3. b. How many litres of Acrylic paint will a painter need to *buy* to paint a wall with an area of 75 m^2?

9 m^2 = 1 litre

→ 1 m^2 = 1 litre ÷ 9

75 m^2 = 1 litre ÷ 9 × 75

≈ 8,333 litres

∴ The painter will need to buy 9 litres since it is impossible to buy a decimal portion of paint. The painter may actually have to buy 10 litres if the paint is only sold in 2 litre, 5 litre or 10 litre tins.

c. How many litres of Enamel paint will a painter need to *buy* to paint a wall with an area of 104,2 m^2?

7,5 m^2 = 1 litre

→ 1 m^2 = 1 litre ÷ 7,5

104,2 m^2 = 1 litre ÷ 7,5 × 104,2

≈ 13,893 litres

∴ The painter will need to buy 14 litres since it is impossible to buy a decimal portion of paint.

3. d. A painter buys 10 litres of Acrylic paint. What is the maximum size wall that he will be able to paint with this tin of paint?

1 litre = 9 m^2

→ 10 litres = 90 m^2

So, as long as the painter does not waste any paint, he can paint a maximum size wall of 90 m^2. In reality, because paint does get wasted when painting, the painter can realistically probably only paint an 85 m^2 with 10 litres of this type of Acrylic paint.

4. The table below shows the number of bags of cement needed to plaster a wall.

WALL AREA (m^2)	No. BAGS CEMENT
60	5
120	10

(Adapted from: PPC Cement, Pamphlet − *The Sure Way to Estimate Quantities*, www.ppccement.co.za)

a. How many bags of cement will a builder need to *buy* to plaster a 30 m^2 wall?

60 m^2 = 5 bags

→ 30 m^2 = 2,5 bags

So, the builder will need to *buy* 3 bags of cement as it is not possible to buy half a bag. The builder may only use 2½ bags though.

b. How many bags of cement will a builder need to *buy* to plaster a 103 m^2 wall?

60 m^2 = 5 bags

1 m^2 = 5 bags ÷ 60

103 m^2 = 5 bags ÷ 60 × 103

= 8,583 bags

So, the builder will need to buy at least 9 bags of cement, but many only use 8 full bags and just over half of the 9th bag.

5. Hamilton is planning a trip from Durban to Johannesburg. The distance is 565 km.

a. If Hamilton's car has an average petrol consumption rate of 8 litres per 100 km, calculate how many litres of petrol he will need to complete the journey.

100 km = 8 litres

1 km = 8 litres ÷ 100

565 km = 8 litres ÷ 100 × 565

= 45,2 litres

5. b. If the current petrol price is R10,30 per litre, show that Hamilton will need to fill R465,56 worth of petrol in his car.

Cost of petrol = 45,2 litres × R10,30 per litre

= R465,56

c. Explain why if Hamilton puts exactly R465,56 worth of petrol into his car then there is a possibility that he could run out of petrol during the trip.

This value is based on the "average" petrol consumption of Hamilton's car. So, depending on how far Hamilton drives, how often he stops, and whether or not he is driving into the wind, will all affect what the actual petrol consumption rate is for the journey.

d. What Rand value of petrol would you suggest Hamilton should fill in his car? Explain.

Maybe R500,00 worth of petrol. This provides for leeway if the petrol consumption rate for the journey is higher than the average consumption rate for the car.

1.2.3 The Impact of Rounding

Practice Exercise: The Impact of Rounding

1. A map has a scale of 1 : 100 000.

a. Bongani measures the distance between two towns on the map to be 11,8 cm.
According to Bongani's measurements, what is the actual distance between the two towns (in km)?

Scale: 1 : 100 000

→ 11,8 cm : 100 000 × 11,8 cm

= 1 180 000 cm

= 11 800 m

= 11,8 km

b. Songi measures the distance between the same two towns to be 11,7 cm.
According to Songi's measurements, what is the actual distance between the two towns (in km)?

Scale: 1 : 100 000

→ 11,7 cm : 50 000 × 11,7 cm

= 1 170 000 cm

= 11 700 m

= 11,7 km

c. Donnie measures the distance between the two towns to be 11,8 cm. If she rounds this value off to 12 cm and then uses 12 cm to determine the actual distance between the two towns, what effect will this have on the accuracy of her answer? You must show all working.

Scale: 1 : 100 000

→ 12 cm : 100 000 × 12 cm

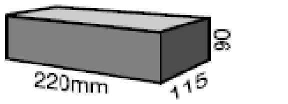
= 1 200 000 cm

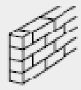
= 12 000 m

= 12 km

So, rounding off by 0,2 units will affect the accuracy of her answer by ,2 km.

2. The picture below shows the number of bricks, bags of cement and m^3 of sand needed to build a wall. Adapted from: PPC Cement, Pamphlet − The Sure Way to Estimate Quantities, www.ppccement.co.za)



MATERIAL REQUIRED FOR 1000 BRICKS			
WALL SIZE	m^2 WALL AREA	No. BAGS SUREBUILD	m^3 SAND
 Single brick wall	20	4	1,0

Mpho calculates that the wall he is planning to build will have an area of 105,4 m^2.

a. Calculate how many bags of cement Mpho will need to build this wall.

20 m^2 = 4 bags

→ 1 m^2 = 4 bags ÷ 20

105,5 m^2 = 4 bags ÷ 20 × 105,5

≈ 21,1 bags (to one decimal place)

So, Mpho needs slightly more than 21 bags and will therefore need to buy 22 bags.

b. If Mpho rounds this value off to 105 m^2 and then calculates the number of bags of cement that he will need, what will his answer be?

20 m^2 = 4 bags

→ 1 m^2 = 4 bags ÷ 20

105 m^2 = 4 bags ÷ 20 × 105

= 21 bags

c. Explain what implications rounding off the area of the wall value will be for Mpho.

Rounding down the area of the wall value could mean that Mpho ends up buying 1 to few bags of cement than he actually needs.

Test Your Knowledge: Rounding

1. a. Round off 2 973 to the nearest:

   i. Ten − **2 970**

   ii. Hundred − **3 000**

   iii. Thousand − **3 000**

b. Round off R134,78 to the nearest rand.

R135,00

c. Round R12 456 987,00 to the nearest million rand.

R12 000 000

d.

i. Round off 3,18 to one decimal place.

3,2

ii. Round off 5,52 to one decimal place.

5,5

iii. Round off 24,148 to two decimal places.

24,15

iv. Round off 3,5 to the nearest whole number.

4

v. Round off 24,145 to one decimal place.

24,1

2. Nomalunge is packing apples into packets to sell at the local market. She has 250 apples and is putting 7 apples into a packet. How many packets containing seven apples will she have?

No. of packets = 250 apples ÷ 7 apples/bag

= 35,7 bags

= 35 bags of seven apples and 1 bag of 5 apples

3. John is making bookcases and is using a plank of wood that is 2,6 m long to make the shelves. If each shelf has a length of 70 cm, how many shelves can he cut from one plank of wood?

Length of each shelf = 70 cm = 0,7 m

No. of shelves = 2,6 m ÷ 0,7 m

= 3,7

= 3 shelves 70 cm long and 1 shelf 50 cm long

4. Your college needs to transport 743 learners. The bus company says that their buses can take a maximum of 60 learners. How many buses does your school need?

No. of busses = 743 learner ÷ 60 learner/bus

= 12,4 (to one decimal place)

∴ **13 busses are needed − 12 might be completely full and 1 will transport the remaining 23 learners.**

5. Your college is expecting about 345 people to attend a fashion show. How many rows of chairs are needed if each row takes 18 chairs?

No. of rows = 345 people ÷ 18 people/row

= 19,2 (to one decimal place)

$\therefore$ **20 rows are needed.**

6. Consider the statement: $\frac{1}{11} \times \frac{3}{11} \times \frac{5}{11} \times \frac{7}{11}$

a. Using your calculator, convert each fraction to its decimal equivalent correct to 3 decimal places and then determine the product. Give your final answer to three decimal places.

$\frac{1}{11} = 0{,}091$

$\frac{3}{11} = 0{,}273$

$\frac{5}{11} = 0{,}455$

$\frac{7}{11} = 0{,}636$

$\therefore \frac{1}{11} \times \frac{3}{11} \times \frac{5}{11} \times \frac{7}{11}$

$= 0{,}091 \times 0{,}273 \times 0{,}455 \times 0{,}636$

$= 0{,}007$

6. b. Repeat the above for 1 decimal place. Give your final answer to three decimal places.

$\frac{1}{11} = 0{,}1$

$\frac{3}{11} = 0{,}3$

$\frac{5}{11} = 0{,}5$

$\frac{7}{11} = 0{,}6$

$\therefore \frac{1}{11} \times \frac{3}{11} \times \frac{5}{11} \times \frac{7}{11}$

$= 0{,}1 \times 0{,}3 \times 0{,}5 \times 0{,}6$

$= 0{,}009$

c. Compare the answers that the calculations above produce and make comment on the differences.

Rounding off individual values within a calculation affects the accuracy of the final answer.

1.3 PERCENTAGE

1.3.2 Typical calculations involving percentage

A. Expressing a Part of a Whole as a Percentage

Practice Questions: Expressing a Part of a Whole as a %

1. Xolani gets 18 out of 30 for a test. What percentage did he get for the test?

Mark as a % = 18 ÷ 30 × 100%

= 60%

2. A town has a total population of 2 450 people. During an election 1 666 people in the town vote. What percentage of the total population of the town voted in the election?

% who voted = 1 666 ÷ 2 450 × 100%

= 68%

3. The table below shows the number of teachers in each province in South Africa in 2005.

Province	No. of Teachers
Eastern Cape	67 230
Free State	23 400
Gauteng	60 121
KwaZulu-Natal	80 979
Limpopo	56 160
Mpumalanga	27 701
North West	27 454
Northern Cape	6 641
Western Cape	32 447

(National Department of Education. 2006. Education Statistics in South Africa at a Glance in 2005. p.4)

3. a. How many teachers were there in South Africa in 2005?

382 133

b. What percentage of the total number of teachers in South Africa teach in:

i. Mpumalanga?

ii. Western Cape?

iii. Gauteng AND KwaZulu-Natal?

(Give your answers to one decimal place)

b. i. **% from Mpumalanga**

= 27 701 ÷ 382 133 × 100%

= 7,2%

ii. **% from Western Cape**

= 32 447 ÷ 382 133 × 100%

= 8,5%

iii. **% from Gauteng & KZN**

= (60 121 + 80 979) ÷ 382 133 × 100%

= 36,9%

B. Determining a Percentage of an Amount

Practice Questions: Determining a % of an Amount

1. Trudy is given a 5% discount on a shirt that costs R125,00. How much *discount* does Trudy receive?

Discount = 5% × R125,00

$$= \frac{5}{100} \times R125{,}00$$

= R6,25

2. Sindiwe earns R4 200,00 per month and receives an 8% increase in salary. How much *increase* does Sindiwe receive?

Increase = 8% × R4 200,00

= R336,00

3. The table below shows the percentage of learners in each province in South Africa in 2005.

Province	Percentage
KwaZulu Natal	20,9%
Gauteng	17,2%
Eastern Cape	17,0%
Limpopo	14,9%
Western Cape	8,5%
Mpumulanga	7,0%
North West	6,9%
Free State	5,9%
Northern Cape	1,6%

(National Department of Education. 2006. Education Statistics in South Africa at a Glance in 2005. p.4)

3 … If there was a total 13 936 737 learners in South Africa in 2005, calculate how many learners there were in:

a. Eastern Cape

Learners = 17% × 13 936 737

= 2 369 245,2

≈ 2 369 245 learners

(rounded off to full people)

b. Western Cape

Learners = 8,5% × 13 936 737

= 1 184 622,6

≈ 1 184 623 learners

(rounded off to full people)

c. North West

Learners = 6,9% × 13 936 737

= 961 634,85

≈ 961 635 learners

d. Northern Cape

Learners = 1,6% × 13 936 737

= 222 987,79

= 222 988 learners

(rounded off to full people)

C. Adding a Percentage of an Amount to an Amount

Practice Questions: Adding a %

1 The price of a can of cool drink that costs R5,50 increases by 5%. What will the new price of the can of cool drink be?

New price = R5,50 + (5% × R5,50)

= R5,50 + R0,275

= R5,78 (to two decimal places)

2. Sindiwe earns R4 200,00 per month and receives an 8% increase in salary. What will Sindiwe's new salary be

New salary = R4 200,00 + (8% × R4 200)

= R4 200,00 + R336,00

= R4 536,00

3. Mandy makes and sells bracelets. It costs her R9,50 to make each bracelet and she sells the bracelet with 110% mark up. Determine how much she sells the bracelets for.

Selling price = R9,50 + (110% × R9,50)

= R9,50 + R10,45

= R19,95

4. A supermarket owner is looking to increase the prices of certain goods in his shop. The table below shows the current price of the goods and the percentage by which the owner wants to increase the prices.

Goods	Current Price	% Increase
Maize-Meal	R55,45	17%
Chicken	R32,99	9%

Calculate the new price of each of the goods.

Maize-Meal:

New price = R55,45 + (17% × R55,45)

= R55,45 + R9,4265

= R64,88 (to 2 decimal places)

Chicken:

New price = R32,99 + (9% × R32,99)

= R32,99 + R2,9691

= R35,96 (to 2 decimal places)

5. Bob is mixing concrete in order to build a wall. He decides to buy slightly more cement, sand and stone than he needs to account for wastage.

Goods	Accurate Quantity Needed	Extra Needed for Wastage
Cement	58 bags	10%
Sand	88 wheelbarrows	15%
Stone	90 wheelbarrows	15%

Determine how many bags of cement and wheelbarrows of sand and stone Bob will need to *buy*.

Cement = 58 bags + 10% extra

= 58 bags + (10% × 58 bags)

= 58 bags + 5,8 bags

= 63,8 bags

∴ Bob must buy 64 full bags of cement.

Sand = 88 wheelbarrows + 15% extra

= 88 wh/barrows + (15% × 88 wh/barrows)

= 88 wh/barrows + 13,2 wh/barrows

= 101,2 wh/barrows

∴ Bob must buy 102 full wheelbarrows of sand.

Stone = 90 wheelbarrows + 15% extra

= 90 wh/barrows + (15% × 90 wh/barrows)

= 90 wh/barrows + 13,5 wh/barrows

= 103,5 wh/barrows

∴ Bob must buy 104 full wheelbarrows of stone.

D. Subtracting a Percentage of an Amount from an Amount

Practice Questions: Subtracting a %

1. Trudy is given a 5% discount on a shirt that costs R125,00. How much will she have to pay for the shirt?

Price = R125,00 − (5% × R125,00)

= R125,00 − R6,25

= R118,75

2. The average rainfall in Mphophomeni decreased by 13% from 2006 to 2007. If the average rainfall in 2006 was 28,3 mm, determine the average rainfall in 2007.

(Give your answer to one decimal place)

Average rainfall in 2007

= 28,3 mm − (13% × 28,3 mm)

= 28,3 mm − 3,679 mm

= 24,6 mm (to one decimal place)

3. Sindi buys a car that costs R75 000,00. The value of her car decreases by 15% per year.

a. How much will the car be worth after 1 year?

Value of car after 1 year

= R75 000,00 − (15% × R75 000,00)

= R75 000,00 − R11 2500,00

= R63 750,00

b. How much will the car be worth after 2 years?

Value of car after 2 years

= R63 750,00 − (15% × R75 000,00)

= R63 750,00 − R9 562,50

= R54 187,50

4. Would it be possible to decrease the price of radio that costs R390,00 by 105%? Explain your answer.

No – decreasing a price by 100% means decreasing the price by the same value as the price, which will always give R0,00. So, decreasing a price by 105% means that the price will decrease by more than the current price of item, which is impossible.

E. Calculating the Original Amount after a Percentage has been Added or Subtracted

Practice Questions: Calculating the Original Amount

1. Donny is given a 7% increase in salary so that she now earns R6 210,00 per month. How much did she earn before the increase?

R6 210,00 represents 107% more than the pre-increase salary.

i.e. R6 210,00 = 107%

→ 1% ≈ R58,0374 (to 4 decimal places)

Since the original salary represents 100%: Original salary = R58,0374 × 100

= R5 803,74

OR:

Original salary = R6 210,00 ÷ 107%

= R6 210,00 ÷ 1,07

= R5 083,74

2. A bicycle costs R755,00 *including* VAT (Value Added Tax). If VAT is 14%, how much VAT is included in the price of the bicycle?

R755,00 represents 114% more than the price excluding VAT.

i.e. R755,00 = 114%

→ 1% ≈ R6,6228 (to 4 decimal places)

Since the original price (excluding VAT) represents 100%:

Original price = R6,6228 × 100

= R662,28

VAT = R755,00 – R662,28 = R92,72

3. Jemima sells necklaces with a 40% mark up on what it costs her to make the necklaces. If she sells the necklaces for R55,00, how much does it cost her to make the necklaces?

R55,00 represents 140% more than the cost price.

i.e. R55,00 = 140%

→ 1% ≈ R0,3929 (to 4 decimal places)

Since the original price represents 100%: Cost price = R0,3929 × 100

= R39,29

4. House prices in KwaZulu-Natal increased on average, by 18% from 2006 to 2007. If a house cost R680 000,00 in 2007, how much would that same house have cost in 2006?

R680 000,00 represents 118% more than the price in 2006.

i.e. R680 000,00 = 118%

→ 1% ≈ R5 762,7119 (4 decimal places)

Since the original price represents 100%: Original price = R5 762,7119 × 100

= R576 271,19

F. Calculating a Percentage Change

Practice Questions: % Change

1. The price of bread increased from R7,20 to R7,80. Calculate the percentage increase in price to one decimal place.

% increase

= (R7,80 − R7,20) ÷ R7,20 × 100%

= R0,60 ÷ R7,20 × 100%

= 8,3% (to 1 decimal place)

2. In 2007, 12 003 people entered the Comrades Marathon and in 2008 11 191 people entered. Calculate the percentage decrease in the number of entrants from 2007 to 2008.

% decrease

= (12 003 − 11 191) ÷ 12 003 × 100%

= 812 ÷ 12 003 × 100%

= 6,8% (to 1 decimal place)

OR

% change

= (11 191 − 12 003) ÷ 12 003 × 100%

= −812 ÷ 12 003 × 100%

= −6,8% (to 1 decimal place)

→ The negative sign indicates that this represents a decrease.

3. The table below shows the number of teachers in South Africa over the period 2001 – 2004.

Year	No. of Teachers
2001	354 201
2002	360 155
2003	362 598
2004	362 042

Calculate the percentage increase per year in the number of teachers in South Africa to one decimal place.

2001 – 2002:

% increase

= (360 155 − 354 201) ÷ 354 201 × 100%

= 5 954 ÷ 354 201 × 100%

= 1,7% (to 1 decimal place)

2002 – 2003:

% increase

= (362 598− 360 155) ÷ 360 155 × 100%

= 2 443 ÷ 360 155 × 100%

= 0,7% (to 1 decimal place)

2003 – 2004:

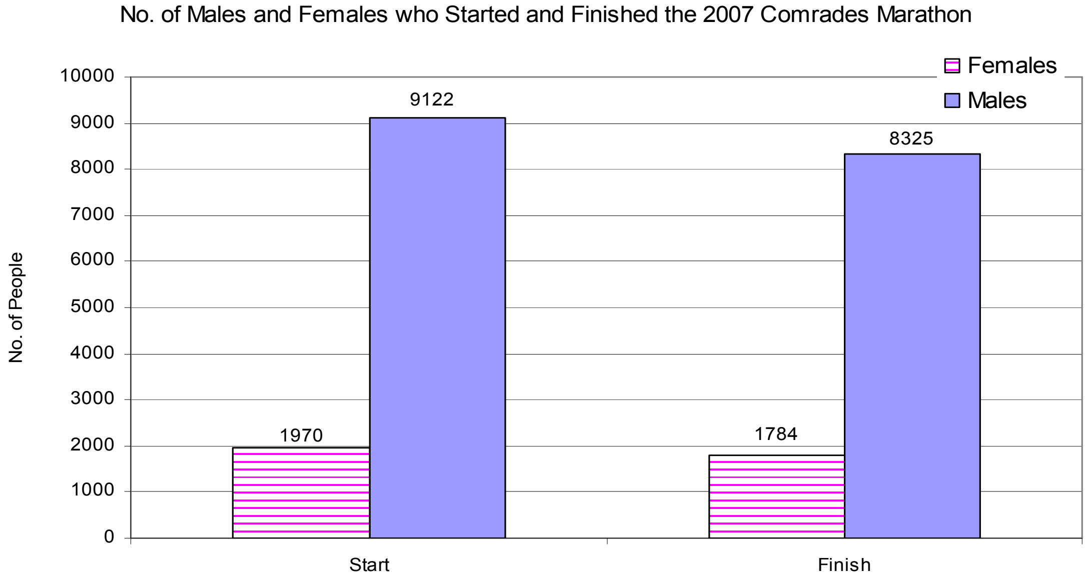
% change

= (362 042**− 362 598) ÷ 362 598 × 100%**

= −556 ÷ 362 598 × 100%

= −0,2% (i.e. decrease) (1 decimal place)

4. The graph below shows the number of females and males who started and finished the 2007 Comrades Marathon.



4. a. What percentage of the females who started the race finished (to one decimal place)?

Females who started = 1 970 **Females who finished = 1 784**

∴ % of females who finished = (1 970 − 1 784) ÷ 1 970 × 100%

= 186 ÷ 1 970 × 100%

= 9,4% (to one decimal place)

b. What percentage of the males who started the race finished (to one decimal place)?

Males who started = 9 122 **Males who finished = 8 325**

∴ % of males who finished = (9 122 − 8 325) ÷ 9 122 × 100%

= 797 ÷ 9 122 × 100%

= 8,7% (to one decimal place)

c. Did the females or males perform better in the 2007 Comrades Marathon? Explain.

It would appear that the males performed better. i.e. A greater percentage of females did not finish compared to males.

Test Your Knowledge: Percentages

1. a.

i. You get $\frac{27}{60}$ for your first Mathematical Literacy test. Express your result as a percentage.

Mark as a % = $\mathbf{\frac{27}{60} \times 100\%}$

= 45%

ii. If you get $\frac{17}{40}$ for your second Mathematical Literacy test, in which test did you do better?

Mark as a % = $\mathbf{\frac{17}{40} \times 100\%}$

= 42,5%

$\therefore$ **You performed better in the 1st test.**

1. b. 26 590 people watched Bafana Bafana play against Ghana. If the stadium can accommodate 30 000 people, what percentage of the stadium was full (to one decimal place)?

% of stadium that was full

= (26 590 ÷ 30 000) × 100

= 88,6% (to one decimal place)

2. a. How much will a waitron receive as a tip if she gets a 10% tip on a bill of R349,56?

Tip = 10% × R349,56

= R34,96 (rounded off to cents)

b. 75% of the money raised at your school's market day was given to charity. How much money went to charity if your school raised R15 486,00?

Money for charity = 75% × R15 486,00

= R11 614,50

2. c. 18% of the 11 046 athletes in the 2007 Comrades Marathon were females. How many females were there in the race?

No. of females = 18% × 11 046

= 1 988,28

= 1 988

(rounded off to full people)

3. a. Jimmy earns R18,00 an hour. How much will Jimmy earn per hour if he gets a 6,5% increase?

New pay = R18,00 + (6,5% × R18,00)

= R18,00 + R1,17

= R19,17 per hour

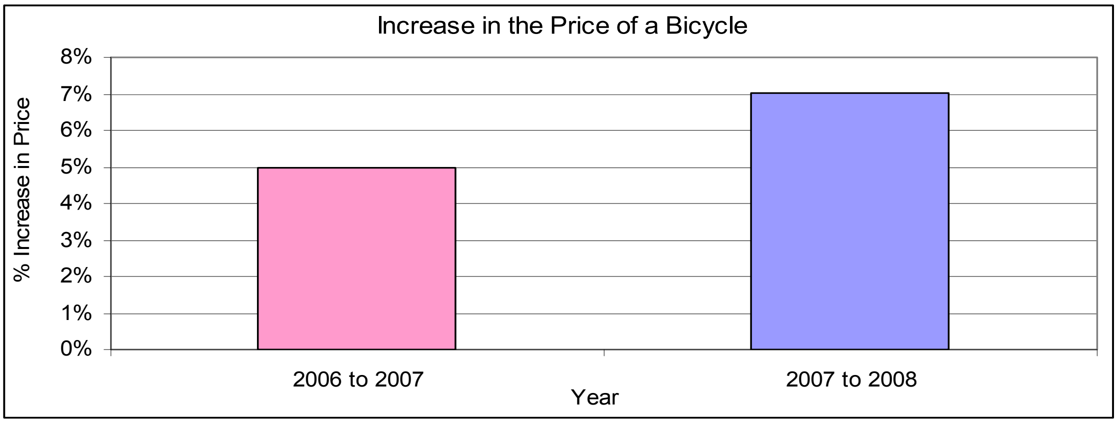
3. b. A new car will cost R179 500,00 without VAT. What will it cost you with 14% VAT included? (VAT = Value Added Tax)

Price with VAT = R179 500,00 + (14% × R179 500,00)

= R179 500,00 + R25 130,00

= R204 630,00

c. The graph below shows how the price of a bicycle increased from 2006 to 2007 and from 2007 to 2008.



If the price of a bicycle in 2006 was R3 200,00, calculate how much that same bicycle would have cost in 2008.

Increase from 2006 to 2007 = 5%

→ Price in 2007 = R3 200,00 + (5% × R3 200,00)

= R3 200,00 + R160,00

= R3 360,00

Increase from 2007 to 2008 = 7%

→ Price in 2007 = R3 360,00 + (7% × R3 360,00)

= R3 360,00 + R235,20

= R3 595,20

4. a. A shop advertises a 33% discount on all goods in the shop. How much would you pay for a pair of pants that was selling for R200,00?

Discounted price

= R200,00 − (33% × R200,00)

= R200,00 − R66,00

= R134,00

b. 15% of the people who had bought tickets to a concert did not arrive. If the organisers had sold 5 880 tickets, how many people were at the concert?

No. who did arrive

= 5 880 − (15% × 5 880)

= 5 880 − 882

= 4 998

4. c. Bongiwe earns R5 460,00 each month. She decides that she wants to save 5% of her salary each month. How much money does she have left after she has banked her savings?

Amount left after savings:

= R5 460,00 − (5% × R5 460,00)

= R5 460,00 − R273,00

= R5 187,00

5. a. The price of milk increased from R6,50 to R7,80 per litre. What was the percentage increase?

% increase = (R7,80 − R6,50) ÷ R6,50 × 100%

= R1,30 ÷ R6,50 × 100%

= 20%

b. A supermarket advertises that they are selling bottles of cooking oil that cost R13,99 at a discounted price of R10,99. Calculate the percentage discount (to one decimal place).

% discount

= (R13,99 − R10,99) ÷ R13,99 × 100%

= R3,00 ÷ R13,99 × 100%

= 21,4% (to one decimal place)

5. c. In 2001 the population of Cape Town was approximately 2 900 000 and in 2007 the population was approximately 3 500 000. What was the percentage increase in the population from 2001 to 2007?

% increase

= (3 500 000 − 2 900 000) ÷ 2 900 000 × 100%

= 600 000 ÷ 2 900 000 × 100%

= 20,7%

6. The VAT inclusive price of a washing machine is R1 580,00. Calculate how much the machine costs without VAT. Take VAT to be 14%.

R1 580,00 represents 114% more than the price excluding VAT.

i.e. R1 580,00 = 114%

→ 1% ≈ R13,8596 (to 4 decimal places)

Since the original price (excluding VAT) represents 100%:

Original price = R13,8596 × 100

= R1 385,96

1.4 RATIO

1.4.5 Typical calculations involving ratio

A. Converting Between Different Forms of a Ratio

Practice Questions: Using Ratios

1. Write the following ratios in simplest form:

a. 20 : 32 **= 10 : 16 (÷ by 2)**

= 5 : 8 (÷ by 2)

b. 72 : 56 **= 36 : 28 (÷ by 2)**

= 18 : 14 (÷ by 2)

= 9 : 7 (÷ by 2)

c. 27 : 81 **= 1 : 3 (÷ by 27)**

2. Write the following ratios in unit form
(i.e. in the form 1 : n or n : 1):

a. 25 : 75 **= 1 : 3 (÷ by 25)**

b. 728 : 91 **= 8 : 1 (÷ by 91)**

c. 12 : 59 **= 1 : 4,9 (÷ by 12)**

d. 107 : 11 = **9,7 : 1 (÷ by 11)**

3. The *pupil : teacher* ratios in two schools are given below.

- School 1 → 782 : 32
- School 2: → 1 328 : 57

Show by calculation which school has the better pupil : teacher ratio.

School 1: 782 : 32 = 24,4 : 1 (÷ by 32)

School 2: 1 328 : 57 = 23,3 : 1 (÷ by 57)

∴ **School 2 has a lower pupil : teacher ratio and therefore is the better ratio.**

4. The scale of a map is 1 : 20 000.

If the distance measured on the map is 24 cm, how far will this be in actual distance (in km)?

Scale: 1 : 20 000

→ 24 cm : 20 000 × 24 cm

= 480 000 cm

= 4 800 m

= 4,8 km

5. The scale of a plan is 1 : 20.

a. If the length of an item on the plan is 185 mm, determine the actual length of this item in metres.

Scale: 1 : 20

→ **185 mm : 20 × 185 mm**

= 3 700 mm

= 370 cm

= 3,7 m

5. b. If the actual length of an item is 5 m, determine how long this item will have been drawn on the plan.

Scale: 1 : 20

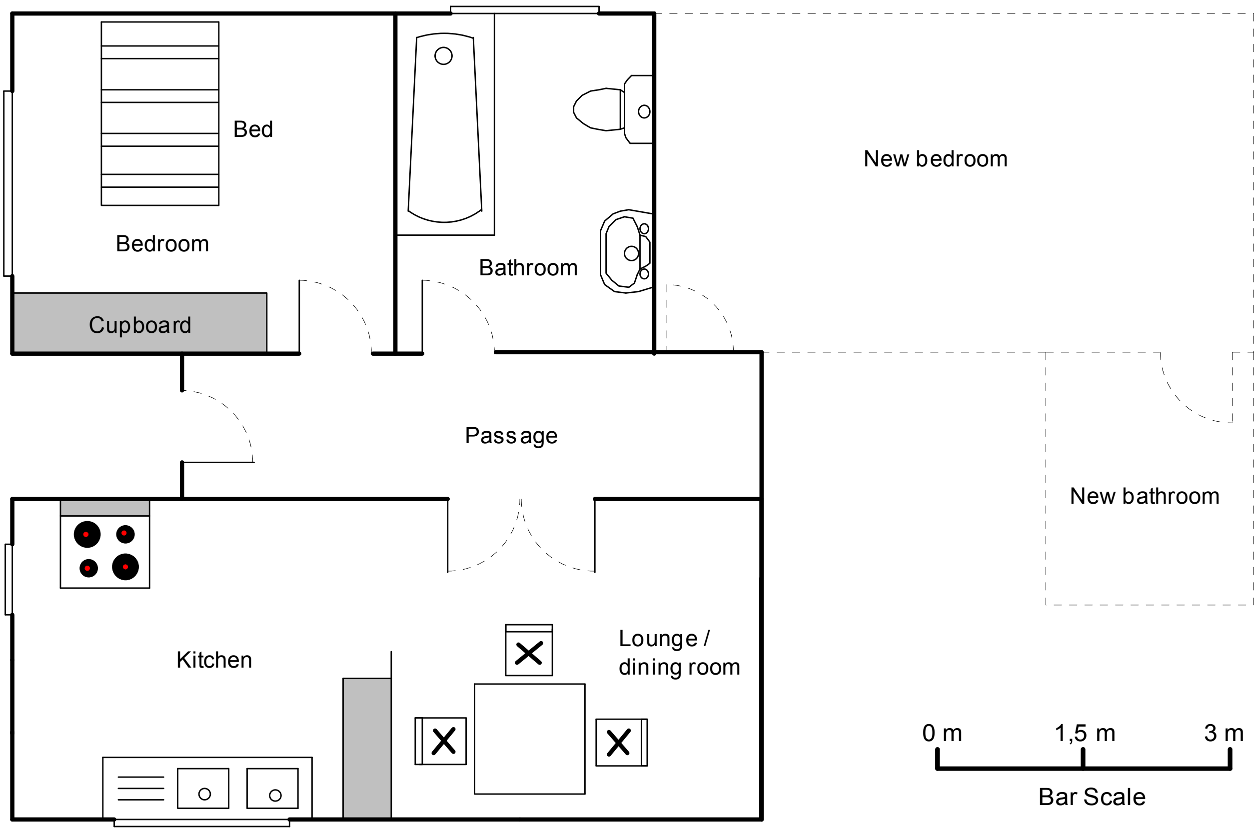
$\rightarrow \mathbf{\frac{1}{20} : 1}$

$\mathbf{(\frac{1}{20} \times 5\ m) : 5\ m}$

0,25 m : 5 m

25 cm : 5 m

6. The picture below shows a 2-dimensional top-view picture of the layout of a house. The owners of the house are planning on building a new bedroom and bathroom on to the house.



Use the bar scale to determine the dimensions (length and width) of the new bedroom and new bathroom. If necessary, give your answers to one decimal place.

On the bar scale: **2 cm = 1,5 m**

→ 1 cm = 0,75 m

New bedroom:

Length on plan = 8 cm **∴ Actual length = 0,75 m × 8 = 6 m**

Width = 4,5 cm **∴ Actual length = 0,75 m × 4,5 = 3,4 m (1 decimal place)**

New bathroom:

Length on plan = 3,3 cm **∴ Actual length = 0,75 m × 3,3 ≈ 2,5 m (1 decimal place)**

Width = 2,8 cm **∴ Actual length = 0,75 m × 2,8 = 2,1 m**

B. Determining Missing Numbers in a Ratio

Practice Questions: Determining Missing Numbers in a Ratio

1. If paint is mixed in the ratio

red : green : blue = 2 : 6 : 9

a. Determine the number of units of green and blue that are needed if 10 units of red are to be used.

Green = 6 × 5 = 30 units

Blue = 9 × 5 = 45 units

b. Determine the number of units of red and blue that are needed if 18 units of green are to be used.

Red = 2 × 3 = 6 units

Blue = 9 × 3 = 27 units

c. Determine the number of units of red and green that are needed if 25 units of blue are to be used.

Red = 2 ÷ 9 × 25 ≈ 5,6 units

Green = 6 ÷ 9 × 25 ≈ 16,7 units

2. Energade concentrate energy drink recommends that 1 unit of concentrate be mixed with 5 units of water.

a. How many ml of water must be added to 50 ml of concentrate?

Concentrate : water = 1 : 5

→ 50 ml concentrate : 5 × 50 ml water

= 250 ml water

b. How many litres of water must be added to 300 ml of concentrate?

Concentrate : water = 1 : 5

→ 300 ml concentrate : 5 × 300 ml water

= 1 500 ml water

= 1,5 litres water

c. How much juice (water & concentrate) will you make with 100 ml of concentrate?

Concentrate : water = 1 : 5

→ 100 ml concentrate : 5 × 100 ml water

= 500 ml water

∴ Total ml of juice = 100 ml + 500 ml

= 600 ml

d. Simphiwe mixes together 400 ml of water with 50 ml of concentrate. Will the juice be too sweet, not sweet enough or just right?

Concentrate : water = 1 : 5

→ 50 ml concentrate : 5 × 50 ml water

= 250 ml water

So, Simphiwe has put too much water in and the juice will not be sweet enough.

3. For making low strength concrete, the ratio of cement : sand : stone is 1 : 4 : 4.

a. How many wheelbarrows of sand and stone will you need if you use 8 wheelbarrows of cement?

cement : sand : stone = 1 : 4 : 4

→ **8 : 32 : 32 (i.e. × by 8)**

So, 32 wheelbarrows each of sand and stone are needed.

b. How many spades of cement will you need to mix with 36 spades of sand?

cement : sand : stone = 1 : 4 : 4

→ **9 : 36 : 36 (i.e. × by 9)**

So, you will need 9 spades of cement.

3. c. How many bags of cement will you need to buy if you use 37 bags of stone?

cement : sand : stone = 1 : 4 : 4

If stone = 37 bags then:

Cement = 1 ÷ 4 × 37 bags

= 9,25 bags

= 10 full bags

C. Dividing or Sharing an Amount in a Given Ratio

Practice Questions: Dividing an Amount in a Given Ratio

1. Sean and Zinhle invest R3 000,00 and R4 200,00 into an investment. After 3 years their combined money has grown to R9 352,00. If they divide the money in the same ratio in which they invested, how much money will each person receive?

Ratio of Sean and Sinhle's investments = 3 000 : 4 200

This gives a total of 7 200 parts.

So, each person will get:
R9 352,00 ÷ 7 200 units = R1,299 per unit (to 3 decimal places)

∴ Sean's units = R1,299/unit × 3 000 units

= R3 897,70

∴ Zinhle's units = R1,299/unit × 4 200 units

= R5455,30

2. Mpho and Sello worked together on a project and received R450,00 for their completed work. Mpho worked for 3 days and Sello worked for 4 days, and they agree to divide the money between them in the ratio 3 : 4. How much should each person receive?

Total days = 7

Fraction Mpho worked = $^{3}/_{7}$

∴ Money Mpho must receive
= 3 ÷ 7 × R450,00

≈ R192,86

∴ Money Sello must receive =

R450,00 − R192,86 = R257,14

3. A hairdresser needs to make up a 40 ml mixture of tint and hydrogen peroxide. The ratio of tint : peroxide is 1 : 2.

How many milliliters of tint and how many milliliters of peroxide will the hairdresser need to use to make the 40 ml mixture.

Total parts = 3

→ Tint = $^{1}/_{3}$ × 40 ml ≈ 13,3 ml

→ Peroxide = $^{2}/_{3}$ × 40 ml ≈ 26,7 ml

OR

→ Tint = $^{1}/_{3}$ × 40 ml ≈ 13,3 ml

∴ Peroxide = 40 ml − 13,3 ml = 26,7 ml

4. Energade concentrate energy drink recommends that 1 unit of concentrate be mixed with 5 units of water.

a. How many ml of concentrate and ml of water must be mixed to make 500 ml of juice?

Total parts = 6

→ Concentrate = $^{1}/_{6}$ × 500 ml ≈ 83,3 ml

→ Water = 500 ml − 83,3 ml = 416,7 ml

4. b. How many ml of concentrate and ml of water must be mixed to make 3 litres of juice?

Total parts = 6

→ Concentrate = $^{1}/_{6}$ × 3 000 ml ≈ 500 ml

→ Water = 3 000 ml − 500 ml = 2 500 ml

5. Three brothers combine their money and then invest the money. The table below shows the amount that each brother invests:

	Amount Invested
Brother 1	R8 000,00
Brother 2	R13 000,00
Brother 3	R20 000,00

a. After 5 years the money has grown by an effective 48% from its original value. Determine how much money there will be in the investment after 5 years.

Total money invested = R41 000,00

Money in investment after 5 years

= R41 000,00 + (48% × R41 000,00)

= R41 000,00 + R19 680,00

= R60 680,00

5. b. If after 5 years the brothers decide to withdraw and divide the money in the ratio of their initial investments, how much will each brother receive?

Total units = 41 000

So, each brother will get:

R60 680,00 ÷ 41 000 units = R1,48 per unit

Brother 1 will get:

R8 000,00 × R1,48 per unit = R11 840,00

Brother 2 will get:

R13 000,00 × R1,48 per unit = R19 240,00

Brother 3 will get:

R20 000,00 × R1,48 per unit = R29 600,00

Test Your Knowledge: Ratios

1. The instructions on the label of an energy drink say that you must dilute the concentrate with water in the ratio of 1 : 4.

a. Explain what this means.

Every 1 unit of concentrate must be mixed with 4 of the same type of units of water.

b. If I have 2 litres of the energy drink concentrate mentioned in question 1 (a), how many litres of water do I need to add to make up the mixture?

1 : 4

→ 2 litres concentrate : 8 litres water
(i.e. multiply by 2)

2. High Strength Concrete is made up of gravel, sand and cement. The mixing ratio is 4 : 2 : 1.

a. If I have 2 wheelbarrows of gravel, how many wheel barrows of sand and cement do I need to make up a batch of concrete?

Gravel : sand : cement = 4 : 2 : 1

→ 2 : 1 : ½ (i.e. divide by 2)

So, you will need 1 wheelbarrow of sand and ½ a wheelbarrow of cement.

2. b. If I have 3 bags of cement, how many bags of gravel and sand of the same size do I need to make up a batch of concrete?

Gravel : sand : cement = 4 : 2 : 1

→ 12 : 6 : 3 (i.e. × by 3)

So, you will need 12 bags of gravel and 6 bags of sand.

c. If I have 3 wheelbarrows full of sand, how many wheelbarrows of gravel and cement do I need to make up a batch of concrete?

Gravel : sand : cement = 4 : 2 : 1

For 3 wheelbarrows of sand:

Gravel → 4 ÷ 2 × 3 = 6 wheelbarrows

Cement → 1 ÷ 2 × 3 = 1,5 wheelbarrows

3. A new green colour of paint is made by mixing blue paint and yellow paint in the ratio 4 : 3.

If I have 12 litres of blue paint, how many litres of green paint do I need to make up the new green colour?

4 : 3

→ 12 : 9 (× by 3)

4. Grace and Nikiswa received a total of R640,00 for the work that they did. Grace worked for 14 hours and Nikiswa worked for 18 hours.

a. Write the hours that they worked as a simplified ratio.

Grace : Nikiswa = 14 : 18

= 7 : 9

b. Calculate how much each of the girls should be paid.

Total parts = 16

→ Grace = $^{7}/_{16}$ × R640,00

= R280,00

→ Nikiswa = R640,00 − R280,00 = R360,00

5. The instructions on the label of an energy drink say that you must dilute the concentrate with water in the ratio of 1 : 4.

If I want to make 6 litres of diluted energy drink, how much concentrate must I use and how much water?

Total parts = 5

Concentrate = 1 part out of 5

→ Concentrate = 1 ÷ 5 × 6 litres

= 1,2 litres

→ Water = 6 litres − 1,2 litres = 4,8 litres

6. The following recipe caters for 6 people.

- $1\frac{1}{2}$ cups cooked rice
- 650 g chicken
- 375 ml chicken stock
- $\frac{1}{2}$ teaspoon salt
- 2 tablespoons flour

Calculate how much of each ingredient you would need to cater for 15 people.

Rice:

6 people = 1,5 cups

→ 1 person = 1,5 cups ÷ 6

15 people = 1,5 cups ÷ 6 × 15

= 3,75 cups

= 3 ¾ cups

Chicken:

6 people = 650 g

→ 1 person = 650 g ÷ 6

15 people = 650 g ÷ 6 × 15

≈ 1 625 g

Stock:

6 people = 375 ml

→ 1 person = 375 ml ÷ 6

15 people = 375 ml ÷ 6 × 15

≈ 937,5 ml

Salt:

6 people = ½ teaspoon

→ 1 person = ½ teaspoon ÷ 6

15 people = ½ teaspoon ÷ 6 × 15

≈ 1,25 teaspoons

= $1\frac{1}{4}$ teaspoons

Flour:

6 people = 2 tablespoons

→ 1 person = 2 tablespoons ÷ 6

15 people = 2 tablespoons ÷ 6 × 15

≈ 5 tablespoons

7. A map is drawn with a scale of 1 : 50 000.

For each of the following distances on the map, calculate the actual distance on the ground (give your answers in kilometres):

a. 2 cm on the map:

Scale: 1 : 50 000

2 cm : 50 000 × 2 cm

= 100 000 cm

= 1 000 m

= 1 km

b. 9 cm on the map:

Scale: 1 : 50 000

9 cm : 50 000 × 9 cm

= 450 000 cm

= 4 500 m

= 4,5 km

c. 30 cm on the map:

Scale: 1 : 50 000

30 cm : 50 000 × 30 cm

= 1 500 000 cm

= 15 000 m

= 15 km

8. A map has a scale of 1 : 200 000. The distance between two towns is 60 km.

How far apart are the towns on the map (in cm)?

Scale: 1 : 200 000

This means that 1 unit on the map = 200 000 units in actual distance.

So: 1 unit in actual distance = 1 ÷ 200 000 units on the map.

∴ 60 km in actual distance = 1 ÷ 200 000 × 60 km on the map

= 0,0003 km on the map

= 0,3 m on the map

= 30 cm on the map

9. On a map, a distance of 5 cm represents an actual distance of 15 km. Determine the scale of the map and write the scale in the form 1 : n.

5 cm on the plan = 15 km in actual distance

→ 5 cm : 15 km

= 15 000 m

= 1 500 000 cm

→ 1 cm on the map = 1 500 000 cm ÷ 5

= 300 000cm actual

∴ Scale of the map = 1 : 300 000

1.5 PROPORTION

1.5.2 Direct Proportion

Practice Questions: Direct Proportion

1. Determine whether or not the following ratios are in proportion:

a. 4 : 10 and 16 : 40

4 : 10 = 2 : 5 (divide by 2)

16 : 40 = 2 : 5 (divide by 8)

∴ Direct proportion.

b. 20 : 220 and 37 : 407

20 : 220 = 1 : 11 (divide by 20)

37 : 407 = 1 : 11 (divide by 37)

∴ Direct proportion.

c. 5 : 17 and 20 : 63

5 : 17 = 1 : 3,4 (divide by 5)

20 : 63 = 1 : 3,15 (divide by 20)

∴ Not a direct proportion.

d. 6 : 7 and 30 : 35 and 102 : 119

6 : 7

30 : 35 = 6 : 7 (divide by 5)

102 : 119 = 6 : 6,158 (divide by 17)

∴ There is a direct proportion between the first two ratios but not with the 3rd ratio.

2. The tables below show the cost of talking on various different cell phone options.

Determine by calculation whether or not the values given in the table are in direct proportion.

a.

Time	0 min	5 min	10 min	15 min
Cost	R0,00	R12,50	R25,00	R37,50

0 min to 5 min the cost increases by R12,50.

5 min to 10 min the cost increases by R25,00 − R12,50 = R12,50.

∴ Direct proportion.

b.

Time	10 min	20 min	30 min	40 min
Cost	R105	R125	R145	R165

0 min to 10 min the cost increases by R105,00.

10 min to 20 min the cost increases by R125,00 − R105,00 = R20,00.

∴ No direct proportion.

2. c.

Time (min)	60	120	240	360
Cost	R168	R336	R504	R672

0 min to 60 min the cost increases by R168,00.

60 min to 120 min the cost increases by R336,00 − R168,00 = R168.

∴ **Direct proportion.**

3. The table below shows the monthly cost of electricity for a user in the Msunduzi Municipality.

Electricity used (kWh)	10	20	30
Monthly Cost	R5,42	R10,84	R16,26

a. Explain why there is a *direct proportion* between the electricity used per month and the cost of that electricity.

For every 10 kWh of electricity used, the monthly cost increases by R5,42.

3. b. Use the fact that there is a direct proportion to calculate the monthly cost of using the following kWh of electricity during the month:

i. 50 kWh

10 kWh = R5,42

→ **50 kWh = R5,42 × 5 = R27,10**

ii. 100 kWh

10 kWh = R5,42

→ **100 kWh = R5,42 × 10 = R54,20**

iii. 372 kWh

10 kWh = R5,42

→ **1 kWh = R5,42 ÷ 10**

372 kWh = R5,42 ÷ 10 × 372

≈ R201,62

iv. 512,7 kWh

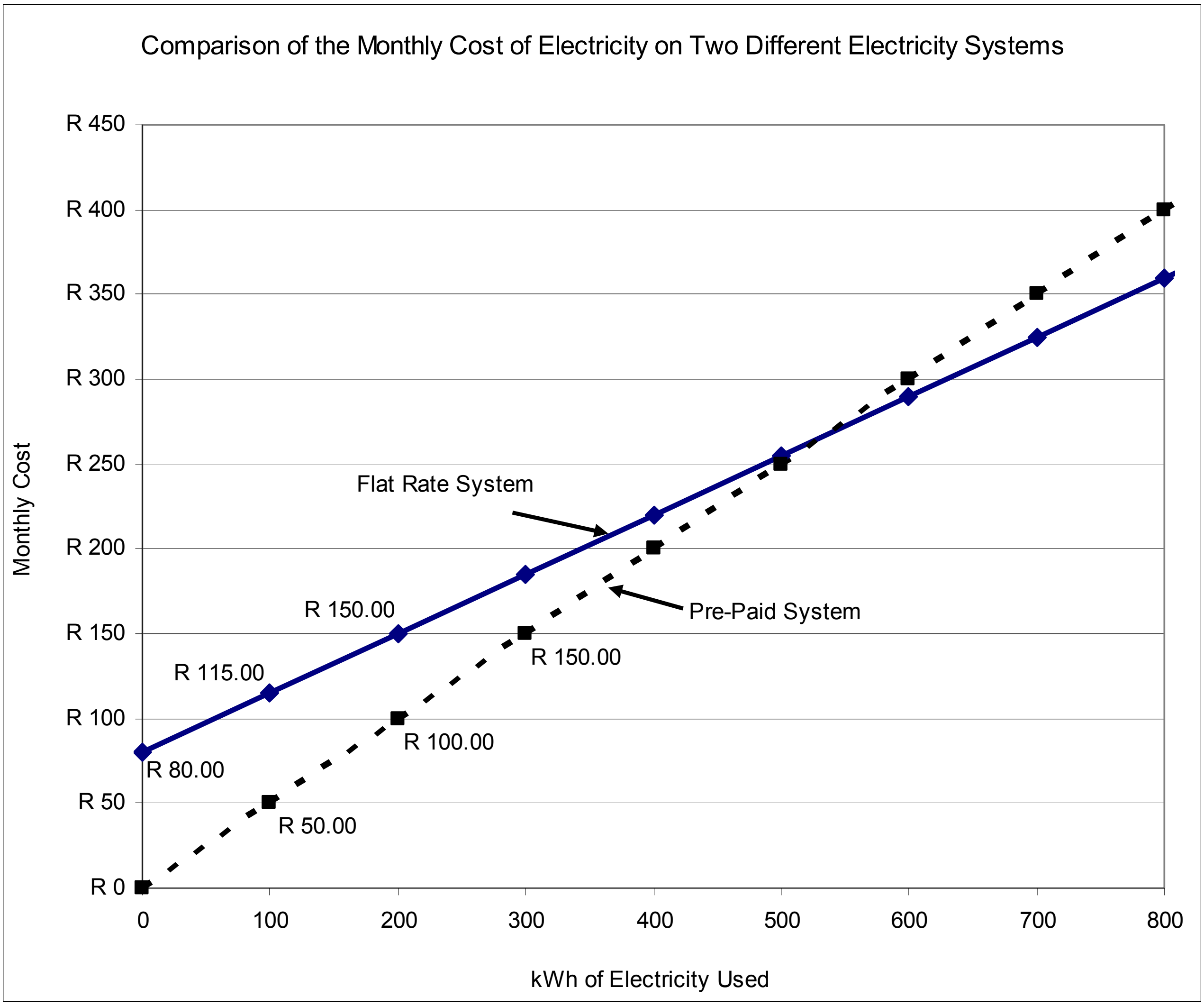
10 kWh = R5,42

→ **1 kWh = R5,42 ÷ 10**

512,7 kWh = R5,42 ÷ 10 × 512,7

≈ R277,88

4. The graphs below show the cost of electricity of two different systems in a municipality.



a. Does the Pre-Paid system or the Flat Rate represent a direct proportion? Explain.

Pre-paid. i.e. For every 100 kwh of electricity used, the monthly cost increases by R50,00.

On the flat rate system, on the other hand, for the first 100 kWh, the monthly cost increases by R115,00, but for the second 100 kWh the monthly cost increases by only R35,00. So, there is no direct proportion on the flat-rate system.

b. On the Flat Rate system there is a fixed monthly service fee as well as a charge per unit (kWh) of electricity used during the month. How much is the fixed monthly service fee?

R80,00

4. c. Calculate the per unit fee (i.e. the cost of using 1 kWh of electricity) for electricity on the:

i. Pre-paid system

100 kWh = R50,00

→ 1 kWh = R50,00 ÷ 100

= R0,50

∴ Per unit fee = R0,50 per kWh

ii. Flat rate system

100 kWh = R35,00

→ 1 kWh = R35,00 ÷ 100

= R0,35

∴ Per unit fee = R0,35 per kWh

4. d. How much would it cost to use 1 000 kWh of electricity on the Pre-Paid system?

Monthly cost = R0,50/kWh × 1 000 kWh

= R500,00

4.e. How much would it cost to use 1 000 kWh of electricity on the Flat Rate system?

Monthly cost

= R80,00 + (R0,35/kWh × 1 000 kWh)

= R430,00

1.5.3 Inverse Proportion

Practice Questions: Inverse Proportion

1. The table below shows the number of days that it takes to build a wall as dependent on the numbers of workers building the wall.

No. of Workers	1	2	3
Days to build the wall	24	12	8

a. Explain why the relationship between the number of workers and the number of days needed to build the wall is an inverse proportion relationship.

As the number of workers increases, so the number of days needed to build the wall decreases.

Or

No. of workers multiplied by Days gives the same constant factor − i.e. 24.

b. What is the constant product?

24

c. Use the constant product to determine how many days it would take to build the wall if there were 6 workers.

Workers × Days = 24

→ 6 Workers × Days = 24

So Days = 4

d. Write down an equation to represent the relationship between the number of workers and the number of days needed to build the wall.

Workers × Days = 24

e. If a graph were to be drawn to represent this situation, would the graph be a straight line or a curved graph? Explain.

The graph would be curved and would slope downwards at a decreasing rate. i.e. The graph would start off curving downwards very fast and then start to level off towards the horizontal axis.

2. Zinzi uses her car to drive to work. If she drives alone, then she has to pay all of the petrol costs. If she finds people to travel with her then they all share the travel costs.

The table below shows Zinzi's petrol costs as dependent on the number of people who travel in the car with her.

No. of People in the Car	1	2	3
Zinzi's Petrol Costs	R380,00	R190,00	R126,67

a. Explain why the relationship between the number of people in Zinzi's car and Zinzi's petrol costs is an inverse proportion relationship? Explain.

As the number of people in the car increases, so the Zinzi's petrol cost decreases.

b. What is the constant product?

R380,00

c. Use the constant product to determine what Zinzi's petrol costs will be if she travels to work with 5 people in the car every month.

Petrol costs = R380,00 ÷ 5 = R76,00

d. Write down an equation to represent Zinzi's petrol costs.

Zinzi's petrol costs

= R380,00 ÷ No. of people in the car

3. The table below shows the cost of travelling in a Yellow Cab taxi.

Distance (km)	10	20	100
Cost	R125	R250	R1 250

a. Is there an inverse proportion relationship between the distance travelled in the taxi and the cost of the trip? Explain.

No, as the distance travelled increases, so the cost of the trip also increases.

3. b. Calculate how much the taxi charges per kilometer travelled.

10 km costs R125,00

Cost per 1 km = R125,00 ÷ 10 = R12,50

c. Calculate the cost of travelling 147 km in this taxi.

Cost = R12,50 per km × 147 km

= R1 837,50

3. d. Write down an equation to represent the cost of a trip in the Yellow Cab Taxi.

Cost = R12,50/km × distance travelled (km)

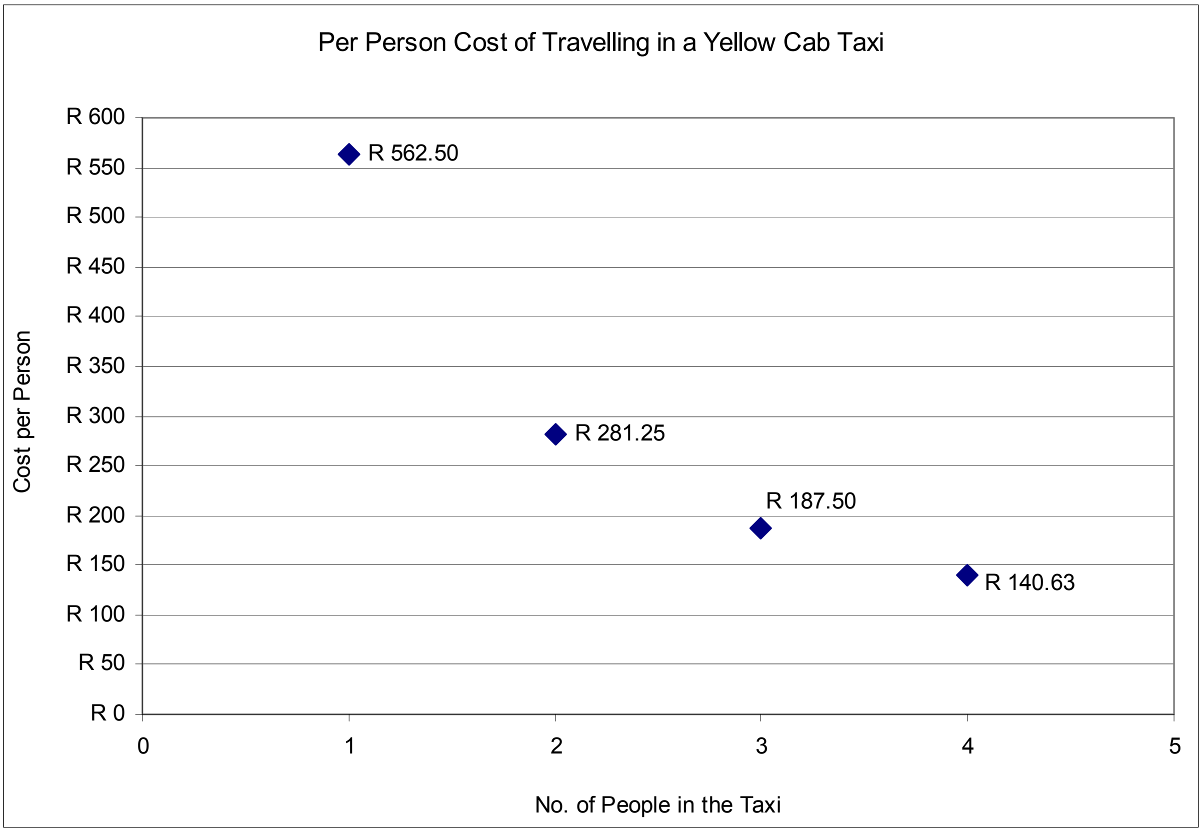
e. If a graph were drawn to represent the cost of a trip in the taxi, what would the graph look like?

i.e. → would the graph be straight or curved;

→ in which direction would the graph go?

The graph will be a straight line increasing by R12,50 on the vertical axis for every 1 km increase on the horizontal axis. The graph will start at the origin (0 km; R0,00) and will increase upwards towards the right.

4. Ryan wants to catch a Yellow Cab Taxi from university to the bus station. If he catches the taxi alone then he will pay R562,50. If he shares the taxi with one friend, each of them will pay R281,25. The graph below illustrates this scenario.



a. Is there an inverse proportion relationship between the number of people in the taxi and the amount that each person has to pay for the trip? Explain.

Yes, as the number of people in the taxi increases so the amount that each person has to pay decreases

b. Why have the points on the graph not been joined?

The points are discrete points. i.e. It is impossible to have 1½ people in the taxi.

c. How much will each person have to pay for the taxi trip if there are 5 people in the taxi?

Cost per person = R562,50 ÷ 5

= R112,50

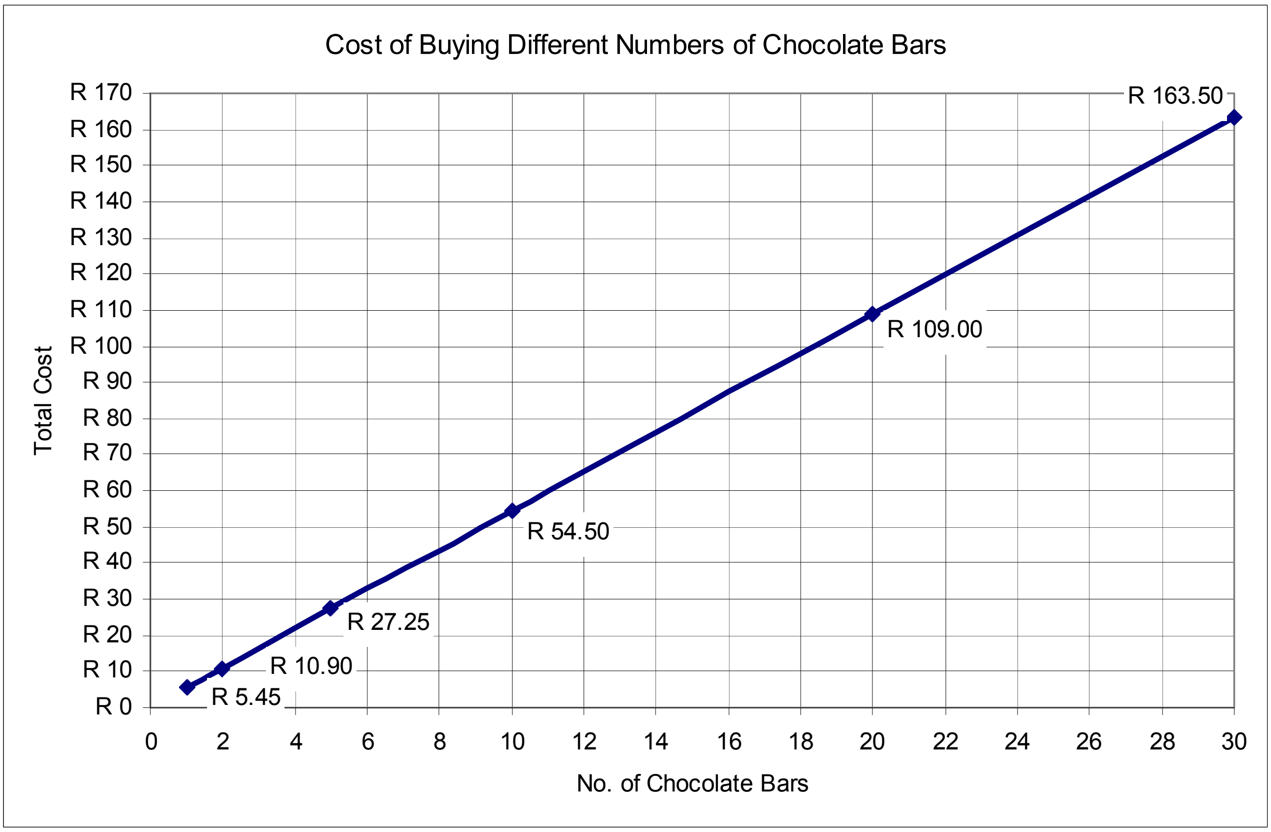
Test Your Knowledge: Proportion

1. A chocolate bar costs R5,45.

a. Complete the following table:

Number of chocolate bars	1	2	5	10	20	30
Cost	**R5,45**	**R10,90**	**R16,35**	**R54,50**	**R119,00**	**R163,50**

b. On the set of axes below, draw a graph to represent the above situation



c. What type of proportional relationship is represented in this situation? Explain.

The situation is a direct proportion. i.e. For every 1 increase in the number of chocolate bars bought, the cost of buying the chocolate bars increases by R5,45.

2. A teacher has 36 learners in her class. She buys enough sweets to give each child 5 sweets.

a. What type of proportional relationship is there between the number of children in the class and the number of sweets that each child receives? Explain.

This is an inverse proportion relationship. i.e. As the number of learners in the class decreases, so the number of sweets that each child receives increases.

b. How many sweets will each learner get if only 30 learners come to school?

Total sweets = 36 × 5 = 180

No. of sweets per child for 30 learners

= 180 ÷ 30 = 6 sweets per child

3. A scout troop wants to go on an expedition. The bus company quotes them R650,00 for a 30-seater bus. The price of hiring the bus stays the same even if not all of the 30 seats are taken.

a. What type of proportional relationship is there between the number of scouts on the bus and the amount that each parent has to pay? Explain.

This is an inverse proportion. i.e. If 30 scouts attend, then the bus hire fee of R650,00 is shared amongst 30 parents. If only 20 scouts attend then the same bus hire fee is now shared amongst 20 scouts.

3. a … So, as the number of scouts who attend increases, so the amount that each parents has to pay decreases.

b. How much will it cost the parents of each scout for transport if only 17 scouts go on the expedition?

Cost per parent = R650,00 ÷ 17 ≈ R38,24

4. Riyaad gets paid R15,50 per hour for his holiday job.

a. Is there an inverse proportion relationship between the amount that Riyaad gets paid and the number of hours that he works? Explain.

No, it is a direct proportion relationship because as the number of hours that he works increases, so the amount that he earns also increases.

Also, for every increase of 1 hour in the amount of time that he works, the amount that he earns increases at the constant rate of R15,50.

b. How much will Riyaad earn in 12 hours?

Amount earned = R15,50/hour × 12 hours

= R186,00

1.6 RATE

1.6.2 Constant Rates

Practice Exercise: Constant Rates

1. Petrol costs R10,50 per litre. How much would it cost to put 40 ℓ of petrol into a car?

Petrol cost = R10,50 per litre

→ Cost of 40 ℓ = R10,50 per ℓ × 40 ℓ

= R420,00

2. Mince is selling for R42,99 per kilogram. How much would it cost to buy 3 kilograms of mince?

Cost = R42,99 per kg

→ Cost of 3 kg = R42,99 per kg × 3 kg

= R128,97

3. The cost of a telephone call on a Telkom landline during peak time is R2,80 per minute. Calculate the cost of making a 17 minute call.

Cost = R2,80 per min × 17 min

= R47,60

4. The cost of a call on a particular cell phone contract is R0,04 per second. How much would it cost to make a call that lasts
6 min 23 seconds?

Time in seconds = (6 × 60) sec + 23 sec

= 360 sec + 23 sec

= 383 sec

4 … → **Cost = R0,04 per sec × 383 sec**

= R15,32

5. Cheese is selling for R58,49 per kilogram. How much would it cost to buy a 400 g block of cheese?

1 kg = R58,49

→ 1 000 g = R58,49

1 g = R58,49 ÷ 1 000

∴ 400 g = R58,49 ÷ 1 000 × 400

≈ R23,40

6. A particular type of paint has a coverage of 9 m^2 per litre. Calculate how many litres of paint will be needed to paint a wall that has a surface area of 23 m^2?

Coverage: 9 m^2 = 1 litre

→ 1 m^2 = 1 litre ÷ 9

∴ 23 m^2 = 1 litre ÷ 9 × 23

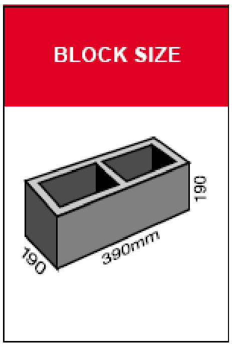
≈ 2,6 litres (to one decimal place)

(= 3 full litres)

1.6.3 Unit Rates

Practice Exercise: Unit Rates

1. The table below shows the number of blocks and bags of cement needed to build a wall.



FOR 1000 BLOCKS		
WALL SIZE m^2	No. BAGS SUREBUILD	m^3 SAND
80	9	2,00

(PPC Cement, Pamphlet – *The Sure Way to Estimate Quantities*, www.ppccement.co.za)

a.

i. How many bags of cement are needed to make a 160 m^2 wall?

80 m^2 wall = 9 bags

∴ 160 m^2 wall = 18 bags (× by 2)

ii. How many bags of cement are needed to make a 40 m^2 wall?

80 m^2 wall = 9 bags

∴ 40 m^2 wall = 4,5 bags (÷ by 2)

iii. How many bags of cement are needed to make a 150 m^2 wall?

80 m^2 wall = 9 bags

→ 1 m^2 wall = 9 bags ÷ 80

150 m^2 wall = 9 bags ÷ 80 × 150

= 16,9 bags (to one decimal place)

Will need to buy 17 full bags

1. b.

i. How many blocks are needed to make a 200 m^2 wall?

80 m^2 wall = 1 000 blocks

1 m^2 wall = 1 000 blocks ÷ 80

200 m^2 wall = 1 000 blocks ÷ 80 × 200

= 2 500 blocks

ii. How many m^3 of sand is needed to make a 150 m^2 wall?

80 m^2 wall = 2 m^3 sand

1 m^2 wall = 2 m^3 sand ÷ 80

150 m^2 wall = 2 m^3 sand ÷ 80 × 150

= 3,75 m^3

c.

i. A builder buys 15 bags of cement to make a wall. How big is the wall?

9 bags cement = 80 m^2

1 bag cement = 80 m^2 ÷ 9

15 bags cement = 80 m^2 ÷ 9 × 15

≈ 133,3 m^2

(to one decimal place)

1. c. ii. A builder buys 250 blocks to make a wall. How many bags of cement will he need to buy?

1 000 blocks = 9 bags of cement

250 blocks = 9 bags of cement ÷ 4

= 2,25 bags of cement

= 3 full bags

2. Which is the better value for money:

a. 300 g box of chocolates that costs R13,05 OR

1 kg box that costs R44,99?

300 g = R13,05

→ 1 g = R13,05 ÷ 300

1 000 g (1 kg) = R13,05 ÷ 300 × 1 000

= R43,50

So, the 300 g box is cheaper per kilogram.

b. 350 ml bottle of juice that costs R6,25

OR

a 1 litre bottle of juice that costs R12,80?

350 ml = R6,25

1 ml = R6,25 ÷ 350

∴ 1 000 ml (1 litre) = R6,25 ÷ 350 × 1 000

= R17,86

So, the 1 litre bottle for R12,80 is cheaper per litre.

c. 200 g packet of biscuits that costs R7,25

OR

1,2 kg box of biscuits that costs R44,50?

200 g packet = R7,25

→ 1 200 g (1,2 kg) = R7,25 × 6

= R43,50

So, the 1,2 kg box is cheaper per kg.

3. Two cars leave Durban at the same time. Car A travels 535 km in 5 hours and Car B travels 980 km in $8\frac{1}{2}$ hours. Which car is travelling the fastest? Explain.

Car A:

5 hours = 535 km

1 hour = 535 km ÷ 5

= 107 km

∴ Average speed = 107 km/h

Car B:

8,5 hours = 980 km

1 hour = 980 km ÷ 8,5

= 115,3 km (to 1 decimal place)

∴ Average speed = 115,3 km/h

So, Car B is travelling the fastest.

1.6.4 Average Rates

Practice Exercise: Average Rates

1. a. A car has an average petrol consumption rate of 8 litres per 100 km.

a. How much petrol will the car use to travel 370 km?

100 km = 8 litres

→ 1 km = 8 litres ÷ 100

370 km = 8 litres ÷ 100 × 370

= 29,6 litres

b. If the current price of petrol is R10,30 per litre, how much will it cost to travel 370 km?

Cost = R10,30 per litre × 29,6 litres

= R304,88

c. If the owner of the car puts R550,00 worth of petrol in his car, how far will hebe able to travel until the petrol runs out?

Cost of petrol = R10,30 per litre

So, with R550,00 the person would be able to travel:

R10,30 = 1 litre

→ R1,00 = 1 litre ÷ 10,3

R550,00 = 1 litre ÷ 10,3 × 550

= 53,398 litres

Consumption rate = 8 litres per 100 km

→ 8 litres = 100 km

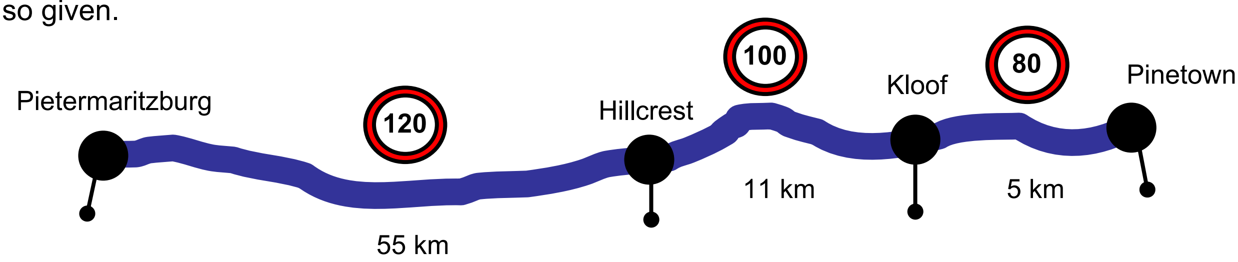
1 litre = 100 km ÷ 8

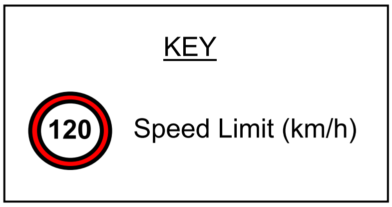
53,398 litres = 100 km ÷ 8 × 53,398

≈ 667,5 km

(to one decimal place)

2. The picture below shows a distance chart with the distances between different towns on route from Pietermaritzburg to Pinetown. The speed limits between the various towns on the route are also given.





a. Mpumi is driving from Pietermaritzburg to Pinetown. If she drives the whole way at the speed limit, calculate how long it will take for her to travel from:

i. Pietermaritzburg to Hillcrest (in minutes and seconds)

Distance = 55 km

Speed = 120 km/h

→ 120 km = 1 hour

1 km = 1 hour ÷ 120

55 km = 1 hour ÷ 120 × 55

= 0,4583 hours

= (0,4583 × 60) minutes

= 27½ minutes

= 27 min 30 sec

ii. Hillcrest to Kloof (in minutes and seconds)

Distance = 11 km

Speed = 100 km/h

→ 100 km = 1 hour

1 km = 1 hour ÷ 100

11 km = 1 hour ÷ 100 × 11

= 0,11 hours

= (0,11 × 60) minutes

= 6,6 minutes

= 6 min + 0,6 min

= 6 min + (0,6 × 60) sec

= 6 min 36 sec

iii. Kloof to Pinetown

Distance = 5 km

Speed = 80 km/h

→ 80 km = 1 hour

1 km = 1 hour ÷ 80

5 km = 1 hour ÷ 80 × 5

= 0,0625 hours

= (0,0625 × 60) minutes

= 3,75 minutes

= 3 min 45 sec

b. Mpumi's car has an average petrol consumption rate of 9 litres per 100 km. If the current petrol price if R10,30 per litre, calculate how much it will cost her in petrol costs to travel from Pietermaritzburg to Pinetown.

Total distance = 71 km

Fuel consumption: 100 km = 9 litres

1 km = 9 litres ÷ 100

71 km = 9 litres ÷ 100 × 71

= 6,39 litres

Cost of petrol = R10,30 per litre × 6,39 litres

= R65,82

3. The table below shows the running times of the winner of the 2008 Comrades Marathon at different places on the route.

Place on the Route	Total Running Time (h : min : sec)	Total Distance Run (km)
Cowies Hill	01:04:50	16,7
Drummond	02:42:44	42,6
Cato Ridge	03:37:43	57,1
Camperdown	04:03:54	63,8
Polly Shorts	04:57:13	79,1
Finish	05:24:46	86,8

a.

i. Calculate how long it took for the athlete to run from the Start to Cowies Hill in minutes. Round off your answer to 3 decimal places.

Time = 1 h 04 min 50 sec

= 60 min + 4 min + (50 ÷ 60) min

= 60 min + 4 min + 0,833 min (to 3 decimal places)

≈ 64,8333 min

ii. Determine the average speed (in *minutes and seconds per kilometer*) at which the athlete ran from the Start to Cowies Hill.

Distance = 16,7 km

Time ≈ 64,8333 min

→ 16,7 km = 64,8333 min

1 km = 64,833 min ÷ 16,7 km

= 3,882 min (to 3 decimal places)

≈ 3 min + (0,882 × 60) sec

≈ 3 min 53 sec

3. b. Determine the average speed (in *minutes and seconds per km*) at which the athlete ran from Polly Shorts to the Finish.

Distance = 86,8 km − 79,1 km = 7,7 km

Total time (in min)

= 5 h 24 min 46 sec − 4 h 57 min 13 sec

= 27 min 33 sec

= 27 min + (33 ÷ 60) min

= 27 min + 0,55 min

= 27,55 min

→ 7,7 km = 27,55 min

1 km = 27,55 min ÷ 7,7 km

= 3,578 min (to 3 decimal places)

≈ 3 min 35 sec

∴ Average speed ≈ 3 min 35 sec per km.

∴ Average speed ≈ 3 min 53 sec per km.

c. Determine the average running speed (in *minutes and seconds per km*) of the athlete over the whole race.

Distance = 86,8 km

Total time = 5 h 24 min 46 sec

= (5 × 60) min + 24 min + (46 ÷ 60) min

= 300 min + 24 min + 0,767 min (to 3 decimal places)

≈ 324,767 min

→ 86,8 km = 324,767 min

1 km = 324,767 min ÷ 86,8 km

= 3,742 min (to 3 decimal places)

≈ 3 min 45 sec

∴ Average speed ≈ 3 min 45 sec per km.

d. Why do we use the word "average" when referring to the running speed of the athlete?

Over the course of one km or over the whole race, the speed of the athlete will continuously change as the athlete speeds up or slows down. As such, when we say that the athlete runs the race at an average speed of 3 min 45 sec per km, this does not mean that he ran every km at this pace. Rather, this speed represents the average of all their speeds for every km of the race.

1.6.5 Constructing Rates to Solve Problems

Practice Exercise: Constructing Rates

1. In a cricket match between South Africa and England, South Africa scored 235 off 50 overs. After 28 overs, England had managed to score 115 runs.

a. Determine South Africa's run rate in runs per over (to one decimal place).

50 overs = 235 runs

→ 1 over = 235 runs ÷ 50

= 4,7 runs

∴ SA's run rate is 4,7 runs per over.

b. Determine England's run rate in runs per over (to one decimal place).

28 overs = 115 runs

→ 1 over = 115 runs ÷ 28

= 4,1 runs (to one decimal place)

∴ England's current run rate is 4,5 runs per over.

c. Based on your answers in a. and b., who do you think might win the match?

At the moment, SA is scoring more runs per over than England. If this continues then SA will win the match.

1. d. At what run rate (in runs per over) must England score runs from now until the end of the game in order to win the match?

Remaining overs = 50 − 28 = 22

Remaining runs needed to *draw* = 235 − 115

= 120

∴ 121 runs needed to *win.*

Run rate required to win:

22 overs = 121 runs

1 over = 121 runs ÷ 22

= 5,5 runs

∴ England must score runs at a rate of 5,5 runs per over from now until the end of the match if they want to win.

2. Trudy is driving from Pietermaritzburg to Durban airport, a distance of 120 km.

After 45 minutes she has travelled 72 km.

a. Determine the average speed (in km/h) at which she has travelled for this part of the journey.

Time = 45 min

= (45 ÷ 60) hours

= 0,75 hours

Distance = 72 km

→ 0,75 hours = 72 km

1 hour = 68 km ÷ 0,75

= 96 km

∴ Average speed = 96 km/h

b. Trudy left home at 9:00 am and she needs to be at the airport by 10:30 am. If she continues to drive at this speed, will she arrive in time?

Distance remaining = 120 km − 72 km

= 48 km

Time taken so far = 45 min

→ Time remaining = 10:30 am − 9:45 am

= 45 min

2. a. …

Time it will take to travel 48 km at 96 km/h:

96 km = 1 h

→ 1 km = 1 h ÷ 96

48km = 1 h ÷ 96 × 2824

= 0,525 h

= 30 min

So, it will take her 30 minutes to reach the airport and she only needs to be there in 45 minutes time. This means that she will make it to the airport on time if she continues to travel at this speed.

3. In 2008 Leonid Shvetsov broke the record for the Comrades Marathon. The table below shows the running time of this athlete at various places along the route.

Place on the Route	Total Running Time (h : min : sec)	Total Distance Run (km)
Cowies Hill	01:04:50	16.7
Drummond	02:42:44	42.6
Cato Ridge	03:37:43	57.1
Camperdown	04:03:54	63.8
Polly Shorts	04:57:13	79.1
Finish	---	86.8

In order to break the record Leonid Shvetsov had to finish in a time faster than 5 hours 25 min and 35 seconds. Calculate how fast (in minutes and seconds per km) Leonid Shvetsov had to run from Polly Shorts to the Finish in order to break the record.

Remaining Distance = 86,8 km − 79,1 km = 7,7 km

Remaining time to *break* the record = 5 h 23 min <u>*36*</u> sec − 4 h 57 min 13 sec

= 26 min 23 sec

= 26,383 min (to 3 decimal places)

∴ Average speed needed to break the record:

7,7 km = 26,383 min

1 km = 26,393 min ÷ 7,7

= 3,426 min (to 3 decimal places)

= 3 min + 0,426 min

= 3 min + (0,426 × 60) sec

≈ 3 min 26 sec

So, Leonid Shvetsov had to run the last 7,7 km at an average speed of 3 min 26 sec per km in order to break the record.

Test Your Knowledge: Rates

1. a. If I bought a packet of apples for R12,99 and there were 9 apples in the packet, what is the cost per apple?

Cost per apple = R12,99 ÷ 9 ≈ R1,44

b. If 1,3 kg of mince costs R42,84, what is the price per kilogram?

1,3 kg = R42,84

→ 1 kg = R42,84 ÷ 1,3

= R32,95

∴ Price per kg = R32,95

c. If I used 22 kℓ of water in June and it cost me R144,98, what is the price of water per kilolitre?

22 kℓ = R144,98

→ 1 kℓ = R144,98 ÷ 22

= R6,59

∴ Cost per kℓ is R6,59.

2. a. If petrol costs R8,24 per litre, how much would it cost to fill a 50 ℓ tank.

Petrol cost = R8,24 per litre

→ 1 litre = = R8,24

∴ Cost of 50 litres = R8,24 per litre × 50 litres

= R412,00

2. b. If you earn R650 per week for working for 5 days in the week, what is your daily rate of pay?

Daily rate of pay = R650,00 ÷ 5

= R130,00

c. Boerewors costs R32,45 / kg.

How much would $3\frac{1}{2}$ kg of boerewors cost me?

1 kg = R32,45

→ 3,5 kg = R32,45 × 3,5

= R113,58

3. The Tariffs for uShaka Sea World are as follows:

Adults:	R98 per person
Senior citizens (aged 60 +):	R85 per person
Children:	R66 per person

Calculate the cost for a family to visit uShaka if the family consists of 2 adults, 1 Grandpa and 3 children.

Cost = (2 × R98,00) + R85,00 + (3 × R66,00)

= R196,00 + R85,00 + R198,00

= R479,00

4. Which of the following items give better value for money?

a. 2,5kg of sugar at R15,69

OR

5kg of sugar at R29,75?

2,5 kg sugar = R15,69

→ 5 kg sugar = R15,69 × 2

= R31,38

∴ The 5 kg bag at R29,75 is cheaper per kilogram.

b. 100 Trinco teabags at R7,89

OR:

80 Freshpak teabags at R6,80?

Trinco:

100 teabags = R7,89

∴ Cost per 1 teabag = R7,89 ÷ 100 = R0,0789

Freshpak:

80 teabags = R6,80

∴ Cost per 1 teabag = R6,80 ÷ 80 = R0,085

So, the Trinco is cheaper per bag.

5. a. If I travel at a constant speed of 80 km/h, how long will it take me to complete a journey of 65 km? Round off your answer to the nearest minute.

80 km = 1 h

→ 1 km = 1 h ÷ 80

65 km = 1 h ÷ 80 × 65

= 0,8125 h

= 48,75 minutes

≈ 49 minutes

b. If my car has a petrol consumption rate of 6 ℓ per 100 km and the cost of petrol is R10,44 per litre, calculate how much it would cost to travel the 65 km journey.

100 km = 6 litres

→ 1 km = 6 litres ÷ 100

65 km = 6 litres ÷ 100 × 65

= 3,9 litres

Cost of petrol = R10,44 per litre × 3,9 litres

= R40,72

TOPIC 2

PATTERNS AND RELATIONSHIPS

2.1 MOVING BETWEEN TABLES, GRAPHS AND EQUATIONS

Practice Exercise: Tables, Equations and Graphs

1. Sipho is planning a birthday party and is looking for a venue to hold the party. A local sports club charge R500,00 per evening for the venue and R50,00 per person.

a. Complete the following table:

Number of guests	10	20	30	40	50	60	70	80	90	100
Cost of the party	**R1000**	**R1500**	**R2000**	**R2500**	**R3000**	**R3500**	**R4000**	**R4500**	**R5000**	**R5500**

b. How much will it cost if 120 people attend the party?

Cost = R5 500,00 + R500,00 + R500,00

= R6 500,00

OR

Cost = R500,00 + (R50,00 × 120)

= R6 500,00

c. Write down an equation to describe the cost of the party.

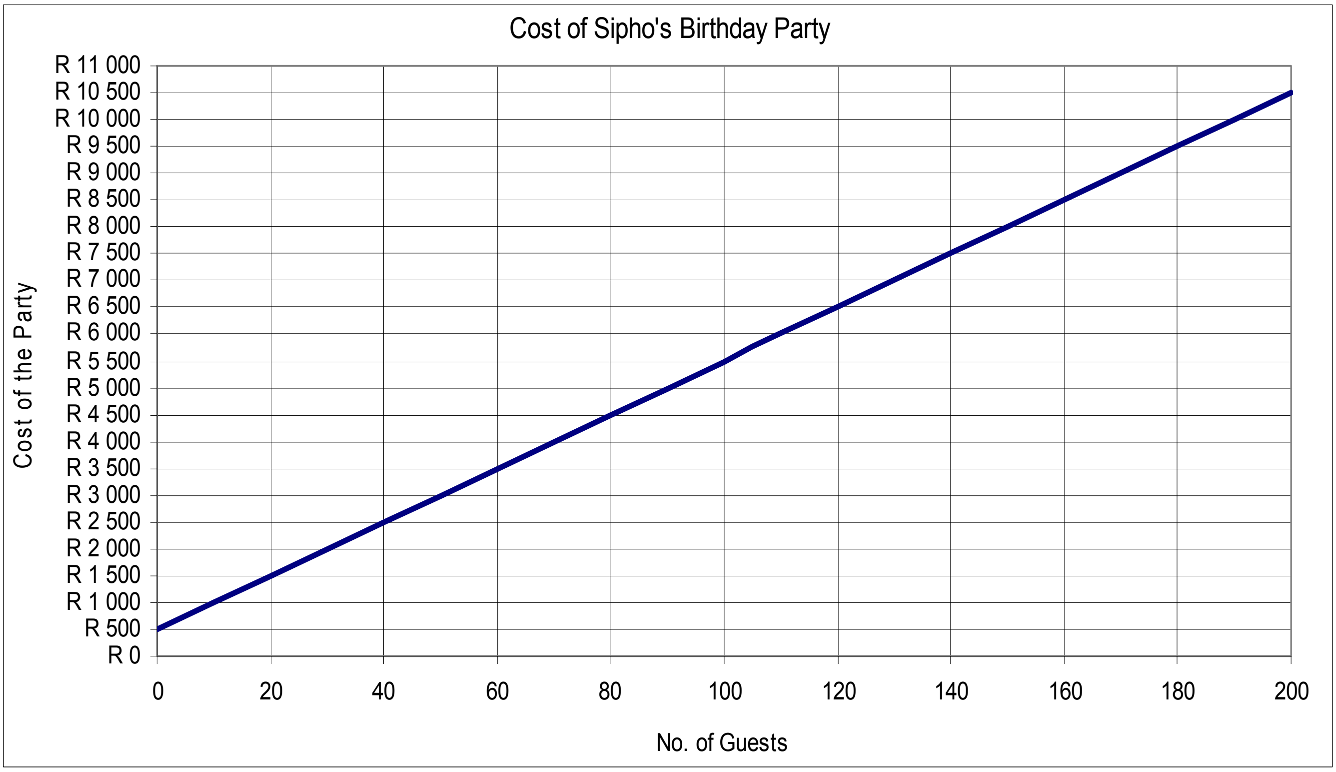
Cost = R7 500,00 + (R50,00 × no. of people)

d. Use the equation to determine the cost of the party if 167 people attend.

Cost = R500,00 + (R50,00 × 167)

= R8 850,00

e. On the set of axes below, draw a graph to show the cost of the party for up to an including 200 people.



f. Use the graph to answer the following questions:

i. How much will it cost if 180 people attend the party?

R9 500,00

ii. How much will it cost of 130 people attend the party?

R7 000,00

iii. If Sipho has to pay R9 000,00 for the party, how many people attended.

170 people

2. A metered taxi has the following rates:

- R3,00 flat-rate
- R8,50 per km travelled.

a. Complete the following table. The first two blocks in the table have been completed for you.

Distance Travelled (km)	1	2	3	4	10	20	30	40
Cost of the ride	**R11,50**	**R20,00**	**R28,50**	**R37,00**	**R88,00**	**R173,00**	**R258,00**	**R343,00**

b. Construct an equation to represent the cost of a trip in this taxi.

Cost = R3,00 + (R8,50 × distance travelled)

c. Use the equation to determine how much it would cost to travel

i. 120 km

Cost = R3,00 + (R8,50 × 120)

= R3,00 + R1 020,00

= R1 023,00

ii. 157 km

Cost = R3,00 + (R8,50 × 157)

= R3,00 + R1 334,50

= R1 337,50

d. How many km did you travel if the ride cost you R215,50?

R215,50 = R3,00 + (R8,50 × distance)

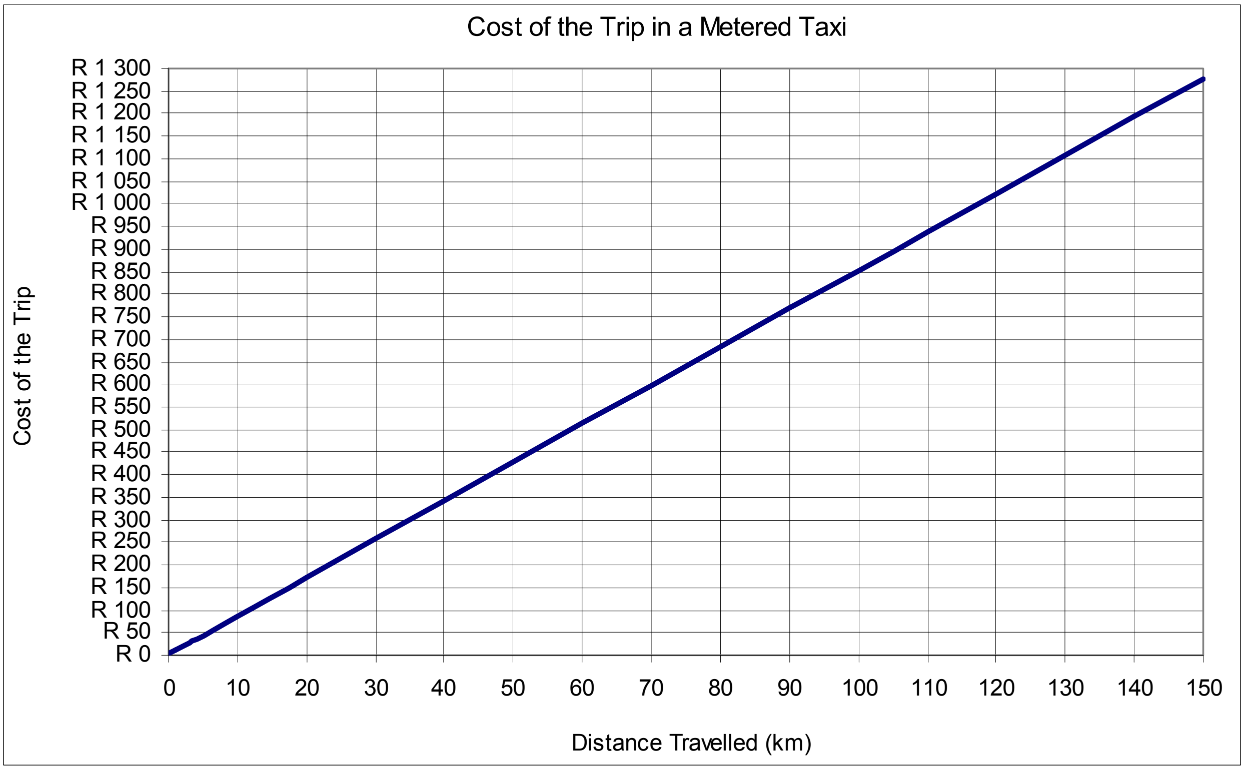
R215,50 − R3,00 = (R8,50 × distance)

R212,50 = (R8,50 × distance)

R212,50 ÷ R8,50 = distance

∴ distance = 25 km

e. On the set of axes below, draw a graph to show the relationship between the cost of a trip in the taxi and the distance travelled by the taxi for up to an including 150 km.



f. Use the graph to answer the following questions:

i. Approximately how much will it cost to travel 130 km in the taxi?

≈ R1 100,00

(accurate answer is R1 108,00)

ii. Approximately how much will it cost to travel 143 km in the taxi?

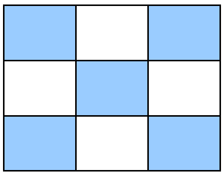
≈ R1 220,00

(accurate answer is R1 218,50)

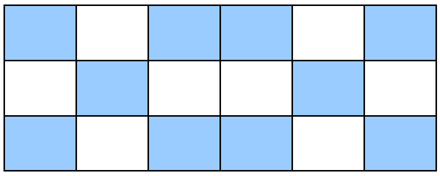
iii. If the cost of a trip in a taxi is R1 023,00, apprximately how far did the taxi travel?

≈ 120 km

3. Moira is tiling the floor in her kitchen. The picture below shows the pattern that she is going to use:



1 repeat

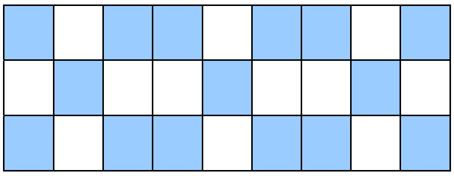


2 repeats

a. Complete the following table:

Repeat of the pattern	1	2	3	4	5	10	20	50
No. of blue tiles	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**250**

b. Draw a picture to show how many extensions there will be in 3 repeats of the pattern.



c. How many blue tiles will Moira need if she repeats the pattern 17 times?

No. of blue tiles = 17 × 5 = 85

d. If Moira were to use 75 blue tiles, how many repeats of the pattern would there be?

No. of repeats = 75 ÷ 5 = 15

e. Write down an equation to represent the relationship between the number of repeats of the pattern and the number of blue tiles in the pattern.

No. of blue tiles = No. of repeats × 5

f. Use this equation to determine how many blue tiles Moira will need if she repeats the pattern 23 times.

No. of blue tiles = 23 × 5 = 115

g. If a graph were drawn to represent this pattern, the graph would be a straight line. Explain why this is the case?

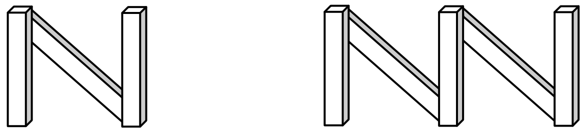
For every 1 repeat of the pattern, the number of blue tiles in the pattern increases by 5. As such, there is a constant increase in the number of blue tiles for every repeat of the pattern.

h. How many *white* tiles will Moira need if she repeats the pattern 13 times?

White tiles = No. of repeats × 4

→ **White tiles = 13 × 4 = 52**

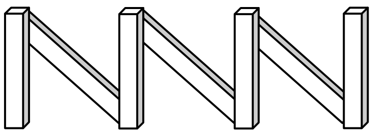
4. Jessi is building a fence around his farm. The picture below shows the design of the fence.



a. Complete the following table:

No. of extensions of the fence	1	2	3	4	10	20
No. of pieces of wood	**3**	**5**	**7**	**9**	**21**	**41**

b. Draw a picture to show how many pieces of wood there will be in 3 extensions of the fence.



c. How many pieces of wood will there be in 7 extensions of the fence?

Pieces of wood = 9 + 2 + 2 + 2 = 15

OR

Pieces of wood = (2 × 7) + 1 = 15

d. Write down an equation to represent the relationship between the number of extensions in the fence and the number of pieces of wood needed.

Pieces of wood = (no. of extensions × 2) + 1

e. Use the equation to determine how many pieces of wood Jessi will need to build a fence that has 18 extensions of the pattern.

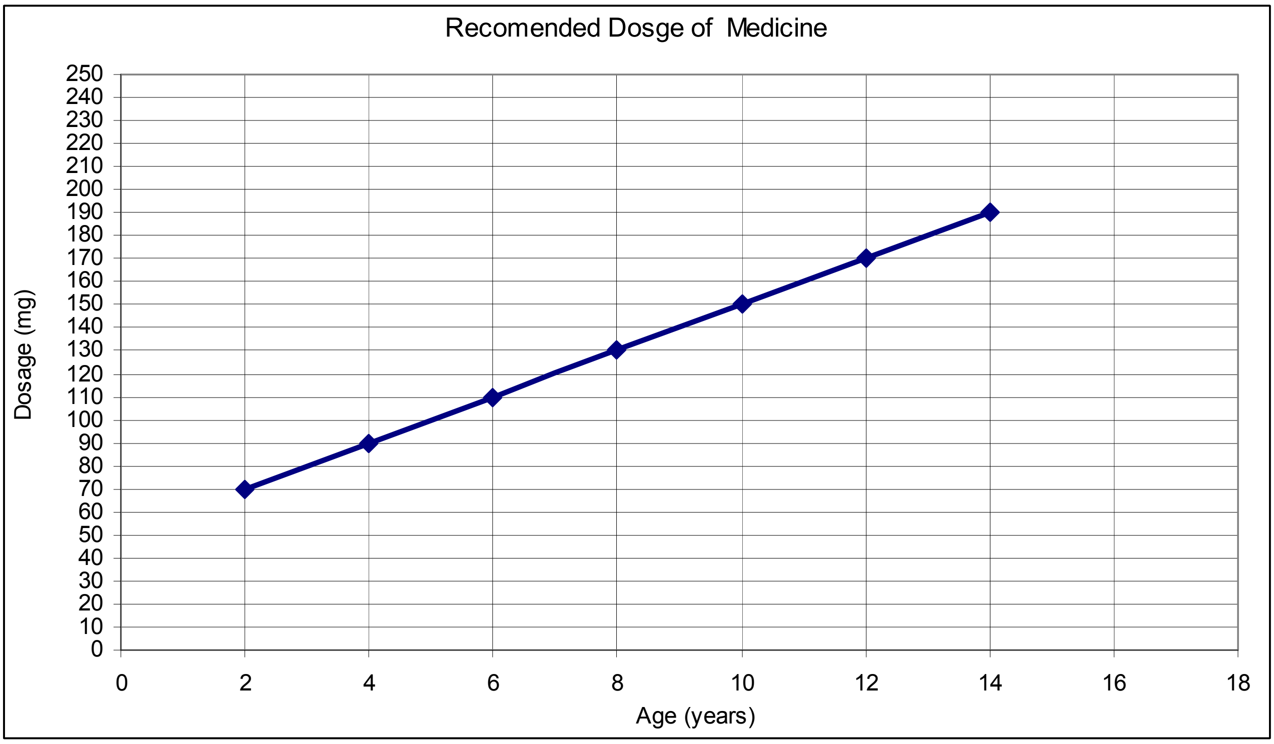
Pieces of wood = (18 × 2) + 1

= 37

f. If a graph were to be drawn to represent the relationship between the number of extensions in the fence and the number of pieces of wood needed to make the fence, what would this graph look like and why?

The graph would be a straight line. This is because for every extension that is added on to the fence, the number of pieces of wood needed increases by 2. So, there is a constant increase in the number of pieces of wood needed.

5. The graph below lists the dosage (in *mg)* of a particular drug that should be administered to children according to their age.



a. How many mg of the drug should be administered to a child who is 6 years old

110 mg

b. How old is a child if a doctor prescribes a dosage of 150mg?

10 years old

c. Estimate the dosage that should be given to a child who is 9 years old.

≈ 140 mg

d. How old do you estimate a child to be if the doctor has prescribed a dosage of 80mg?

≈ 3 years old

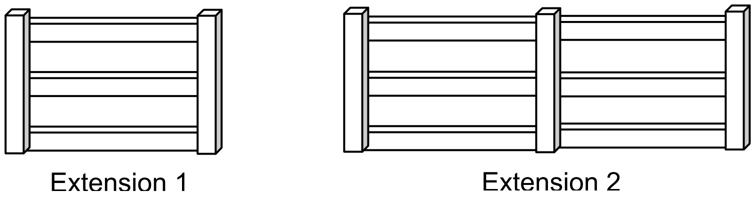
e. Extend the graph to determine the reccomended dosage of medicine for a person who is 17 years old.

≈ 220 mg

f. Write down an equation to represent the recommended dosage of medicine as dependent on the age of the child.

Dosage (mg) = (childs age × 10) + 50

6. Muchacha is building a fence around his house. The picture below shows the design of the fence:



a. Use any method to determine how many pieces of wood Muchacha will need to build a fence with 37 extensions.

To help students to see the pattern they can either construct a table of values or go straight to constructing an equation.

Pieces of wood = (no. of extensions × 4) + 1

→ wood = (37 × 4) + 1

= 149

b. If Muchacha were to use 41 pieces of wood, how many extensions of the fence would there be?

Pieces of wood = (no. of extensions × 4) + 1

41 = (no. of extensions × 4) + 1

41 − 1 = (no. of extensions × 4)

40 ÷ 4 = no. of extensions

∴ no. of extensions = 10

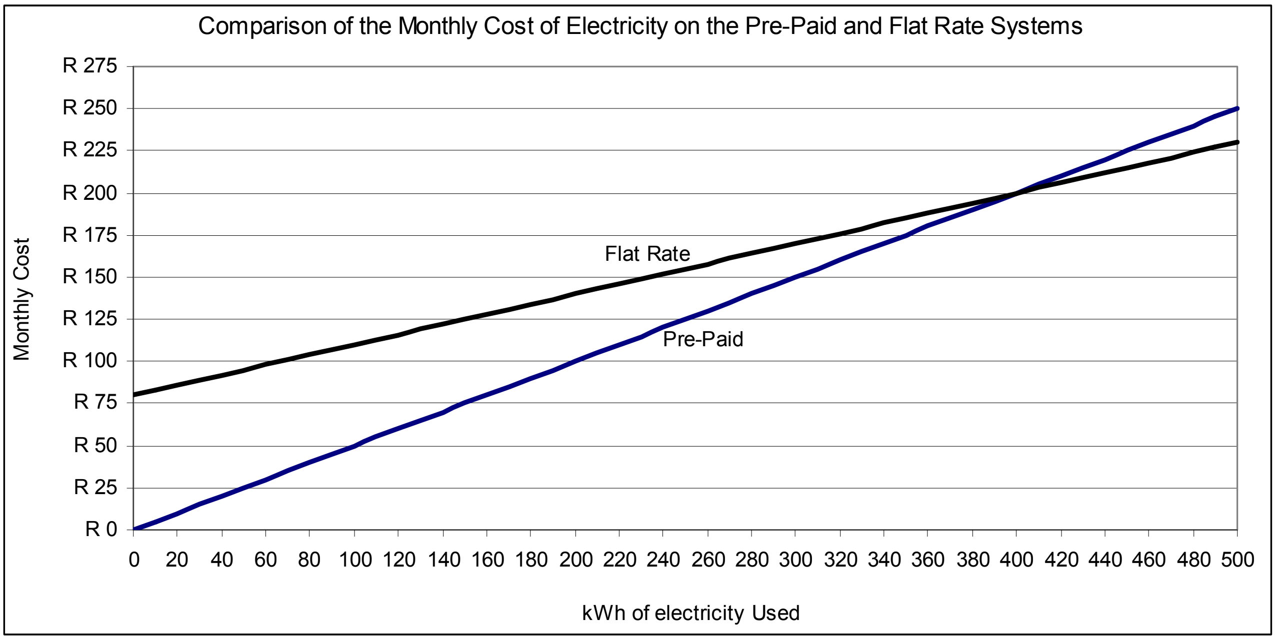
7. The table below shows the cost of pre-paid electricity and flat-rate electricity in the Mtuntili Municipality:

System	Fixed Fee	Charge per kWh
Pre-Paid	None	R0,50
Flat-Rate	R80,00	R0,30

a. Use the table below to show the difference in cost between electricity on the pre-paid system and the flat-rate system for up to and including *50 kWh* of electricity. Use an interval of 10 in the table.

System	**0**	**10**	**20**	**30**	**40**	**50**
Pre-paid	**R0,00**	**R5,00**	**R10,00**	**R15,00**	**R20,00**	**R25,00**
Flat-Rate	**R80,00**	**R83,00**	**R86,00**	**R89,00**	**R92,00**	**R95,00**

b. Use the table to help you to draw two separate graphs on the same set of axes to represent the cost of electricity on the pre-paid and flat-rate systems for up to and including *500 kWh* of electricity. You need to construct your own set of axes.



c. If a person uses an average of 320 kWh of electricity per month, should they be on the pre-paid system or the flat-rate system?

Pre-paid system

d. If a person uses an average of 450 kWh of electricity per month, should they be on the pre-paid system or the flat-rate system?

Flat rate system

e. How many kWh must a person be using every month for the cost of being on the pre-paid system to be the same as being on the flat-rate system?

400 kWh

f.

i. Write down separate equations to represent the monthly cost of electricity on the pre-paid and flat-rate systems.

Pre-paid:

Monthly cost = R0,50 × kWh

Flat-rate:

Monthly cost = R80,00 + (R0,30 × kWh)

ii. Use the equations to determine how much it would cost on both systems to use 257,3 kWh of electricity in a month.

Pre-paid: **Cost = R0,50 × 257,3 = R128,65**

Flat-rate:

Monthly cost = R80,00 + (R0,30 ×

257,3)

= R157,19

2.2 SUBSTITUTION AND SOLVING EQUATIONS

2.2.3 Substitution

Practice Exercise: Substitution

1. If $p = 5$ and $q = 4$, determine the value of each of the following:

a. $p + 2 \times q$

= 5 + 2 × 4

= 5 + 8

= 13

b. $3 \times (p + q) + p \times q$

= 3 × (5 + 4) + 5 × 4

= 3 × 9 + 20

= 27 + 20

= 47

c. $q \div 3 + 1$

= 4 ÷ 3 + 1

= 1,333 + 1

= 2,333

2. The equation below represents the cost of pre-paid electricity in a particular municipality:

Monthly Cost = R0,72 × kWh of electricity used

a. How much will it cost to use 200 kWh of electricity?

2. b. How much will it cost to use 418,7 kWh of electricity?

Monthly Cost = R0,72 × 418,7

= R301,46

3. The equation below represents the transaction fee charged for withdrawing money from a bank account over the counter at a branch.

Fee = R20,00 + (0,95% × amount withdrawn)

a. How much will it cost in transaction fees to withdraw R100,00 from the bank account at the branch?

Fee = R20,00 + (0,95% × R100,00)

$$= \text{R}20{,}00 + \left(\frac{0{,}95}{100} \times \text{R}100{,}00\right)$$

= R20,00 + R0,95

= R20,95

b. How much will it cost in transaction fees to withdraw R1 550,00 from the bank account at the branch?

Fee = R20,00 + (0,95% × R1 550,00)

= R20,00 + R14,73

Monthly Cost = R0,72 × 200

= R144,00

3\. c. Sindi withdraws R620,00 from her bank account at the branch and is charged R32,00 in transaction fees. Has she been charged the correct fee?

For a R620,00 withdrawal, the fee should be:

Fee = R20,00 + (0,95% × R620,00)

= R20,00 + R5,89

= R25,89

So, she has not been charged correctly – she has been overcharged.

4\. The formula below is used to determine the Body Mass Index (BMI) of an adult.

$$BMI\ (kg/m^2) = \frac{weight\ (kg)}{[height\ (m)]^2}$$

a. Determine the BMI of an adult who weighs 62 kg and is 1,65 m tall.

$$\textbf{BMI} = \frac{62\,\text{kg}}{(1{,}65\,\text{m})^2}$$

$$= \frac{62\,\text{kg}}{2{,}7225\,\text{m}^2}$$

$$\approx 22{,}8\ \text{kg/m}^2$$

b. Determine the BMI of an adult who weighs 92 kg and is 1,73 m tall.

$$\textbf{BMI} = \frac{92\,\text{kg}}{(1{,}73\,\text{m})^2}$$

= R34,73

4\. c. An adult who weighs 75 kg and is 2,1 m tall works out that their BMI is 17 kg/m^2. Are they correct?

The correct BMI for this person is:

$$\textbf{BMI} = \frac{75\,\text{kg}}{(2{,}1\,\text{m})^2}$$

$$= \frac{75\,\text{kg}}{4{,}41\,\text{m}^2}$$

$$\approx 17\ \text{kg/m}^2$$

So, they are correct.

d. This BMI of a person is used to determine the weight status of the adult according to the following categories.

BMI	Weight Status
<18.5	Underweight
>= 18.5 and < 25	Normal
>= 25 and < 30	Overweight
> 30	Obese

Determine the weight status of the adults with the following weights and heights:

i. Weight – 73 kg; height − 1,68 m

$$\textbf{BMI} = \frac{73\,\text{kg}}{(1{,}68\,\text{m})^2}$$

$$= \frac{73\,\text{kg}}{2{,}8224\,\text{m}^2}$$

$$\approx 25{,}9\ \text{kg/m}^2$$

∴ Weight status = overweight

$$= \frac{92\,\text{kg}}{2{,}9929\,\text{m}^2}$$

≈ 30,7 kg/m^2

ii. Weight – 105 kg; height − 1,7 m

$$\textbf{BMI} = \frac{105\,\text{kg}}{(1{,}7\,\text{m})^2}$$

$$= \frac{105\,\text{kg}}{2{,}89\,\text{m}^2}$$

≈ 36,3 kg/m^2

∴ Weight status = obese

iii. Weight – 41 kg; height − 1,55 m

$$\textbf{BMI} = \frac{41\,\text{kg}}{(1{,}55\,\text{m})^2}$$

$$= \frac{41\,\text{kg}}{2{,}4025\,\text{m}^2}$$

≈ 17,1 kg/m^2

∴ Weight status = underweight

5. To calculate the monthly repayment on a bank loan the following formula can be used:

Repayment = (loan amount ÷ 1 000) × factor

The "factor" is a value that is determined by the length of the loan and the current interest rate − various factors are given in the table below:

Factor Table					
Length	**13.5%**	**14%**	**15%**	**15.5%**	**16%**
15	12.98	13.32	14	13.34	14.69
20	12.07	12.44	13.17	13.54	13.91
25	11.66	12.04	12.81	13.20	13.59

5. a. Calculate the monthly repayment on a R200 000 loan if the length of the loan is 20 years and the interest rate is 15%.

Repayment = (loan amount ÷ 1 000) × factor

= (R200 000 ÷ 1 000) × 13,17

= R200 × 13,17

= R2 634,00

b. Calculate the monthly repayment on a R725 500,00 loan if the length of the loan is 25 years and the interest rate is 16%.

Repayment = (R725 500 ÷ 1 000) × 13,59

= R725,5 × 13,59

= R9 859,55

c. Calculate the monthly repayment on a R2 150 000,00 loan if the length of the loan is 20 years and the interest rate is 14%.

Repayment = (R2 150 000 ÷ 1 000) × 12,44

= R2 150 × 12,44

= R26 746,00

d. Calculate the monthly repayment on a R1,25 million loan if the length of the loan is 25 years and the interest rate is 15,5%.

Repayment = (R1 250 000 ÷ 1 000) × 13,20

= R1 250 × 13,20

= R16 500,00

5. e. Based on the information presented in the table and on your answers above:

i. What effect does a longer loan length have on the monthly repayments of a loan?

For a fixed loan amount: the longer the loan period, the smaller the monthly repayment.

ii. What effect do changes in the interest rate have on the monthly repayments of a loan?

For a fixed loan amount: the higher the interest rate the higher the monthly repayment; the lower the interest rate, the lower the monthly repayment.

2.2.4 Solving Equations

Practice Exercise: Solving Equations

1. Determine the value of *p* in each of the following equations:

a. *p* + 7 = 15

p = 8

b. 3 × *p* – 8 = 28

p = (28 + 8) ÷ 3

p = 36 ÷ 3

p = 12

c. 2 × *(p* – 2) = 14

p = (14 ÷ 2) + 2

= 7 + 2

= 9

2. a. The equation below represents the cost of pre-paid electricity in a particular municipality:

Monthly Cost = R0,72 × kWh of electricity used

i. If a person spends R250,00 on electricity, how many kWh of electricity have they used?

Monthly Cost = R0,72 × kWh

R250,00 = R0,72 × kWh

→ kWh = R250,00 ÷ R0,72

≈ 347,2 kWh

2. a. ii. If a person spends R317,50 on electricity, how many kWh of electricity have they used?

Monthly Cost = R0,72 × kWh

R317,50 = R0,72 × kWh

→ kWh = R317,50 ÷ R0,72

= 440,97

≈ 441 kWh

b. The equation below represents the cost of flat-rate electricity in the same municipality:

Monthly Cost = R92,00 + (R0,55 × kWh)

i. What is the fixed monthly service fee on the flat-rate system?

R92,00 → i.e. If no kWh of electricity are used a person will still pay R92,00.

ii. What is the per kWh charge for electricity on the flat-rate system?

R0,55

iii. If a person receives an electricity bill for R300,00, how many kWh of electricity have they used during the month?

Monthly Cost = R92,00 + (R0,55 × kWh)

R300,00 = R92,00 + (R0,55 × kWh)

R300,00 − R92,00 = (R0,55 × kWh)

R208,00 ÷ R0,55 = kWh

∴ kWh ≈ 378,2

iv. If a person receives an electricity bill for R412,27, how many kWh of electricity have they used during the month?

Monthly Cost = R92,00 + (R0,55 × kWh)

R412,27 = R92,00 + (R0,55 × kWh)

R412,27 − R92,00 = (R0,55 × kWh)

R320,27 ÷ R0,55 = kWh

∴ kWh ≈ 582,3

2. c. A person uses an average of 420 kWh of electricity per month. Should they be on the pre-paid system or the flat-rate system? Explain.

Pre-paid:

Monthly cost = R0,72 × 420

= R302,40

Flat-rate:

Monthly cost = R92,00 + (R0,55 × 420)

= R92,00 + R231,00

= R323,00

∴ The person should be on the pre-paid system.

3. The equation below represents the transaction fee charged for withdrawing money from a bank account over the counter at a branch.

Fee = R20,00 + (0,95% × amount withdrawn)

a. Write 0,95% as a decimal value.

0,0095

b. If a person pays R24,75 in transaction fees, how much have they withdrawn from the bank?

Fee = R20,00 + (0,95% × amount withdrawn)

R24,75 = R20,00 + (0,0095 × withdrawal)

R24,75 − R20,00 = (0,0095 × withdrawal)

R4,75 = 0,0095 × withdrawal

R4,75 ÷ 0,0095 = withdrawal

∴ Withdrawal = R500,00

3. c. If a person pays R110,25 in transaction fees, how much have they withdrawn from the bank?

Fee = R20,00 + (0,95% × amount withdrawn)

R110,25 = R20,00 + (0,0095 × withdrawal)

R110,25 − R20,00 = (0,0095 × withdrawal)

R90,25 = 0,0095 × withdrawal

R90,25 ÷ 0,0095 = withdrawal

∴ Withdrawal = R9 500,00

4. The formula below is used to determine the Body Mass Index (BMI) of an adult.

$$BMI\ (kg/m^2) = \frac{weight\ (kg)}{[height\ (m)]^2}$$

a. If a person is 1,68 m tall and has a BMI of 21,05 kg/m^2, how much do they weigh?

$$\mathbf{21{,}05\ kg/m^2 = \frac{weight\ (kg)}{[1{,}68\ m]^2}}$$

21,05 kg/m^2 × [1,68 m]^2 = weight (kg)

21,05 kg/m^2 × 2,8224 m^2 = weight (kg)

∴ weight (kg) ≈ 59,4 kg

b. If a person is 1,77 m tall and has a BMI of 25,86 kg/m^2, how much do they weigh?

$$\mathbf{25{,}86\ kg/m^2 = \frac{weight\ (kg)}{[1{,}77\ m]^2}}$$

25,86 kg/m^2 × [1,77 m]^2 = weight (kg)

25,86 kg/m^2 × 3,1329 m^2 = weight (kg)

∴ weight (kg) ≈ 81 kg

4. c. If a person weighs 75 kg and has a BMI of 25,95 kg/m^2, how tall are they?

$$\mathbf{25{,}95\ kg/m^2 = \frac{75\ kg}{[height\ (m)]^2}}$$

25,95 kg/m^2 × [height (m)]^2 = 75 kg

→ [height (m)]^2 = 75 kg ÷ 25,95 kg/m^2

[height (m)]^2 = 2,890 m^2

∴ height = $\sqrt{2{,}890\ m^2}$

= 1,7 m

4. d. If a person weighs 61 kg and has a BMI of 28,23 kg/m^2, how tall are they?

$$\mathbf{28{,}23\ kg/m^2 = \frac{61\ kg}{[height\ (m)]^2}}$$

28,23 kg/m^2 × [height (m)]^2 = 61 kg

→ [height (m)]^2 = 61 kg ÷ 28,23 kg/m^2

[height (m)]^2 = 2,161 m^2

∴ height = $\sqrt{2{,}161\ m^2}$

≈ 1,47 m

5. To calculate the monthly repayment on a bank loan the following formula can be used:

Repayment = (loan amount ÷ 1 000) × factor

The "factor" is a value that is determined by the length of the loan and the current interest rate − various factors are given in the table below:

Factor Table					
Length	**13.5%**	**14%**	**15%**	**15.5%**	**16%**
15	12.98	13.32	14	13.34	14.69
20	12.07	12.44	13.17	13.54	13.91
25	11.66	12.04	12.81	13.20	13.59

a. On a R800 000,00 loan at an interest rate of 15,5%, a person pays R10 832,00 in monthly repayments. What is the length of the loan?

Repayment = (loan amount ÷ 1 000) × factor

R10 832,00 = R800 000,00 ÷ 1 000 × factor

R10 832,00 = R800,00 × factor

R10 832,00 ÷ R800,00 = factor

→ factor = 13,54

With an interest rate of 15,5%, this factor gives a length of 20 years.

b. On a R1 250 000,00 loan at an interest rate of 14%, a person pays R15 050,00 in monthly repayments. What is the length of the loan?

R15 050,00 = R1 250 000,00 ÷ 1 000 × factor

R15 050,00 = R1 250,00 × factor

R15 050,00 ÷ R1 250,00 = factor

→ factor = 12,04

With an interest rate of 14%, this factor gives a length of 25 years.

5. c. On a R320 000,00 loan with a length of 15 years, a person pays R4 480,00 in monthly repayments. What is the interest rate on the loan?

R4 480,00 = R320 000,00 ÷ 1 000 × factor

R4 480,00 = R320,00 × factor

R4 480,00 ÷ R320,00 = factor

→ factor = 14

With a length of 15 years, this factor gives an interest rate 15%.

d. On a R2 500 000 loan with a length of 20 years, a person pays R34 775,00 in monthly repayments. What is the interest rate on the loan?

R34 775,00 = R2 500 000,00 ÷ 1 000 × factor

R34 775,00 = R2 500,00 × factor

R34 775,00 ÷ R2 500,00 = factor

→ factor = 13,91

With a length of 20 years, this factor gives an interest rate 16%.

e. A person pays R5 045,26 in monthly repayments on a loan. If the interest rate on the loan is 13,5% and the length of the loan is 20 years, calculate the size of the loan.

R5 045,26 = (loan amount ÷ 1000) × 12,07

R5 045,26 ÷ 12,07 × 1 000 = loan amount

∴ loan amount = R418 000,00

6. The table below shows the transfer fees that have to be paid when buying a house.

Property Value	Transfer Fee
≤ R500 000	0%
R500 001 to R1 Million	R25 000,00
Above R1 000 001	R25 000,00 + 8% on the value above R1 Million

a. Calculate the transfer fee on a R1 250 000,00 house.

Transfer duty = R25 000,00 + (8% × value above R1 million)

= R25 000,00 + (8% × [R1 250 000 − R1 000 000])

= R25 000,00 + (8% × R250 000,00)

= R25 000,00 + R20 000,00

= R45 000,00

b. Calculate the transfer fee on a R2 425 500,00 house.

Transfer duty = R25 000,00 + (8% × [R2 425 500 − R1 000 000])

= R25 000,00 + (8% × R1 425 500,00)

= R25 000,00 + R114 040,00

= R139 040,00

6. c. If the transfer duty on a house amounts to R65 000,00, what is the price of the house?

Transfer duty = R25 000,00 + (8% × value above 1 million

R65 000,00 = R25 000,00 + (8% × value above 1 million

R65 000,00 − R25 000,00 = 8% × value above 1 million

R40 000,00 = 8% × value above R1 million

R40 000,00 = 0,08 × value above R1 million

R40 000,00 ÷ 0,08 = value above R1 million

R500 000,00 = value above R1 million

∴ Price of the house = R1 500 000,00

c. If the transfer duty on a house amounts to R201 000,00, what is the price of the house?

R201 000,00 = R25 000,00 + (8% × value above 1 million

R201 000,00 − R25 000,00 = 8% × value above 1 million

R176 000,00 = 8% × value above R1 million

R176 000,00 = 0,08 × value above R1 million

R176 000,00 ÷ 0,08 = value above R1 million

R2 200 000,00 = value above R1 million

∴ Price of the house = R3 200 000,00

Test Your Knowledge: Tables, Equations and Graphs

The table below shows the electricity tariffs for pre-paid electricity and flat-rate electricity in a municipality.

Electricity System	Fixed Monthly Service Fee	Charge per kWh
Pre-Paid	Nil	R0,75
Flat-Rate	R75,00	R0,50

1. Calculate the cost of using 317,2 kWh of electricity on the:

a. Pre-paid system

Cost = R0,75/kWh × 317,2 kWh

= R237,90

b. Flat-rate system

Cost = R75,00 + (R0,50/kWh × 317,2 kWh)

= R75,00 + R158,60

= R233,60

2. Write down separate equations to represent the monthly cost of pre-paid electricity and the monthly cost of flat-rate electricity.

Pre-paid:

Monthly cost = R0,75/kwh × kWh used

Flat-rate:

Monthly cost = R75,00 + R0,50/kwh × kWh used

3. a. Use the appropriate equation to determine how many kWh of electricity a person has used during the month if they spend R325,00 on pre-paid electricity.

Monthly cost = R0,75/kwh × kWh used

R325,00 = R0,75/kWh × kWh used

R325,00 ÷ R0,75/kWh = kWh used

∴ kWh used ≈ 433,3

b. Use the appropriate equation to determine how many kWh of electricity a person has used during the month if they spend R516,29 on flat-rate electricity.

Monthly Cost = R75,00 + (R0,50/kWh × kWh)

R516,29 = R75,00 + (R0,50/kWh × kWh)

R516,29 − R75,00 = R0,50/kWh × kWh

R441,29 = R0,50/kWh × kWh

R441,29 ÷ R0,50/kWh = kWh

∴ kWh used ≈ 882,6 kWh

4. Use the equations to construct a table of values showing the cost of pre-paid and flat-rate electricity. Use the table below to help you.

kWh of Electricity Used	Pre-Paid Monthly Cost	Flat-Rate Monthly Cost
0	**R 0,00**	**R 75,00**
50	**R 37,50**	**R 100,00**
100	**R 75,00**	**R 125,00**
200	**R 150,00**	**R 175,00**
300	**R 225,00**	**R 225,00**
400	**R 300,00**	**R 275,00**
500	**R 375,00**	**R 325,00**

5. Use the table to draw two separate graphs on the same set of axes to represent the cost of pre-paid electricity and flat-rate electricity. Use the blank set of axes below.

(see below for the solution)

Use the *graph* to answer the following questions:

6. a. If a person uses 250 kWh of electricity, approximately how much will they pay in electricity costs on the:

i. Pre-paid system?

≈ R188,00 (accurate answer is R187,50)

ii. Flat-rate system?

Exactly R200,00

6. b. If a person spends on average R260,00 on electricity every month, how many kWh of electricity would they be using on the:

i. Pre-paid system?

≈ 345 kWh (accurate answer is 346,7 kWh)

ii. Flat-rate system?

Exactly 370 kWh

c. If a person uses an average of 450 kWh of electricity per month, should they be on the pre-paid or the flat-rate system? Explain.

Definitely on the flat-rate system. On the pre-paid system this usage would cost ≈ R340,00 per month, while on the flat-rate system it would cost R40,00 less at R300,00 per month.

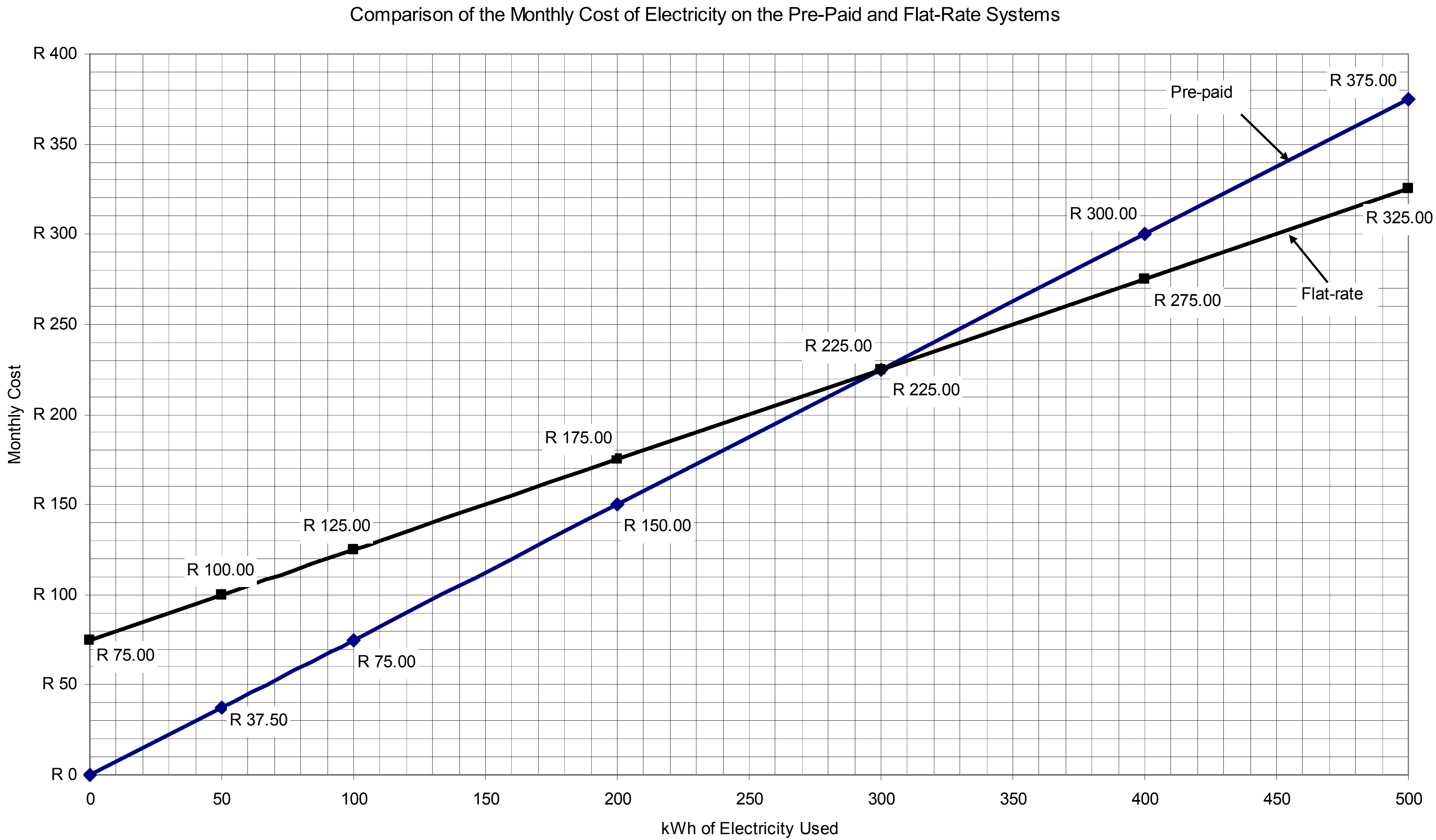
d. Approximately how many kWh of electricity must a person be using every month in order for it to be more expensive to be on the pre-paid system rather than the flat-rate system?

Less than 300 kWh

e. Approximately how much money must a person be spending on electricity every month in order for it to be more expensive to be on the pre-paid system rather than the flat-rate system?

More than R255,00

5.


Comparison of the Monthly Cost of Electricity on the Pre-Paid and Flat-Rate Systems
Monthly Cost
R 400
R 350
R 300
R 250
R 200
R 150
R 100
R 50
R 0
0
50
100
150
200
250
300
350
400
450
500
kWh of Electricity Used
Pre-paid
Flat-rate
R 375.00
R 325.00
R 300.00
R 275.00
R 225.00
R 225.00
R 175.00
R 150.00
R 125.00
R 100.00
R 75.00
R 75.00
R 37.50

TOPIC 3

SPACE, SHAPE & ORIENTATION

3.1 CONVERTING UNITS OF MEASUREMENT

Practice Exercise: Converting Units of Measurement

1. Use the table below to answer the convert the given values to the given unit of measurement.

Length	Volume	Weight
1 km = 1 000 m	1 litre = 1 000 ml	1 kg = 1 000 g
1 m = 100 cm	1 m^3 = 1 000 litres	1 g = 1 000 mg
1 cm = 10 mm	1 ml = 1 cm^3	1 tonne = 1 000 kg

1. a. 1 500 m = **1,5** km

b. 15,325 km = **15 325** m

c. 165 mm = **16,5** cm = **0,165** m

d. 1,25 m = **125** cm = **1 250** mm

e. 1,275 litres = **1 275** ml

f. 723 ml = **0,723** litres

g. 450 g = **0,45** kg

h. 312 kg = **0,312** tones = **312 000** grams

i. 575 cm^3 = **575 ml**

= 0,575 litres

=0,000575 m^3

2. The table below shows the conversion ratios for converting from metric to imperial measurements.

Length	Capacity	Weight
1 mile = 1,609 km	1 gallon = 4,5461 litres	1 pound = 0,4536 kg
1 foot = 0,3048 m		
1 inch = 25,4 mm		

a. 3 miles = **1,609 km × 3 = 4,827 km**

b. 8,5 miles = **1,609 km × 8,5 = 13,6765 km**

c. 5 feet = **0,3048 m × 5 = 1,524 m**

d. 143 pounds = **0,4536 kg × 143**

= 64,8648 kg

e. 1 km = **1 mile ÷ 1,609 = 0,622 miles**

f. 1 m = **1 foot ÷ 0,3048 = 3,2808 feet**

g. 1 litre = **1 gallon ÷ 4,5461 ≈ 0,22 gallons**

h. 125 mm = **1 inch ÷ 25,4 × 125**

= 4,92 inches

i. 72 kg = **1 pound ÷ 0,4536 × 72**

= 158,73 pounds

j. 3 feet = **0,3048 m × 3**

= 0,9144 m

=91,44 cm

k. 1 572 ml =**1,572 litres**

= 1 gallon ÷ 4,5461 × 1,572

= 0,346 gallons

l. 3 500 m = **3,5 km**

= 1 mile ÷ 1,609 × 3,5 = 2,175 miles

3. The table below shows the conversion ratios for converting from ml to grams and grams to ml for different cooking ingredients.

Ingredients	5 ml	12,5 ml	25 ml	100 ml
Flour	3 g	8 g	15 g	60 g
Margarine	5 g	12,5 g	25 g	100 g
Mealie Meal	3 g	6 g	12 g	50 g
Rice	4 g	10 g	20 g	80 g
Brown & White Sugar	4 g	10 g	20 g	80 g

1 cup = 250 ml 1 tablespoon = 15 ml 1 teaspoon = 5 ml

a. How many ml of flour is equal to 6 g of flour?

3 g = 5 ml

∴ 6 g = 10 ml

b. How many ml of sugar is equal to 40 g of sugar?

20 g = 25 ml

∴ 40 g = 50 ml

c. How many grams of margarine is equal to 100 ml of margarine?

100 g

d. How many ml of mealie meal is equal to 112 g of mealie meal?

112 g = 50 g + 50 g + 12 g

= 100 ml + 100 ml + 25 ml

= 225 ml

e. How many ml of sugar is equal to 130 g of sugar?

130 g = 80 g + 20 g + 20 g + 10 g

= 100 ml + 25 ml + 25 ml + 12,5 ml

= 162,5 ml

f. How many ml of rice is equal to 450 g of rice?

450 g rice = (80 g × 5) + (20 g × 2) + 10 g

= (100 ml × 5) + (25 ml × 2) + 12,5 ml

= 500 ml + 50 ml + 12,5 ml

= 562,5 ml

g. How many ml of mealie meal is equal to 280 grams of mealie meal?

280 g = (50 g × 5) + (3 g × 10)

= (100 ml × 5) + (5 ml × 10)

= 500 ml + 50 ml

= 550 ml

h. How many grams of flour is equal to 290 ml of flour?

290 ml =

(100 ml × 2) + (25 ml × 3) + (5 ml × 3)

= (60 g × 2) + (15 g × 3) + (3 g × 3)

= 120 g + 45 g + 9 g

= 174 g

3. i. How many grams of rice is equal to 2 cups of rice?

2 cups = 2 × 250 ml = 500 ml

100 ml = 80 g

∴ 500 ml = 80 g × 5

= 400 g

j. How many grams of sugar is equal to 3 tablespoons of sugar?

3 Tbsp = 15 ml × 3 = 45 ml

5 ml = 4 g

∴ 45 ml = 4 g × 9

= 36 g

k. How many cups of flour is equal to 450 g of flour?

3 g = 5 ml

→ 450 g = 5 ml ÷ 3 × 450

= 750 ml

= 3 cups

(i.e. 250 ml × 3 = 750 ml)

L. How many tablespoons of sugar is equal to 60 g of sugar?

60 g sugar = 20 g × 3

= 25 ml × 3

= 75 ml

1 Tbsp = 15 ml

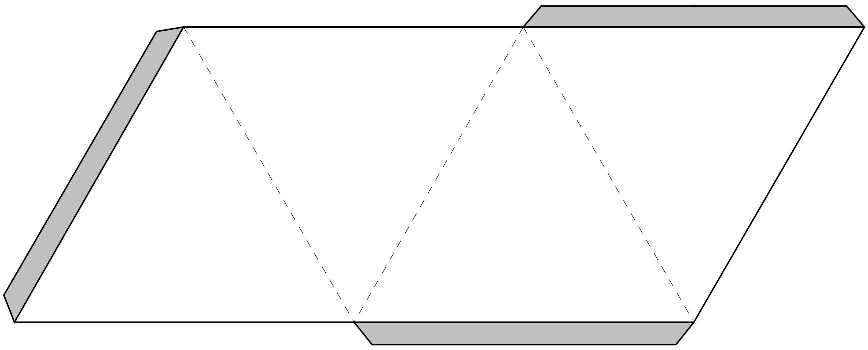
∴ 75 ml = 5 tablespoons

3.2 WORKING WITH 2-D PICTURES AND 3-D SHAPES

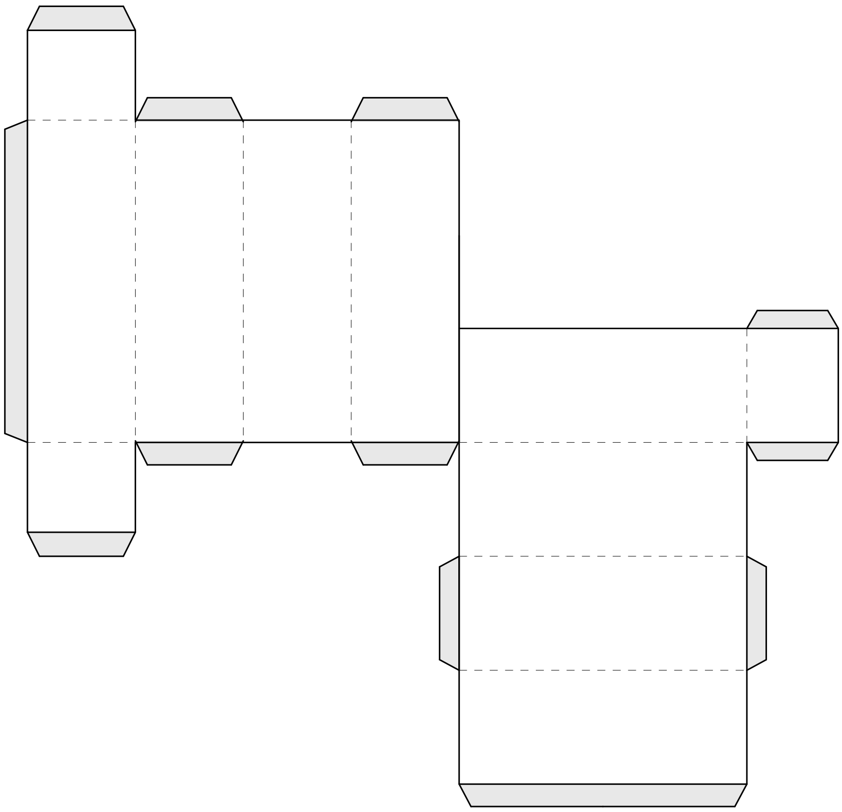
3.2.2 Moving from 3-D Shapes to 2-D Pictures

Activity 1: Constructing nets

1.

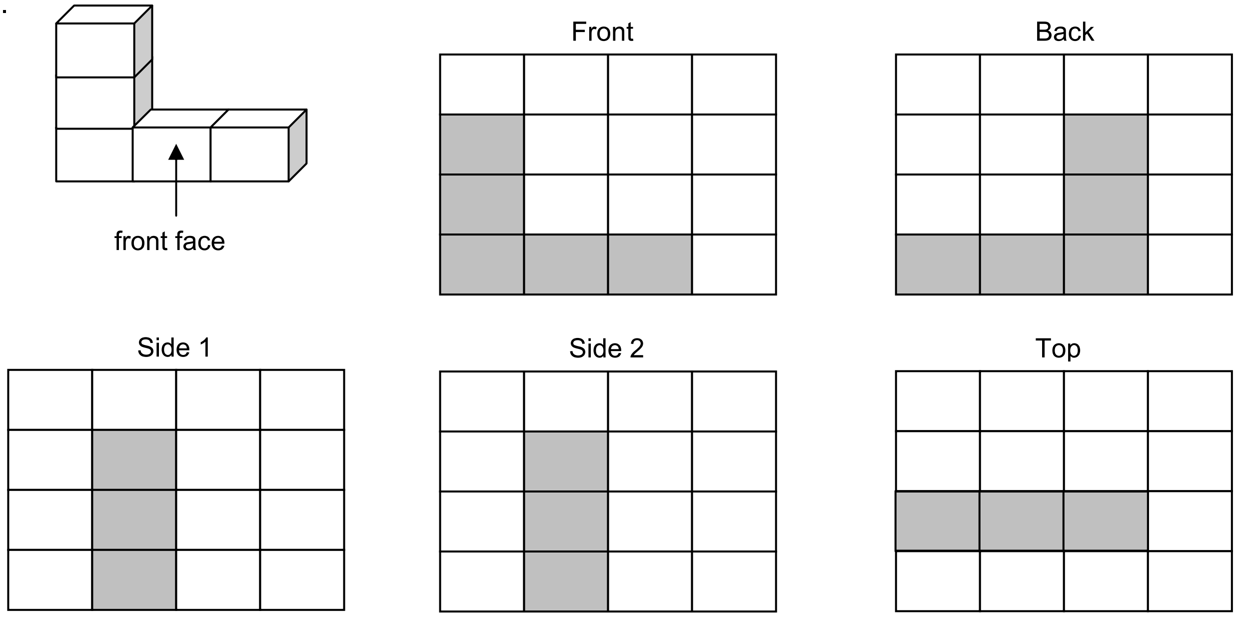


2.

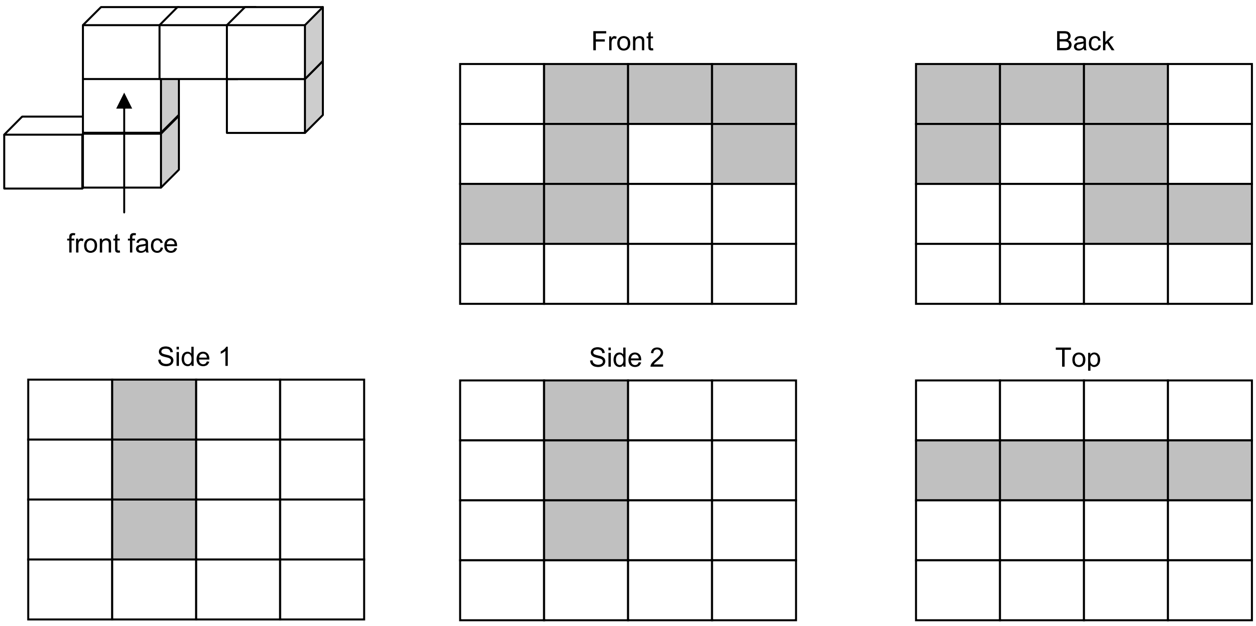


Activity 2: Drawing different perspectives

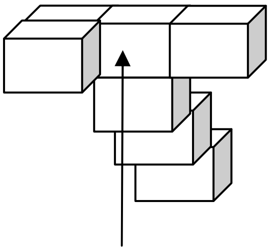
1.



2.

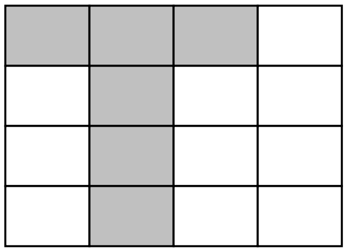


3.

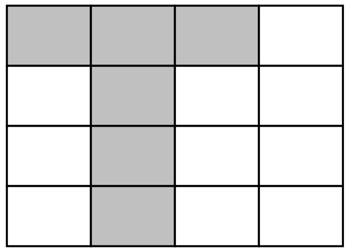


front face

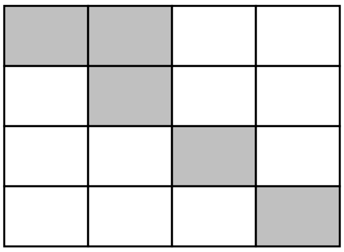
Front



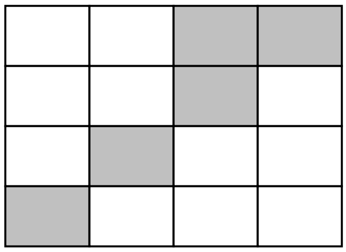
Back



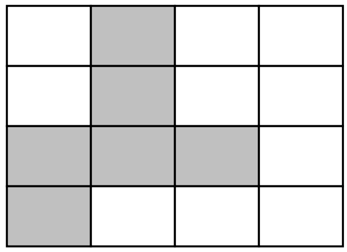
Side 1



Side 2



Top



3.3 AREA

3.3.2 Discovering Area Formulas

A. Area of a Rectangle / Square

Activity:

1.

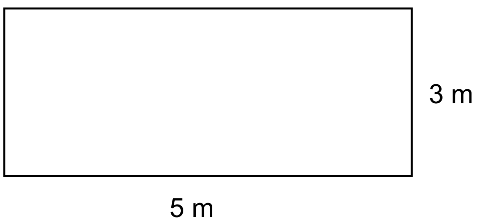
Shape number	1	2	3	4	5	6	7	8	9
Length	4	8	12	6	3	24	5	10	4
Breadth	6	3	2	4	7	1	3	2	4
Number of squares	24	24	24	24	21	24	15	20	16

2. No. of blocks = no. of blocks along length × no. of blocks along breadth

Practice Exercise: Area of Rectangles

1. Calculate the areas of the following shapes:

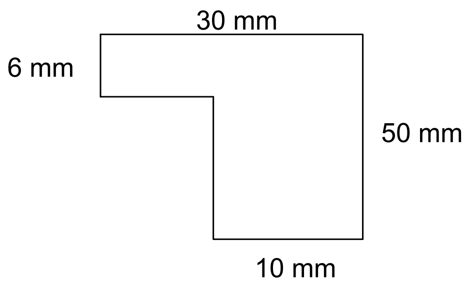
a.



Area = 5 m × 3 m

= 15 m^2

b.

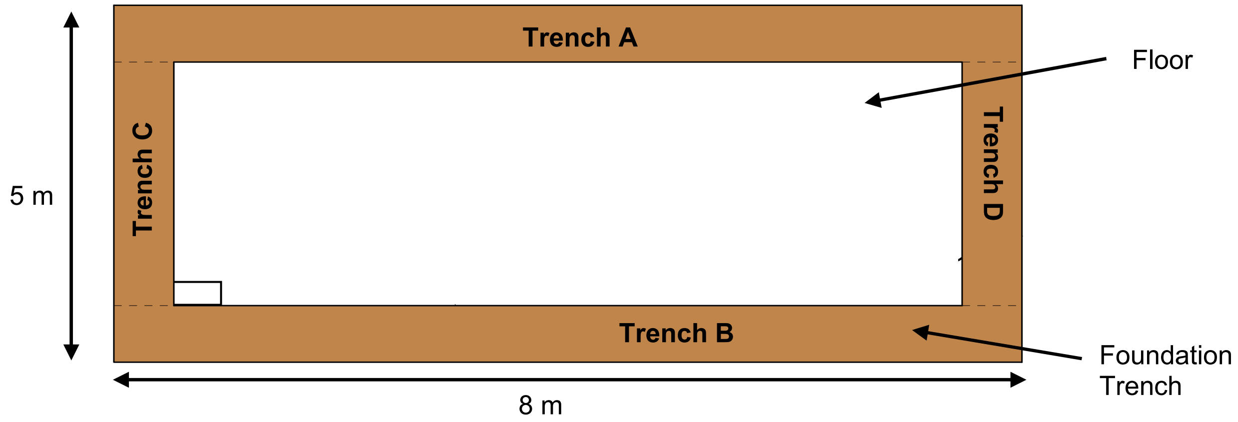


Area = (6 mm × 30 mm) + (10 mm × 44 mm)

= 180 mm^2 + 440 mm^2

= 620 mm^2

2. Zipho is building a house. The picture below shows the dimensions of the floor and foundation trench of the house.



a. Determine the area of the floor.

Length of floor = 5 m − 1,2 m − 1,2 m

= 2,6 m

Width of floor = 8 m − 1,2 m − 1,2 m

= 5,6 m

∴ Area of floor = 2,6 m × 5,6 m = 14,56 m^2

b. Once the floor has been built, Zipho plans to tile the floor with square tiles that are 0,8 m long and 0,8 m wide. Approximately how many tiles will Zipho need for the floor?

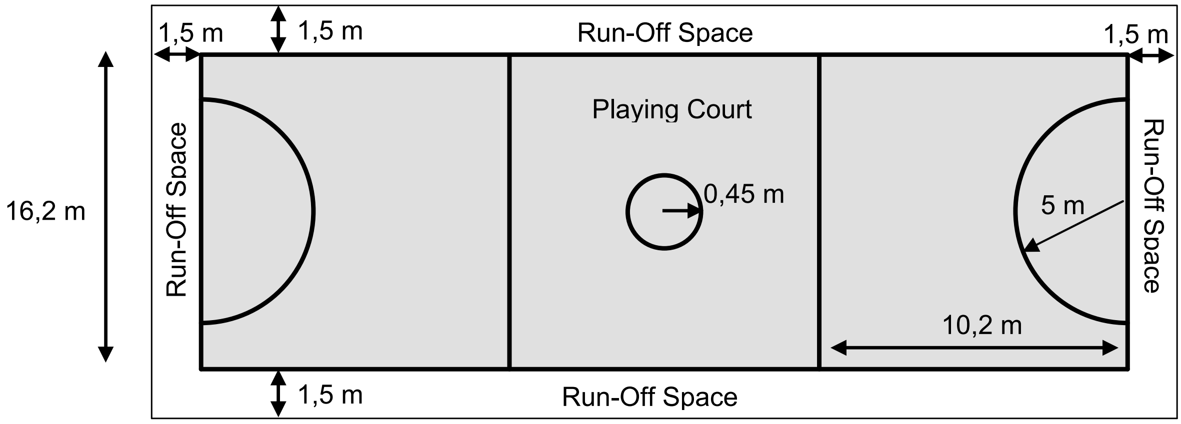
Area of 1 tile = 0,8 m × 0,8 m = 0,64 m^2

∴ No. of tiles needed = 14,56 m^2 ÷ 0,64 m^2

= 22,75 tiles

= 23 full tiles

3. The picture below shows the dimensions of a netball court. The court is surrounded by a “runoff space”. This is extra space around the side of the playing court so that the players have space to run if they leave the court.



a. The caretaker wants to repaint the *playing court*.

i. Calculate the surface area of the playing court.

Length of court = 10,2 m × 3 = 30,6 m

Width of playing court = 16,2 m

∴ Area of playing court = 16,2 m × 30,6 m

= 495,72 m^2

ii. If the paint that the caretaker will use has a coverage of 4 m^2 per litre, calculate how many litres of paint the caretaker will need to buy.

Paint coverage: 4 m^2 = 1 litre

→ 1 m^2 = 1 litre ÷ 4

495,72 m^2 = 1 litre ÷ 4 m × 495,72

= 123,93 litres

= 124 full litres

b. The caretaker also wants to repaint the lines on the playing court. Calculate how many metres of lines he needs to repaint.

(You may need to use the following formula: *Perimeter of a circle = 2 × π × radius of circle*)

Total length of straight lines

= [(10,2 m × 3) × 2] + (16,2 m × 4)

= 61,2 m + 64,8 m

= 126 m

Total circular areas

= (2 × π × 0,45 m) + (2 × π × 5 m)

= 2,8278 m + 31,42 m

= 34,2478 m

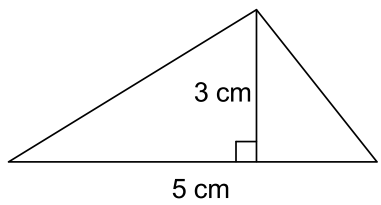
∴ Total lines = 126 m + 34,2478 m ≈ 160,2 m

B. Area of a Triangle

Practice Exercise: Area of Triangles (+ Rectangles)

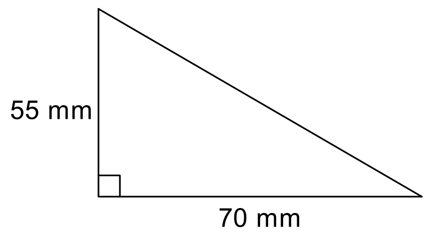
1. Calculate the areas of the following triangles:

a.



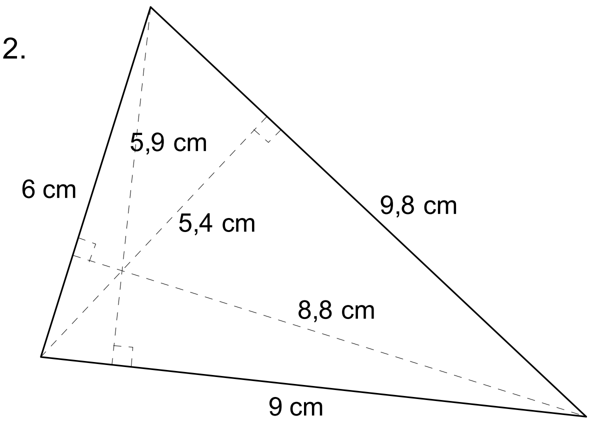
Area = ½ × 5 cm × 3 cm = 7,5 cm^2

b.



Area = ½ × 70 mm × 55 mm = 1 925 mm^2

2.



a. Determine the area of the triangle using:

i. Height 5,9 cm and base 9 cm

Area = ½ × 9 cm × 5,9 cm = 26,55 cm^2

ii. Height 8,8 cm and base 6 cm

Area = ½ × 6 cm × 8,8 cm = 26,4 cm^2

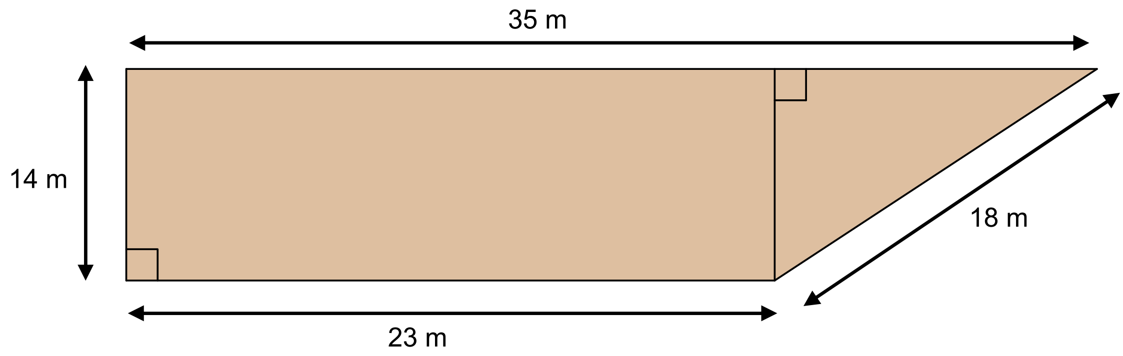
iii. Height 5,4 cm and base 9,8 cm

Area = ½ × 9,8 cm × 5,4 cm = 26,46 cm^2

b. Compare the areas that you calculated in (a). What do you notice?

The areas are almost identical. Every triangle has 3 heights and 3 bases, and using any base and its perpendicular height will give the same area for the triangle.

3. Imraan owns the piece of land pictured below.



a. Imraan needs to work out the area of the land so that he knows how much land he has to buy fertiliser for.

i. Calculate the area of the rectangular portion of the piece of land.

Area = 23 m × 14 m = 322 m^2

ii. Calculate the area of the triangular portion of the piece of land.

Base = 35 m − 23 m = 12 m

Height = 14 m

∴ Area = ½ × 12 m × 14 m = 84 m^2

iii. Calculate the total area of the piece of land.

Total area = 322 m^2 + 84 m^2 = 406 m^2

iv. The fertilizer that Imraan intends to use has a coverage of 1,5 m^2 per bag. How many bags of fertilizer will Imraan need to fertilise the whole plot of land?

Coverage: 1,5 m^2 = 1 bag

1 m^2 = 1 bag ÷ 1,5

406 m^2 = 1 bag ÷ 1,5 × 406

≈ 270,7 bags

= 271 full bags

b. Imraan wants to erect a fence around the outside of the piece of land. The fence will be supported by wooden poles that will be spaced 2 m apart from each other.
How many wooden poles will Imraan for the whole fence?

Total perimeter
= 35 m + 18 m + 23 m + 14 m = 90 m

∴ No. of poles = 90 m ÷ 2 m spacing

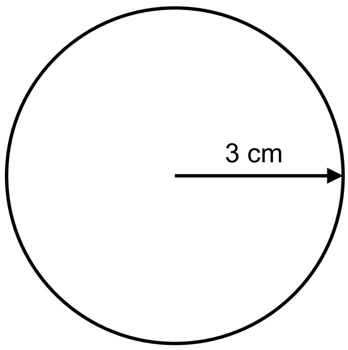
= 45 poles

C. Area of a Circle

Practice Exercise: Area of Circles (+ Rectangles)

1. Calculate the areas of the following circles:

a.

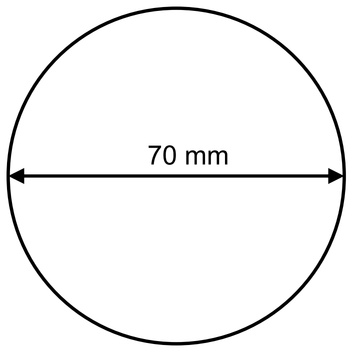


Area = $\pi \times (3\text{ cm})^2$

$= \pi \times 9\text{ cm}^2$

$= 28{,}278\text{ cm}^2$

b.



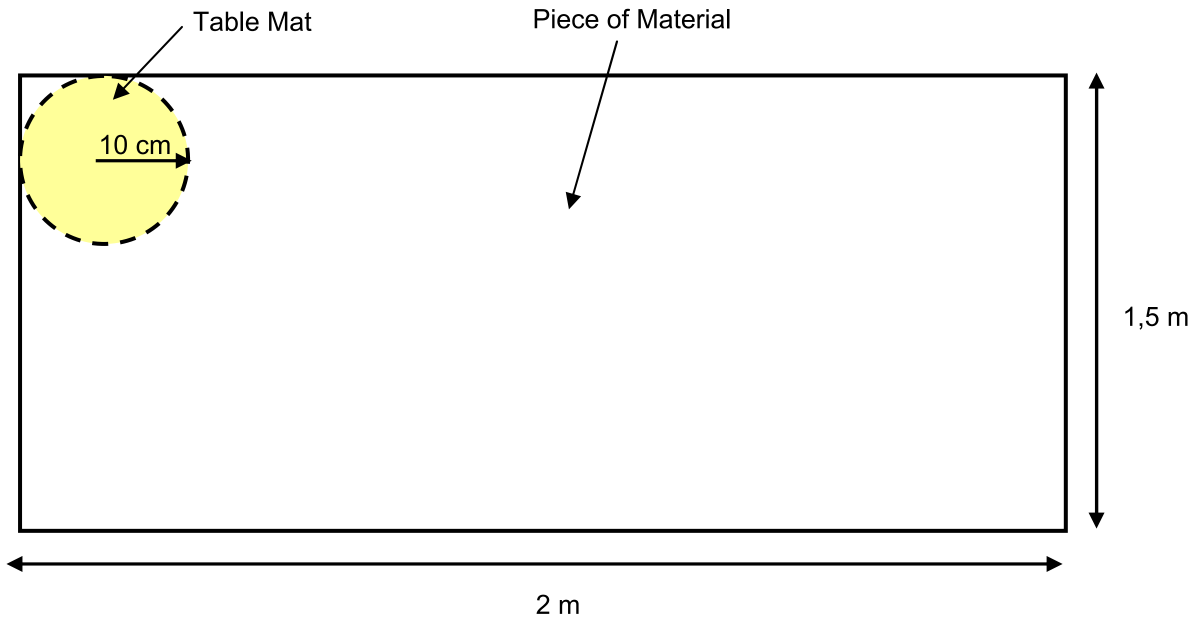
Radius = 35 mm

Area = $\pi \times (35\text{ mm})^2$

$= \pi \times 1225\text{ mm}^2$

$= 3\ 848{,}95\text{ mm}^2$

2. Luanda makes circular table mats. She cuts the mats out from a rectangular piece of material. The picture below shows the dimensions of each table mat and the dimensions of the rectangular piece of material out of which she cuts the circular mats.



a.

i. Calculate the area of each circular table mat.

Area of mat (in m) = $\pi \times (0{,}1\ m)^2$

$= \pi \times 0{,}01\ m^2$

$= 0{,}03142\ m^2$

(or $314{,}2\ cm^2$)

ii. Calculate the area of the rectangular piece of material.

Area = $2\ m \times 1{,}5\ m = 3\ m^2$

iii. Use both methods discussed in Section 3.3.2 calculate how many table mats Luanda will be able to cut from the rectangular material.

Method 1.

No. of mats = $3\ m^2 \div 0{,}03142\ m^2$

$\approx$ 95,5 mats

= 95 full mats

Method 2.

Each mat is 20 cm long and 20 cm wide.

→ No. of mats along length = 2 m ÷ 20 cm

= 2 m ÷ 0,2 m

= 10

→ No. of mats along width = 1,5 m ÷ 20 cm

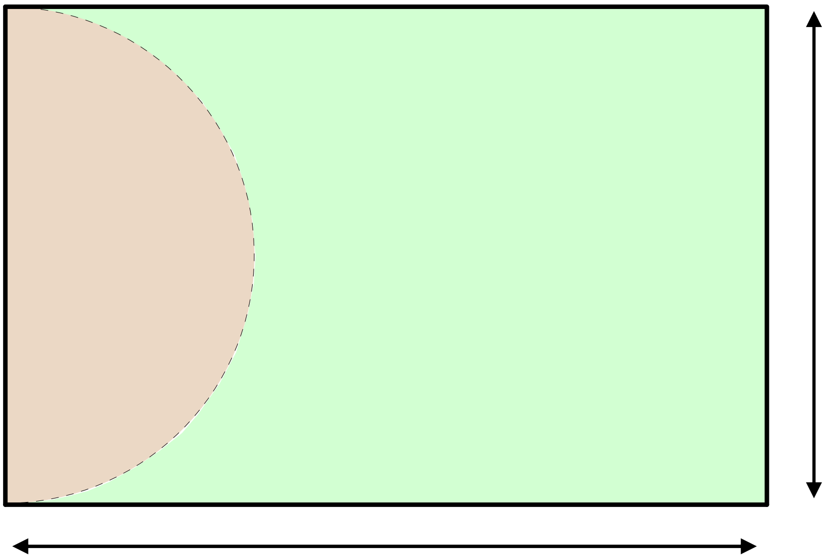
= 1,5 m ÷ 0,2 m

= 7,5

= 7 full mats

∴ Total no. of mats = 10 × 7 = 70

3. Bulelwa is landscaping a garden. She wants to create a semi-circular flower bed at one end of the garden and then plant grass for the rest of the garden.



a. Determine how much top-soil Bulelwa will need for the flower bed.

Radius of semi-circle = 2,7 m

Area of soil (semi circle) = $[\pi \times (2,7 \text{ m})^2] \div 2$

$= [\pi \times 7,29 \text{ m}^2] \div 2$

$\approx 11,453 \text{ m}^2$

(to three decimal places)

b. Determine how much grass Bulelwa will need for the rest of the garden.

Area of whole garden = 5,4 m × 6,2 m

$= 33,48 \text{ m}^2$

$\therefore$ Area of grass = $33,48 \text{ m}^2 - 11,453 \text{ m}^2$

$\approx 22 \text{ m}^2$

(to one decimal place)

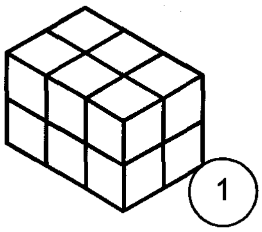
3.4 VOLUME

3.4.2 Discovering Volume Formulas

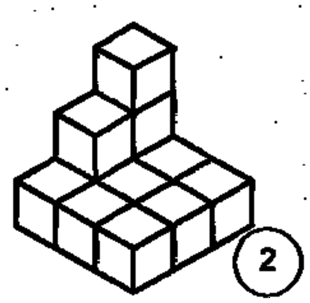
A. Volume of a Rectangular Box

Activity:

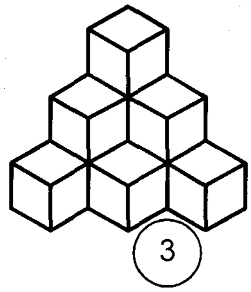
1. Determine the volume of the following 5 shapes by counting the number of unit blocks in each shape.



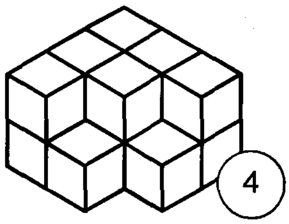
12 blocks



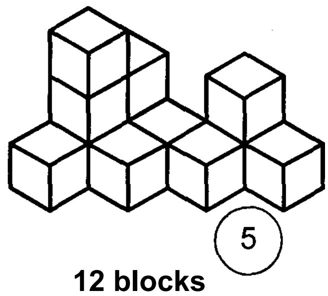
12 blocks



10 blocks

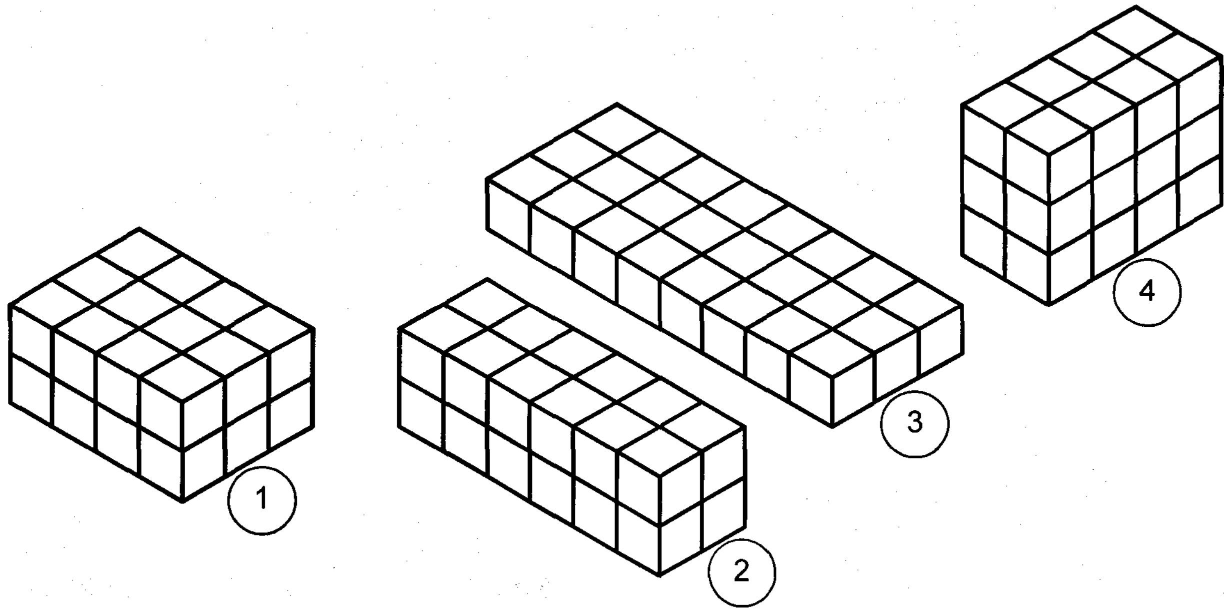


14 blocks



12 blocks

2. For each of the following objects, complete the table of values given below:



Shape number	1	2	3	4
Length	**4**	**6**	**8**	**2**
Breadth	**3**	**2**	**3**	**4**
Height	**2**	**2**	**1**	**3**
Number of cubes	**24**	**24**	**24**	**24**

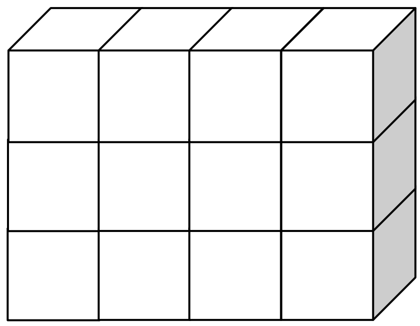
3. Based on your answers in the table, can you think of an equation that could be used to describe the relationship between the length, breadth and height of a rectangular object and the volume of that object? Write your answer below.

No. of blocks = blocks along length × blocks along width × blocks along height

Practice Exercise: Volume of Rectangular Boxes

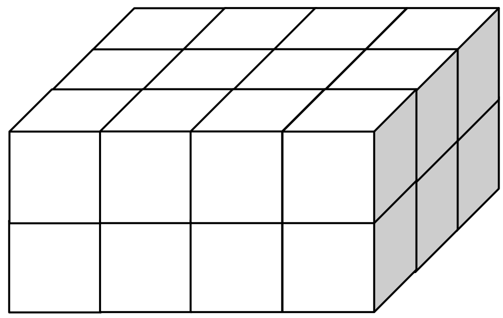
1. The boxes below are made from unit cubes. Calculate the volumes of the boxes.

a.



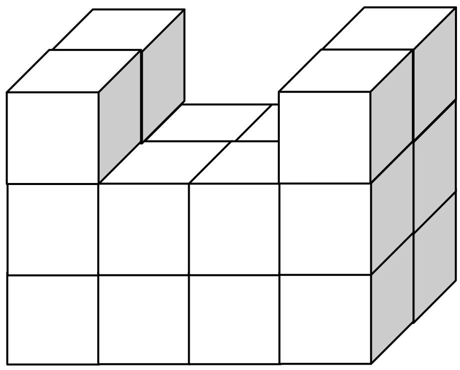
12 blocks

b.



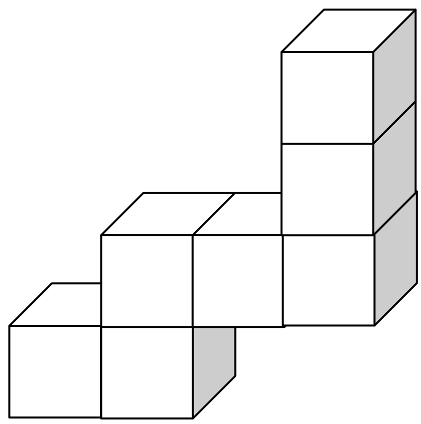
24 blocks

c.



20 blocks

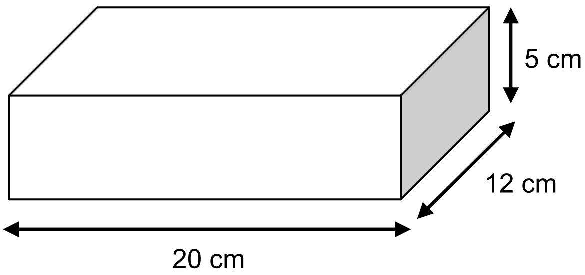
d.



7 blocks

2. Calculate the volumes of the following boxes:

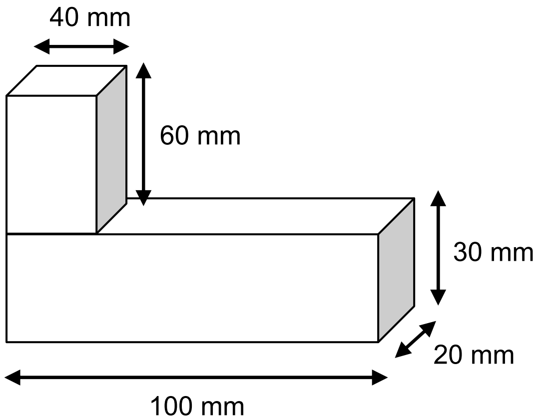
a.



Volume = 20 cm × 12 cm × 5 cm

= 1 200 cm^3

b.



Volume of small box

= 40 mm × 60 mm × 20 mm

= 48 000 mm^3

Volume of big box

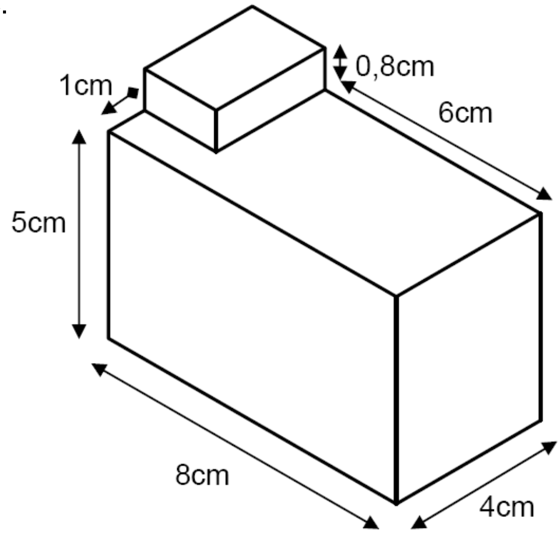
= 100 mm × 20 mm × 30 mm

= 60 000 mm^3

Total volume = 48 000 mm^3 + 60 000 mm^3

= 108 000 mm^3

c.



Volume of big box = 8 cm × 4 cm × 5 cm

= 160 cm^3

Small box:

Length = 4 cm − 1 cm = 3 cm

Width = 8 cm − 6 cm = 2 cm

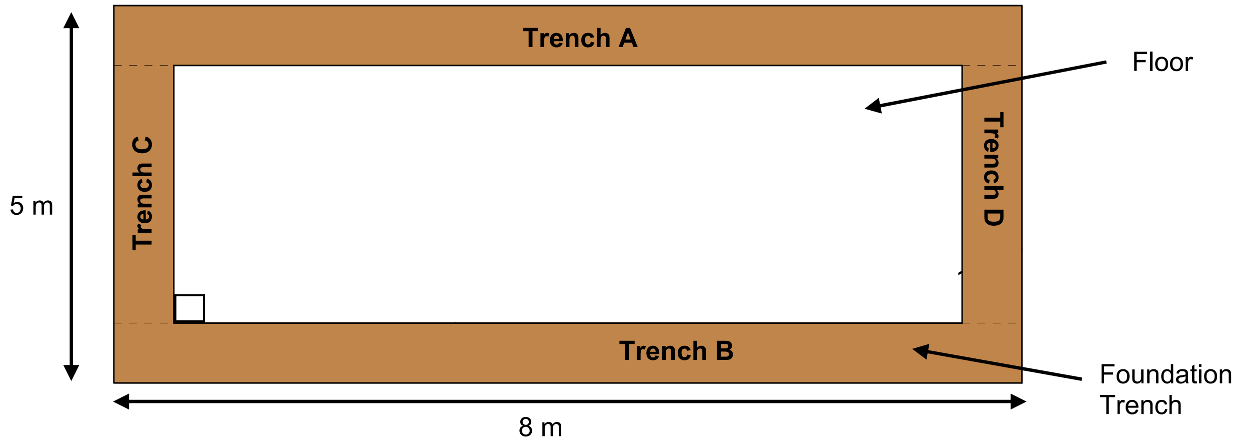
Height = 0,8 m

→ Volume of small box = 3 cm × 2 cm × 0,8 cm

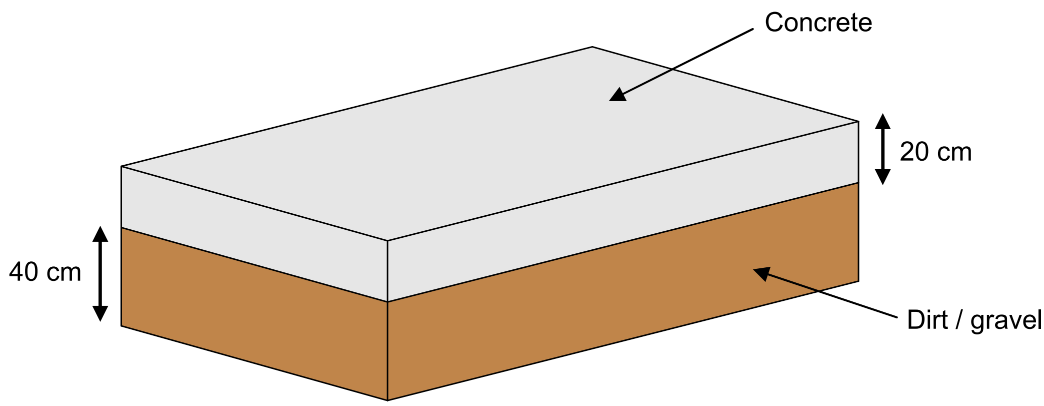
= 4,8 cm^3

Total volume = 160 cm^3 + 4,8 cm^3 = 164,8 cm^3

3. Zipho is building a house. The picture below shows the dimensions of the floor and foundation trench of the house.



a. The picture below shows a 3-D picture of the *floor*.



Determine the volume of concrete needed for the floor.

Length of the floor = 8 m − 1,2 m − 1,2 m = 5,6 m

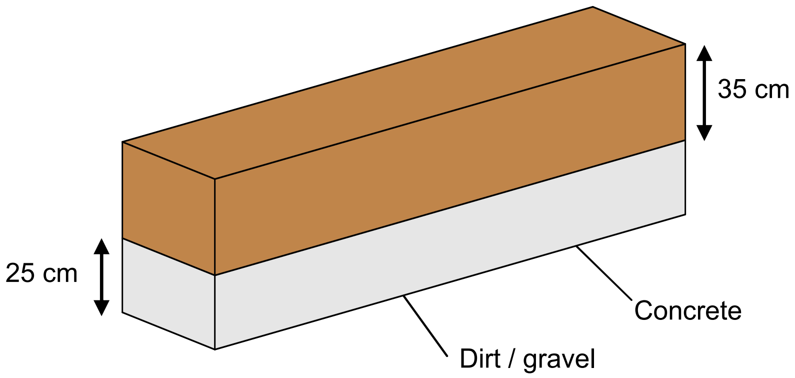
Width of the floor = 5 m − 1,2 m − 1,2 m = 2,6 m

Depth of the concrete floor = 20 cm = 0,2 m

∴ **Volume of concrete needed for the floor = 5,6 m × 2,6 m × 0,2 m**

= 2,912 m^3

b. The picture below shows a 3-D picture of a segment of the *foundation trench*.



Determine the volume of concrete needed for the foundation trench.

Trench A & B:

Length = 8 m **Width = 1,2 m** **Depth = 0,25 m**

→ Volume of A & B = (8 m × 1,2 m × 0,25 m) × 2 = 4,8 m^3

Trench C & D:

Length = 5 m − 1,2 m − 1,2 m = 2,6 m **Width = 1,2 m** **Depth = 0,25 m**

→ Volume of C & D = (2,6 m × 1,2 m × 0,25 m) × 2 = 1,56 m^3

∴ Total volume = 4,8 m^3 + 1,56m^3 = 6,36 m^3

c. PPC Cement provides the following guideline for the number of bags of cement, m^3 of sand and m^3 of stone needed to make a particular quantity of concrete.

(PPC Cement, Pamphlet – The Sure Way to Estimate Quantities, www.ppccement.co.za)

LOW STRENGTH CONCRETE

Foundations, Post Holes, Paths & Patios

Strength Guide	*10 - 15 Mpa*
Mix Proportions	*1:4:4 (OPC use 1:5:5)*
Water Guide	*80ℓ per 2 bags of cement*

MIX QUANTITY m^3	No.BAGS SUREBUILD	SAND m^3	STONE m^3
0,2	1	0,1	0,1
0,4	2	0,3	0,3
0,6	3	0,4	0,4
0,7	4	0,5	0,5
0,9	5	0,6	0,7
1,1	6	0,8	0,8
1,8	10	1,3	1,3
3,7	20	2,6	2,7
7,5	40	5,2	5,4
15,0	80	10,4	10,7

i. Use the guideline to determine how many bags of cement Zipho will need to buy to make enough concrete for the *foundations* of the house.

Volume of concrete needed ≈ 6,4 m^3

6,4 m^3 = 3,7 m^3 + 1,8 m^3 + 0,7 m^3 + 0,2 m^3

= 20 bags + 10 bags + 4 bags + 1 bag

= 35 bags

ii. If the ratio of *cement : sand : stone* is *1 : 4 : 4* and if *1 wheelbarrow of cement = 2 bags of cement*, determine how many wheelbarrows of sand and stone Zipho will need for the concrete for the *foundations* of the house.

1 wheelbarrow = 2 bags of cement

→ 1 bag of cement = ½ wheelbarrow of cement

→ 35 bags of cement = 17,5 wheelbarrows of cement

Ratio of cement : sand : stone = 1 : 4 : 4

∴ 17,5 wh/barrows of cement : (4 × 17,5) wh/barrows of sand : (4 × 17,5) wh/barrows of stone

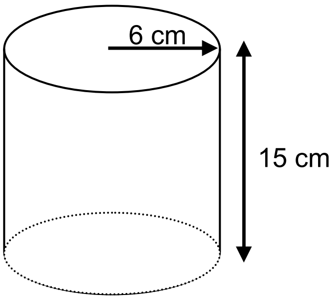
= 17,5 wh/barrows of cement : 70 wh/barrows of sand : 70 wh/barrows of stone

C. Volume of a Cylinder

Practice Exercise: Volume of Cylinders

1. Calculate the volumes of the following shapes:

a.

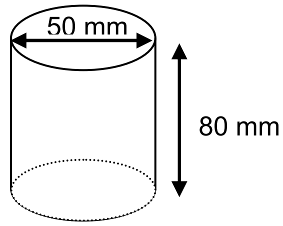


Volume = $\pi \times (6 \text{ cm})^2 \times 15$ cm

$= \pi \times 36 \text{ cm}^2 \times 15$ cm

$\approx 1\,696{,}7 \text{ cm}^3$

b.



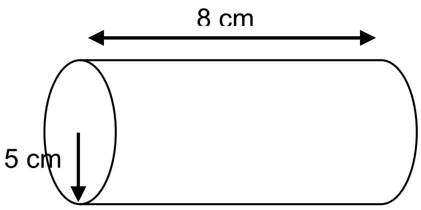
Radius of cylinder = 50 mm ÷ 2 = 25 mm

Volume = $\pi \times (25 \text{ mm})^2 \times 80$ mm

$= \pi \times 625 \text{ mm}^2 \times 80$ mm

$= 157\,100 \text{ mm}^3$

c.

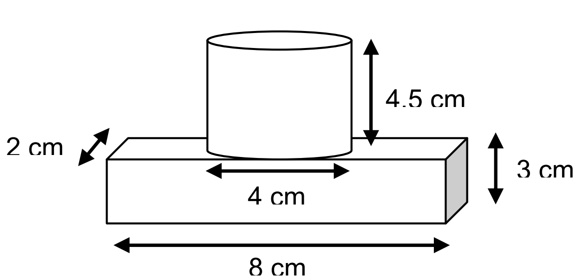


Volume = $\pi \times (5 \text{ cm})^2 \times 8$ cm

$= \pi \times 25 \text{ cm}^2 \times 8$ cm

$= 628{,}4 \text{ cm}^3$

d.



Volume of box = 8 cm × 3 cm × 2 cm

$= 48 \text{ cm}^3$

Volume of cylinder = $\pi \times (2 \text{ cm})^2 \times 4{,}5$ cm

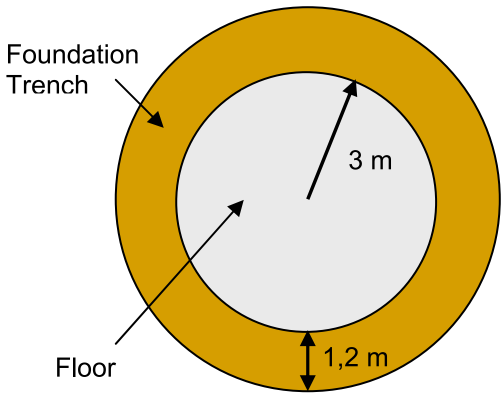
$= \pi \times 4 \text{ cm}^2 \times 4{,}5$ cm

$= 56{,}556 \text{ cm}^3$

$\therefore$ Total volume = $48 \text{ m}^3 + 56{,}556 \text{ cm}^3$

$\approx 104{,}6 \text{ cm}^3$

2. The picture below shows the radius of the floor and the width of the foundation trench for a circular house (rondavel).



a. If the floor is going to be *25 cm thick*, calculate the volume of concrete that the builder will need to make for the floor.

Volume = $\pi \times (3\text{ m})^2 \times 0{,}25\text{ m}$

$= \pi \times 9\text{ m}^2 \times 0{,}25\text{ m}$

$= 7{,}0695\text{ m}^3$

b. The foundation trench will be filled with concrete that is *30 cm thick*. Calculate the volume of concrete that the builder will need to make for the foundations of the house.

Volume of whole shape (floor + trench) = $\pi \times (3\text{ m} + 1{,}2\text{ m})^2 \times 0{,}3\text{ m}$

$= \pi \times (4{,}2\text{ m})^2 \times 0{,}3\text{ m}$

$= \pi \times 17{,}64\text{ m}^2 \times 0{,}3\text{ m}$

$\approx 16{,}627\text{ m}^3$

Volume of floor *if it was 30 cm thick* = $\pi \times (3\text{ m})^2 \times 0{,}3\text{ m}$

$= \pi \times 9\text{ m}^2 \times 0{,}3\text{ m}$

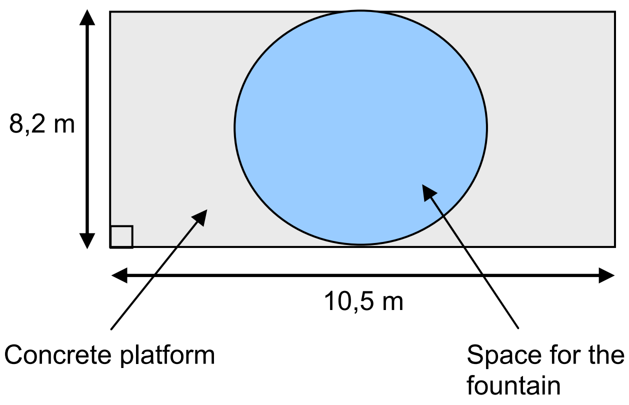
$= 8{,}483\text{ m}^3$

$\therefore$ Volume of trench = volume of whole shape − volume of floor

$= 16{,}627\text{ m}^3 - 8{,}483\text{ m}^3$

$\approx 8{,}14\text{ m}^3$

3. Vilikazi is landscaping a garden and decides to build a circular fountain in the middle of a concrete platform.



a. If the circular fountain is going to be 20 cm deep, calculate what volume of dirt Vilikazi will remove from the ground to make space for the fountain.

Diamtere of circle = 8,2 m

→ **Radius of circle = 4,1 m**

$\therefore$ **Volume** $= \pi \times (4{,}1\text{ m})^2 \times 0{,}2\text{ m}$

$= \pi \times 16{,}81\text{ m}^2 \times 0{,}2\text{ m}$

$= 10{,}563\text{ m}^3$

b. If the concrete platform will be 20 cm deep, calculate the volume of concrete that Vilikazi will need to make the platform.

Total volume of whole rectangular shape $= 10{,}5\text{ m} \times 8{,}2\text{ m} \times 0{,}2\text{ m} = 17{,}22\text{ m}^3$

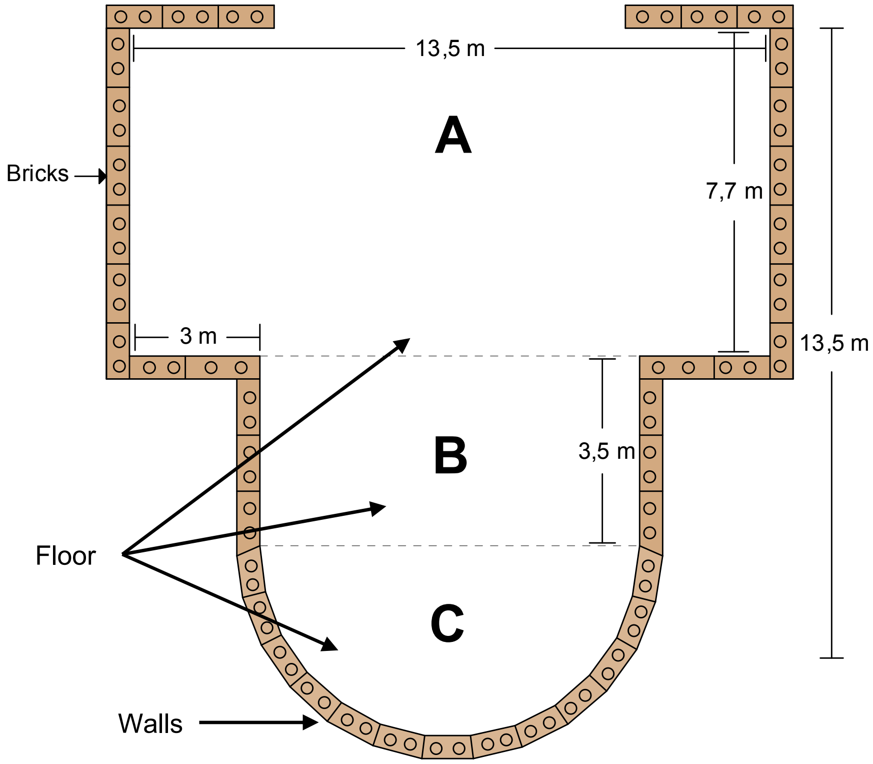
Volume of concrete for platform = whole shape − circular fountain

$= 17{,}22\text{ m}^3 - 10{,}563\text{ m}^3$

$\approx 6{,}7\text{ m}^3$ **(to one decimal place)**

Test Your Knowledge: 2-D & 3-D Pictures, Area and Volume

The picture below shows the outline of a building.



1. a. Calculate the area of Part A of the floor.

Area of A = 13,5 m × 7,7 m = 103,95 m^2

b. Calculate the area of Part B of the floor.

Length of B = 13,5 m − 3 m − 3 m

= 7,5 m

∴ Area of B = 7,5 m × 3,5 m = 26,25 m^2

1. c. Calculate the area of Part C of the floor.

Diameter of C = 7,5 m

→ Radius of C = 3,75 m

Area of C = π × (3,75 m)^2 ÷ 2 = 22,092 m^2

d. The builder plans to carpet the floor. If the cost of carpeting is R85,00 per m^2, calculate how much it will cost to carpet this building.

Total area of the floor

= 103,95 m^2 + 26,25 m^2 + 22,092 m^2

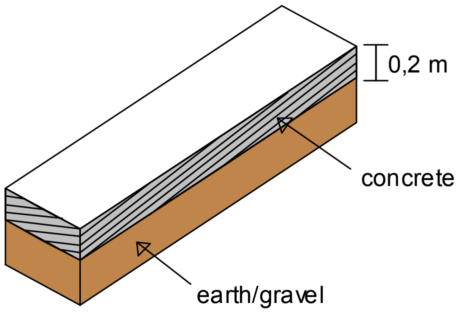
= 152,292 m^2

Cost: 1 m^2 = R85,00

→ 152,292 m^2 = R85,00 × 152,292

= R12 944,82

2. The picture below shows a 3-D cross-section of the floor.



a. Calculate the volume of concrete needed for Part A of the floor.

Length = 13,5 m

Width = 7,7 m

Depth = 0,2 m

∴ Volume of Part A = 13,5 m × 7,7 m × 0,2 m

= 20,79 m^3

b. Calculate the volume of concrete needed for Part B of the floor.

Length = 7,5 m

Width = 3,5 m

Depth = 0,2 m

∴ Volume of Part B = 7,5 m × 3,5 m × 0,2 m

= 5,25 m^3

c. Calculate the volume concrete needed for Part C of the floor.

Diameter = 7,5 m

→ Radius = 3,75 m

Depth = 0,2 m

∴ Volume = [π × (3,75 m)^2 ÷ 2] × 0,2 m

= [π × 14,0625 m^2 ÷ 2] × 0,2 m

≈ 4,418 m^3 (to 3 decimal places)

d. The table below shows the number of bags of cement needed for making different volumes of concrete.

Concrete (m^3)	Bags of Cement
0,1	1
0,3	2
0,6	4
1,5	10
3	20
15	100

Use the table to determine how many bags of cement the builder will need to make the floor.

Total volume of the floor

= 20,79 m^3 + 5,25 m^3 + 4,418 m^3
= 30,458 m^3

≈ 30,5 m^3

30,5 m^3 = (15 m^3 × 2) + 0,3 m^3 + 0,1 m^3 + 0,1 m^3

= 200 bags + 2 bags + 1 bag + 1 bag

= 204 bags