# CHAPTER 11
# Financial mathematics

In this chapter you will:

- Calculate interest using the simple interest formula ($SI = P \times i \times n$)
- Use the simple interest formula to calculate the principal (P)
- Use the simple interest formula to calculate the interest rate (i)
- Use the simple interest formula to calculate the investment period (n)
- Compare offers from different banks
- Calculate interest using the compound interest formula ($A = P(1 + i)^n$ )
- Use the compound interest formula to calculate interest
- Calculate different time periods and their corresponding interest rates
- Calculate compound interest over different time periods
- Compare interest rates
- Use the compound interest formula to calculate the principal (P)
- Use the compound interest formula to calculate the interest rate (i)
- Use the compound decrease formula to calculate final amounts
- Use the compound decrease formula to calculate population decreases
- Calculate repayments on hire purchase agreements
- Calculate percentage profit and percentage loss

***This chapter covers material from Topic 5: Financial Mathematics***

**SUBJECT OUTCOME 5.2:**

**Use simple and compound interest to explain and define a variety of situations**

| | |
|---|---|
| **Learning Outcome 1:** | Differentiate between simple and compound interest and extrapolate the advantages and disadvantages of each in specific situations |
| **Learning Outcome 2:** | Calculate simple and compound interest over different periods at specific rates |
| **Learning Outcome 3:** | Do calculations using computational tools efficiently and correctly and verify solutions in terms of the context |
| **Learning Outcome 4:** | Use solutions to calculations effectively to define the changes that occur over a period |

## § 11.1 THE SIMPLE INTEREST FORMULA

- We calculate the amount of *simple interest* earned by using the formula $\boldsymbol{SI = P \times i \times n}$ where: ***SI*** = simple interest, ***P*** = the principal (the money invested), ***i*** = the interest rate p.a., written as a decimal, and ***n*** = the number of time periods
- If we want to calculate the *final amount received at the end of the time period*, we use the formula $\boldsymbol{A = P + SI}$ where: ***A*** = the final amount received at the end of the time period, ***P*** = the principal or the amount invested and ***SI*** = simple interest.
- Note that p.a. means *per annum* or *per year*.
- Note also that we always round amounts of money off to 2 decimal places.

| EXAMPLES | SOLUTIONS |
|---|---|
| 1) Write each of the following as a decimal:<br>a) 9% | $9\% = \frac{9}{100} = 0{,}09$ |
| b) 14,5% | $14{,}5\% = \frac{14{,}5}{100} = 0{,}145$ |
| 2)<br>a) Calculate SI, the simple interest earned when R5 430 is invested for 5 years at an interest rate of 9,5% p.a. | P = R5 430<br>$n$ = 5 year<br>$i = 9{,}5\% = i = \frac{9{,}5}{100} = 0{,}095$ p.a.<br>$\boldsymbol{SI = P \times i \times n}$<br>= R5 430 × 0,095 × 5<br>= R2 579,25 |
| b) Calculate the final amount received at the end of 5 years. | $\boldsymbol{A = P + SI}$<br>= R5 430 + R2 579,25<br>= R8 009,25 |

**Exercise 11.1**

1) Write the following percentages as decimals:

   a) 11,25% ……

   b) 10,4% ……..

   c) 8,9% ……

2) A principal of R7 395 is invested for 6 years at an interest rate of 10,25% p.a.

   a) Calculate SI, the simple interest earned

   P = ……  $n$ = ……  $i$ = …….

   $\boldsymbol{SI = P \times i \times n}$

   …….

   …….

   b) Calculate A, the amount received at the end of 6 years.

   $\boldsymbol{A = P + SI}$

   ……

   ……

## § 11.2 USING THE SIMPLE INTEREST FORMULA TO CALCULATE THE PRINCIPAL (P)

| EXAMPLE | SOLUTION |
|---|---|
| A sum of money is invested at a simple interest rate of 10,7% p.a. for 5 years. | $i$ = 10,7 % p.a. = $\frac{10,7}{100}$ p.a.= 0,107 p.a.<br>$n$ = 5 years |
| 1) How much needs to be invested in order to make R550 interest?<br>$P = ?$ | $SI$ = R550 $P$ = ?<br>$SI = P \times i \times n$<br>$550 = P \times 0,107 \times 5$<br>$\frac{550}{0,107\times5} = \frac{P\times0,107\times5}{0,107\times5}$<br>R1 028,04 = $P$<br>So R1 028,04 needs to be invested. |
| 2) What amount is received at the end of 5 years?<br>$A = ?$ | $A = P + SI$<br>= 1 028,04 + 550<br>= 1 578,04<br>R1 578,04 is received at the end of 5 years. |

**Exercise 11.2**

1) A sum of money is invested at an interest rate of 11,6 % p.a. simple interest for 6 years.
   a) How much needs to be invested in order to make R1055 interest?

   $i$ = …… $n$ = …… $SI$ = ……

   $SI = P \times i \times n$

   ……

   ……

   ……

   So ……. needs to be invested.

   b) What amount will be received at the end of 6 years?

   $A = P + SI$

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   ……

2) A sum of money is invested at an interest rate of 10,75% p.a. for 4 years.
   a) How much must be invested in order to make R950 $SI$ on the investment?

   $i$ = …… $n$ = …… $SI$ = ……

   $SI = P \times i \times n$

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   So ……. needs to be invested.

   b) How much is saved in total at the end of 4 years?

   $A = P + SI$

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## § 11.3 USING THE SIMPLE INTEREST FORMULA TO CALCULATE THE INTEREST RATE (*i*)

| EXAMPLE | SOLUTION |
|---|---|
| R12 457 is invested for 6 years. At the end of this time, an amount of R18 278 is paid out. | $P$ = R12 457 $A$ = R18 278 $n$ = 6 years |
| 1) Calculate the simple interest, S I, received<br><br>$SI = ?$ | $A = P + SI$<br>18 278 = 12 457 + $SI$<br>$SI$ = 18 278 – 12 457<br>$SI$ = R5 821 |
| 2) At what % interest rate p.a. was the money invested? Answer correct to 2 decimal places.<br><br>$i = ?$ | $SI = P \times i \times n$<br>5 821 = 12 457 × $i$ × 6<br>$\frac{5\,821}{12\,457 \times 6} = \frac{12\,457 \times i \times 6}{12\,457 \times 6}$<br>$i$ = 0,02436…<br>$i$ = 0,07788… × 100 %<br>$i$ = 7,79 % p.a. |

**Exercise 11.3**

1) Convert the following decimals to a percentage (correct to 2 decimal places);
   a) 0,08 …….
   b) 0,12 …….
   c) 0,325 …….
   d) 0,615 …….
   e) 0,022 5 …….
   f) 0,0257788… …….

2) R15 901 is invested for 4 years. At the end of this time, an amount of R19 780 is paid out.
   a) Calculate the simple interest earned.

      $A$ = ……... $P$ = …….. $n$ = ……. $SI$ = ……..

      $A = P + SI$

      …….

      …….

      …….

   b) Calculate the interest rate and write your answer as a percentage correct to 2 decimal places.

      $SI = P \times i \times n$

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## § 11.4 USING THE SIMPLE INTEREST FORMULA TO CALCULATE THE INVESTMENT PERIOD ($n$)

| EXAMPLE | SOLUTION |
|---|---|
| R6 058 is invested at a simple interest rate of 9,25% p.a.<br><br>How many years will it take in order to receive an amount of R10 540,92 at the end of the investment period? | $i$ = 9,25% p.a. = $\frac{9,25}{100}$ p.a. = 0,0925 p.a.<br>$\boldsymbol{A}$ = R10 540,92 $\boldsymbol{P}$ = R6 058<br>$\boldsymbol{A = P + SI}$<br>10 540,92 = 6 058 + $SI$<br>10 540,92 – 6 058 = $SI$ – 6 058<br>$SI$ = R4 482,92<br>$\boldsymbol{SI = P \times i \times n}$<br>4 482,92 = 6 058 × 0,0925 × $n$<br>$\frac{4482,92}{6\,058 \times 0,0925} = \frac{6\,058 \times 0,0925 \times n}{6\,058 \times 0,0925}$<br>$n$ = 8<br>The money must be invested for 8 years. |

**Exercise 11.4**

1) R10 000 is invested at a simple interest rate of 9,35% p.a. How many years will it take in order to receive an amount of R14 675 at the end of the investment period?

***A*** **=** ……... ***P*** **=** …….. ***i*** **=** ……..

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2) R8 990 is invested at a simple interest rate of 10,25% p.a. How many years will it take in order to receive an amount of R12 675,90 at the end of the investment period?

***A*** **=** ……... ***P*** **=** …….. ***n*** **=** ……. ***S I*** **=** ……..

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## § 11.5 COMPARING OFFERS FROM DIFFERENT BANKS

| EXAMPLE | |
|---|---|
| Mpho has R5 980 to invest for 6 years.<br>• Bank A offers him a fixed simple interest rate of 9,5% p.a. PLUS a bonus of 5% of the principal.<br>• Bank B offers him a fixed simple interest rate of 10,5% p.a.<br>Which bank offers the better investment? | |
| **SOLUTION** | |
| **BANK A** | **BANK B** |
| $P$ = R5 980 $n$ = 6 years<br>$i$ = 9,5 % p.a. = $\frac{9,5}{100}$ p.a. = 0,095 p.a.<br>$SI = P \times i \times n$<br>= 5 980 × 0,095 × 6<br>= R3 408,60<br><br>Bonus = 5% of R5 980<br>= 0,05 × 5 980<br>= R299<br><br>At the end of 6 years Mpho would receive R3 408,60 + R299 = ***R3 707,60 from Bank A***. | $P$ = R5 980 $n$ = 6 years<br>$i$ = 10,5 % p.a. = $\frac{10,5}{100}$ p.a. = 0,105 p.a.<br>$SI = P \times i \times n$<br>= 5 980 × 0,105 × 6<br>= R3 767,40<br><br>At the end of 6 years Mpho would receive ***R3 767,40 from Bank B***, which is more than the R3 707,60 he would receive from Bank A<br><br>So Bank B offers the better investment. |

**Exercise 11.5**

Patricia has R6 400 to invest for 4 years.

- Bank A offers her a fixed simple interest rate of 9,75 % p.a. plus a bonus of 2% of the principal.
- Bank B offers her a fixed simple interest rate of 10% p.a.

In which bank should she invest her money?

**BANK A**

***P*** = ……... ***n*** = …….. ***i*** = ……..

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**BANK B**

***P*** = ……... ***n*** = …….. ***i*** = ……..

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## § 11.6 MIXED EXERCISE

✧ Remember that $SI = P \times i \times n$ and $A = P + SI$

1) Jean invests R10 450 for 5 years at 9,5% p.a. *S I*.
   a) How much interest will she receive at the end of this period?

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   b) How much will she have saved at the end of this period?

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***A*** = …….
***P*** = .....
***i*** = ……
***n*** = ……...
***S I*** = ……

2) How much must Sipho invest at 9,5% *S I* p.a. for 6 years in order to earn interest of R2 500?

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   ……...
   ……

***A*** = …….
***P*** = .....
***i*** = ……
***n*** = ……...
***S I*** = ……

3)
   a) Calculate the number of whole years needed for an investment of R5 990 to grow to R7 876,85 if the interest rate is 9% *S I* p.a.

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   b) At what interest rate (correct to 2 decimal places) must it be invested for R5 990 to grow to R8 500 after 5 years?

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***A*** = …….
***P*** = .....
***i*** = ……
***n*** = ……...
***S I*** = ……

4) For how long (whole years) must John invest R11 545 at 9,5% p.a. *S I* to save R16 000 in total?

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***A*** = …….
***P*** = .....
***i*** = ……
***n*** = ……...
***S I*** = ……

## § 11.7 COMPOUND INTEREST

- When a person is paid ***interest*** at regular intervals on a sum of money invested, without the interest being added to the sum invested, the investment is called ***simple interest***.
  The formula for calculating simple interest is $SI = P \times i \times n$.
  We use the formula $A = P + SI$ to calculate the final amount (A).
- When the interest is added to the sum invested, and ***interest is calculated on this new amount***, the investment is called ***compound interest***.
  The formula used for calculating compound interest is $A = P(1+i)^n$, where
  ***A*** is the final amount obtained after the investment period
  ***P*** is the principal or the amount invested
  ***i*** is the interest rate (written as a decimal)
  ***n*** is the number of time periods of the investment

| **EXAMPLE** | |
|---|---|
| James invests R4 000 at 8% p.a. for 5 years. Should he choose:<br>1) Simple interest or<br>2) Compound interest? | |
| SOLUTION | |
| SIMPLE INTEREST | COMPOUND INTEREST |
| $P$ = 4 000<br>$i$ = 8% p.a. = 0,08 p.a.<br>$n$ = 5 years<br>$SI = P \times i \times n$<br>$= 4\,000 \times 0{,}08 \times 5$<br>= R1 600<br>$A = P + SI$<br>= 4 000 + 1 600<br>= R5 600 | $P$ = 4 000<br>$i$ = 8% p.a. = 0,08 p.a.<br>$n$ = 5 years<br>$A = P(1+i)^n$<br>$= 4\,000\,(1 + 0{,}08)^5$<br>$= 4\,000\,(1{,}08)^5$<br>= R5 877,31 |
| • James should choose compound interest as he would end up with a larger final amount. | |

**Exercise 11.7**

Paula invests R5 500 at 10% p.a. for 4 years. Should she choose ***simple interest*** or ***compound interest***?

SIMPLE INTEREST

$P$ = ……

……

$i$ = ……

……

$n$ = ……

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……

……

COMPUND INTEREST

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CONCLUSION: ……

## § 11.8 USING THE COMPOUND INTEREST FORMULA TO CALCULATE INTEREST (*i*)

- We use the formula $\boldsymbol{A = P(1+i)^n}$ to calculate the final amount received at the end of the investment period.
- We use the formula $\boldsymbol{CI = A - P}$ to calculate the interest obtained at the end of the investment period.

| EXAMPLE | SOLUTION |
|---|---|
| 1) Use the *CI* formula to calculate the amount received at the end of 8 years if R7 000 is invested at 5% p.a. | $\boldsymbol{P}$ = R7 000<br>$\boldsymbol{n}$ = 8<br>$\boldsymbol{i}$ = 5 % = 0,05 p.a.<br>$A = P(1+i)^n$<br>$= 7\,000(1 + 0{,}05)^8$<br>$= 7\,000(1{,}05)^8$<br>= R10 342,19 |
| 2) How much interest is received on this investment? | $\boldsymbol{CI = A - P}$<br>= R10 342,19 – R7 000<br>= R3 342,19 |

**Exercise 11.8**

1) Use the *CI* formula to calculate the amount paid out at the end of an investment of R6 955 at an interest rate of 10,4% p.a. compounded annually for 7 years.

   $\boldsymbol{P}$ = …… $\boldsymbol{n}$ = …… $\boldsymbol{i}$ = ……

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   ……

2) How much of this amount is interest?

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   ……

## § 11.9 TIME PERIODS AND INTEREST RATES

- Interest on an investment is calculated at the end of a given time period. This time period is not necessarily one year. The interest can be compounded (calculated) ***annually***, ***half-yearly***, ***quarterly***, ***monthly*** and sometimes ***daily***.
- To find the ***interest rate*** when the interest is compounded more than once per year, ***divide*** the interest rate p.a. by the number of time periods in one year
- To find the number of ***time periods***, ***multiply*** the number of years by the required number of time periods in one year

**EXAMPLE**

Suppose the investment period is **6 years** at an **interest rate of 9% p.a.**
Suppose the interest is compounded more than once per year,

- we divide the interest rate up into the different time periods
- we adjust the time periods accordingly.

**SOLUTION**

| TIME PERIOD | INTEREST RATE PER TIME PERIOD WRITTEN AS A DECIMAL ($i$) | NUMBER OF TIME PERIODS ($n$) |
|---|---|---|
| **annually** | 9% p.a. = 0,09 per year | $n = 6\times1 = 6$ years |
| **half-yearly** | 9% p.a. = $\frac{0,090}{2} = 0,045$ per half year | $n = 6\times2 = 12$ half-years |
| **quarterly** | 9% p.a. = $\frac{0,0900}{4} = 0,0225$ per quarter | $n = 6\times4 = 24$ quarters |
| **monthly** | 9% p.a. = $\frac{0,090000000}{12} = 0,008\dot{3}$ per month | $n = 6\times12 = 72$ months |

**Exercise 11.9**

1) Complete the table for an investment period of 7 years and an interest rate of 4% p.a.

| Time period | Interest rate per time period as a decimal ($i$) | Number of time periods ($n$) |
|---|---|---|
| **annually** | | |
| **half-yearly** | | |
| **quarterly** | | |
| **monthly** | | |

2) Complete the table for an investment period of 8 years and an interest rate of 12,5% p.a.

| Time period | Interest rate per time period as a decimal ($i$) | Number of time periods ($n$) |
|---|---|---|
| **annually** | | |
| **half-yearly** | | |
| **quarterly** | | |
| **monthly** | | |

## § 11.10 CALCULATING COMPOUND INTEREST OVER DIFFERENT TIME PERIODS

| EXAMPLE | SOLUTION |
|---|---|
| R8 550 is invested at a financial institution for 6 years at an interest rate of 7,2 % p.a. compounded **quarterly**. | ***P*** = R8 550<br>***i*** = 7,2% p.a. = 0,072 p.a.<br>$= \frac{0,072}{4}$ per quarter = 0,018 per quarter<br>***n*** = 6 × 4 = 24 quarters |
| 1) What amount is due after 6 years? | $A = P(1+i)^n$<br>$= 8\,550(1 + 0,018)^{24} = 8\,550(1,018)^{24}$ = R13 119,36 |
| 2) How much of this is interest? | ***CI = A – P***<br>= R13 119,36 – R8 550 = R4 569,36 |

**Exercise 11.10**

1) An investment of R8 550 is made at a financial institution. The interest rate is 7,2 % p.a. compounded annually for 6 years.
   a) What amount is due after 6 years?

      ***P*** = …… ***i*** = …… ***n*** = ……..

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      ……

      ……

      ……

   b) How much is interest?

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      ……

   c) Compare your answers with those in the example above. Do you earn more interest by compounding annually or compounding quarterly?

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2) Use the numbers in question 1 to compound the investment half-yearly.
   a) What amount is due after 6 years?

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   b) How much is the interest?

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3) Is compounding an investment more frequently an advantage? ……

## § 11.11 COMPARING INTEREST RATES

**EXAMPLE**

Joe wants to invest R13 450 for 6 years.

- Bank A offers him 9,6% p.a. interest compounded monthly.
- Bank B offers 10% p.a. interest compounded quarterly.

1) Calculate the amount received from each bank.
2) How much interest does each option yield?
3) Which offer should Joe take?

**SOLUTION**

| BANK A | BANK B |
|---|---|
| $P$ = R13 450<br>$i$ = 9,6% p.a. = 0,096 p.a.<br>$= \frac{0,096}{12}$ per month = 0,008 per month<br>$n = 6 \times 12 = 72$ months<br>1) $A = P(1+i)^n$<br>$= 13\,450(1 + 0,008)^{72}$<br>$= 13\,450(1,008)^{72}$<br>= R23 871,55<br>2) $CI = A - P$<br>= R23 871,55 – R13 450<br>= R10 421,55 | $P$ = R13 450<br>$i$ = 10% p.a. = 0,1 p.a.<br>$= \frac{0,1}{4}$ per quarter = 0,025 per quarter<br>$n = 6 \times 4 = 24$ quarters<br>1) $A = P(1+i)^n$<br>$= 13\,450(1 + 0,025)^{24}$<br>$= 13\,450(1,025)^{24}$<br>= R24 327,36<br>2) $CI = A - P$<br>= R24 327,36 – R13 450<br>= R10 877,36 |

3) Joe should take the offer by Bank B, since he receives more interest there.

**Exercise 11.11**

Vusi wants to invest R9 560 for 5 years.
Bank A offers him 10,2% p.a. interest compounded monthly.
Bank B offers 10,8 % p.a. interest compounded quarterly.

| | BANK A | BANK B |
|---|---|---|
| 1) Calculate the amount received from each bank. | ***P*** = ……..……<br>***i*** = ……..….<br>……..……..<br>***n*** = ……..……<br>……..……<br>……<br>……<br>…….. | ***P*** = ……..……<br>***i*** = ……..….<br>……..……..<br>***n*** = ……..……<br>……..……<br>……<br>……<br>…….. |
| 2) How much interest will each option yield? | ……..<br>……...<br>……... | ……..<br>……...<br>……... |
| 3) Which offer should Vusi take? | …… | |

## § 11.12 USING THE COMPOUND INTEREST FORMULA TO CALCULATE THE PRINCIPAL (P)

**EXAMPLE**

How much money must be invested in order to receive an amount of R14 600 in 5 years
1) at an interest rate of 10,4 % p.a. compounded annually?
1) at an interest rate of 10,4 % p.a. compounded quarterly?

**SOLUTION**

1) $A$ = R14 600

$i = 10{,}4\ \% = \frac{10{,}4}{100} = 0{,}104$ p.a.

$n = 5$

$$A = P(1+i)^n$$

$14\ 600 = P(1 + 0{,}104)^5$

$14\ 600 = P(1{,}104)^5$

$\frac{14\ 600}{(1{,}104)^5} = \frac{P(1{,}104)^5}{1{,}104^5}$

$8\ 902{,}41 = P$

R8 902,41 must be invested.

| Key Sequence | | | | | | |
|---|---|---|---|---|---|---|
| 14 600 | ÷ | 1.104 | $x^{\blacksquare}$ | 5 | = | |

2) $A$ = R14 600

$i = 0{,}104$ p.a. $= \frac{0{,}103}{4} = 0{,}026$ per quarter

$n = 5 \times 4 = 20$ quarters

$$A = P(1+i)^n$$

$14\ 600 = P(1 + 0{,}026)^{20}$

$14\ 600 = P(1{,}026)^{20}$

$\frac{14\ 600}{1.026^{20}} = \frac{P(1{,}026)^{20}}{1{,}026^{20}}$

$8\ 737{,}87 = P$

R8 737,87 must be invested.

| Key Sequence | | | | | | |
|---|---|---|---|---|---|---|
| 14 600 | ÷ | 1.026 | $x^{\blacksquare}$ | 20 | = | |

**Exercise 11.12**

An investment amounted to R24 987,50 at the end of 5 years.

1) How much was the initial investment at

a) 9,5% p.a. compounded annually?

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b) 9,5% p.a. compounded half-yearly ?

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2) For which one would you have to invest less?

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## § 11.13 CALCULATING THE INTEREST RATE (*i*)

**EXAMPLE**

An investment of R17 000 grows to R25 000 in 5 years.

1) What was the percentage interest rate p.a. compounded annually (correct to 2 decimal places)?
2) What was the percentage interest rate p.a. compounded half-yearly (correct to 2 decimal places)?

**SOLUTION**

1) $P$ = R17 000
$A$ = R25 000
$n$ = 5 years
$i$ = ? % p.a.

$$A = P(1+i)^n$$
$$25\,000 = 17\,000(1+i)^5$$
$$\frac{25\,000}{17\,000} = \frac{17\,000(1+i)^5}{17\,000}$$
$$1{,}470\,588\ldots = (1+i)^5$$
$$\sqrt[5]{1{,}47\ldots} = \sqrt[5]{(1+i)^5}$$
$$1{,}08018\ldots = 1 + i$$
$$1{,}080\,18\ldots - 1 = 1 + i - 1$$
$$0{,}080\,18\ldots = i$$
$$i = 0{,}080\,18\ldots \times 100\%$$
$$= 8{,}02\ \%$$

**Key Sequence**

$\sqrt{\square}$ 5 ▶ 25 000 ÷ 17 500 =

2) $P$ = R17 000
$A$ = R25 000
$n$ = 5 × 2 = 10 half-years
$i$ = ? % p.a.

$$A = P(1+i)^n$$
$$25\,000 = 17\,000(1+i)^{10}$$
$$\frac{25\,000}{17\,000} = \frac{17\,000(1+i)^{10}}{17\,000}$$
$$1{,}470\,588\ldots = (1+i)^{10}$$
$$\sqrt[10]{1{,}470588\ldots} = \sqrt[10]{(1+i)^{10}}$$
$$1{,}039\,319\ldots = 1 + i$$
$$1{,}039\,319\ldots - 1 = 1 + i - 1$$
$$i = 0{,}039\,319\ldots \text{per half year}$$
$$i = 0{,}039\,319\ldots \times 100\%$$
$$= 3{,}9319\ \ldots\% \text{ per half year}$$
$$= 7{,}86\ \%\ \text{p.a.}$$

**Key Sequence**

$\sqrt{\square}$ 10 ▶ 25 000 ÷ 17 500 =

**Exercise 11.13**

An investment of R5 000 increases to R7 500 in 6 years
Calculate the interest rate as a percentage per annum, correct to 1 decimal place

1) when interest is compounded annually

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2) when interest is compounded half-yearly

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## § 11.14 A MIXED EXERCISE

1) Calvin's overseas trip in five years time will probably cost R23 000. How much must he invest now at 12,25% p.a. compounded annually to fund the trip?

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2) Doris has R10 450 to invest and requires R20 000 at the end of 6 years. What interest rate compounded half-yearly will she need? Answer correct to 1 decimal place.

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3) Sophie invested R15 500 and received R13 546,70 in interest at the end of 5 years.
   a) How much has she saved in total after 5 years?
   b) At what interest rate did she invest her money if it was compounded monthly for 5 years?

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## § 11.15 THE COMPOUND DECREASE FORMULA

- An investment which **grows or increases** in value, over time, is said to **appreciate** in value.
- An investment which **decreases** in value, over time, is said to **depreciate** in value. Office equipment and motor cars are investments that depreciate with time and become worth less than what was paid for them. The purchasing power of your money decreases owing to **inflation**. The price of most goods and services show a steady, compounded increase, from year to year.
- For depreciation we use the formula $A = P(1-i)^n$

  where $A$ = the amount received at the end of the investment time
  $P$ = the principal (the money invested)
  $i$ = the interest rate p.a. written as a decimal
  $n$ = the number of time periods of the investment

| EXAMPLE | SOLUTION |
|---|---|
| A new car cost R98 500 in 2004. The car depreciates by 11% of its value each year | $P$ = R98 500<br>$i$ = 11 % p.a. = 0,11 |
| 1) How much will the car be worth in 2008? Give the answer correct to the nearest rand. | $n$ = 2008 – 2004 = 4 years<br>$A = P(1-i)^n$<br>$= 98\ 500(1 - 0{,}11)^4$<br>$= 98\ 500(0{,}89)^4$<br>= 61 801<br>The car is worth R61 801 in 2008. |
| 2) By how much will the car have depreciated? | The car has depreciated by R98 500 – R61 801<br>= R36 699 in four years. |

**Exercise 11.15**

A motorcar cost R71 000 in July 2002. Its value depreciates by 10% p.a. compounded annually.

1) How much will the motorcar be worth in July 2008?

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2) By how much will the car have depreciated?

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## § 11.16 POPULATION DECREASE

✧ The compound decrease formula may also be used for calculating population decrease.

| EXAMPLE | SOLUTION |
|---|---|
| A town has a population of 78 895 in January 2008. An annual average rate of compound decrease of 6% p.a. is expected in the population after this date. | $P$ = 78 895<br>$i$ = 6% p.a. = 0,06 p.a. |
| 1) What will be the expected population in January 2012? | $n$ = 2012 – 2008 = 4 years<br>$A = P(1-i)^n$<br>$A = 78\,895(1-0{,}06)^4$<br>$= 78\,895(0{,}94)^4 = 61\,597$<br>The expected population in January 2012 is 61 597. |
| 2) Calculate the population decrease. | Population decrease = 78 895 – 61 597 = 17 297 |

**Exercise 11.16**

1) A town has a population of 54 678 in January 2008. An annual average rate of compound decrease of 8% p.a. is expected in the population after this date.
   a) What will be the expected population in January 2010?

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   b) Find the population decrease.

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2) The population of a city in July 2008 is 98 979 people. The city has experienced an annual average rate of compound decrease of 7% p.a..
   a) What was the population in July 2004?

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   b) Calculate the population decrease.

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## § 11.17 MIXED EXERCISE

1) Equipment depreciates at 11% p.a. In June 2008, the equipment was worth R230 000.
   a) What will the equipment be worth in June 2011?

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   b) By how much will it have depreciated?

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2) If the inflation rate is steady at 6% p.a. for 5 years compounded annually, what is the purchasing power of R100 after 5 years?

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3) Machinery worth R256 300 depreciates at a rate of 12% p.a. How much is it worth after 6 years if the rate of depreciation is compounded annually?

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## § 11.18 HIRE PURCHASE (H.P.) AGREEMENTS

- Goods bought on HP are taken from the store **before the full purchase price is paid**.
- A deposit may be paid at the time of purchase and interest is charged on the balance. The equal repayments made may be weekly or monthly.
- These agreements often charge *S I* (***S I = P × i × n***) on the full loan for the full period, not just on the amount still to be paid.

**EXAMPLE**

Musa buys goods for R34 600 on H.P. He puts down a 15% deposit and repays the balance at 16% p.a. *S I* in equal monthly instalments over 5 years.

1) Calculate the deposit
2) Calculate the balance owing after interest has been included
3) How much will he pay per month?

**SOLUTION**

| | | |
|---|---|---|
| 1) Deposit<br>= 15% of R34 600<br>= 0,15 × R34 600<br>= R5 190 | 2) Balance owing after paying deposit<br>= R34 600 – R5 190<br>= R29 410<br><br>$P$ = R29 410 $n$ = 5 years<br>$i$ = 16% p.a. = 0,16 p.a.<br><br>$SI = P \times i \times n$<br>= 29 410 × 0,16 × 5<br>= R23 528<br><br>Total amount owing,<br>$A = P + SI$<br>= R23 528 + R29 410<br>= R52 938 | 3) $n$ = 5 years<br>= 5 × 12<br>= 60 months<br><br>Monthly repayments<br>$= \frac{52\,938}{60}$<br>= R882,30 per month |

**Exercise 11.18**

Neo purchases a motorcar for R71 650. She pays a deposit of 15% and pays the rest off in equal monthly instalments at an interest rate of 12,5% p.a. *SI* over 5 years.

1) How much is her deposit?

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2) How much does of the original price does she still owe?

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3) How much does she owe in total once interest has been added?

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4) What is her monthly repayment?

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## § 11.19 PROFIT AND LOSS

- The profit ($P$) or the loss ($L$) is the difference between the selling price ($SP$) and the cost price ($CP$). The profit will be a positive amount and the loss will be negative.

$$P = SP - CP \qquad\qquad L = CP - SP$$

- The profit or loss is usually calculated as a percentage of the cost price.

$$\textit{Percentage Profit} = \frac{P}{CP} \times 100\% \qquad\qquad \textit{Percentage Loss} = \frac{L}{CP} \times 100\%$$

| EXAMPLE | SOLUTION |
|---|---|
| 1) A baker sells biscuits for R12,55 per bag. He calculates that each bag of biscuits costs him R6,50 to produce.<br>a) What is his profit on each bag?<br>b) What is his percentage profit (correct to 2 decimal places) on each bag? | a) $P = SP - CP$<br>$= \text{R}12{,}55 - \text{R}6{,}50 = \text{R}6{,}05$<br>b) Percentage Profit<br>$= \frac{6{,}05}{6{,}50} \times 100\% = 93{,}08\%$ |
| 2) The baker sells his old stock at half-price, at R6,28 per bag.<br>a) What is his loss per bag?<br>b) What is his percentage loss (correct to 2 decimal places) on each bag? | a) $L = CP - SP$<br>$= \text{R}6{,}50 - \text{R}6{,}28 = \text{R}0{,}22$<br>His loss per bag is 22c<br>b) Percentage Loss<br>$= \frac{0{,}22}{6{,}50} \times 100\% = 3{,}38\%$ |

**Exercise 11.19**

1) A bookshop sells a book marked R149,50. The cost of the book to the shop is R75,50.

   a) Calculate the profit made on each book sold.

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   b) Find the percentage profit (correct to 1 decimal place) made by the shop on each book sold.

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2) Owing to poor sales, the bookshop decides to sell the book in 1) for R109,20.

   a) What discount are they giving on the original selling price?

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   b) Calculate the profit made on each book sold at the new selling price

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   c) Calculate the new percentage profit, correct to the nearest whole number.

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