

CHAPTER 10

Data handling

In this chapter you will:

- Distinguish between discrete and continuous data
- Organise data into lists and into frequency tables
- Calculate the range, mode, median and mean of ungrouped data
- Group data in a frequency table
- Calculate the mode, median and mean of grouped data
- Calculate the lower quartile (Q_1), upper quartile (Q_3) and the Interquartile Range (IQR)
- Draw vertical bar graphs, horizontal bar graphs and compound bar graphs
- Draw pie charts
- Draw line graphs and broken line graphs
- Draw histograms of grouped data and frequency polygons

This chapter covers material from Topic 4: Statistical and Probability Models

SUBJECT OUTCOME 4.1:

Collect, organise and interpret univariate numerical data

Learning Outcome 1: Calculate measures of central tendency, namely mean, median and mode, of grouped and ungrouped data

Learning Outcome 2: Calculate measures of dispersion including range, percentiles, quartiles, interquartile and semi-interquartile range

SUBJECT OUTCOME 4.2

Represent data effectively and appropriately

Learning Outcome 1: Create and interpret bar and compound bar graphs

Learning Outcome 2: Create and interpret histograms of grouped data

Learning Outcome 3: Create and interpret frequency polygons

Learning Outcome 4: Create and interpret pie charts

Learning Outcome 5: Create and interpret line and broken line graphs

§ 4.1 TYPES OF DATA

- ✦ Data handling deals with:
 - a) the collection,
 - b) the organisation,
 - c) the presentation,
 - d) the analysis and
 - e) the interpretation of data.
- ✦ Data can be collected from questionnaires, interviews and experiments. Books, magazines, and the internet are also sources of data.
- ✦ Collected data, called **raw data**, has to be **organised** and **presented** in such a way that it makes sense and so that some deduction may be drawn from it.
- ✦ The data may be **discrete** or **continuous**.
 - With **discrete data**, each item of data is exact and distinct. This data is often a total of counted objects.

Examples of discrete data are:

 - a) Number of children (no fractions)
 - b) Shoe size (half sizes included, but no smaller fractions)
 - c) Number of objects
 - With **continuous data**, the data is measured to some degree of accuracy, e.g. to 1 decimal place. It is an approximation and not necessarily exact. This data is usually a measurement.

Examples of continuous data are

 - a) Age (measured to the nearest year or month)
 - b) Time (measured to the nearest hour, minute, second)
 - c) Length (measured to the nearest cm)

Exercise 4.1

Classify each of the following examples of data as a discrete or continuous:

- 1) The number of goals scored in successive football matches
- 2) Shirt sizes of the men in an office
- 3) The height of each child in a class
- 4) The temperature of an oven in cake baking
- 5) The volume of water in each tank at a recycling centre
- 6) The number of children in each classroom in a school
- 7) The set of mathematics test marks obtained by a class
- 8) The heights of the mealies in a field
- 9) The number of sweets in various boxes
- 10) The mass of each of the players in a rugby team

§ 4.2 ORGANISING DATA

- ✦ If data is not organised, no pattern or trend can be seen. One way to organise data is to arrange it in descending or ascending order.

EXAMPLE

Listed below are the results (raw data) of an experiment in which a dice is thrown 40 times and the number on the upper face is recorded after each throw.

2 3 1 5 4 2 3 5 2 1 6 2 4 5 2 3 5 2 2 5
5 6 5 4 2 2 5 2 6 3 4 5 5 2 1 3 2 6 5 4

Arrange this data in descending order.

SOLUTION

6 6 6 6 5 5 5 5 5 5 5 5 5 5 5 4 4 4 4 4
3 3 3 3 3 2 2 2 2 2 2 2 2 2 2 2 2 1 1 1

- ✦ Organising the data in an array still does not give a clear picture. A **frequency table** shows better organisation.

EXAMPLE

- 1) Organise the above data (the dice being thrown 40 times) using a frequency table.
- 2) What conclusion can be drawn from the data?
- 3) Which of the two forms of organisation presents the data more clearly?

SOLUTION

1)

Number	Tally	Frequency
6		4
5	 	11
4		5
3		5
2	 	12
1		3
TOTAL		40

- 2) Conclusion: There are more 2's and 5's than of any other number. A fair dice would show about the same frequency for each number, especially with a greater number of throws. The dice is possibly biased (weighted).
- 3) The frequency table presents the data more clearly than a list.

Note:

- By adding the frequencies we can check that all the data is entered on the table

Exercise 4.2

A dice is thrown 20 times and the number is recorded after each throw. Write the data in ascending order and also in a frequency table.

6	3	1	6	4	6	1	1	2	1	6	2	4	5	6	3	6	1	6	1

Face	1	2	3	4	5	6	Total
Frequency							

§ 4.3 THE RANGE AND THE MODE

- ✦ The **range** is a measure of spread. It is the difference in value between the highest and the lowest values. **Range = highest value – lowest value.**
- ✦ The **mode** is the value that occurs most often amongst all the data items. It is the value having the greatest frequency.

EXAMPLES	SOLUTIONS												
1) Find the range and the mode of the following raw data: 2, 3, 1, 2, 4, 4, 1, 3, 2, 4, 2, 2, 1	We can arrange the data in ascending order: 1, 1, 1, <u>2</u> , <u>2</u> , <u>2</u> , <u>2</u> , 3, 3, 4, 4, 4 OR we can use a frequency table: <table><tr><td>data</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>frequency</td><td>3</td><td>4</td><td>2</td><td>3</td></tr></table> Range = highest value – lowest value = 4 – 1 = 3 The data item occurring most often (having the greatest frequency) is 2, so the Mode = 2 .	data	1	2	3	4	frequency	3	4	2	3		
data	1	2	3	4									
frequency	3	4	2	3									
2) Find the range and the mode of the following data: 3, 4, 6, 4, 1, 5, 6, 1, 3, 5	Arrange the data in ascending order: 1, 1, 3, 3, 4, 4, 5, 5, 6, 6, Range = highest value – lowest value = 6 – 1 = 5 There is no mode , since no one data item occurred more frequently than any of the others.												
3) Find the range and the mode of the data on the frequency table: <table><tr><td>data</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>frequency</td><td>2</td><td>5</td><td>1</td><td>5</td><td>4</td></tr></table>	data	1	2	3	4	5	frequency	2	5	1	5	4	Range = highest data value – lowest data value = 5 – 1 = 4 The outcomes 2 and 4 each have the greatest frequency of 5 , so there are two modes. We say that the set of data is bimodal . The Modes are 2 and 4 .
data	1	2	3	4	5								
frequency	2	5	1	5	4								

Note:

- It is not the frequency that is the mode, it is the data item

Exercise 4.3

1) Find the range and the mode of the following set of raw data: 4, 2, 6, 3, 3, 2, 4, 3, 7, 2, 7, 6, 6, 7, 6

2) Use the frequency table to find the range and the mode of the data:

data	1	2	3	4	5	6
frequency	3	2	4	3	1	2

3) Organise the following raw data in a frequency table and then find the mode:

A, B, B, D, C, E, B, D, C, C, A, B, A, C, D, D, D, E, A, D, B, A, B

§ 4.4 THE MEDIAN

- ✦ The **median** is the value of the middle item in a distribution once all the data has been arranged in order of size.
- ✦ The median does not have to be one of the data items.

EXAMPLES	SOLUTIONS														
1) Find the median of the following set of data: 6, 3, 4, 7, 2, 5, 3, 4, 5	Arrange the data in ascending order: 2, 3, 3, 4, 4 , 5, 5, 6, 7 There are 9 data items, an odd number. The 5th data item , the middle item, is the median = 4														
2) Find the median of the following set of data: 5, 9, 1, 7, 2, 6, 9, 3.	Arrange the data in ascending order: 1, 2, 3, 5 , 6 , 7, 9, 9 There are 8 data items, an even number. The middle item is the average of the 4th and 5th terms So the median = $\frac{5+6}{2} = \frac{11}{2} = 5,5$														
3) Find the median of the data on the frequency table: <table border="1"><tr><td>Data</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>Total</td></tr><tr><td>Frequency</td><td>2</td><td>4</td><td>1</td><td>3</td><td>2</td><td>12</td></tr></table>	Data	1	2	3	4	5	Total	Frequency	2	4	1	3	2	12	We can write the data items in a list: 1, 1, 2, 2, 2, 2 , 3 , 4, 4, 4, 5, 5. There are 12 data items, an even number. The middle item is the average of the 6th and 7th terms So the median = $\frac{2+3}{2} = \frac{5}{2} = 2,5$
Data	1	2	3	4	5	Total									
Frequency	2	4	1	3	2	12									

Exercise 4.4

- 1) Find the median of the raw data: 3, 6, 1, 0, 2, 5, 2, 3, 1

Write the data in numerical order:

Number of data items =

Median =

- 2) Find the median of the raw data: 9, 2, 1, 9, 7, 4, 2, 3

Write the data in numerical order:

Number of data items =

Median =

- 3) Find the median of the data in the frequency table :

DATA	FREQUENCY
0	2
1	2
3	2
4	5
5	3
TOTAL	

List of data items:

Number of data items =

Median =

§ 4.5 THE MEAN

- ✦ The **mean** is found by dividing the sum of all the data by the number of data items.
- ✦ It is called the **equal shares** average.
- ✦ When finding the mean, it is not necessary to first arrange the data items in order.

EXAMPLES	SOLUTIONS												
1) Find the mean of the data: 2, 3, 4, 5, 6, 7, 8	$\text{Mean} = \bar{x} = \frac{2+3+4+5+6+7+8}{7} = \frac{35}{7} = 5$												
2) Find the value of x so that the mean of the given data: 8, 3, 1, x , 4, 6, 2, 6, is 4	$\text{Mean} = \bar{x} = \frac{8+3+1+x+4+6+2+6}{8} = 4$ $\frac{30+x}{8} = 4$ $\frac{30+x}{8} \times 8 = 4 \times 8$ $30+x = 32$ $x = 2$												
3) Find the mean, correct to 1 decimal place, of the data in the following frequency table: <table border="1"><tr><td>data</td><td>2</td><td>3</td><td>4</td><td>5</td><td>Total</td></tr><tr><td>frequency</td><td>4</td><td>1</td><td>2</td><td>2</td><td>9</td></tr></table>	data	2	3	4	5	Total	frequency	4	1	2	2	9	Raw data: 2, 2, 2, 2, 3, 4, 4, 5, 5 $\bar{x} = \frac{2+2+2+2+3+4+4+5+5}{9} = \frac{29}{9} = 3.\dot{2}$ OR $\bar{x} = \frac{(4 \times 2) + (1 \times 3) + (2 \times 4) + (2 \times 5)}{9} = \frac{29}{9} = 3.\dot{2}$
data	2	3	4	5	Total								
frequency	4	1	2	2	9								

Exercise 4.5

- 1) Calculate the mean of the data: 7, 3, 5, 2, 9, 3, 1, 2

- 2) Find the mean, to 1 decimal place, of the data whose frequency table is given.

data	1	2	3	4	5	Total
frequency	3	1	4	2	4	

List of raw data:

Mean = $\bar{x} =$

- 3) The following data items are given: 7, 3, 1, x , 6. The mean of the data is 5. Find the value of x .

- 4) Find the range, mode, median and mean of the following set of data correct to 1 decimal place:
32; 32; 33; 35; 37; 40; 40; 42; 46; 54; 57; 57; 57; 57; 61; 62; 62; 63; 65; 66; 70

§ 4.6 GROUPING DATA

✦ When data is being collected, it is sometimes convenient to record it in groups or **class intervals**.

EXAMPLE

The test marks, out of 40, for of a class of 20 students are recorded below.

30 22 18 17 28 40 39 13 6 19
39 12 16 37 29 9 17 20 1 28

Collect the data items on a grouped frequency table.

SOLUTION

- ✦ Presenting these marks on a frequency distribution table would need 17 rows.
- ✦ We can group the marks in equal intervals for fewer rows and a more compact presentation.

Interval	Tally	Frequency
1 – 10		3
11 – 20	 	8
21 – 30	 	5
31 – 40		4
TOTAL		20

Note:

By adding the frequencies we can check that all 20 pieces of data are on the table.

Exercise 4.6

The number of runs scored by the batsmen in the 18 cricket matches played by a team are:

35 0 12 34 21 6 15 39 44 7 19 33 34 40 29 14 22 39

- 1) What is the highest score?
- 2) What is the lowest score?
- 3) Use a suitable grouping of this data to complete the frequency distribution table below.

Interval	Tally	Frequency
0 – 9		
TOTAL		

§ 4.7 FINDING THE MODE AND MEDIAN OF GROUPED DATA

- ✦ The interval with the largest frequency is called the **modal class** or **modal group** or **modal interval**.
- ✦ With a grouped frequency table it is not possible to find a single value that is the median. We can, however, find the interval in which the median lies.

EXAMPLE	SOLUTION												
Find the mode and the median of the following set of data: <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Mark interval</th><th>Frequency</th></tr> </thead> <tbody> <tr> <td>1 – 10</td><td>3</td></tr> <tr> <td>11 – 20</td><td>8</td></tr> <tr> <td>21 – 30</td><td>5</td></tr> <tr> <td>31 – 40</td><td>4</td></tr> <tr> <td>TOTAL</td><td>20</td></tr> </tbody> </table>	Mark interval	Frequency	1 – 10	3	11 – 20	8	21 – 30	5	31 – 40	4	TOTAL	20	The modal class or modal group or modal interval is a score of 11 – 20 marks as this is the interval with the greatest frequency. There are 20 data values. Half of 20 is 10. The median lies between the 10th and the 11th values. Add up the frequencies either from the top or from the bottom. Both the 10th and the 11th values lie in the interval 11 – 20. The median lies in the interval 11 – 20.
Mark interval	Frequency												
1 – 10	3												
11 – 20	8												
21 – 30	5												
31 – 40	4												
TOTAL	20												

Exercise 4.7

For each of the following, find

- a) the modal class
- b) the interval in which the median lies

1)

Class interval	Frequency
0 – 4	2
5 – 9	5
10 – 14	6
TOTAL	

- a)
- b)

2)

Age group	Number of people
0 – 9	6
10 – 19	7
20 – 29	10
30 – 39	3
40 – 49	1
TOTAL	

- a)
- b)

§ 4.8 FINDING THE MEAN OF GROUPED DATA

✦ Since you do not know the exact data when data is grouped, you can only find an **estimate** of the mean

EXAMPLE	SOLUTION																																			
Find the mean (correct to 2 decimal places) of the following set of data	<u>STEP 1:</u> Find the midpoint of the class interval: <i>Mid-point of the class interval</i> = $\frac{1}{2}$ (smallest value + greatest value)																																			
<table><tr><th>Class interval</th><th>Frequency</th></tr><tr><td>0 – 4</td><td>2</td></tr><tr><td>5 – 9</td><td>5</td></tr><tr><td>10 – 14</td><td>6</td></tr><tr><td>15 – 19</td><td>5</td></tr><tr><td>20 – 24</td><td>6</td></tr><tr><td>TOTAL</td><td>24</td></tr></table>	Class interval	Frequency	0 – 4	2	5 – 9	5	10 – 14	6	15 – 19	5	20 – 24	6	TOTAL	24	<table><tr><th>Class interval</th><th>mid-point</th><th>Frequency</th></tr><tr><td>0 – 4</td><td>$\frac{0+4}{2} = \frac{4}{2} = 2$</td><td>2</td></tr><tr><td>5 – 9</td><td>$\frac{5+9}{2} = \frac{14}{2} = 7$</td><td>5</td></tr><tr><td>10 – 14</td><td>$\frac{10+14}{2} = \frac{24}{2} = 12$</td><td>6</td></tr><tr><td>15 – 19</td><td>$\frac{15+19}{2} = \frac{34}{2} = 17$</td><td>5</td></tr><tr><td>20 – 24</td><td>$\frac{20+24}{2} = \frac{44}{2} = 22$</td><td>6</td></tr><tr><td>TOTAL</td><td></td><td>24</td></tr></table>	Class interval	mid-point	Frequency	0 – 4	$\frac{0+4}{2} = \frac{4}{2} = 2$	2	5 – 9	$\frac{5+9}{2} = \frac{14}{2} = 7$	5	10 – 14	$\frac{10+14}{2} = \frac{24}{2} = 12$	6	15 – 19	$\frac{15+19}{2} = \frac{34}{2} = 17$	5	20 – 24	$\frac{20+24}{2} = \frac{44}{2} = 22$	6	TOTAL		24
Class interval	Frequency																																			
0 – 4	2																																			
5 – 9	5																																			
10 – 14	6																																			
15 – 19	5																																			
20 – 24	6																																			
TOTAL	24																																			
Class interval	mid-point	Frequency																																		
0 – 4	$\frac{0+4}{2} = \frac{4}{2} = 2$	2																																		
5 – 9	$\frac{5+9}{2} = \frac{14}{2} = 7$	5																																		
10 – 14	$\frac{10+14}{2} = \frac{24}{2} = 12$	6																																		
15 – 19	$\frac{15+19}{2} = \frac{34}{2} = 17$	5																																		
20 – 24	$\frac{20+24}{2} = \frac{44}{2} = 22$	6																																		
TOTAL		24																																		
	<u>STEP 2:</u> For each interval multiply together the frequency and the midpoint																																			
	Estimate of $\bar{x}$																																			
	$= \frac{(2 \times 2) + (5 \times 7) + (6 \times 12) + (5 \times 17) + (6 \times 22)}{2 + 5 + 6 + 5 + 6}$																																			
	$= \frac{4 + 35 + 72 + 85 + 132}{24} = \frac{328}{24} = 13,67$																																			

Exercise 4.8

The following table of grouped data gives the ages (in years) of a number of people:

Age group in years	Mid point of age group	Number of people
0 – 9		5
10 – 19		4
20 – 29		10
30 – 39		5
40 – 49		3
TOTAL		

- 1) Complete the table
- 2) Use the table to find:
 - a) The modal class
 - b) The median class
 - c) The estimate of the mean, correct to the nearest whole year

§ 4.9 FINDING QUARTILES

- ✦ In many experimental situations, data extremes are often discarded as inaccurate measurements. The middle section of the data set, where most of the data lies, gives the best picture of the group they represent.
- ✦ The **median** is the value that divides the data into two halves.
- ✦ **Quartiles** are the values that divide the data into four quarters, each quarter containing the same number of data items
 - The **median** is the middle quartile.
 - The **lower quartile (Q_1)** is the median of the lower half of the values.
 - The **upper quartile (Q_3)** is the median of the upper half of the values.

EXAMPLE	SOLUTION
For the following set of data 7 1 3 9 8 4 4 3 2 7 9 Find 1) The range 2) The median 3) The lower quartile, Q_1 4) The upper quartile, Q_3	First write the data in ascending order: 1 2 3 3 4 4 7 7 8 9 9 Q_1 Median Q_3 1) Range = $9 - 1 = 8$ 2) There are 11 data items, an odd number. The median is the 6th data item. The median = 4, since there are 5 data items below it and 5 above. 3) $Q_1 = 3$ is the median of the first 5 data items, since there are 2 data items on either side of it. 4) $Q_3 = 8$ is the median of the upper 5 data values, since there are 2 data items on either side of it.

Exercise 4.9

For the following sets of data:

- 1) a) Arrange the following set of data in ascending order:

4; 2; 3; 4; 1; 5; 5; 1; 6; 6; 1; 4; 3; 4; 2

.....

- b) Find

i) The range

ii) The median

iii) The lower quartile Q_1

iv) The upper quartile Q_3

- 2) a) Arrange the following set of data in ascending order:

13; 14; 3; 6; 23; 78; 20; 22; 21; 35; 62; 93; 4; 67; 54; 4; 21; 30; 44; 54

.....

- b) Find

i) The range

ii) The median

iii) The lower quartile Q_1

iv) The upper quartile Q_3

§ 4.10 THE INTERQUARTILE RANGE (IQR)

✦ The **interquartile range** (IQR) is a measure of spread of a set of statistical data. It is the difference between the upper and lower quartiles. We write: $\mathbf{IQR = Q_3 - Q_1}$

✦ The **semi-interquartile range (Semi-IQR)** is one half of the interquartile range.

We write: $\mathbf{Semi-IQR = \frac{IQR}{2} = \frac{Q_3 - Q_1}{2}}$

EXAMPLE

The following set of data is a set of marks, out of 10, of a class of learners:

3 6 4 4 5 9 1 3 2 5 3 7 2 7 3 1 9 1

1) For this set of marks find

- a) The median b) The lower quartile, Q_1 c) The upper quartile, Q_3
 d) The interquartile range, the IQR f) The semi-IQR range

2) Explain what each of these statistics mean practically with respect to the class.

SOLUTION

1) Data arranged in order:

1;1;1;2;2;3;3;3;3;4;4;5;5;6;7;7;9;9

a) There are 18 data items, so

$$\text{The median} = \frac{3+4}{2} = \frac{7}{2} = 3,5$$

b) $Q_1 = 2$

c) $Q_3 = 6$

d) $\mathbf{IQR = Q_3 - Q_1 = 6 - 2 = 4}$

e) $\mathbf{Semi-IQR = \frac{IQR}{2} = \frac{4}{2} = 2}$

2) Nobody got full marks

a) The middle mark of the group is 3,5 out of 10, even though nobody actually got this mark. The learners as a class have not done well.

b) $\mathbf{Q_1}$ shows that $\frac{1}{4}$ of the learners scored less than 2 marks. They scored 1's and 2's. $\frac{1}{4}$ of the class did badly.

c) $\mathbf{Q_3}$ shows that $\frac{3}{4}$ of the learners scored less than 6 marks.

d) The **IQR** shows that the marks of half the learners lie within 4 marks of each other.

e) The **semi-IQR** shows that the marks of half the learners lie within 2 marks of the median.

Exercise 4.10

The class wrote another test out of 10 marks, covering the same work. Their marks were:

2; 9; 0; 2; 3; 10; 2; 1; 9; 8; 4; 1; 8; 8; 10; 8; 3; 9

1) Find the range, median, Q_1 , Q_3 , IQR, and the semi-IQR range of this set of marks.

2) Use these statistics to determine whether the class did better in this test.

§ 4.11 QUARTILES AND FREQUENCY TABLES

✦ We can find the **Interquartile Range (IQR)** of data listed in a **frequency table**.

EXAMPLE	SOLUTION																										
The test scores, out of 10 marks, of 29 learners is listed in the following frequency table: <table border="1"> <thead> <tr> <th>SCORE</th><th>FREQUENCY</th></tr> </thead> <tbody> <tr><td>0</td><td>0</td></tr> <tr><td>1</td><td>2</td></tr> <tr><td>2</td><td>4</td></tr> <tr><td>3</td><td>0</td></tr> <tr><td>4</td><td>5</td></tr> <tr><td>5</td><td>2</td></tr> <tr><td>6</td><td>6</td></tr> <tr><td>7</td><td>3</td></tr> <tr><td>8</td><td>3</td></tr> <tr><td>9</td><td>2</td></tr> <tr><td>10</td><td>1</td></tr> <tr> <td>TOTAL</td><td>29</td></tr> </tbody> </table> For this set of marks, calculate: 1) The range 2) The median 3) Q_1 4) Q_3 5) The IQR 6) The semi-IQR range	SCORE	FREQUENCY	0	0	1	2	2	4	3	0	4	5	5	2	6	6	7	3	8	3	9	2	10	1	TOTAL	29	1) The range = $10 - 1 = 9$ 2) There are 29 data items. The median is the 15th item. Add the frequencies to find this item. Median = 6, since there are 14 items on either side of it. 3) Q_1 is the mean of the 7th and 8th data items, since there are 6 items on either side of these. Both of these items are 4, so $Q_1 = 4$. 4) Q_2 is the mean of the 22nd and 23rd data items, since there are 6 items on either side of these. Both of these items are 7, so $Q_2 = 7$ 5) $IQR = Q_3 - Q_1 = 7 - 4 = 3$. 6) $\text{Semi-IQR} = \frac{IQR}{2} = \frac{3}{2} = 1,5$.
SCORE	FREQUENCY																										
0	0																										
1	2																										
2	4																										
3	0																										
4	5																										
5	2																										
6	6																										
7	3																										
8	3																										
9	2																										
10	1																										
TOTAL	29																										

Exercise 4.11

The test scores, out of 10 marks, of 29 learners are recorded in the following frequency table:

Score	Frequency
0	1
1	2
2	2
3	1
4	3
5	6
6	7
7	3
8	2
9	1
10	1
TOTAL	

For this set of marks, calculate

- 1) The total number of scores and write it on the table
- 2) The range
- 3) The median
- 4) Q_1
- 5) Q_3
- 6) The IQR
- 7) The semi-IQR range

§ 4.12 DRAWING BAR GRAPHS

- ✦ We can represent numeric or non-numeric discrete data on a bar graph. A bar graph gives a better visual picture than a frequency table.
- ✦ The bar length represents the frequency of the data item.
- ✦ The bars are not joined together as the data is discrete.

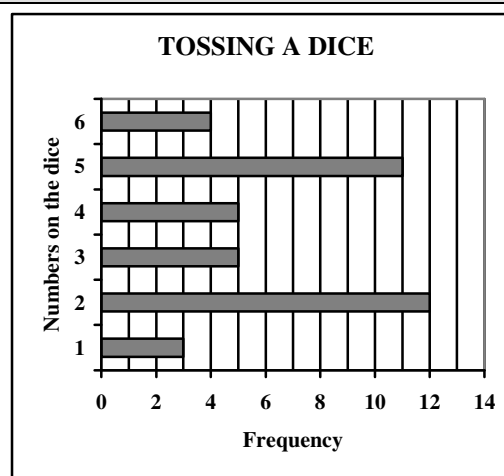
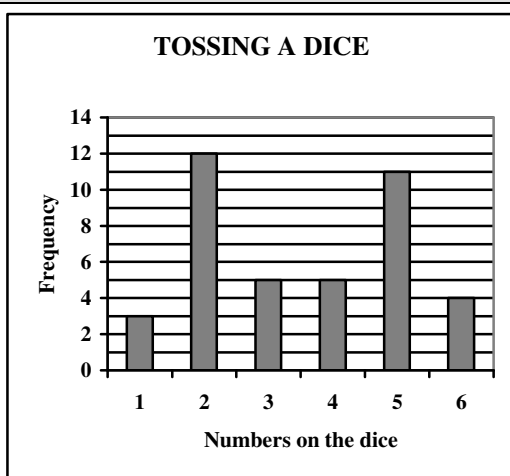
EXAMPLE

A dice is thrown 40 times and the number is recorded after each throw.

Represent the given data: **a)** as a vertical bar graph **b)** as a horizontal bar graph

Number on dice	1	2	3	4	5	6
Frequency	3	12	5	5	11	4

SOLUTION

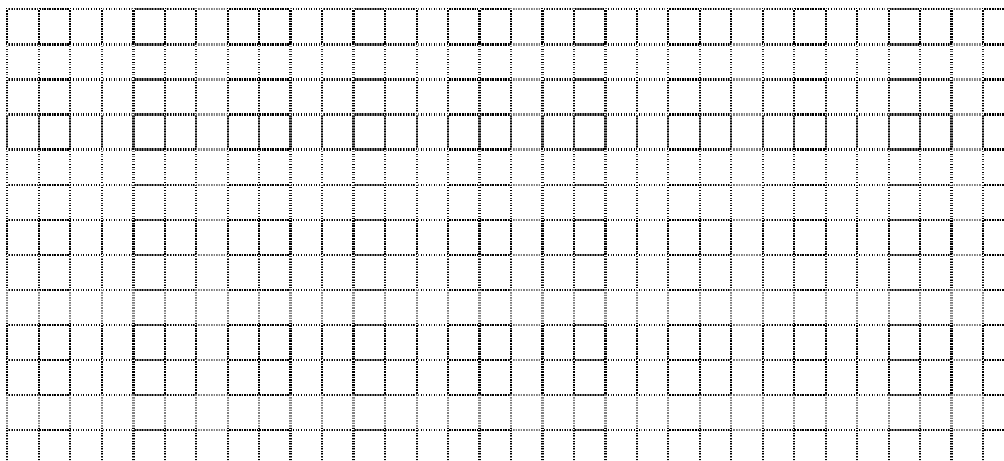


Exercise 4.12

A bag contains red, black, yellow and white balls that are all identical in size. A ball is chosen at random, its colour recorded and the ball is returned to the bag, which is shaken. This process is repeated 34 times, and the following results are obtained:

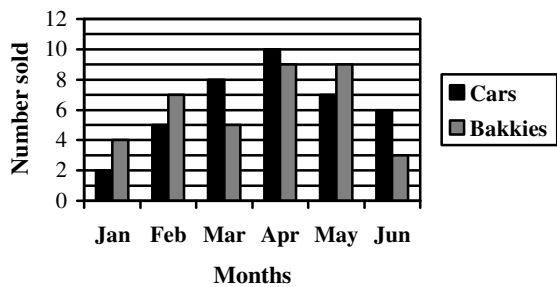
Ball colour	Red (R)	Black (B)	Yellow (Y)	White (W)
Frequency	7	9	10	8

Represent the data: **a)** as a vertical bar graph **b)** as a horizontal bar graph.



§ 4.13 COMPOUND BAR GRAPHS

- ✦ Two or more sets of data can be compared by drawing them side by side on a compound bar graph.

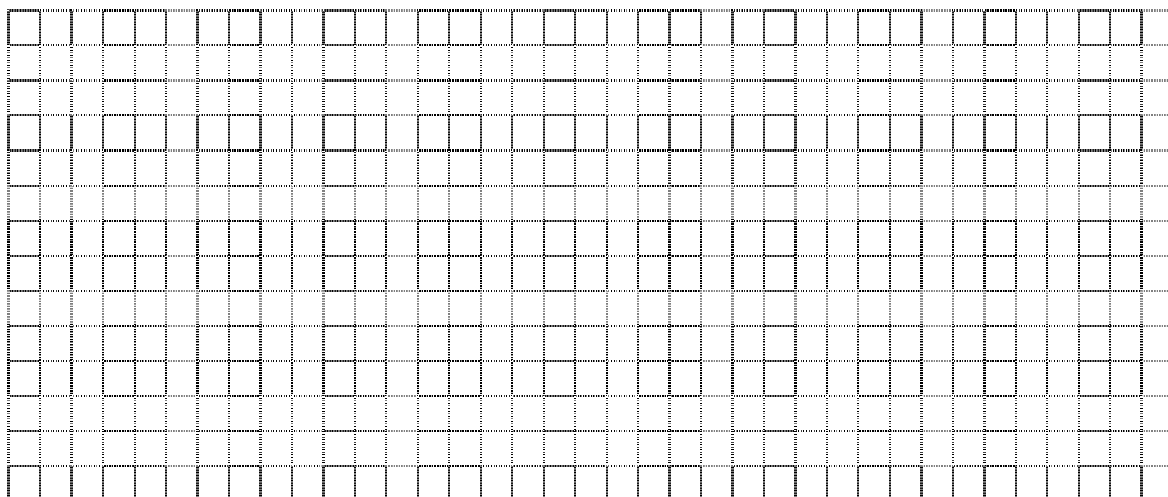
EXAMPLE	SOLUTION																					
The table shows the number of cars and bakkies sold by a garage in each of the first six months of 2008: <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <thead> <tr> <th>Month</th> <th>Jan</th> <th>Feb</th> <th>Mar</th> <th>Apr</th> <th>May</th> <th>Jun</th> </tr> </thead> <tbody> <tr> <td>Cars</td> <td>2</td> <td>5</td> <td>8</td> <td>10</td> <td>7</td> <td>6</td> </tr> <tr> <td>Bakkies</td> <td>4</td> <td>7</td> <td>5</td> <td>9</td> <td>9</td> <td>3</td> </tr> </tbody> </table> 1) Draw a compound bar graph to illustrate the data 2) Use the graph to answer the following: - Which was the best month for car sales? How many were sold? - Which was the worst month for bakkie sales? - In which months were more cars than bakkies sold?	Month	Jan	Feb	Mar	Apr	May	Jun	Cars	2	5	8	10	7	6	Bakkies	4	7	5	9	9	3	NUMBER OF CARS AND BAKKIES SOLD FROM JANUARY TO JUNE 2008  a) April was the best month. 10 cars were sold. b) June was the worst month c) March, April and June
Month	Jan	Feb	Mar	Apr	May	Jun																
Cars	2	5	8	10	7	6																
Bakkies	4	7	5	9	9	3																

Exercise 4.13

The number of cars and bakkies sold by a garage in each of the last 6 months of 2007:

Month	July	Aug.	Sept.	Oct.	Nov.	Dec.
Cars	6	9	8	8	12	6
Bakkies	4	3	9	10	11	4

- 1) Represent the data as a compound bar graph.



- 2) Use the graph to answer the questions:
- Which was the best month for car sales? How many were sold?
 - Which was the worst month for bakkie sales?
 - In which months were more cars than bakkies sold?

§ 4.14 PIE CHARTS

- ✦ To represent data in a pie chart, we divide a circle into 'slices' like a pie.
- ✦ Each slice or sector represents the frequency of the data item.

EXAMPLE

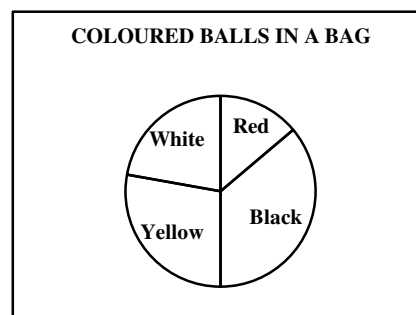
Suppose there are 36 different coloured balls of identical size in a bag.

Ball colour	Red	Black	Yellow	White	TOTAL
Frequency	5	13	10	8	36

Draw a pie chart to represent this data

SOLUTION

- The angle at the centre of a circle (one revolution) is 360°
- There is a total of 36 balls in the frequency table, so each ball is represented by $\frac{360^\circ}{36} = 10^\circ$ at the centre of the pie.
- Red balls are represented by $5 \times 10^\circ = 50^\circ$.
Black balls are represented by $13 \times 10^\circ = 130^\circ$.
Yellow balls are represented by $10 \times 10^\circ = 100^\circ$.
White balls are represented by $8 \times 10^\circ = 80^\circ$.
- Note that these angles add up to 360°
($50^\circ + 130^\circ + 100^\circ + 80^\circ = 360^\circ$)



Exercise 4.14

A dice is thrown 40 times and the number is recorded after each throw.

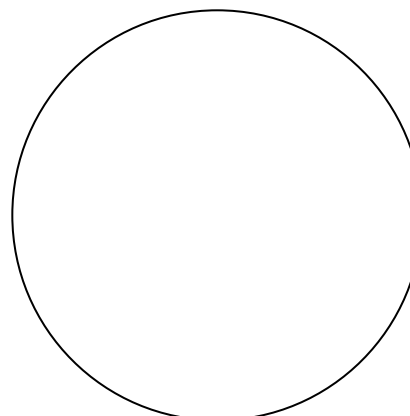
Number	1	2	3	4	5	6	TOTAL
Frequency	4	11	6	5	12	2	
Angle at the centre							

- 1) What angle represents each throw?

.....

- 2) Calculate the angles at the centre of the circle and then complete the above table. Check that the angles add to 360° .

.....



- 3) Draw in a radius of the circle. Use it to begin measuring the angles with your protractor. Complete your pie chart by accurately drawing in each sector or slice of the circle.
- 4) Label each sector and include a title for your pie chart.

§ 4.15 DRAWING LINE GRAPHS

✦ To represent data using a line graph, the data on the horizontal axis must be **continuous**.

EXAMPLE	SOLUTION																
One kilogram (kg) is approximately equal to 2,2 pounds (lb) 1) Complete the following table and use it to draw a conversion graph. <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr> <td style="padding: 2px 10px;">Kilograms (kg)</td> <td style="padding: 2px 10px;">5</td> <td style="padding: 2px 10px;"></td> <td style="padding: 2px 10px;">15</td> </tr> <tr> <td style="padding: 2px 10px;">Pounds (lb)</td> <td style="padding: 2px 10px;"></td> <td style="padding: 2px 10px;">22</td> <td style="padding: 2px 10px;"></td> </tr> </table> 2) Use your graph to: a) Convert 9 kg to lb b) Convert 24 lb to kg	Kilograms (kg)	5		15	Pounds (lb)		22		<table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr> <td style="padding: 2px 10px;">Kilograms (kg)</td> <td style="padding: 2px 10px;">5</td> <td style="padding: 2px 10px;">10</td> <td style="padding: 2px 10px;">15</td> </tr> <tr> <td style="padding: 2px 10px;">Pounds (lbs)</td> <td style="padding: 2px 10px;">11</td> <td style="padding: 2px 10px;">22</td> <td style="padding: 2px 10px;">33</td> </tr> </table> CONVERSIONS BETWEEN KG AND LBS a) 9 kg $\approx$ 20 lb b) 24 lb $\approx$ 11 kg	Kilograms (kg)	5	10	15	Pounds (lbs)	11	22	33
Kilograms (kg)	5		15														
Pounds (lb)		22															
Kilograms (kg)	5	10	15														
Pounds (lbs)	11	22	33														

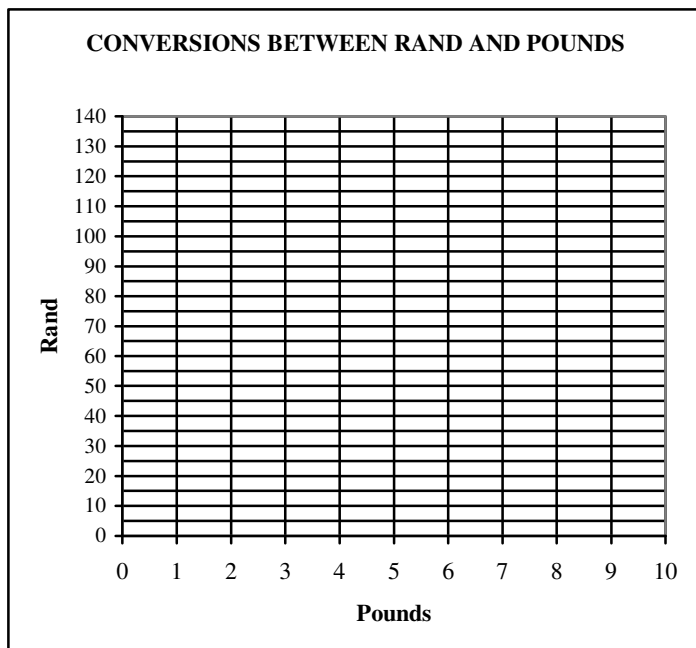
Exercise 4.15

On a certain day the conversion rate between the South African rand and the British pound is £1 = R15

- 1) Use the conversion rate to complete the given table:

Rand (R)			
Pounds (£)	1	2	9

- 2) Draw a graph showing the conversion between rand and pounds on the following grid:



- 3) Use your graph to:

- a) Convert £5 to rand
.....
- b) Convert £8 to rand
.....
- c) Convert R60 to £
.....
- d) Convert R100 to £
.....

§ 4.16 BROKEN LINE GRAPHS

✦ Broken line graphs are used to represent **discrete** data.

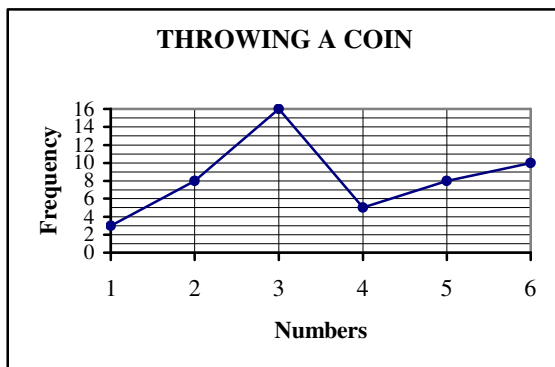
EXAMPLE

A dice is thrown 50 times and the numbers are recorded.

Number	1	2	3	4	5	6
Frequency	3	8	16	5	8	10

Represent this data on a broken line graph.

SOLUTION



Note:

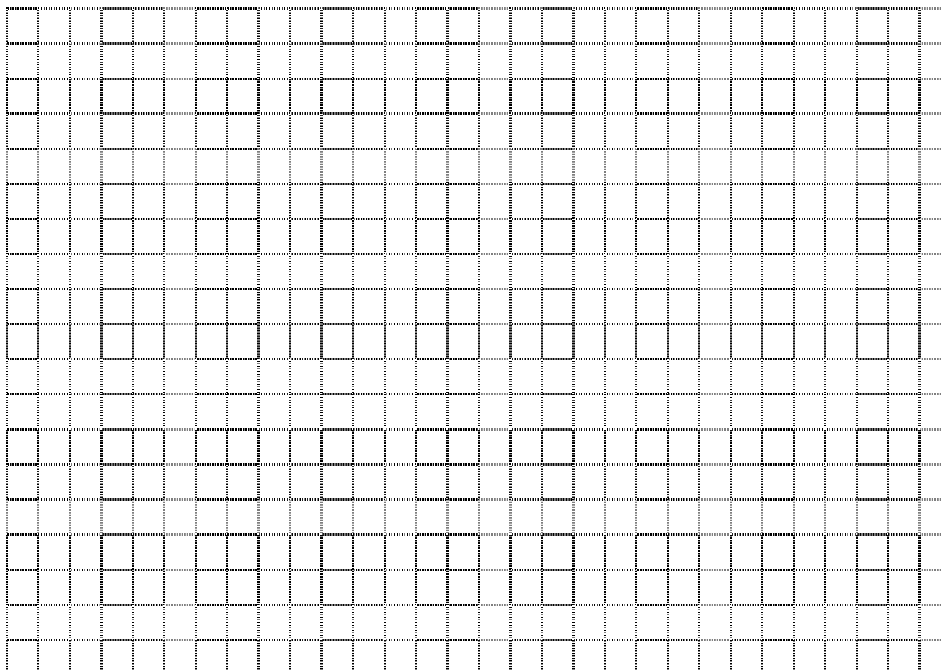
- The data is discrete, so plotted points are joined with a broken line to show the trend.
- There are no values between the plotted points
- It would seem from the frequencies that the die is weighted

Exercise 4.16

A class of learners were asked to choose whether they preferred chocolate bar A, B, C or D. The results of the survey are recorded on a frequency table.

Chocolate bar	A	B	C	D
Frequency	6	13	16	10

Draw a broken line graph to represent this data.



§ 4.17 DRAWING A HISTOGRAM

- ✦ Continuous data can be represented by using a histogram.
- ✦ If the width of each column is the same, the height of the column indicates the frequency.
- ✦ A histogram has **no gaps** between the rectangular bars.

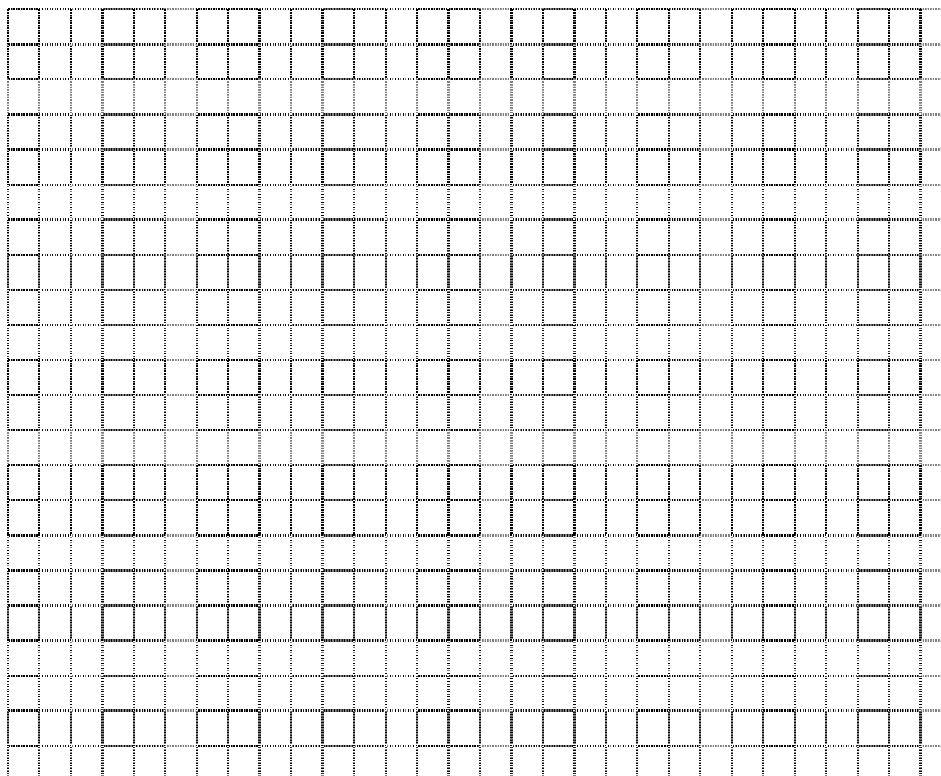
EXAMPLE	SOLUTION																
A dentist recorded the number of fillings in the teeth of a group of college students. He listed the results in a grouped frequency table <table><tr><td>Number of fillings</td><td>0 – 1</td><td>2 – 3</td><td>4 – 5</td></tr><tr><td>Frequency</td><td>22</td><td>18</td><td>4</td></tr></table> Draw a histogram to represent the data.	Number of fillings	0 – 1	2 – 3	4 – 5	Frequency	22	18	4	STUDENTS' FILLINGS <table><caption>Data for Students' Fillings Histogram</caption><tr><th>Number of fillings</th><th>Frequency</th></tr><tr><td>0 to 1</td><td>22</td></tr><tr><td>2 to 3</td><td>18</td></tr><tr><td>4 to 5</td><td>4</td></tr></table>	Number of fillings	Frequency	0 to 1	22	2 to 3	18	4 to 5	4
Number of fillings	0 – 1	2 – 3	4 – 5														
Frequency	22	18	4														
Number of fillings	Frequency																
0 to 1	22																
2 to 3	18																
4 to 5	4																

Exercise 4.17

The following frequency table gives the percentage obtained in a test by 38 students in a Level 2 class:

Marks	1 – 20	21 – 40	41 – 60	61 – 80	81 – 100
Frequency	4	6	9	11	8

Draw a histogram to represent the data:



§ 4.18 DRAWING MORE HISTOGRAMS

- ✦ The horizontal axis of a histogram is a **continuous number line**.
- ✦ The vertical axis show the **frequency**.
- ✦ There are no gaps between the bars of the histogram.
- ✦ The class intervals are all the **same** width, so the bars are the same width.

Exercise 4.18

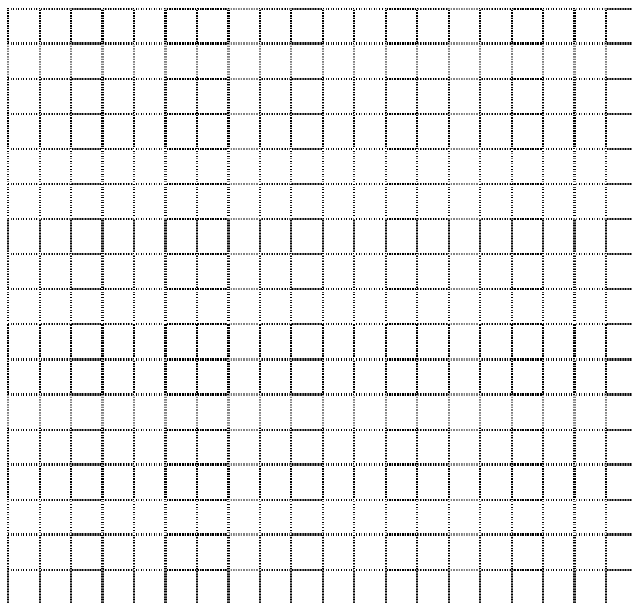
- 1) A farmer checks the masses (in grams) of a number of tomatoes for grading purposes. The table shows the results:

Mass (m) in grams	Frequency
$60 \leq m < 65$	9
$65 \leq m < 70$	20
$70 \leq m < 75$	29
$75 \leq m < 80$	15
$80 \leq m < 85$	7

- a) Draw a histogram to illustrate the data
- b) Comment on the distribution of the masses of the tomatoes checked.

.....

.....



- 2) Dungile recorded the following temperatures at noon for one month in whole degrees Celsius:

26 34 24 24 32 18 12 24 12 14
 12 30 24 20 26 34 16 24 42 16
 12 24 32 34 16 24 12 20 26 8

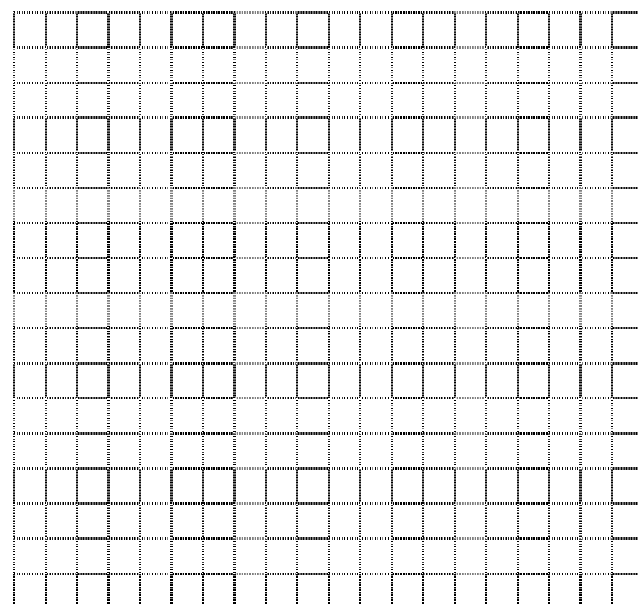
- a) Complete the following grouped frequency table:

Temperature in $^{\circ}\text{C}$	Frequency
$1 \leq t < 10$	
$10 \leq t < 20$	
$20 \leq t < 30$	
$30 \leq t < 40$	
$40 \leq t < 50$	

- b) Draw a histogram to illustrate the information
- c) Comment on any features you notice about this distribution of temperatures.

.....

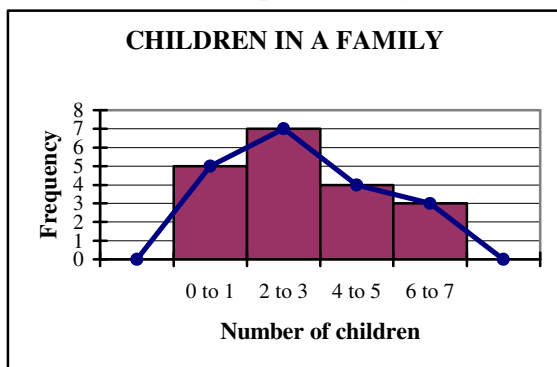
.....



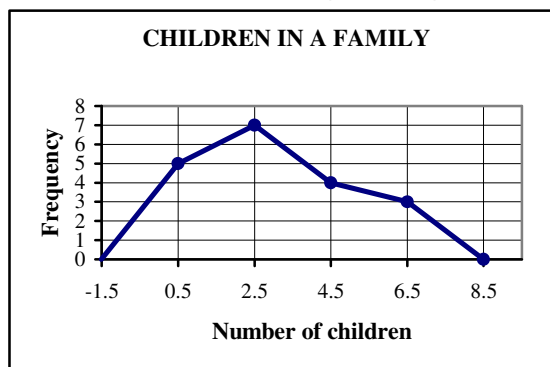
§ 4.19 FREQUENCY POLYGONS

- ✦ A histogram shows frequencies.
- ✦ A frequency polygon is a line graph showing frequencies. It is formed by joining the frequencies at the mid-points of each interval of a histogram with straight lines.

A frequency polygon can be drawn on top of a histogram



A frequency polygon can be drawn without first drawing a histogram.



Notice the following about a frequency polygon

- i. The **midpoints** of the intervals are joined
- ii. An **extra interval** is added to the left and the right of the histogram which has a frequency of 0. These two intervals are necessary to create a **polygon** (a closed shape made up of line segments).
- iii. A frequency polygon can be drawn on top of a histogram or by itself.

Exercise 4.19

The following frequency table gives the percentage obtained in a test by 38 students in a Level 2 class:

Percentages		1 – 20	21 – 40	41 – 60	61 – 80	81 – 100	
Midpoint of the interval							
Frequency	0	4	6	9	11	8	0

- 1) Complete the table by finding the midpoint of each interval.
- 2) Draw a frequency polygon to represent the data:

