TRANSFORMATION GEOMETRY (2)

Learning Outcomes and Assessment Standards

Learning Outcome 3: Space, shape and measurement
Assessment Standards AS 4(a) and AS 4(b)
Investigate, generalise and apply the effect on the coordinates of:

- A point after rotation around the origin through an angle of 90° or 180°.
- The vertices of a polygon after enlargement through the origin by a constant factor.

Overview

In this lesson you will:

- Learn about rotations of 180° clockwise and anti-clockwise
- Learn about enlargements and reductions.

Lesson

Summary of the 90° rotation rules

Rotation of 90° anti-clockwise:

$$(x; y) \rightarrow (-y; x)$$

To get the coordinates of the image points:

First swop around the first and second coordinates.

Then change the sign of the newly formed first coordinate.

Rotation of 90° clockwise:

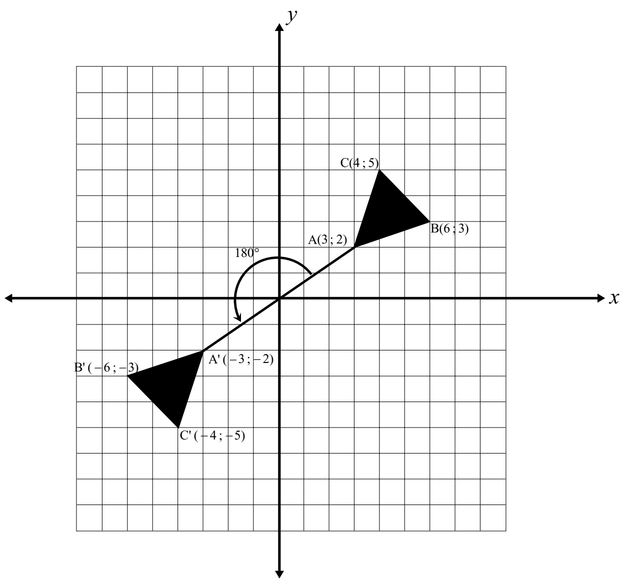
$$(x; y) \rightarrow (-y; -x)$$

To get the coordinates of the image points:

First swop around the first and second coordinates.

Then change the sign of the newly formed second coordinate.

ROTATIONS OF 180° CLOCKWISE AND ANTI-CLOCKWISE



Using the same approach as in the previous episode, rotate △ABC 180° in a clockwise (or anti-clockwise) direction. Draw the newly formed triangle and then complete the table which follows.

INDEPENDENT NEWSPAPERS

Now complete the following table:

Point	x-coordinate	y-coordinate
A		
A′		
B		
B′		
C		
C′		

The algebraic rule for rotating the point $(x; y)$ 180° in a clockwise or anti-clockwise direction is given by $(x; y) \rightarrow (-x; -y)$.

Summary of the 180° rotation rule

Rotation of 180° clockwise or anti-clockwise: $(x; y) \rightarrow (-x; -y)$

To get the coordinates of the image points:

- Change the signs of the original first and second coordinates.

Example

Example 1

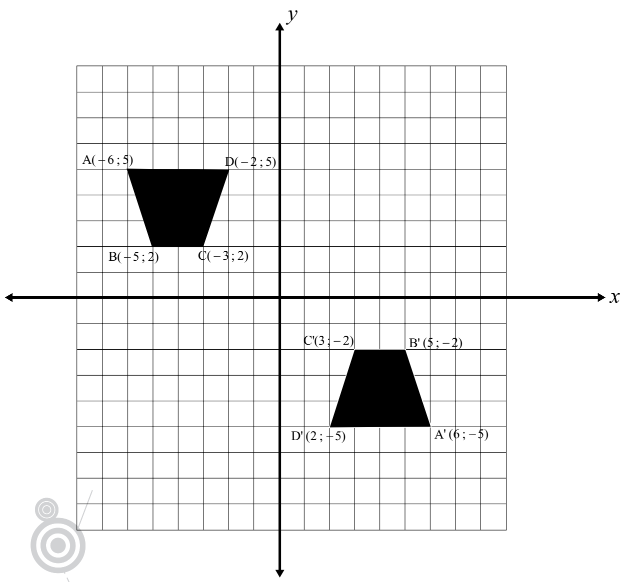
Determine the coordinates of the image of the following points under the transformation $(x; y) \rightarrow (-x; -y)$.

(1) A(2; 5) (2) B(–3; 4)

(1) A(2; 5) → A′ (–2; –5) (2) B(–3; 4) → B′(3; – 4)

Example

Example 2



Draw the image of figure ABCD under the rule $(x; y) \rightarrow (-x; -y)$.

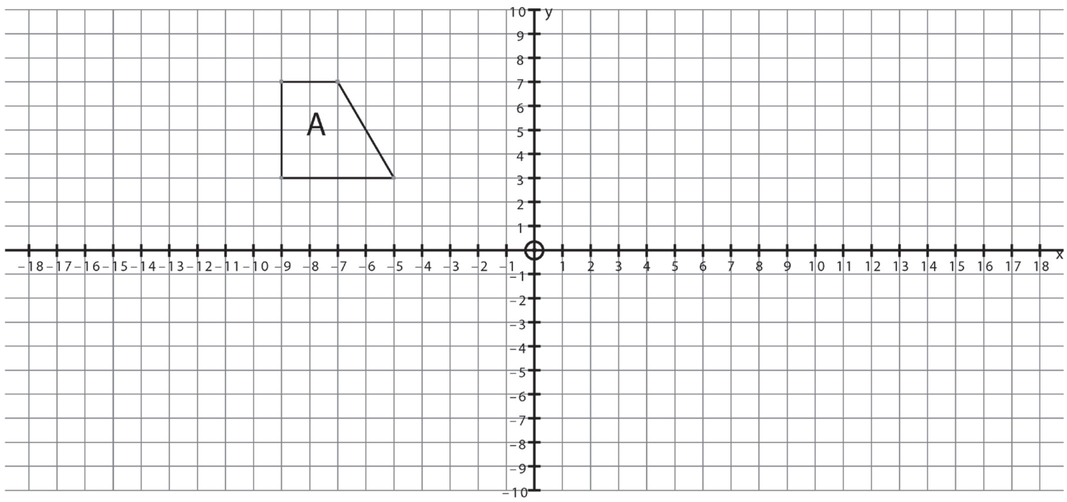
Describe the transformation.

Rotation of 180°

Activity 1

On the axes provided below, draw the images of the shaded figure A, under the following transformations. Describe each transformation.

1. $(x; y) \rightarrow (-y; -x)$ (Call the image B)
2. $(x; y) \rightarrow (-y; x)$ (Call the image C)
3. $(x; y) \rightarrow (-x; -y)$ (Call the image D)

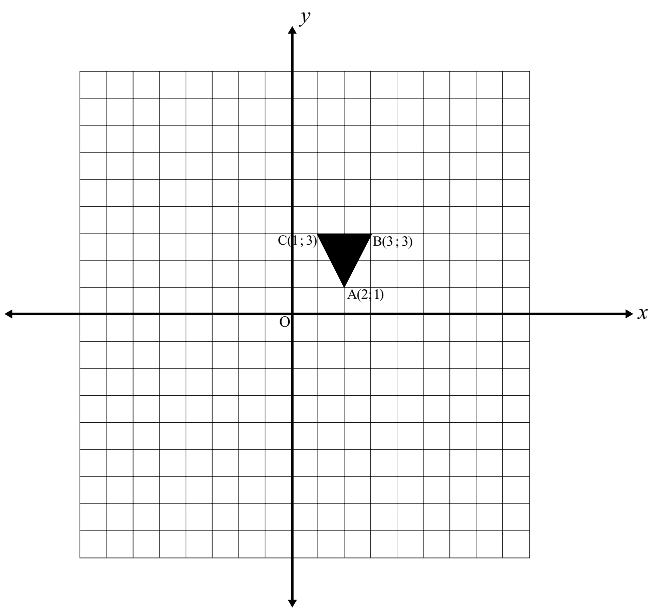


Lesson

ENLARGEMENTS AND REDUCTIONS

A. ENLARGEMENTS OF THE FORM $(x; y) \rightarrow (kx; ky)$ where k>1

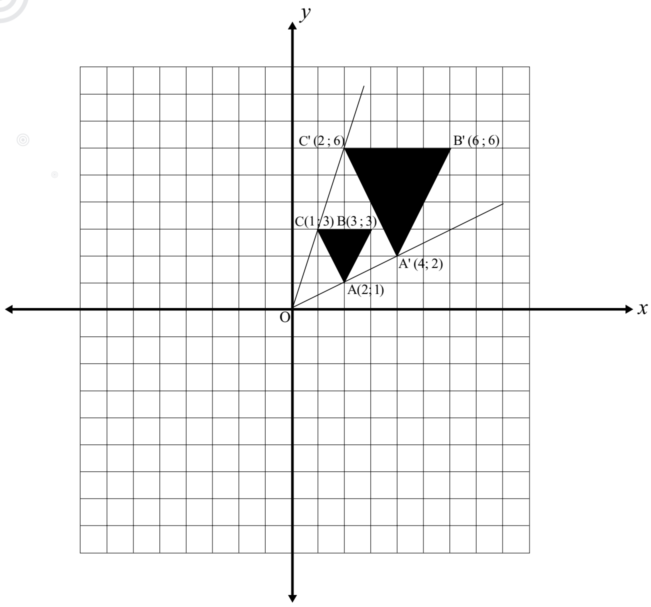
Example 1



Consider △ABC with vertices A(2; 1), B(3; 3) and C(1; 3)

We will now enlarge △ABC by applying a scale factor of 2 to the coordinates of the vertices using the rule $(x; y) \rightarrow (2x; 2y)$.

The coordinates of the newly formed enlargement of $\triangle$ABC are:



A(2; 1) $\rightarrow$ A′ (4; 2)

B(2; 1) $\rightarrow$ B′ (6; 6)

C(1; 3) $\rightarrow$ C′ (2; 6)

Some interesting facts emerge from this enlargement:

(a) $\triangle ABC ||| \triangle A'B'C'$ since their corresponding sides are in proportion.

This can be verified by calculating the lengths of the sides and showing that $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{AC}{A'C'}$

(b) $\frac{\text{Area } \triangle A'B'C'}{\text{Area } \triangle ABC} = \frac{1}{2}(\text{base } B'C')(\text{height})/\frac{1}{2}(\text{base})(\text{height}) = \frac{\frac{1}{2}(4)(4)}{\frac{1}{2}(2)(2)} = 4 = 2^2$

This means that the area of $\triangle A'B'C'$ is 2^2 times larger than the area of $\triangle ABC$.

In general, then, the area of the newly formed triangle under the rule$(x; y) \rightarrow (kx; ky)$ where $k > 1$ is k^2 times larger than the area of the original triangle.

B. REDUCTIONS OF THE FORM $(x; y) \rightarrow (kx; ky)$ where 0 < k < 1

Example

Example 2

Consider $\triangle$ABC with vertices A(5; 2), B(6; 6) and C(2; 6)

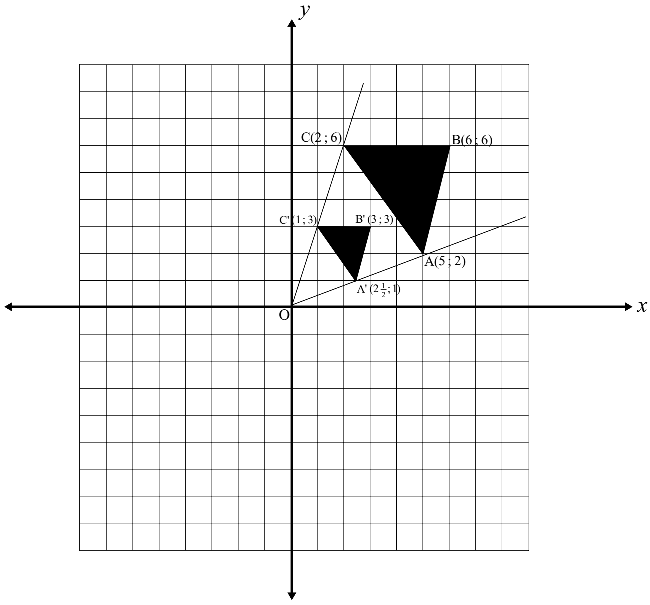
We will now reduce $\triangle$ABC by using the rule $(x; y) \rightarrow \left(\frac{1}{2}x; \frac{1}{2}y\right)$.

The coordinates of the newly formed $\triangle$A′B′C′ are:

A(5; 2) $\rightarrow$ A′($2\frac{1}{2}$; 1)

B(6; 6) $\rightarrow$ B′ (3; 3)

C(2; 6) $\rightarrow$ C′(1; 3)



C. TRANSFORMATIONS OF THE FORM $(x; y) \rightarrow (kx; ky)$ where k<0

These transformations combine an enlargement or reduction with a rotation of 180°.

Example

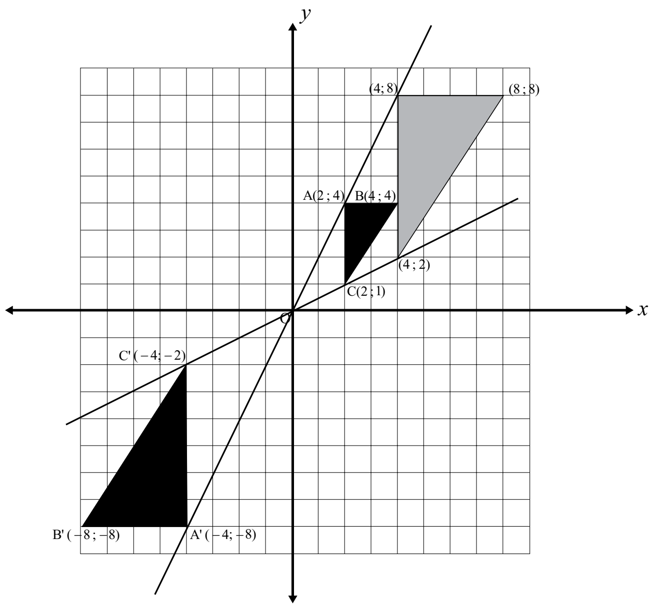
Example 3

Draw the image of A(2; 4)B(4; 4) and C(2; 1) under the transformation rule $(x; y) \rightarrow (-2x; -2y)$.

This transformation involves two different types:

Enlargement by a scale factor of 2: $(x; y) \rightarrow (2x; 2y)$
Followed by a:

Rotation of 180°: $(2x; 2y) \rightarrow (-2x; -2y)$



SUMMARY OF THE ENLARGEMENT AND REDUCTION RULES

$(x; y) \rightarrow (kx; ky)$

- If k>1, the image is an enlargement of the original figure. Multiply the original first and second coordinates by k units to get the coordinates of the image.
- If 0<k<1, the image is a reduction of the original figure. Multiply the original first and second coordinates by k units to get the coordinates of the image.
- If k<0, the image is a rotation of 180° of the original figure followed by an enlargement of the original figure. Multiply the original first and second coordinates by k units to get the coordinates of the image.

Activity

Activity 2

On the axes provided below, draw the images of the shaded figure A, under the following transformations.

1. $(x; y) \rightarrow (2x; 2y)$ (Call the image B)
2. $(x; y) \rightarrow (-x; -y)$ (Call the image C)
3. $(x; y) \rightarrow (-2x; -2y)$ (Call the image D)

Describe each of the above transformations in words.