LESSON 39

TRANSFORMATION GEOMETRY (1)

Learning Outcomes and Assessment Standards

Learning Outcome 3: Space, shape and measurement
Assessment Standard AS 4(a)
Investigate, generalise and apply the effect on the coordinates of a point after rotation around the origin through an angle of 90°.

Overview

Overview

In this lesson you will:

- Revise translations and reflections from Grade 10
- Learn about rotations of 90° clockwise and anti-clockwise.

Lesson

Lesson

Revision of translations and reflections (Grade 10)

Translations

A **translation** is a horizontal or vertical "slide" from one position to another. The object translated doesn't change its shape or orientation.

If **the point $(x; y)$ is translated to the point $(x + a; y + b)$ where a is a horizontal translation and b is a vertical translation then:**

if $a > 0$, the horizontal translation is to the right.

if $a < 0$, the horizontal translation is to the left.

if $b > 0$, the vertical translation is upward.

if $b < 0$, the vertical translation is downward.

Reflections

A **reflection** is a mirror image of a shape about a line of reflection.

Summary of the rules for reflection

Reflection about the y-axis:

$(x; y) \rightarrow (-x; y)$ **(The first coordinates differ in sign)**

Reflection about the x-axis:

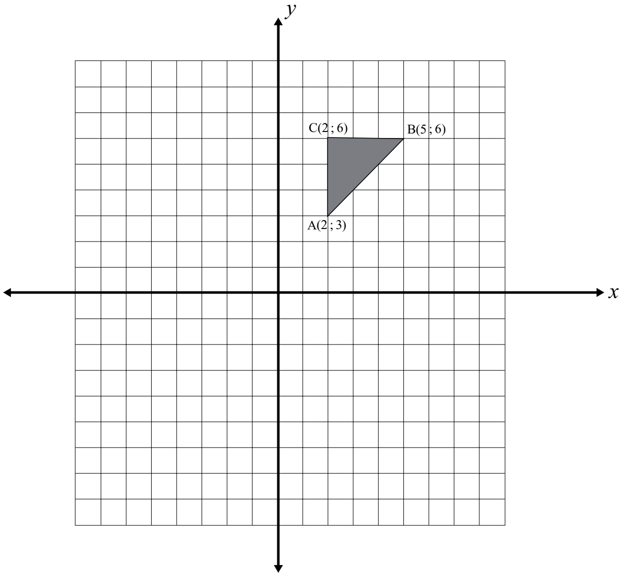
$(x; y) \rightarrow (x; -y)$ **(The second coordinates differ in sign)**

Reflection about the line $y = x$:

$(x; y) \rightarrow (y; x)$ **(The first and second coordinates have interchanged)**

Examples

Consider △ABC in the figure below with the given coordinates.



(a) Draw the image of △ABC under the transformation rule $(x; y) \rightarrow (x - 8; y + 2)$. Call the image △A′B′C′.

Describe the transformation in words.

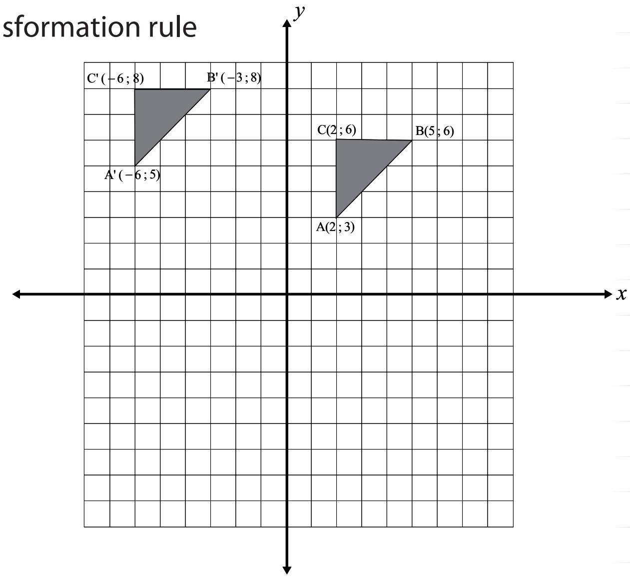
This transformation is a translation of 8 units to the left (horizontally) and then 2 units upwards (vertically).

The coordinates of the image △A′B′C′ can be calculated by substituting the coordinates of A, B and C into the rule $(x; y) \rightarrow (x - 8; y + 2)$ as follows:

A(2; 3) → A′(2 – 8; 3 + 2) = A′(–6; 5)

B(5; 6) → B′(5 – 8; 6 + 2) = B′(–3; 8)

C(2; 6) → C′(2 – 8; 6 + 2) = C′(–6; 8)



(b) Draw the image of △A′B′C′ under the transformation rule $(x; y) \rightarrow (x; -y)$. Call the image △A″B″C″.

Describe the transformation in words.

This transformation is a reflection about the x-axis.

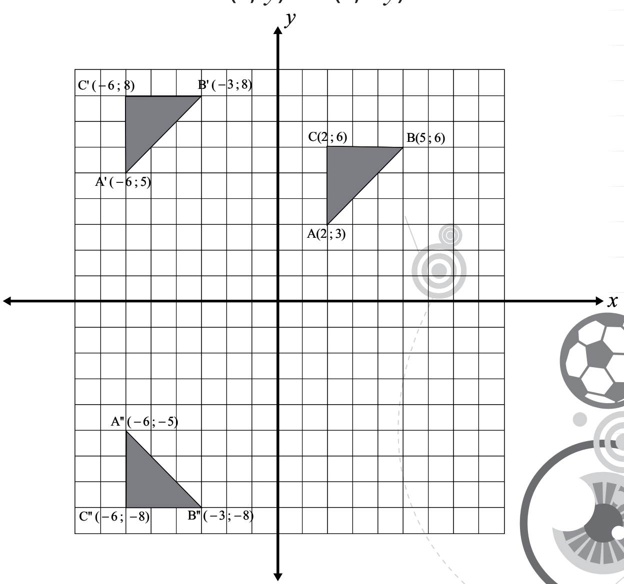
The y-coordinates of the image differ in sign from the original figure.

The coordinates of the image △A″B″C″ can be calculated by substituting the coordinates of A’, B’ and C’ into the rule $(x; y) \rightarrow (x; -y)$ as follows:

A′ (–6; 5) → A″ (–6; –5)

B′ (–3; 8) → B″ (–3; –8)

C′ (–6; 8) → C″ (–6; –8)



Example

(c) Draw the image of $\triangle A''B''C''$ under the transformation rule $(x; y) \rightarrow (-x; y)$. Call the image $\triangle A'''B'''C'''$.

Describe the transformation in words.

This transformation is a reflection about the y-axis.

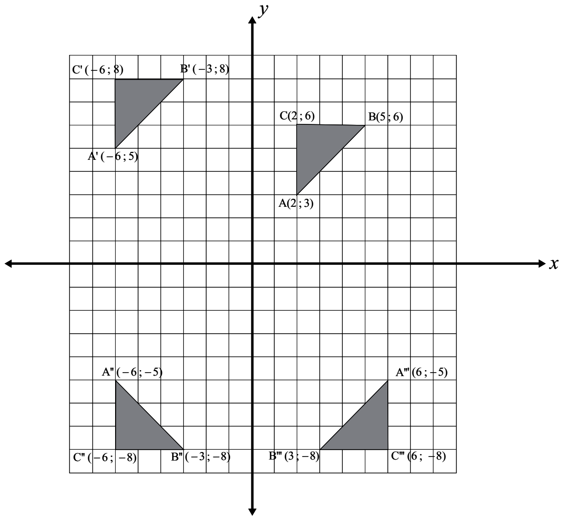
The x-coordinates of the image differ in sign from the original figure.

The coordinates of the image $\triangle A'''B'''C'''$ can be calculated by substituting the coordinates of A', B' and C' into the rule $(x; y) \rightarrow (-x; y)$ as follows:

$A''\ (-6; -5) \rightarrow A'''\ (6; -5)$

$B''\ (-3; -8) \rightarrow B'''\ (3; -8)$

$C''\ (-6; -8) \rightarrow C'''\ (6; -8)$

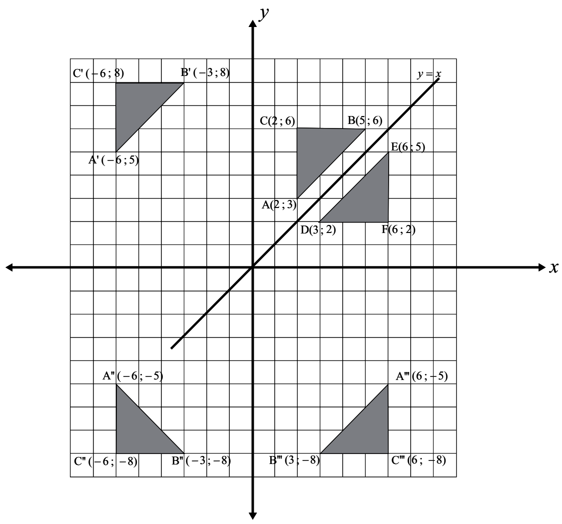


(d) Draw the image of $\triangle ABC$ under the transformation rule $(x; y) \rightarrow (y; x)$. Call the image $\triangle DEF$.

Describe the transformation in words.

This transformation is a reflection about the line $(y = x)$.

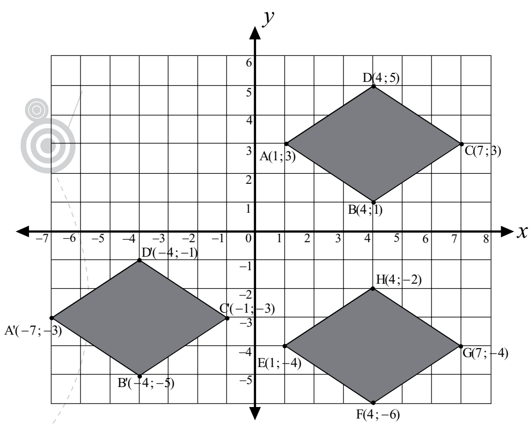
The x and y coordinates of the original are interchanged in the image points.



Activity

Activity

1.



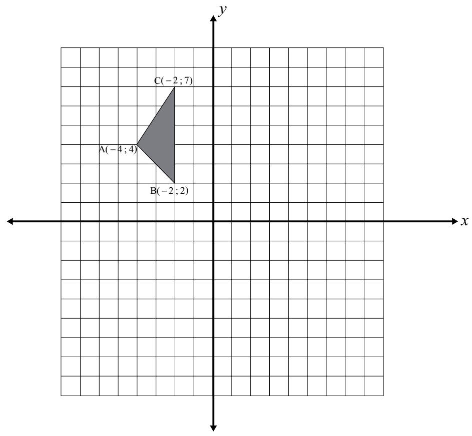
Region EFGH can been seen as the image of either region ABCD or region $A'B'C'D'$.

Determine the transformation rule if:

(a) EFGH is the image of ABCD

(b) EFGH is the image of $A'B'C'D'$

2.



On the set of axes provided alongside, draw the image of $\triangle ABC$ under each of the following transformations:

(a) $(x; y) \rightarrow (-x; y)$

(b) $(x; y) \rightarrow (x; -y)$

(c) $(x; y) \rightarrow (y; x)$

(d) $(x; y) \rightarrow (x + 8; y)$

Lesson

Lesson

Rotations of 90° clockwise and anti-clockwise

Rotations of 90° anti-clockwise

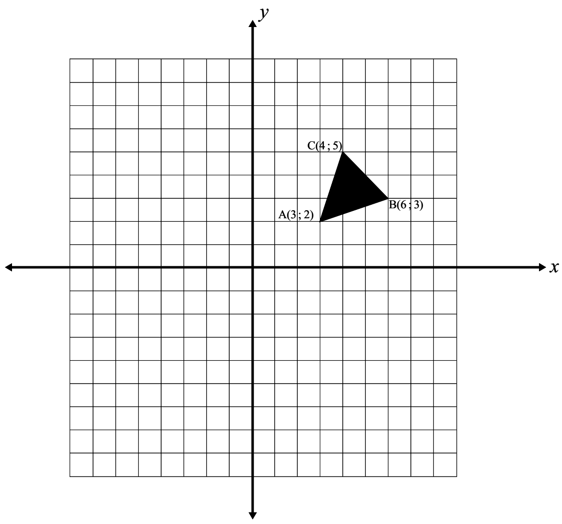
Consider $\triangle$ABC with vertices A(3; 2), B(6; 4) and C(4; 5).

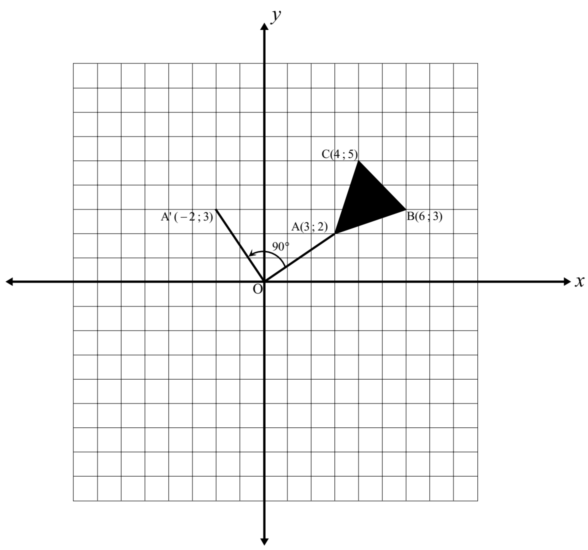
We will now rotate $\triangle$ABC 90° in an anti-clockwise direction by using the following approach:

(a) Join the origin to point A by means of a line.

(b) Measure the length of line OA.

(c) Use a protractor and a ruler to construct line OA′, which is the rotation of the line OA by 90° anti-clockwise.





Then do the same with line OB:

(d) Join the origin to point B by means of a line.

(e) Measure the length of line OB.

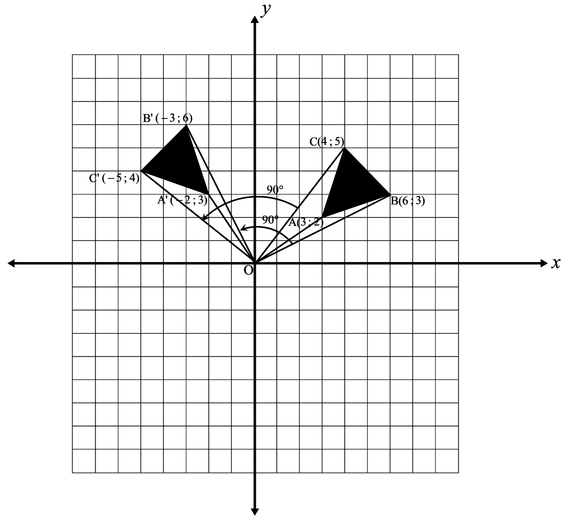
(f) Use a protractor and a ruler to construct line OB′, which is the rotation of the line OB by 90° anti-clockwise.

Then do the same with line OC:

(g) Join the origin to point C by means of a line.

(h) Measure the length of line OC.

(i) Use a protractor and a ruler to construct line OC′, which is the rotation of the line OC by 90° anti-clockwise.



Now complete the following table:

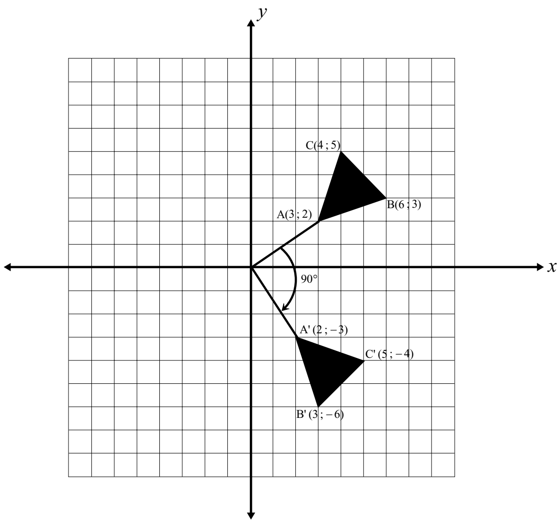
Point	x-coordinate	y-coordinate
A	3	2
A′	–2	3
B	6	3
B′	–3	6
C	4	5
C′	–5	4

The algebraic rule for rotating a point $(x; y)$ 90° in an anti-clockwise direction is given by: $(x; y) \rightarrow (-y; x)$

Rotations of 90° clockwise

Using the same approach, rotate △ABC 90° in a clockwise direction.

Draw the newly formed triangle and then complete the table which follows.



Now complete the following table:

Point	x-coordinate	y-coordinate
A	3	2
A′	2	–3
B	6	3
B′	3	–6
C	4	5
C′	5	–4

The algebraic rule for rotating a point $(x; y)$ 90° in a clockwise direction is given by: $(x; y) \rightarrow (y; -x)$

Summary of the 90° rotation rules
Rotation of 90° anti-clockwise: $(x; y) \rightarrow (-y; x)$
To get the coordinates of the image points:
First swop around the first and second coordinates.
Then change the sign of the newly formed first coordinate.
Rotation of 90° clockwise: $(x; y) \rightarrow (y; -x)$
To get the coordinates of the image points:
First swop around the first and second coordinates.
Then change the sign of the newly formed second coordinate.

Example

Example 1

(a) Determine the coordinates of the image of the point A(2; 5) under the following transformations. Describe each transformation.

(1) $(x; y) \rightarrow (-y; x)$ (2) $(x; y) \rightarrow (y; -x)$

Solution

Solution

(1) A (2; 5) A′(–5; 2) Rotation of 90° anti-clockwise

(2) A (2; 5) A′(5; –2) Rotation of 90° clockwise

(b) Determine the coordinates of the image of the point A(–2; 5) under the following transformations. Describe each transformation.

(1) $(x; y) \rightarrow (-y; x)$ (2) $(x; y) \rightarrow (y; -x)$

Solution

Solution

(1) A (–2; 5) → A′(–5; –2) Rotation of 90° anti-clockwise

(2) A (–2; 5) → A′(5; 2) Rotation of 90° clockwise

(c) Determine the coordinates of the image of the point A(– 2; – 5) under the following transformations. Describe each transformation.

(1) $(x; y) \rightarrow (-y; x)$ (2) $(x; y) \rightarrow (y; -x)$

Solution

Solution

(1) A(–2; –5) → A′(5; –2) Rotation of 90° anti-clockwise

(2) A(–2; –5) → A′(–5; 2) Rotation of 90° clockwise

(d) Determine the coordinates of the image of the point A(2; –5) under the following transformations. Describe each transformation.

(1) $(x; y) \rightarrow (-y; x)$ (2) $(x; y) \rightarrow (y; -x)$

Solution

Solution

(1) A(2; –5) → A′(5; 2) Rotation of 90° anti-clockwise

(2) A(2; –5) → A′(–5; –2) Rotation of 90° clockwise

Example

Example 2

Draw the image of the shaded figure ABCD under the following transformations. Describe each transformation.

(1) $(x; y) \rightarrow (-y; x)$ (2) $(x; y) \rightarrow (y; -x)$

Solution

Solution