

FINANCIAL MATHEMATICS (4)

Learning Outcomes and Assessment Standards

Learning Outcome 1: Number and number relationships

When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.

Assessment Standard AS 5

Demonstrate an understanding of different periods of compounding growth and decay.

Overview



Overview

In this lesson you will learn about:

- Future and present value calculations involving additional payments and withdrawals.

Lesson



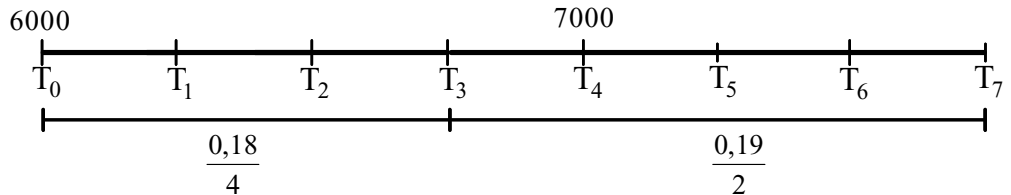
Lesson

Example

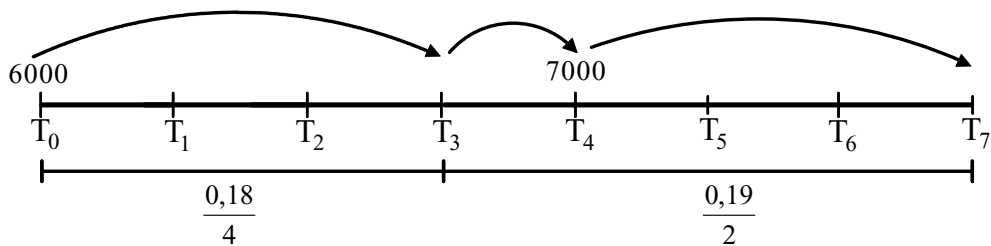


Example 1

R6 000 is deposited into a savings account. Four years later, R7 000 is added to the savings. The interest rate for the first three years is 18% per annum compounded quarterly. Thereafter, the interest rate changes to 19% per annum compounded semi-annually. Calculate the value of the savings at the end of the seventh year.



Method 1 (Moving from L → R on the time line)



Value at T_3 :

$$6\ 000 \left(1 + \frac{0,18}{4}\right)^{12} = 10\ 175,2886$$

Value at T_4 :

$$10\ 175,2886 \left(1 + \frac{0,19}{2}\right)^2 + 7\ 000 = 19\ 200,42541$$

Value at T_7 :

$$19\ 200,42541 \left(1 + \frac{0,19}{2}\right)^6$$



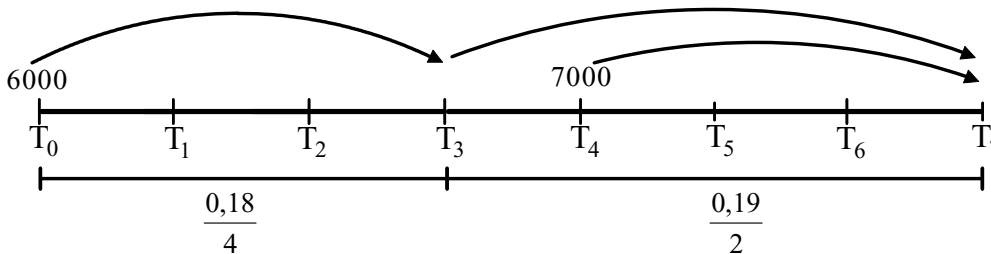
= R33 097,53

Method 2 (No partial calculation)

$$A = \left\{ 6\,000 \left(1 + \frac{0,18}{4} \right)^{12} \left(1 + \frac{0,19}{2} \right)^8 + 7\,000 \right\} \left(1 + \frac{0,19}{2} \right)^6$$

A = R33 097,53

Method 3 (Recommended) (Moving each entry to T₇)



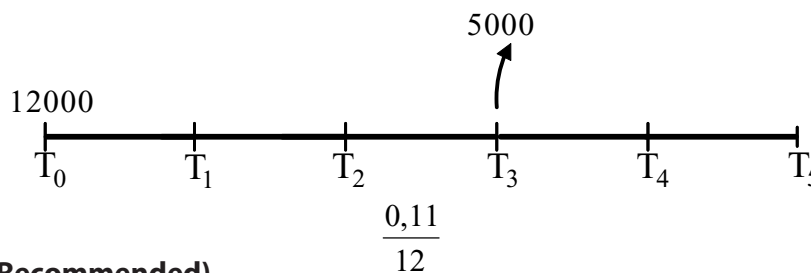
$$A = 6\,000 \left(1 + \frac{0,18}{4} \right)^{12} \left(1 + \frac{0,19}{2} \right)^8 + 7\,000 \left(1 + \frac{0,19}{2} \right)^6$$

A = R33 097,53

[Link to Activity 1](#)

Example 2

Peter invests R12 000 in a savings account in order to save up for an overseas trip in five years time. The interest rate for the five-year period is 11% per annum compounded monthly. At the end of the third year, he runs into financial difficulty and withdraws R5 000 from the account. How much money will he have saved at the end of the five-year period?



Method (Recommended)

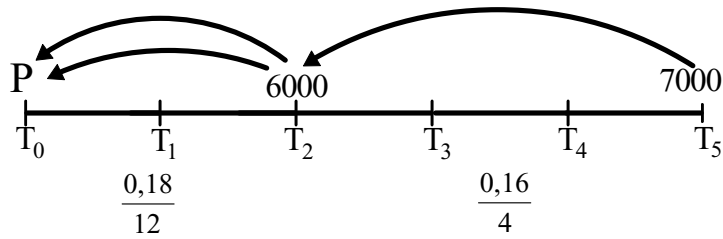
$$A = 12\,000 \left(1 + \frac{0,11}{12} \right)^{60} - 5\,000 \left(1 + \frac{0,11}{12} \right)^{24}$$

∴ A = R14 522,85

[Link to Activity 2](#)

Example 3

Sean takes out a loan. He repays the loan by means of a payment of R6 000 two years after the granting of the loan. Three years later, he repays a final amount of R7 000. The interest rate during the first two years of the loan is 18% per annum compounded monthly. For the remaining three years, the interest rate changes to 16% per annum compounded quarterly. How much money did Sean originally borrow?



$$P = 6\,000 \left(1 + \frac{0,18}{12}\right)^{-24} + 7\,000 \left(1 + \frac{0,16}{4}\right)^{-12} \cdot \left(1 + \frac{0,18}{12}\right)^{-24}$$

$$\therefore P = 4\,197,263517 + 3\,058,531477$$

$$\therefore P = R7\,255,79$$

Note: 4 197,263517 is the present value of 6 000 at T_0 (interest is removed from 6 000 to get 4197,263517).

3 058,531477 is the present value of 7 000 at T_0 (interest is removed from 7 000 to get 3058,531477).

If you now add the interest free amounts, you get the value of the original loan.

Link to Activity 3

Activity 1

- R3 500 is deposited into a savings account. Three years later, R4 000 is added to the savings. The interest rate for the two years is 8% per annum compounded monthly. Thereafter, the interest rate changes to 10% per annum compounded semi-annually. Calculate the value of the savings at the end of the sixth year.
- Thabo deposits R5 000 into a savings account. Three years later, he deposits a further R4 000 into the account. Four years after this, he deposits a further R6 000 into the account. The interest rate for the first 4 years is 13% per annum compounded semi-annually. For the next 3 years, the interest rate increases to 14% per annum compounded quarterly. Calculate the future value of the savings at the end of the seven year period.
- It is the 1st January 2007. Leah decides to deposit R2 000 into a savings account at the end of the year. At the end of 2008, she deposits double her first amount into the account. At the end of the 2009, she deposits double her second amount into the account. The interest rate for the first two years (2007-2008) is 9% per annum compounded monthly. The interest rate for the remaining two years (2009-2010) is 8% per annum compounded monthly. Calculate the value of her investment at the end of 2010.
- Joseph deposits Rx into a savings account. Two years later, he deposits a further $R2x$ into the account. Three years after this, he deposits $R3x$ into the account. The interest rate for the five years is 18% per annum compounded monthly. He receives R60 000 at the end of the five year period. Calculate the value of x .

Example

Activity

Example

Activity 2

Activity

1. A man invests R15 000 in a savings account in order to save up for an overseas trip in six years time. The interest rate for the six-year period is 13% per annum compounded half-yearly. At the end of the fourth year, he runs into financial difficulty and withdraws R3 000 from the account. How much money will he have saved at the end of the six-year period?
2. Moira starts a savings account and deposits R5 000 into the account. Twenty four months later, she deposits a further R6 000 into the account. Three years later, she withdraws R3 000 to buy a gift for her husband. The interest rate during the first three years is 8% per annum compounded monthly. The interest rate then changes to 9% per annum compounded quarterly. Calculate the value of Moira's investment two years after her withdrawal of R3 000.
3. Michelle deposits R4 300 into a savings account. Three years later, she deposits a further R7 000 into the account. Two years later, she withdraws R2 000. Two years after this, she deposits a final amount of R1 000 into the account. The interest rate during the first three years is 13% per annum compounded monthly. For the remaining four years, the interest rate changes to 14% per annum compounded quarterly. Calculate the future value of the savings at the end of the seven year period.

Activity 3

Activity

1. A certain amount deposited in a savings account now will grow to R13 000 after 7 years. The interest rate during the first 4 years is 9% per annum compounded monthly and for the remaining 3 years is 12% per annum compounded quarterly. How much is this amount?
2. A seven-year loan was repaid by means of R2 300 four years after the granting of the loan and R4 200 three years thereafter. The interest rate for the first five years was 10% per annum compounded half-yearly and 12% per annum compounded monthly for the remaining period. What was the amount borrowed?
3. Samantha buys a car and pays an initial deposit of R6 000, R8 000 after 2 years and a further R9 000 two years later. Interest is 18% per annum compounded monthly for the first two years. For the next two years, the interest rate is 19% per annum compounded monthly. Calculate the original price of the car.
4. Thabo pays a deposit of R30 000 for a new car. He repays the remaining loan by paying a further R20 000 in 2 years' time and R100 000 6 years thereafter. Interest is 18% p.a. compounded monthly during the first 3 years, and, 32% p.a. effective for the remaining 5 years. What did Mbeki originally pay for the car?