LESSON 37

FINANCIAL MATHEMATICS (3)

Learning Outcomes and Assessment Standards

Learning Outcome 1: Number and number relationships
When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.

Assessment Standard AS 5
Demonstrate an understanding of different periods of compounding growth and decay.

Overview

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In this lesson you will learn about:

- Future and present value calculations involving changing interest rates.

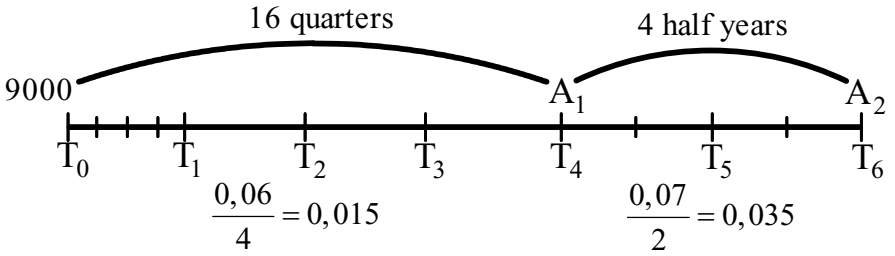
Lesson

Lesson

Example

Example 1

An investment of R9 000 earns 6% per annum compounded quarterly for a period of four years. Thereafter, the interest rate changes to 7% per annum compounded semi-annually for a further two years. Calculate the future value of the investment at the end of the six-year period.



In order to calculate the future value of the investment at $\mathbf{T}_6$, we need to grow R9 000 through two interest rates. To do this, we first grow the money to $\mathbf{T}_4$ through the 6% interest rate to obtain $\mathbf{A}_1$. We then grow this amount further through the 7% interest rate in order to obtain $\mathbf{A}_2$, the future value after 6 years.

Method 1 (Long way)

$A_1 = 9\,000\left(1 + \frac{0,06}{4}\right)^{16} = 11420,86993 \ \ (\text{At } T_4)$

$\therefore A_2 = 11420,86993.\left(\frac{1 + 0,07}{2}\right)^4$

$\therefore A_2 = \text{R}13\,105,71$

$(\text{At } T_6)$

Use the method you prefer

Method 2 (Recommended)

$A_1 = 9\,000\left(1 + \frac{0,06}{4}\right)^{16} \cdot .\left(\frac{1 + 0,07}{2}\right)^4$

$\therefore A_2 = 9\,000(1,015)^{16} \cdot 1,035^4$

$\therefore A_2 = \text{R}13\,105,71$

Link to Activity 1

A formula for calculating the present value (p)

Consider the formula $A = P(1 + i)^n$.

This formula can be rearranged as follows:

$A = P(1 + i)^n$

$\therefore \frac{A}{(1 + i)^n} = P$

$\therefore A(1 + i)^{-n} = P$ (use the exponent definition $\frac{1}{a^n} = a^{-n}$)

$\therefore P = A(1 + i)^{-n}$

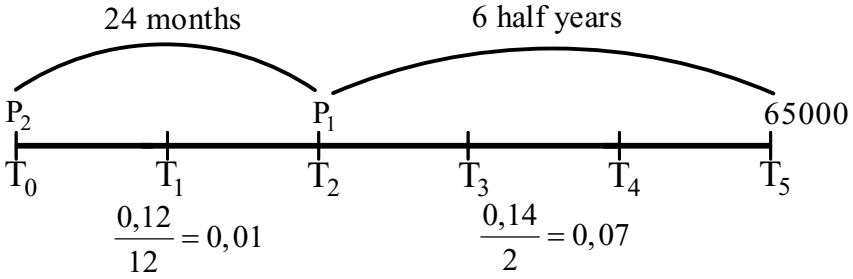
Therefore, we now have two formulae available to us:

$A = P(1 + i)^n$ (to find A given P)
$P = A(1 + i)^{-n}$ (to find P given A)

In the next example, we focus on a quicker way of doing the calculation, involving the formula for P.

Example 2

Peter invests a certain sum of money for 5 years at 12% per annum compounded monthly for the first two years and 14% per annum compounded semi-annually for the remaining term. The money grows to R65 000 at the end of the 5-year period. How much did Peter originally invest?



(Using: $P = A(1 + i)^{-n}$ because we are moving backwards on the time line)

$x = 65\,000(1{,}07)^{-6} \times (1{,}01)^{-24}$

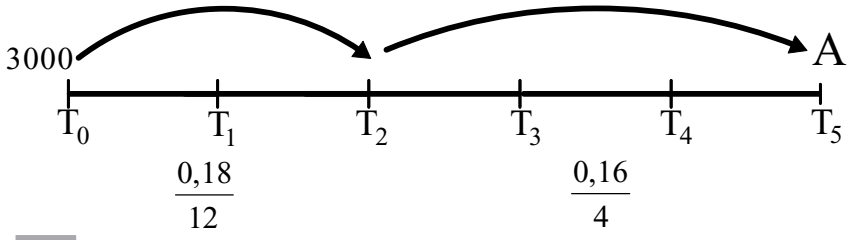
$x = \text{R}34\,111,26$

Summary

The following example will clearly illustrate the use of the two formulae.

- Using the formula $A = P(1 + i)^n$

Suppose that R3 000 is invested for 5 years. The interest rate for the first two years is 18% per annum compounded monthly. For the remaining three years, the interest rate changes to 16% per annum compounded quarterly. Calculate the value of the investment at the end of the five year period.

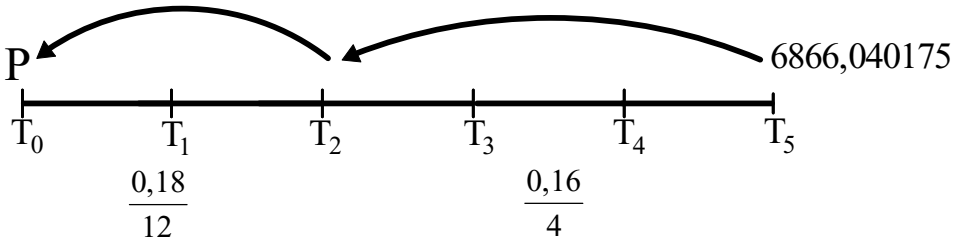


$$A = 3000\left(1+\frac{0,18}{12}\right)^{24}\left(1+\frac{0,16}{4}\right)^{12} = 6866,040175$$

Note: The exponents are **positive** and the movement on the time-line is from **left to right** because interest on the R3000 is growing.

- Using the formula $P = A(1 + i)^{-n}$

Suppose that R6866,040175 is received if a certain amount of money was invested 5 years ago. The interest rate for the first two years was 18% per annum compounded monthly. For the remaining three years, the interest rate changed to 16% per annum compounded quarterly. What was the original amount invested?



$$P = 6866,040175\left(1+\frac{0,16}{4}\right)^{-12}\left(1+\frac{0,18}{12}\right)^{-24} = R3\ 000$$

Example

Notice: The exponents are negative and the movement on the time-line is from right to left because interest on the R6866,040175 is being removed.

Link to Activity 2

Activity

Activity 1

1. R5 000 is deposited into a savings account. The interest rate for the first four years is 7% per annum compounded quarterly. Thereafter, the interest rate changes to 8% per annum compounded semi-annually. Calculate the value of the investment at the end of the tenth year.
2. Mark invests R2 000 for a period of seven years. During the first four years, the interest rate is 18% p.a. compounded monthly. Thereafter, interest changes to 24% p.a. compounded semi-annually. Calculate the future value of the investment after seven years.
3. Simphiwe deposits R3 000 into a savings account paying 13% per annum compounded monthly. After five years, the interest rate increases by 1%. Three years later, the interest rate decreases by 2%. Calculate the value of her investment after ten years.
4. Mvelo invests R6 000 into an account for a period of 12 years. The interest rate for the first seven years is 8% per annum compounded monthly. For the next five years, the interest rate changes to 10% per annum compounded half-yearly.
   (a) Convert the nominal rates to annual effective rates.
   (b) Use the effective rates and calculate the future value of the savings at the end of the 12-year period.

Activity 2

1. Malibongwe deposits a certain amount in a savings account. It grows to an amount of R13 000 after seven years. The interest rate during the first four years is 9% per annum compounded annually and for the remaining three years is 12% per annum compounded monthly. How much is this amount?

2. Simone wants a sum of R10 000 000 in eight years from now. How much must she invest now if interest is 15% p.a. compounded monthly for the first six years, and 20% per annum compounded quarterly for the remaining two years?

3. Mark wants to save for an overseas trip in three years' time. He will need an amount of R50 000 for the trip. The interest rate during the first year is 14% per annum compounded quarterly. For the remaining two years, the interest rate is 11% per annum compounded monthly. What must Mark invest now in order to receive R50 000 in three years' time?

4. Justine receives a certain amount of money as a birthday gift. She wants to invest this money in a saving account in order to buy a motor car when she matriculates four years from now. The expected cost of the motor car in four years' time is R100 000. The interest rate during the first two years of the savings period is 14% per annum compounded monthly. For the remaining two years, the interest rate changes to 13% per annum compounded half-yearly.

   (a) By using the nominal rates, calculate the amount of money Justine received as a birthday gift.

   (b) Convert the nominal rates to effective annual rates.

   (c) By using the effectives rates, calculate the amount of money Justine received as a birthday gift. What do you notice?