LESSON 35

FINANCIAL MATHEMATICS (1)

Learning Outcomes and Assessment Standards

Learning Outcome 1: Number and number relationships
When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.

Assessment Standard AS 4

- Use simple and compound decay formulae to solve problems, including straight line depreciation and depreciation on a reducing balance.

Overview

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In this lesson you will:

- Revise Simple and Compound interest from Grade 10
- Focus on the two types of Depreciation – Linear and Reducing Balance.

Lesson

Lesson

Prior knowledge

Simple interest

Consider an amount of R1 000 invested at 10% per annum **simple** interest:

Accumulated amount after one year:

$A_1 = 1\ 000 + 0{,}10 \times 1\ 000 = R1\ 100$ (Interest received is R100)

Accumulated amount after two years:

$A_2 = 1\ 100 + 0{,}10 \times 1\ 000 = R1\ 200$ (Interest received is R100)

Accumulated amount after three years:

$A_3 = 1\ 200 + 0{,}10 \times 1\ 000 = R1\ 100$ (Interest received is R100)

Accumulated amount after four years:

$A_4 = 1\ 300 + 0{,}10 \times 1\ 000 = R1\ 400$ (Interest received is R100)

Accumulated amount after five years:

$A_5 = 1\ 400 + 0{,}10 \times 1\ 000 = R1\ 500$ (Interest received is R100)

Note: The interest received each year is calculated using the original amount invested (R1000). This means that the interest received each year will always be the same (R100). The graph of this relationship will be a **linear function** (see diagram which follows).

Compound interest

Consider an amount of R1 000 invested at 10% per annum **compound** interest:

Accumulated amount after one year:

$A_1 = 1\ 000 + 0{,}10 \times 1\ 000 = R1\ 100$

Accumulated amount after two years:

$A_1 = 1\ 000 + 0{,}10 \times 1\ 000 = R1\ 100$

Accumulated amount after three years:

$A_3 = 1\ 210 + 0{,}10 \times 1\ 210 = R1\ 331$ (Interest received is R121)

Accumulated amount after four years:

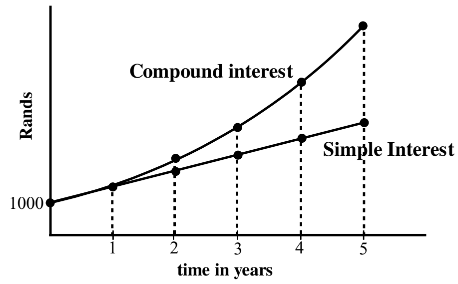
$A_4 = 1\,331 + 0{,}10 \times 1\,331 = R1\,464{,}10$ (Interest received is R133,10)

Accumulated amount after five years:

$A_5 = 1\,464{,}10 + 0{,}10 \times 1\,464{,}10 = R1\,610{,}51$ (Interest received is R146,41)

Note: The interest received at the end of the first year is the same as that received using the simple interest approach. However, at the end of the second year, the interest received is higher when using the compound interest approach because interest for the second year was calculated using the accumulated amount of the first year (R1 100). This will apply to each successive year and therefore the accumulated amount each year will increase exponentially. This is the case because both rates were 10 per annum. The graph of this relationship will therefore be an **exponential function** (see diagram which follows).

The graphical relationship between simple and compound Interest



Formulae for simple and compound interest

Simple interest

The formula which helps us to calculate the accumulated amount (future value) if an original amount is invested or loaned for n years at a rate of r % simple interest is given by:

$A = P(1 + in)$

where:

P = present value of the investment or loan (original amount at the beginning)

A = accumulated amount or future value of the investment or loan after n periods

n = time period in years

$i = \frac{r}{100}$ for the simple interest rate r % (i is in decimal form)

Compound interest

The formula which helps us to calculate the accumulated amount (future value) if an original amount is invested or loaned for n years at a rate of r % compound interest is given by:

$A = P(1 + i)^n$

where:

P = present value of the investment or loan (original amount at the beginning)

A = accumulated amount or future value of the investment or loan after n periods

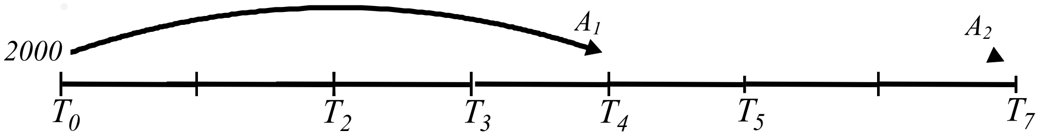
n = time period

$i = \frac{r}{100}$ for the simple interest rate r% (i is in decimal form)

Example

Example

An amount of R2 000 is invested for 4 years at 12% per annum simple interest. Thereafter, the accumulated amount is invested for another 3 years at 11% per annum compound interest. Calculate how much money will have been saved at the end of the seven year period.



For the first four years:

$A_1 = P(1 + in)$

$\therefore A_1 = 2\,000(1 + 0{,}12 \times 4)$

$\therefore A_1 = R2\,960$

For the remaining three years:

$A_2 = P(1 + i)^n$ where $P = A_1$

$\therefore A_2 = 2\,960(1 + 0{,}11)^3$

$\therefore A_2 = R4\,048{,}19$

Note:

You could have done the calculation in one line as follows:

$\therefore A = 2\,000(1 + 0{,}12 \times 4)\,.\,(1 + 0{,}11)3$

$\therefore A = R4\,048{,}19$

Link to Activity 1

Depreciation

When equipment loses value over time, we say that the equipment is depreciating in value. For example, it is generally the case that the moment a new motor car is driven out of the shop, its value depreciates substantially. Obviously, due to wear and tear, the car will lose its value over time.

Book value: is the value of equipment at a particular time after depreciation has been taken into account.

Scrap value:is the book value of the equipment at the end of its useful life.

There are two types of depreciation:

Linear depreciation and reducing balance depreciation

Linear depreciation (straight-line depreciation)

With straight-line depreciation, equipment is depreciated by a percentage of its original value. It works in the same way as simple interest, but the value **decreases** rather than increases as with simple interest.

Consider, for example, the value of a car costing R200 000 after 6 years if it depreciates at a rate of 10% per annum using linear depreciation.

$A_1 = 200\,000 - 0{,}10 \times 200\,000$

$\therefore A_1 = R180\,000$ (The car's value depreciated by R20 000)

$A_2 = 180\ 000 - 0{,}10 \times 200\ 000$

$\therefore\ A_2 = R160\ 000$ (The car's value depreciated by a further R20 000)

$A_3 = 160\ 000 - 0{,}10 \times 200\ 000$

$\therefore\ A_3 = R140\ 000$ (The car's value depreciated by a further R20 000)

$A_4 = 140\ 000 - 0{,}10 \times 200\ 000$

$\therefore\ A_4 = R120\ 000$ (The car's value depreciated by a further R20 000)

$A_5 = 120\ 000 - 0{,}10 \times 200\ 000$

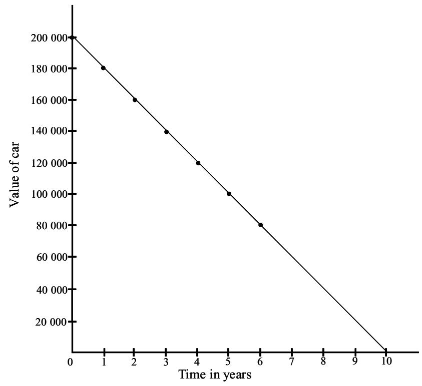
$\therefore\ A_5 = R100\ 000$ (The car's value depreciated by a further R20 000)

$A_6 = 100\ 000 - 0{,}10 \times 200\ 000$

$\therefore\ A_6 = R80\ 000$ (The car's value depreciated by a further R20 000)

Note: The car's value depreciates by the same amount of R20 000 each year. After 6 years, the car is worth R80 000.

We can represent this information on a graph as follows:



Note: The graph of this depreciation takes on the shape/trend of a ***linear function*** (straight-line graph). The value of the car will eventually be zero, because the graph will cut the horizontal axis where the value of the car as read from the vertical axis will be zero.

A useful formula to calculate linear depreciation is:

Consider, for example, the value of a car costing R200 000 after 6 years if it depreciates at a rate of 10% per annum using linear depreciation.

$A = P(1 - in)$

$\therefore A = 200\ 000(1 - 0{,}10 \times 6)$

$\therefore\ A = R80\ 000$

Reducing-balance depreciation

With reducing-balance depreciation, equipment is depreciated by a percentage of its previous value. It works in the same way as compound interest, but the value **decreases** rather than increases as with compound interest.

Consider, for example, the value of the car costing R200 000 after 4 years if it depreciates at a rate of 10% per annum using reducing balance depreciation.

$A_1 = 200\ 000 - 0{,}10 \times 200\ 000$

$\therefore A_1 = R180\ 000$ (The car's value depreciated by R20 000)

$A_2 = 180\ 000 - 0{,}10 \times 180\ 000$

$\therefore\ A_2 = R162\ 000$ (The car's value depreciated by a further R18 000)

$A_3 = 162\ 000 - 0{,}10 \times 162\ 000$

$\therefore\ A_3 = R145\ 800$ (The car's value depreciated by a further R16 200)

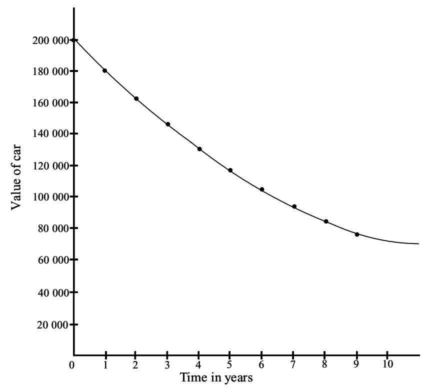
$A_4 = 145\ 800 - 0{,}10 \times 145\ 800$

$\therefore\ A_4 = R131\ 220$ (The car's value depreciated by a further R14 580)

Note: The car's value depreciates by different lesser amounts each year. After 4 years, the car is worth R131 220. Depreciation is calculated on the reducing balance each year.

We can represent this information on a graph as follows:

Note: The graph of this depreciation takes on the same shape/trend as a decreasing ***exponential function***. The car will always have some value as the years progress. This is because the exponential graph never cuts the horizontal axis so as to produce a value of zero rands (as read off the vertical axis).



Link to Activity 2

A useful formula to calculate reducing balance depreciation is:

Consider, for example, the value of the car costing R200 000 after 4 years if it depreciates at a rate of 10% per annum using reducing balance depreciation.

$A = P(1 - i)^n$

$\therefore A = 200\ 000(1 - 0{,}10)^4$

$\therefore A = R131\ 220$

Example

Example 1

What will the book value of a car be after 5 years if the rate of depreciation is 16,1% p.a. and the car's original purchase price was R72 000 where depreciation is based on:

(a) the straight-line method?

(b) the reducing-balance method?

Solution

Solution

(a) $A = 72\ 000(1 - 0{,}161 \times 5)$

$\therefore A = R14\ 040$

(b) $A = 72\ 000(1 - 0{,}161)^5$

$\therefore\ A = R29\ 932{,}45$

Example

Example 2

Calculate the original price of a laptop computer if its depreciated value after 6 years is R1 200 and the rate of depreciation was 14% per annum calculated using:

(a) the straight-line method?

(b) the reducing-balance method?

Solution

(a) $A = P(1 - i)^n$

$\therefore\ 1\,200 = P(1 - 0{,}14 \times 6)$

$\therefore\ 1\,200 = P(0{,}16)$

$\therefore\ \frac{1\,200}{0{,}16} = P$

$\therefore\ P = R7\,500$

(b) $A - P(1 - in)$

$\therefore\ 1\,200 = P(1 - 0{,}14)^6$

$\therefore\ 1\,200 = P(0{,}86)^6$

$\therefore\ \frac{1\,200}{(0{,}86)^6} = P$

$\therefore\ P = R2\,966{,}13$

Example 3

A car costs R69 000 and, after 8 years, has a scrap value of R7000. Find the annual depreciation rate if it is calculated using:

(a) the straight line method.

(b) the reducing balance method.

Solution

(a) $7\,000 = 69\,000(1 - 8i)$

$\therefore\ 7\,000 = 69\,000 - 552\,000i$

$\therefore\ 552\,000i = 69\,000 - 7\,000$

$\therefore\ 552\,000i = 62\,000$

$\therefore\ i = \frac{62\,000}{552\,000}$

$\therefore\ i = 0{,}1123188406$

$\therefore\ r = 11{,}2\%$ per annum

(b) $7\,000 = 69\,000(1 - i)^8$

$\therefore \frac{7\,000}{69\,000} = (1 - i)^8$

$\therefore \left(\frac{7\,000}{69\,000}\right)^{\frac{1}{8}} = 1 - i$

$\therefore\ i = 1 - \left(\frac{7\,000}{69\,000}\right)^{\frac{1}{8}}$

$\therefore\ i = 0{,}248755823$

$\therefore\ r = 24{,}9\%$ per annum

Link to Activity 3

Summary

In this lesson, we looked at the differences between simple and compound interest. Clearly, there is more money to be made if you invest your money using the compound interest option.

We then discussed linear and reducing balance depreciation, showing how things can lose value over time.

In the next lesson, we will focus on different compounding periods as well as nominal and effective interest rates.

Activity 1

1. Find the future value of R4 000 invested for 5 years at
   (a) 14% p.a. simple interest.
   (b) 14% p.a. compound interest.
2. Find the present value of an amount which accumulated to R13 000 in 6 years if the interest was
   (a) 12% p.a. simple interest.
   (b) 12% p.a. compound interest.

3. R70 000 is invested at 9% p.a. simple interest for 3 years. Thereafter, the total amount is reinvested in a different financial institution at 8% p.a. compound interest for 2 more years. What is the future value of the investment after the five-year period?

Activity

Activity 2

Use the graph to answer the following questions:

(a) Determine the value of the car after 7 years: ___________________

(b) Determine the value of the car after 9 years: ___________________

(c) How long did the car take to depreciate to R60 000? _________________

(d) How long did the car take to depreciate to R40 000? _________________

(e) What will the value of the car be after 10 years? ________________

Activity

Activity 3

1. A car is purchased for R250 000. The car depreciates at a rate of 12% per annum. What is the car worth after 5 years if depreciation is calculated using:
   (a) the straight-line method.
   (b) the reducing-balance method.
2. Calculate the original price of a laptop computer if its depreciated value after 7 years is R3 200 and the rate of depreciation was 12% per annum calculated using:
   (a) the straight-line method?
   (b) the reducing-balance method?
3. A photocopying machine costs R140 000 and has a scrap value of R19 000 after 10 years. Find the annual rate of depreciation if it is calculated using:
   (a) the straight-line method.
   (b) the reducing-balance method.
4. A motor vehicle currently has a book value of R56 000. The rate of depreciation was 14% per annum using the reducing balance method. Calculate the original price of the motor vehicle, if it was bought 5 years ago.
5. The computers for a business are currently worth R400 000. Calculate the value of these computers after 6 years, if the rate of depreciation is 16% per annum calculated on a linear basis.
6. A school buys a photocopying machine for R750 000. Calculate the scrap value of the machine after 6 years if the rate of depreciation is 13% per annum calculated on the reducing balance scale.