

APPLICATION OF GRAPHS

Learning outcomes and assessment standards

Learning Outcome 2: Functions and algebra Assessment Standard AS 2

Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures about the effect of the parameters k , p , a and q for the functions including:

$$\begin{aligned} y &= \sin kx & y &= \cos kx \\ y &= \tan kx & y &= \sin(x+p) \\ y &= \cos(x+p) & y &= \tan(x+p) \\ y &= a(x+p)^2 + q \\ y &= a.b^{x+p} + q & y &= \frac{a}{x+p} + q \end{aligned}$$

Overview

In this lesson you will:

- Analyse graphs
- Apply the graphs you have learnt to real life problems.



Overview

Lesson



Lesson

Examples



Example

Look at this graph

1. Find the equations of f and g .

For f put in the x -intercepts

$$y = a(x+3)(x-1)$$

$$y = a(x^2 + 2x - 3)$$

Substitute point $(0; 6)$

$$6 = a(-3)$$

$$a = -2$$

$$f(x) = -2x^2 - 4x + 6$$

For the straight line we use $y = mx + c$

$$c = 2 \quad m = -2$$

$$g(x) = -2x + 2$$

2. Find the coordinates of the turning point P .

axis of symmetry $x = \frac{-b}{2a}$

$$f(x) = -2x^2 - 4x + 6$$

$$x = \frac{-(-4)}{2(-2)}$$

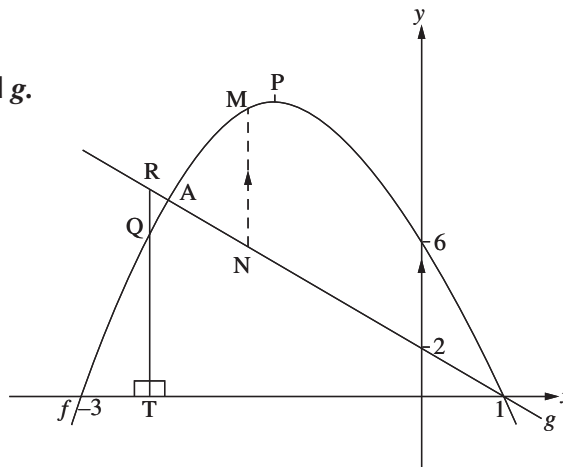
$$x = 1$$

Substitute into f to find the maximum value.

$$f(-1) = -2(1) - 4(-1) + 6$$

$$= 8$$

$$P(-1; 8)$$



3. **Find the coordinates of A.** At A the two graphs intersect.

$$-2x + 2 = -2x^2 - 4x + 6$$

$$2x^2 + 2x - 4 = 0$$

$$x^2 + x - 2 = 0$$

$$(x + 2)(x - 1) = 0$$

$$x = -2$$

$$y = 6$$

$$A(-2; 6)$$

4. **If $OT = 2\frac{1}{2}$ units, find the length of RQ.**

$$RQ = \text{Top graph} - \text{bottom graph}$$

$$RQ = -2x + 2 - (-2x^2 - 4x + 6)$$

$$RQ = -2x + 2 + 2x^2 + 4x - 6$$

$$RQ = 2x^2 + 2x - 4$$

$$\text{Now substitute } x = -\frac{5}{2}$$

$$RQ = 3,5 \text{ units}$$

5. **If MN is a variable line parallel to the x-axis between A and B, find the maximum length of MN.**

$$MN = \text{Top} - \text{Bottom}$$

$$MN = -2x^2 - 4x + 6 - (-2x + 2)$$

$$MN = -2x^2 - 4x + 6 + 2x - 2$$

$$MN = -2x^2 - 2x + 4$$

We need the maximum value of this parabola.

Remember: Turning point is $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$

so $\frac{4ac - b^2}{4a}$ is the maximum value

$$a = -2 \quad b = -2 \quad c = 4$$

$$\text{Maximum value is } \frac{-36}{-8} = 4\frac{1}{2}$$

Activity



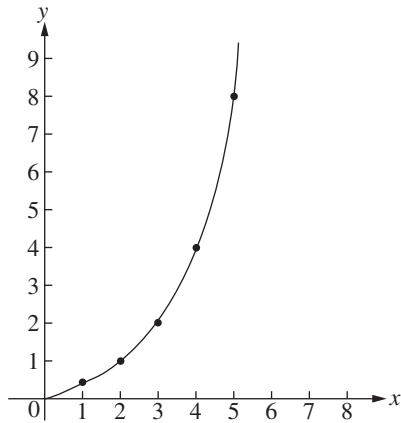
Activity

A nutritionist measures the mass of a specimen of yeast every hour.

His observations are given in the following table.

Hours since 1 st observation (x)	Mass of yeast in m.g. (y)
1	$\frac{1}{2}$
2	1
3	2
4	4
5	
6	

- a) **Plot the points of the graph and join them with a smooth curve.**



- b) **Give an equation for the graph.**

This is a geometric sequence.

$$T_n = ar^{n-1}$$

$$f(x) = \frac{1}{2}(2)^{x-1}$$

- c) **Interpret the graph in your own words.**

- The initial mass of the yeast is $\frac{1}{4}$ m.g.
- After each hour the mass of the yeast doubles.

- d) **How will the original equation change if the graph is moved 3 units upwards and explain it in your own words?**

Graph $y = \left(\frac{1}{2}\right)(2)^{x-1} + 3$

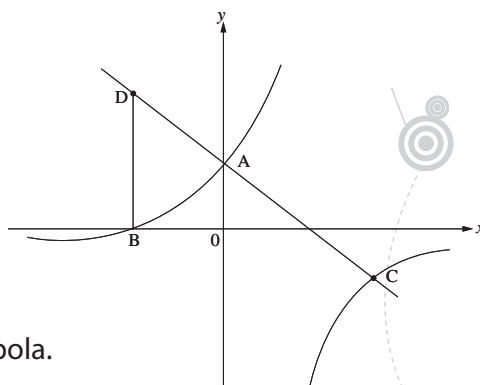
- The initial mass will be $3\frac{1}{4}$ m.g.
- After 1 hour the mass will be $3\frac{1}{2}$ m.g.

1. a) Find $f(x)$ and $g(x)$.
 b) For which values of x if $f(x) > 0$
 c) For which values of x if $f(x) > g(x)$
 d) Find the length of AD
 e) Find the maximum length of PQ if $PQ \perp x$ -axis between T and C.

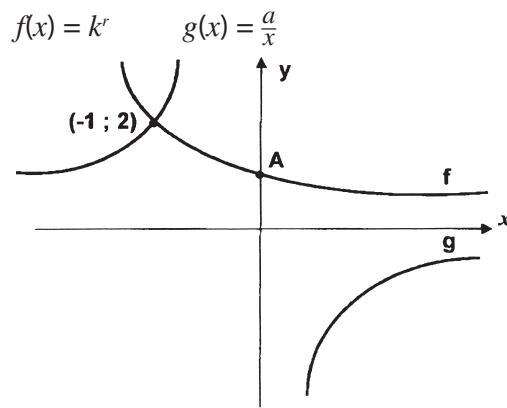
2. $f(x) = \frac{-4}{x-2} - 1$ $g(x) = mx + k$

Calculate

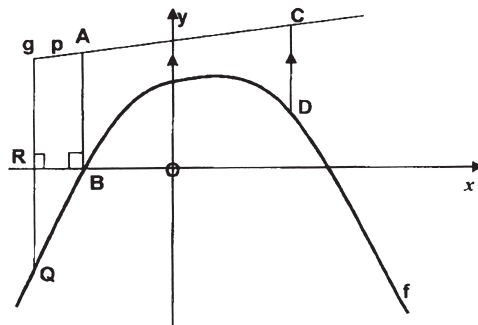
- a) The asymptotes of the hyperbola.
 b) The coordinates of A and B.
 c) The values of m and k if $BD = 5$ units.
 d) The coordinates of C.
 e) The distance DC.
 f) The axis of symmetry of the hyperbola.



3.



- Find the values of k and a .
 - Find the coordinates of A .
 - Find the equation of the reflection of $f(x)$ across the x -axis.
 - Find the equation of the reflection of $f(x)$ across the y -axis.
 - Find the equation of the reflection of $g(x)$ across the x -axis.
 - Find the equation of the reflection of $g(x)$ across the x -axis.
 - Find the equation of $h(x)$ if $h(x)$ is $g(x)$ moved 2 units down and 1 unit to the left.
 - Find the axis of symmetry of h .
4. The sketch represents the graphs of $f(x) = -x^2 + 2x + 3$ and $g(x) = x + 4$.



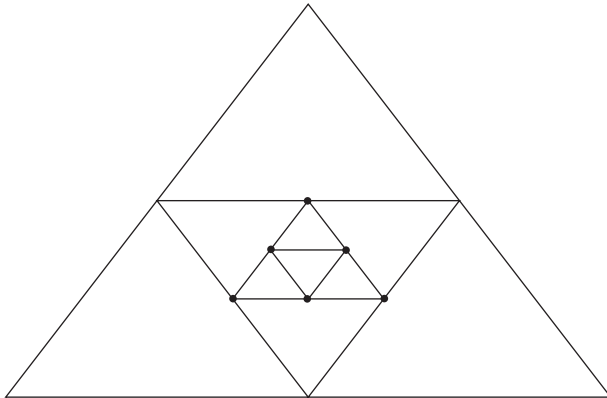
- Find the length of AB .
- If $PQ = 7$ units, find the length of OR .
- If CD is a variable line, parallel to the y -axis, find the minimum value of CD .

Problem solving using graphs and properties of graphs.

5. The equation $M = 1,4(1,2)^x$ can be used to estimate the mass of a baby baboon up to 6 months.
- M is the mass in kg.
 x is the age in months.
- Sketch the graph (you may want to use your calculator in table mode).
 - Show on your graph where you would estimate the mass of the baboon at the age of $4\frac{1}{2}$ months.
 - Use your calculator to find the correct answer to (b).
 - At what age would the baboon weigh 3,5 kg.

- e) If another baby baboon weighed 1,8 kg at birth and its rate of growth is the same, determine the equation to suit this data.

6. This diagram shows an equilateral \triangle with sides 12 cm. The mid-points of the sides are joined to form another \triangle and so on.



- a) Write down a formula for $P_n = \dots$ where P_n is the perimeter and n is the number of the triangle.
- b) Draw the graph with n along the x -axis and P_n on the y -axis.
- c) What do you think eventually happens?
7. $f(x) = \frac{4}{x-4} + 2$
- a) For what values of x is $f(x) = 0$?
- b) Find P if $A(p; 0)$ lies on the graph.
- c) Draw $f(x)$
- d) What is the range of f .
- e) Draw in the axis of symmetry of x and write down the equation.
8. Two painters can paint a room in one work shift of six hours.
- a) How long will the work shift be if four painters complete the work at the same rate?
- b) If the room needs to be painted in a work shift of 2,4 hours, how many painters are then needed if they work at the same rate.
- c) Set up a table of values where x is the number of workers and y is the number of hours to complete the work.
- d) Write down the equation that relates to y to x .
- e) A two hour lunch break has to be included in any work shift. Write down the new equation.
- f) A foreman who manages the team and does not paint, is also included in the formula. Write down the new equation.
- g) How does the equation in (f) compare with the equation in (d)?
- h) Draw the graph of (f).
- i) Estimate from your graph (and show where you will read) how many hours, including lunch, are required if the work needs to be completed by a team of five people (including the foreman).