TRANSLATION OF GRAPHS

Drawing parabolas

LESSON 33

Learning Outcomes and Assessment Standards

Learning Outcome 2: Functions and algebra
Assessment Standard AS 2
Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures about the effect of the parameters k, p, a and q for the functions including:

$y = \sin kx$ $\quad$ $y = \cos kx$
$y = \tan kx$ $\quad$ $y = \sin (x + p)$
$y = \cos (x + p)$ $\quad$ $y = \tan (x + p)$
$y = a(x + p)^2 + q$
$y = a \cdot b^{x+p} + q$ $\quad$ $y = \frac{a}{x+p} + q$

Overview

Overview

In this lesson you will:

- Draw the parabola by completing the square
- Draw the parabola by a suitable formula
- Use substitution to draw the parabola
- Find the equation of a parabola when given x-intercepts.

Lesson

Lesson

$y = ax^2 + bx + c$ is the graph of a quadratic function

To draw the graph we need to write the equation in the form $y = a(x - p)^2 + q$ by completing the square

a tells us the shape

If $a < 0$ ☹ sad. If $a > 0$ ☺ happy.

$(p ; q)$ is the turning point.

y–intercept make $x = 0$

You can use symmetry to find another point.

To find the x-intercepts (if possible) make $y = 0$.

Examples

Example

1. if $y = x^2 - 6x + 8$
   a. Find the turning point
   b. Find the y-intercepts
   c. Find the x-intercepts
   d. Draw the graph

Solutions

Solution

a. We need to complete the square

$y = x^2 - 6x \quad +8$

Add and subtract half the co-efficient of x

$y = x^2 - 6x + (-3)^2 - 9 + 8$

Factorise the square trinomial

$y = (x - 3)^2 - 1$

Turning point (3 ; –1) Shape $\vee$

b. y-intercepts $x = 0$

(0 ; 8)

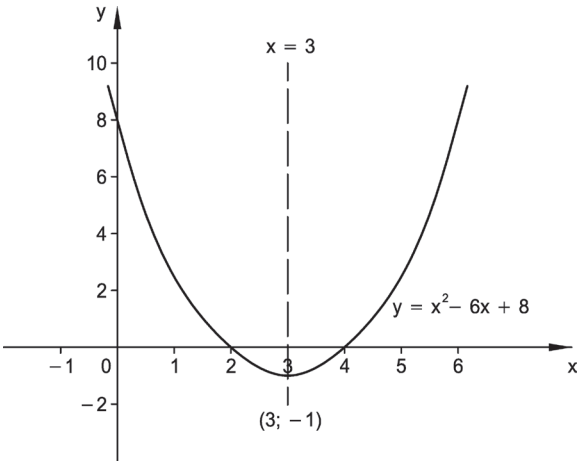
c. x-intercepts $y = 0$

$x^2 - 6x + 8 = 0$ OR $(x - 3)^2 - 1 = 0$

$(x - 4)(x - 2) = 0$ $x - 3 = \pm 1$

(4 ; 0) (2 ; 0) $x = 3 \pm 1 \Rightarrow x = 4 \text{ or } x = 2$

d. Graph



$y = x^2 - 6x + 8$

Let's get a formula

1. This is what we did each time $y = ax^2 + bx + c$

Divide by a $\frac{y}{a} = x^2 + \frac{b}{a}x + \frac{c}{a}$

Complete the square $\frac{y}{a} = x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a}$

$\frac{y}{a} = \left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a}$

Put a back $y = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$

So the turning point is: $\left(-\frac{b}{2a} ; c - \frac{b^2}{4a}\right)$

$x = \frac{-b}{2a}$ is the axis of symmetry

$\frac{4ac - b^2}{4a}$ is the maximum value if $a < 0$

$\frac{4ac - b^2}{4a}$ is the minimum value if $a > 0$

2. We will use the formula to draw the graph.

Sketch $y = -x^2 - 4x + 12$ and show the turning point and intercepts with the axes.

Solution

Shape $\wedge$ Turning point: $\left(-\frac{b}{2a}\,;\frac{4ac-b^2}{4a}\right)$

$a=-1\ b=-\ 4c=12$

Turning point: $\left(\frac{-(-4)}{-2}\,;\frac{4(-1)(12)-16}{-4}\right)$

$(-2\,;18)$

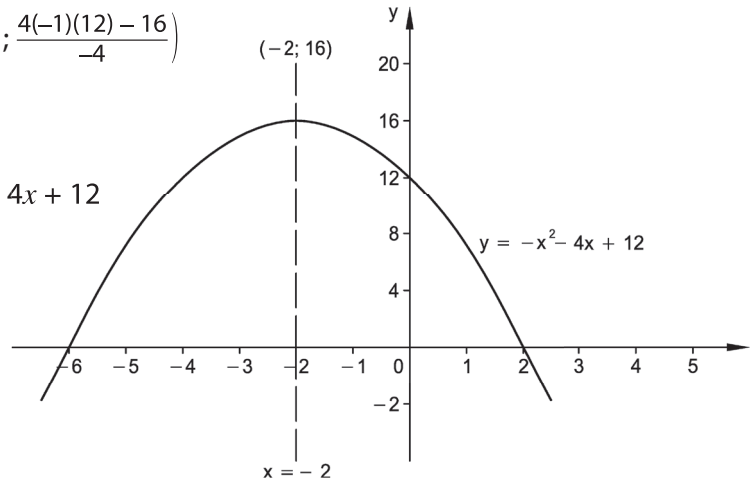
y-intercepts $(0\,;12)$

x-intercepts $0=-x^2-4x+12$

$x^2+4x-12=0$

$(x+6)(x-2)=0$

$(-6\,;0),(2\,;0)$



3. Sketch $y=2x^2+x-6$

Solution

Shape $\vee$

Axis of symmetry: $x=-\frac{b}{2a}$

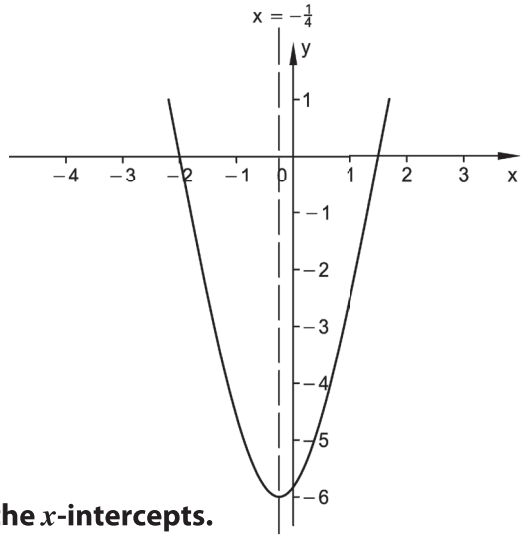
$x=-\frac{1}{4}$

Now instead of using the formula for the minimum value substitute into the original equation.

$y=-\frac{49}{8}$

$y=-6\left(\frac{1}{8}\right)$

Turning point: $\left(-\frac{1}{4}\,;-6\left(\frac{1}{8}\right)\right)$ Graph:



y-intercepts $(0\,;-6)$

x-intercepts $0=2x^2+x-6$

$0=(2x-3)(x+2)$

$\left(\frac{3}{2}\,;0\right)(-2\,;0)$

Finding the equation when you are given the x-intercepts.

Example

$y=a(x-1)(x+6)$

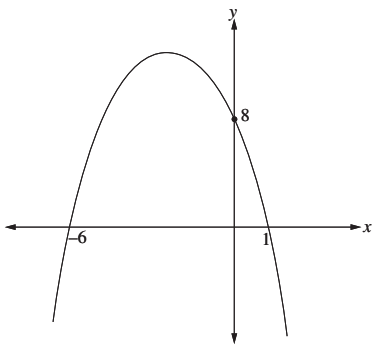
Then $8=a(-1)(6)$

$8=-6a$

$\therefore a=-\frac{3}{4}$

So: $y=-\frac{3}{4}(x^2+5x-6)$

$y=-\frac{3x^2}{4}-\frac{15}{4}x+\frac{9}{2}$



Activity

Activity 1

Sketch each of the following graphs by completing the square to find the turning point and find the y-intercepts and the x-intercepts if possible.

1. $y = 2x^2 - 7x - 4$
2. $y = -x^2 + 8x - 12$
3. $y = -x^2 + 2x - 6$

Activity

Activity 2

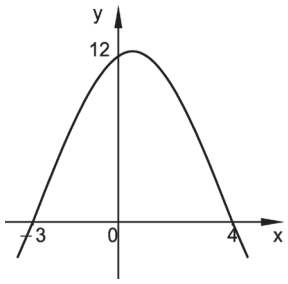
1. Sketch the following functions:

   a) $f : x \to (-x^2 + 3x - 2)$ b) $g : x \to (2x + 1)(x - 3)$

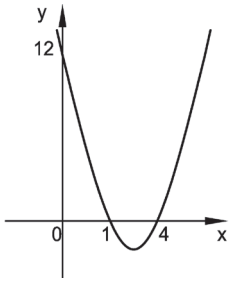
   c) $h : x \to -2x^2 + 5x$

2. Find the equations of the following functions:

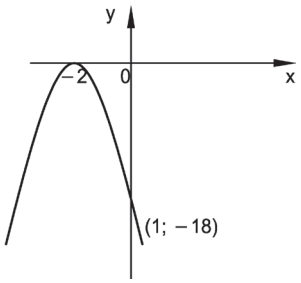
   a)



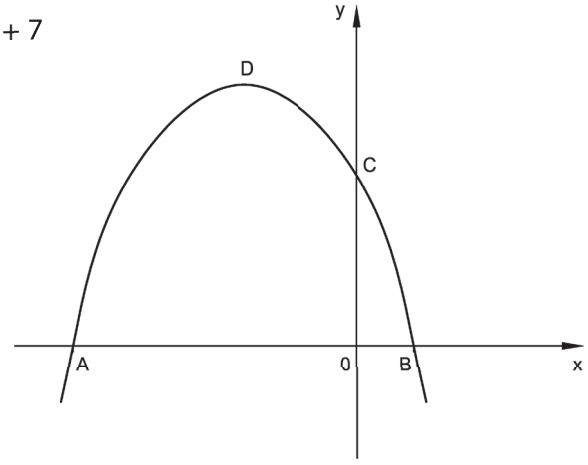
   b)



   c)



3. This is the graph of f; $x \to -x^2 - 6x + 7$



   a) Find the co-ordinates of A, B, C and D (the turning point)

   b) If $-x^2 - 6x + k = 0$ has no real roots find k.

   c) If $-x^2 - 6x + k = 0$ has two negative roots find the possible values of k.

4. a) Show that $2p^2x^2 - 2px + 1$ is always positive if $p > 0$

   b) Sketch $y = 2p^2x^2 - 2px + 1$ and write down the co-ordinates of the turning point in term of p if $p > 0$