FINDING EQUATIONS OF FUNCTIONS

LESSON 32

Learning Outcomes and Assessment Standards

Learning Outcome 2: Functions and algebra
Assessment Standard AS 2
Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures about the effect of the parameters k, p, a and q for the functions including:

$y = \sin kx$ $\quad y = \cos kx$
$y = \tan kx$ $\quad y = \sin (x + p)$
$y = \cos (x + p)$ $\quad y = \tan (x + p)$
$y = a(x + p)^2 + q$
$y = a \cdot b^{x+p} + q$ $\quad y = \frac{a}{x+p} + q$

Overview

Overview

- Find the equation of parabola by using the turning point
- Find the equation of hyperbola by identifying the asymptotes and substituting a point
- Find the equation of an exponential graph by identifying the asymptote and substituting a point.

Lesson

Lesson

Equation: $y = a(x - p)^2 + q$

Remember the turning point is $(p\,;q)$

To find the equation we need:

(1) Turning point (2) One other point to get the value of a

1. The parabola

Example

Example 1

Find the equation of the following parabola

Solution

Solution

Step 1: Fill in the turning point

$y = a(x - 2)^2 - 3$

Step 2: Use the other point on the graph to substitute

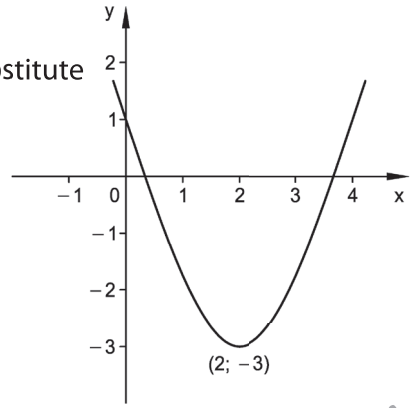
$x = 0 \quad y = 1$

$1 = a(-2)^2 - 3$

$4 = 4a$

$a = 1$

Equation: $y = (x - 2)^2 - 3$

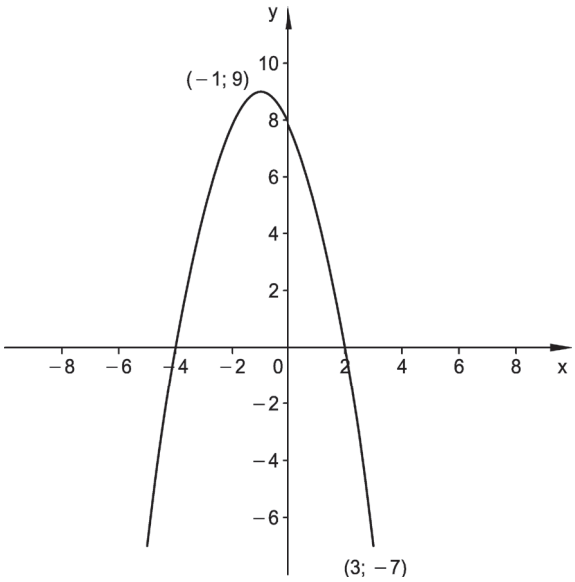


Example

Example 2

a) Find the equation of the following parabola and write it in the form $y = ax^2 + bx + c$

b) Find the x and y intercepts



Put in the turning point

$y = a(x + 1)^2 + 9$

Substitute the point (3 ; –7)

$x = 3 \quad y = -7$

$-7 = a(4)^2 + 9$

$-16 = 16a$

$a = -1$

Equation: $y = -(x + 1)^2 + 9$

$y = -(x^2 + 2x + 1) + 9$

$y = -x^2 - 2x + 8$

(0 ; 8)

y–intercept $x = 0$

x–intercept $y = 0$

$0 = -x^2 - 2x + 8$

$x^2 + 2x - 8 = 0$

$(x + 4)(x - 2 = 0$

x–intercepts (–4 ; 0) (2 ; 0)

2. The hyperbola

Form of equation: $y = \frac{k}{x-p} + q$

Remember $x = p$ and $y = q$ are asymptotes

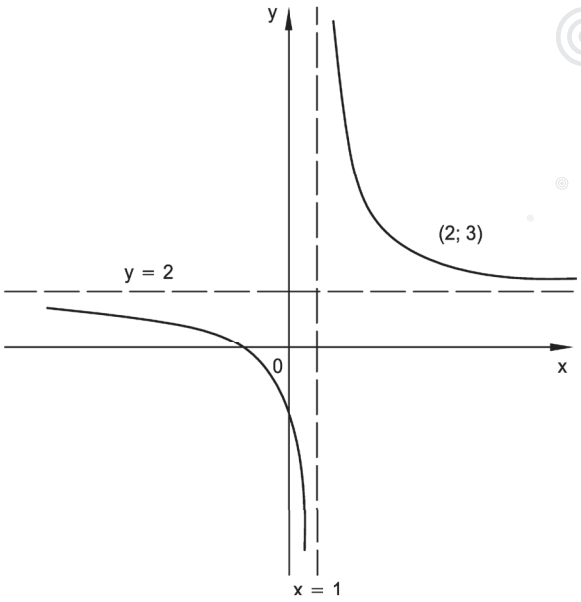
First fill in the asymptotes

Substitute a point to find k

Example 1

a) Find the equation of the parabola.

b) Find any intercepts with the axes.



Put in the asymptotes first

$y = \frac{k}{x-1} + 2$

Substitute point (2 ; 3) to find k

$3 = \frac{k}{2-1} + 2$

$3 = k + 2$

$k = 1$

so

$y = \frac{1}{x-1} + 2$

x–intercept $y = 0$

$0 = \frac{1}{x-1} + 2$

$-2 = \frac{1}{x-1}$

$-2x + 2 = 1$

$-2x + 2 = 1$

$-2x = -1$

$x = \frac{1}{2}$

x–intercepts $\left(\frac{1}{2}; 0\right)$

y–intercept $x = 0$

$y = \frac{1}{-1} + 2$

$y = 1$

y intercept $(0 ; 1)$

3. Exponential graphs

Equation: $y = a^{x+p} + q$

$y = q$ is a horizontal asymptote

If $a > 1$ the function is increasing

If $0 < a < 1$ the function is decreasing

To find the equation first put in the asymptote and then substitute the point.

Example

Example 1

If f : $x \rightarrow 2^{x+p} + q$ find the equation.

Solution

Solution

$y = 2^{x+p} - 2$

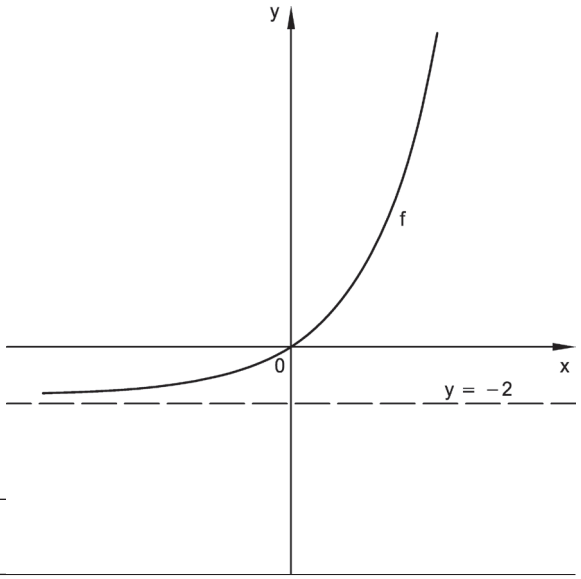
Substitute $(0\ ;\ 0)$ $\quad x = 0 \quad y = 0$

$0 = 20 + p - 2$

$2 = 2p$

$p = 1$

$y = 2^{x+1} - 2$



Example

Example 2

Find the equation of f if f(x) = $\left(\frac{1}{3}\right)^{x+p} + 1$ and find the y–intercept

Solution

Solution

$y = \left(\frac{1}{3}\right)^{x+p} + 1$

Substitute $x = -1 \quad y = 10$

$10 = \left(\frac{1}{3}\right)^{-1+p} + 1$

$9 = \left(\frac{1}{3}\right)^{-1+p}$

Make the base the same

$3^2 = (3^{-1})^{-1+p}$

$3^2 = (3)^{1-p}$

$\therefore 2 = 1 - p$

$p = -1$

$y = \left(\frac{1}{3}\right)^{x+p} + 1$

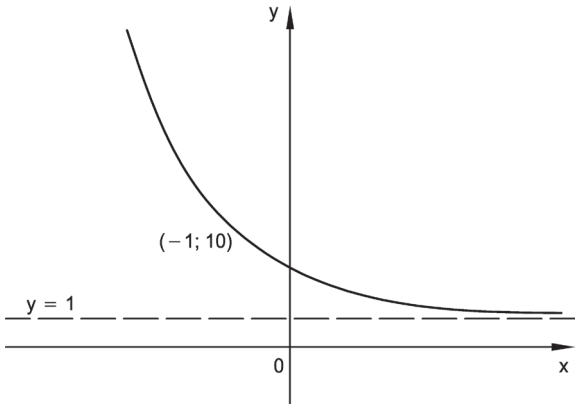
y–intercept make $x = 0$

$y = \left(\frac{1}{3}\right)^{-1} + 1$

$y = (3–1)–1 + 1$

$y = 4$

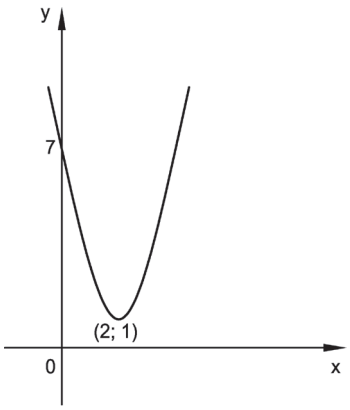
y–intercept $(0\ ;\ 4)$



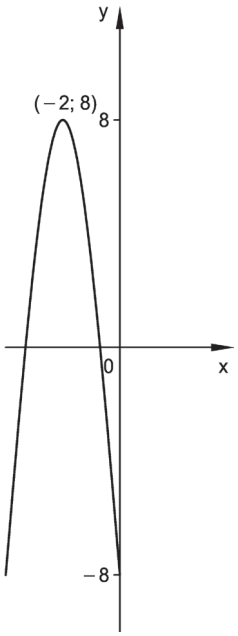
Activity 1

1, Find the equations of the following parabolas

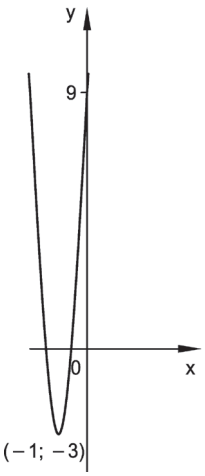
(a)



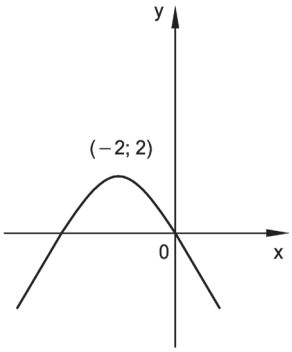
(b)



(c)

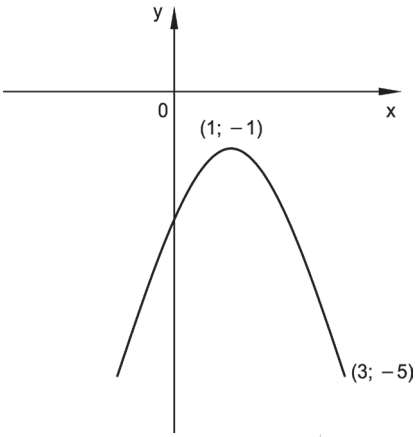


(d)



2. a) Find the equation of the following parabola

b) If the graph is translated 4 units up and 4 units to the left, write down the equation of the function.

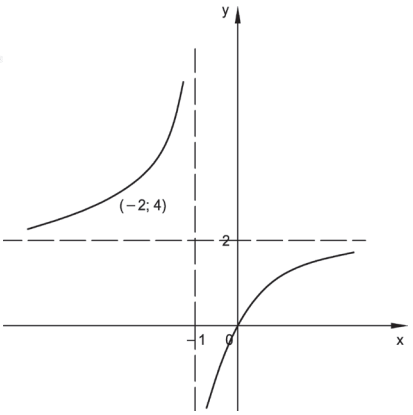


Activity

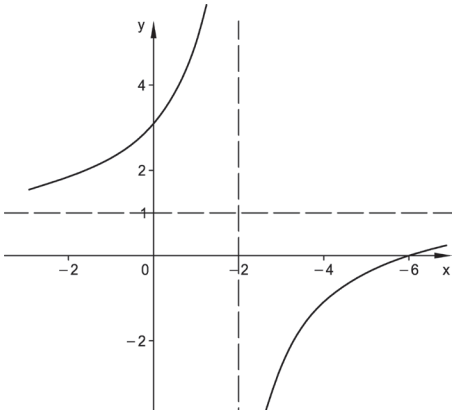
Activity 2

Find the equations of each of the following hyperbolas and, if necessary, write down the x and y–intercepts.

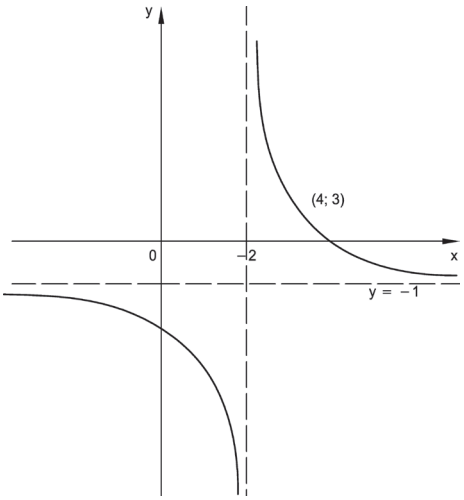
1.



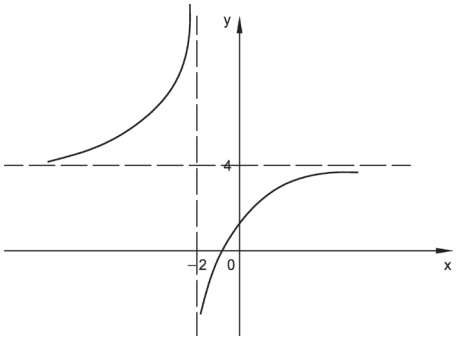
2.



3.



4.

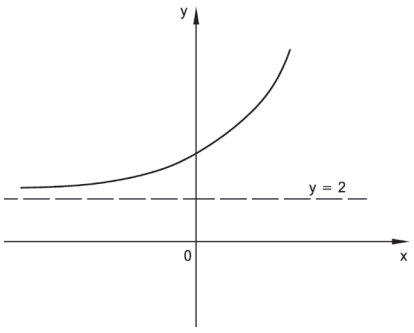


Activity

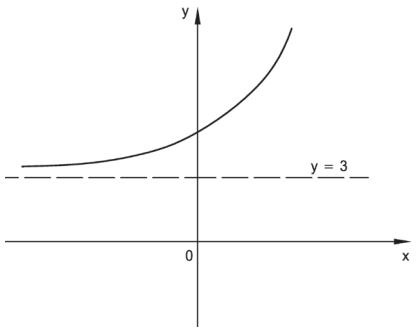
Activity 3

In each of the following graphs, find the value(s) of p and q.

1.

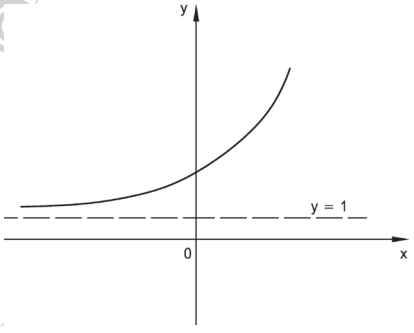


2.

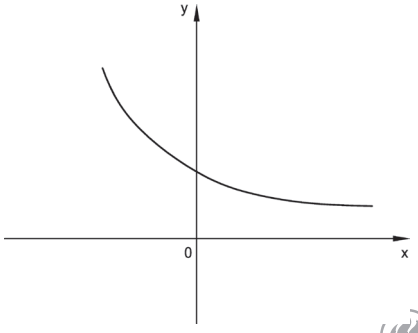


Find the y–intercept if $y = 2^{x+p} + q$

3.



4. If $y = a.b^x$, find the value of a and b.



Find the equation and the y–intercept.