TRANSLATION OF GRAPHS (2)

The exponential function and trigonometric function

LESSON 31

Learning Outcomes and Assessment Standards

Learning Outcome 2: Functions and algebra
Assessment Standard AS 2
Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures about the effect of the parameters k, p, a and q for the functions including:

$y = \sin kx$ $\quad$ $y = \cos kx$
$y = \tan kx$ $\quad$ $y = \sin(x + p)$
$y = \cos(x + p)$ $\quad$ $y = \tan(x + p)$
$y = a(x + p)^2 + q$
$y = a \cdot b^{x+p} + q$ $\quad$ $y = \frac{a}{x+p} + q$

Overview

In this lesson you will:

- Revise translating the exponential graph vertically
- Learn to translate the exponential graph horizontally
- Learn to translate the exponential graph both vertically and horizontally
- Revise translating trigonometric graphs vertically
- Learn to translate trigonometric graphs horizontally
- Learn to draw trigonometric graphs when the period changes.

Lesson

The exponential function

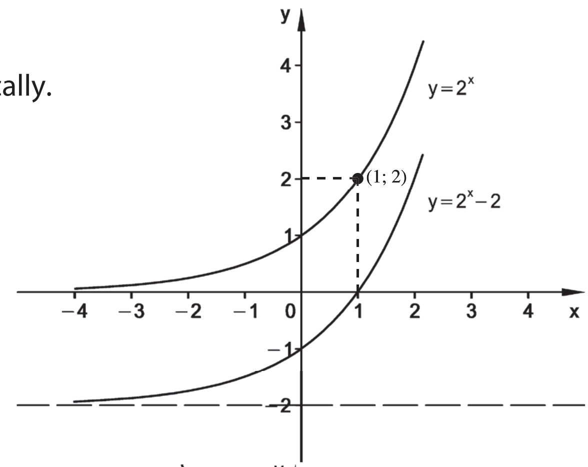
Last year you translated the graph vertically.

Let's revise it.

Draw: $y = 2^x$

Note: There is an asymptote at $y = 0$ (x–axis) so when we translate the graph, we first translate the asymptote.

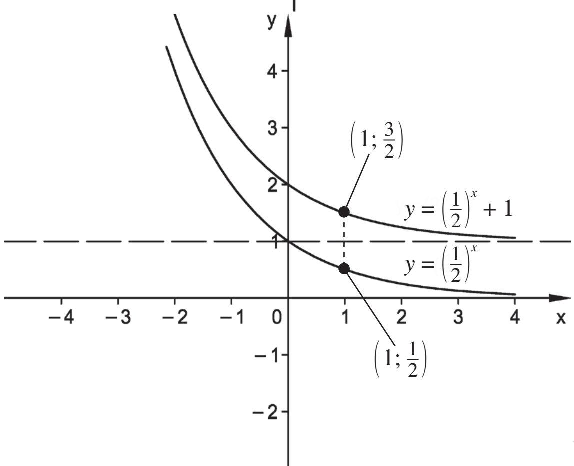
Draw: $y = 2^x - 2$



Draw: $y = \left(\frac{1}{2}\right)^x$

It is a decreasing function because the gradient is negative.

Draw: $y = \left(\frac{1}{2}\right)^x + 1$ (translate the asymptote first)



INDEPENDENT NEWSPAPERS

Investigation

On the same set of axes sketch:

$y = 2^{x+1}$ $y = 2^{x-1}$ $y = 2^{x+3}$ $y = 2^{x-4}$

You can put your calculator into table mode and select negative and positive values. If you do not have a calculator with table mode make a table of values.

x					
y					

Choose positive values and negative values and remember 2(0) = 1 is very important.

What did you learn from this investigation?

If $y = 2^{x-p}$, the graph is translated horizontally p units to the left or right.

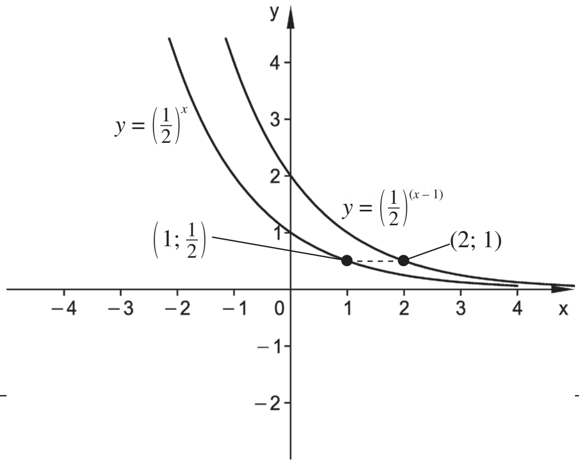
The line $y = 0$ (the x–axis) is the horizontal asymptote.

Example

Example 1

Draw $y = \left(\frac{1}{2}\right)^{x-1}$ is translated 1 unit to the right.

Translation: The graph $y = \left(\frac{1}{2}\right)^{x}$ is translated 1 unit to the right.

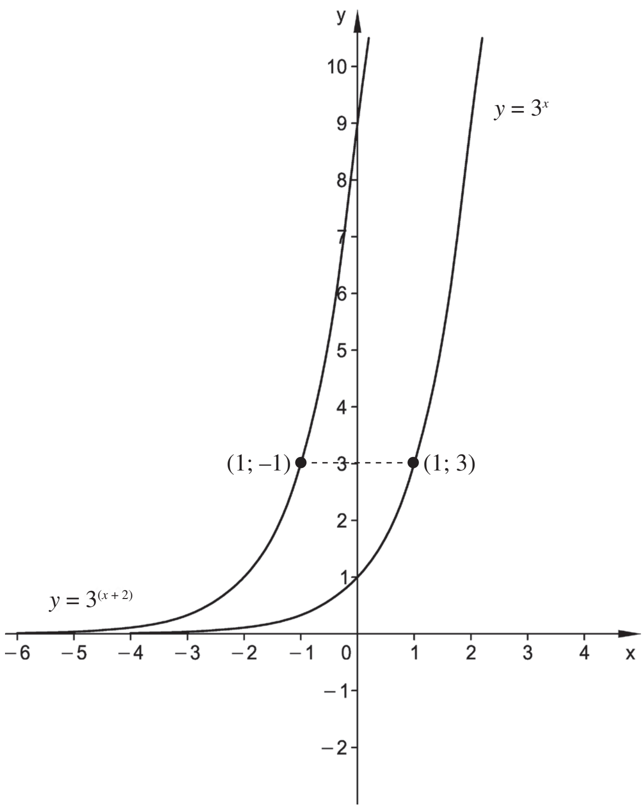


Example

Example 2

Draw $y = 3^{x+2}$

Translation: The graph $y = 3x$ is translated 2 units to the left.

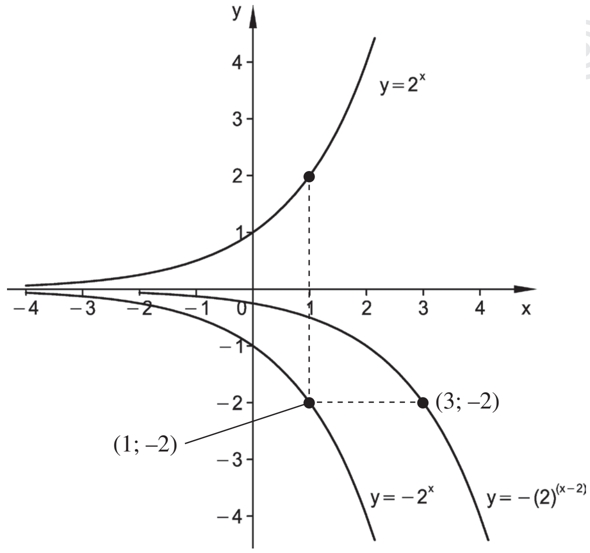


Example 3

What about $y = -2^{x-2}$

Translation: Draw $y = -2^{x}$

Reflect it across the x–axis to get $y = -2^{x}$

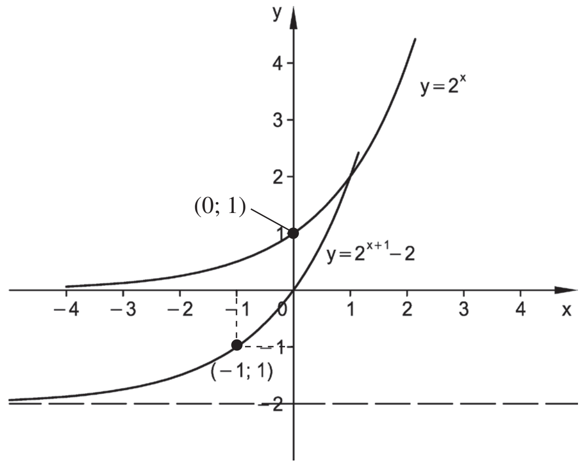


Example 4

Vertical and horizontal shift:
Sketch: $y = 2^{x+1} - 2$

Translation: Draw $y = 2x$

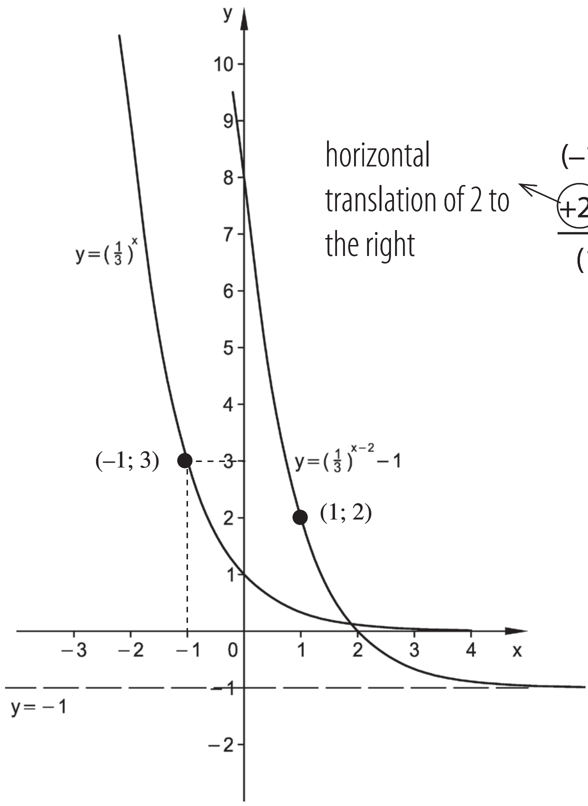
Translate it 1 unit to the left and 2 units down



Example 5

One more: Sketch $y = \left(\frac{1}{3}\right)^{x-2} - 1$

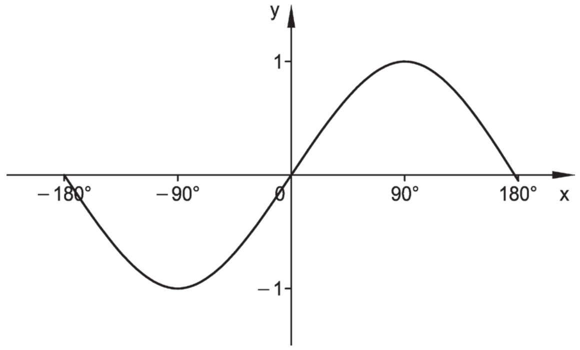
Translation: Draw $y = \left(\frac{1}{3}\right)^{x}$ translate it 2 units right and 1 unit down.



horizontal translation of 2 to the right ← (–1; 3) / (+2; –1) / (1; 2) → vertical translation of one unit down

Trigonometry graphs

Revision

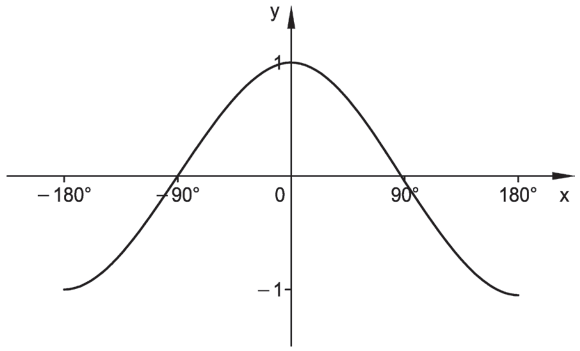


$y = \sin x$

$x \in [-180°; 180°]$

Amplitude: 1

Period: 360°



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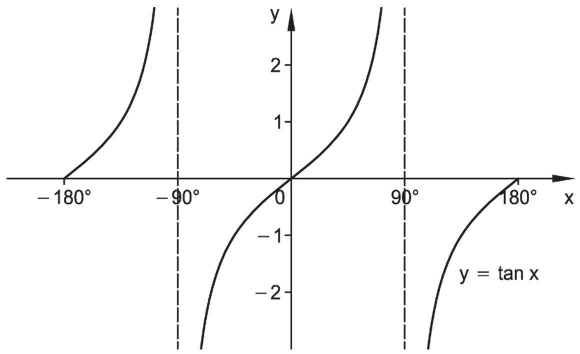


$y = \tan x$

$x \in [-180°; 180°]$

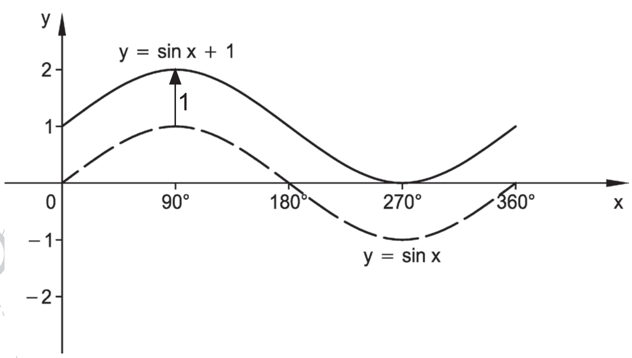
Period: 360°

Asymptotes: $90° + k180°, k \in Z$

Vertical translation

$y = \sin x + 1, x \in [0° ; 360°]$

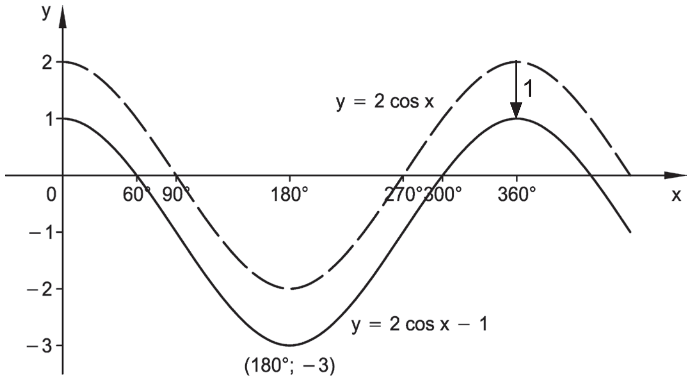
$y = \sin x$ translates up 1 unit



$y = 2 \cos x - 1$ $x \in [0° ; 360°]$

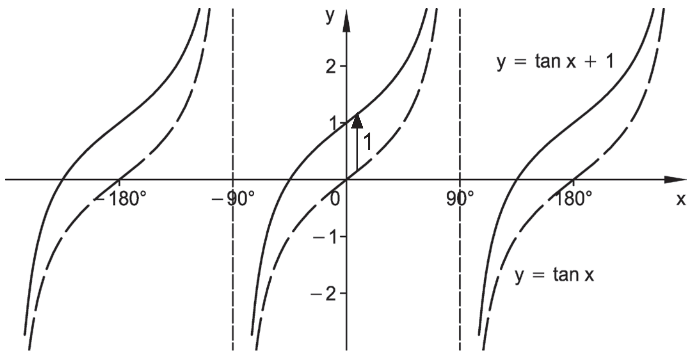
Translation $y = \cos x$ has an amplitude of 2 – stretches 2 up and 2 down.

$y = 2 \cos x$ translates 1 down.



$y = \tan x + 1$, $x \in [-180° ; 180°]$

Translation: $y = \tan x$ translates up 1 unit



Investigation: Use your calculator in table mode to sketch the following graphs if $x \in [0° ; 360°]$

Investigation

$y = \sin 2x$	$y = \sin 3x$	$y = \sin 2x$	
$y = \cos 2x$	$y = \cos 2x$	$y = \cos 4x$	if $x \in [0° ; 360°]$
$y = \tan 2x$	$y = \tan 3x$	$y = \tan 4x$	if $x \in [0° ; 90°]$

What happened to the graphs?

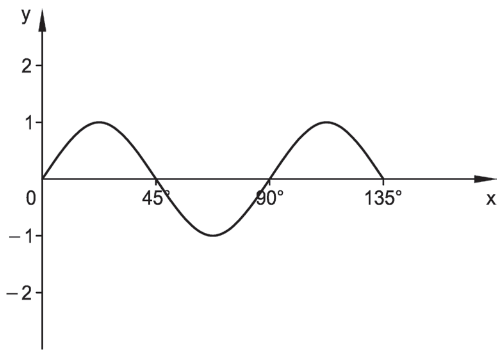
They either shrunk or stretched horizontally.

We say that the period of the graph changed.

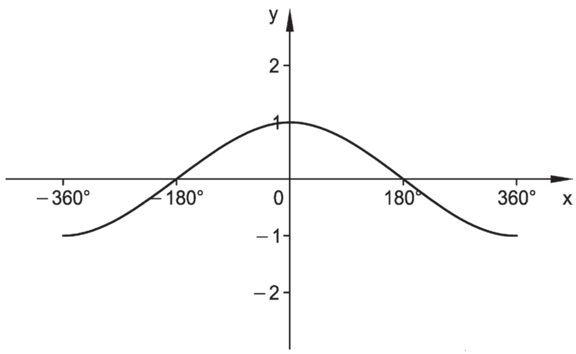
For $y = \sin bx$ and $y = \cos bx$ the period is $\frac{360°}{b}$.

For $y = \tan bx$ the period is $\frac{180°}{b}$.

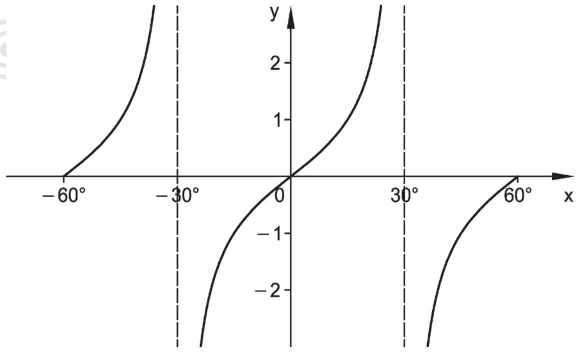
Identify the following graphs:



$y = \sin 4x$ if $x \in [0° ; 135°]$



$y = \cos \frac{1}{2}x$; if $x \in [-360° ; 360°]$

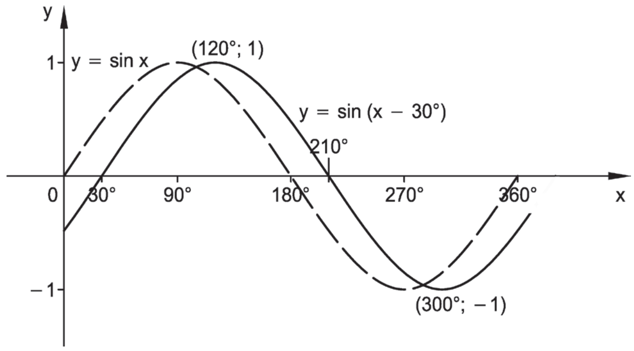


$y = \tan 3x$ if $x \in [-60° ; 60°]$

Horizontal Shift

$y = \sin(x - 30°)$ $x \in [0° ; 360°]$

Translation: $y = \sin x$ translates 30° to the right.

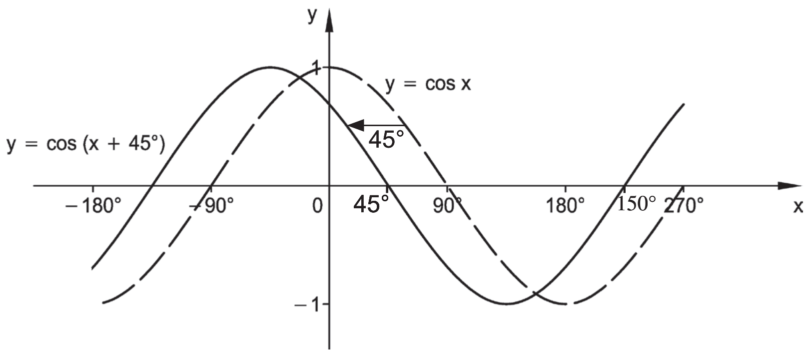


y–intercept $x = 0°$ $\sin(-30°) = -\frac{1}{2}$

The following cos and tan graphs are additional examples for you to work through.

Sketch: $y = \cos(x + 45°)$ $x \in [-180° ; 270°]$

Translation: $y = \cos x$ translates 45° to the left.



$y = \cos(x + 45°)$

$y = \cos x$

Sketch: $y = \tan(x + 30°)$; $x \in [180° ; 180°]$

Translation: $y = \tan x$ translates 60° to the right.

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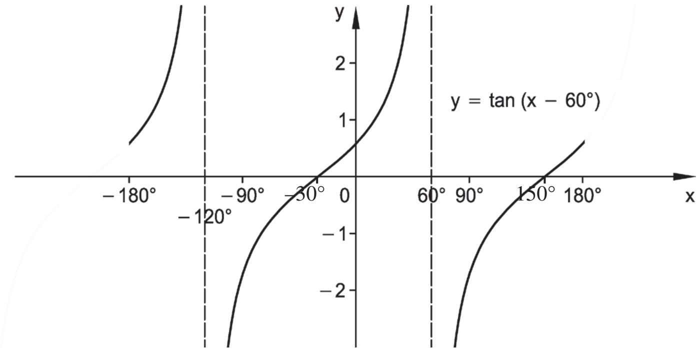


$y = \tan(x - 60°)$

Activity 1

Activity

1. a) Sketch $y = 3^x$

   b) Reflect $y = 3^x$ across the y–axis and write down the equation.

   c) Reflect $y = 3^x$ across the x–axis and write down the equation.

2. a) Sketch $y = \left(\frac{1}{4}\right)^x$

   b) Reflect $y = \left(\frac{1}{4}\right)^x$ across the y–axis and write down the equation.

   c) Reflect $y = \left(\frac{1}{4}\right)^x$ across the x–axis and write down the equation.

Activity 2

Activity

A. Sketch each of the following graphs on a separate set of axes and describe the translation you used.

1. $y = 3^x + 1$
2. $y = \left(\frac{1}{2}\right)^x - 2$
3. $y = -2^{x-1}$
4. $y = 2^x - 2$
5. $y = \left(\frac{1}{3}\right)^{x-1}$
6. $y = 2^{x+1} - 2$
7. $y = \left(\frac{1}{3}\right)^{x-1} - 2$
8. $y = -2^{x+1} - 1$
9. $y = \left(\frac{1}{3}\right)^{x-1} + 1$
10. $y = \sin(x + 30°)$ if $x \in [0\,;\,360°]$
11. $y = 2\cos(x - 45°)$ if $x \in [0\,;\,360°]$
12. $y = \tan(x + 60°)$ if $x \in [0°\,;\,360°]$

B. 1. Sketch $f(x) = 2^{x+1} - 1$

   2. Sketch g if g is the reflection of f across the y–axis.

   3. Sketch h if h is the reflection of f across the x–axis.

C. 1. Sketch $y = \cos(x + 30°)$ if $x \in [-180°\,;\,180°]$

   2. If the y–axis is translated 60° to the right, give the equation of the new graph.

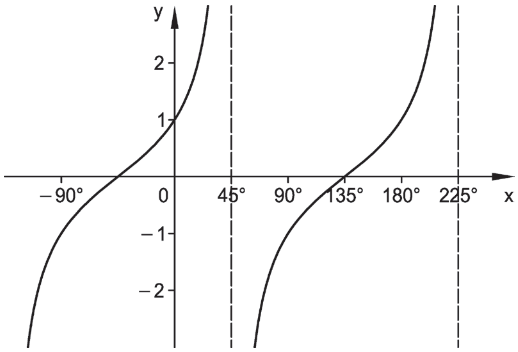
   3. If the graph is translated 2 units vertically down, give the equation of the new graph.

   4. If the graph is translated horizontally 20° to the left, find the new equation of the graph.

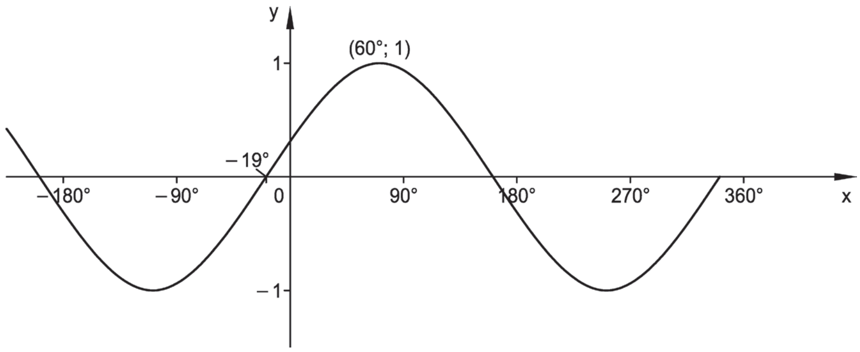
D. 1. Sketch $y = \sin(x - 60°)$ if $x \in [-180°\,;\,360°]$

   2. Name this graph in 3 different ways.

E. Give an equation for the following graph.



F.



1. Find the equation of *g*.
2. Find the co-ordinates of A.
3. Find another equation for *g*.

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