

TRANSLATION OF GRAPHS (1)

The parabola and hyperbola



LESSON

Learning Outcomes and Assessment Standards

Learning Outcome 2: Functions and algebra Assessment Standard AS 2

Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures about the effect of the parameters k , p , a and q for the functions including:

$$\begin{array}{ll}
 y = \sin kx & y = \cos kx \\
 y = \tan kx & y = \sin(x + p) \\
 y = \cos(x + p) & y = \tan(x + p) \\
 y = a(x + p)^2 + q & \\
 y = a \cdot b^{x+p} + q & y = \frac{a}{x+p} + q
 \end{array}$$

Overview

In this lesson you will:

- Draw the functions you learnt in Grade 10 and exercise vertical translation and reflection across both the x - and y -axes
- Translate graphs horizontally along the x -axes
- Translate graphs both vertically and horizontally.



Overview

Lesson



Lesson

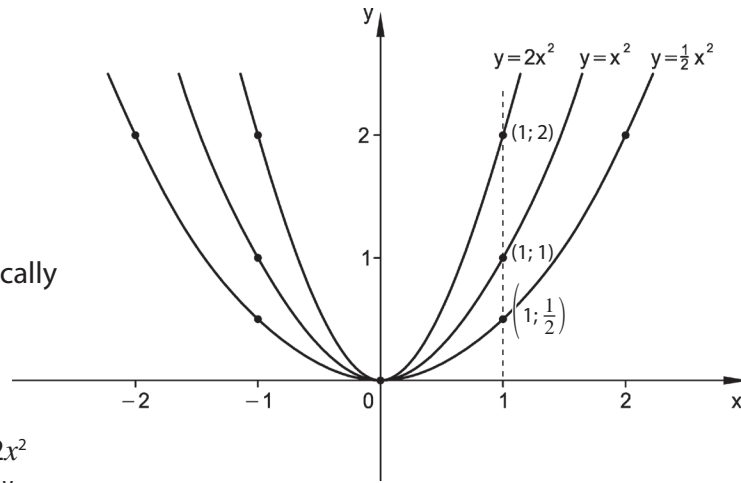
Revision

Let's look at $y = x^2$; $y = \frac{1}{2}x^2$

and $y = 2x^2$ on the same set of axes.

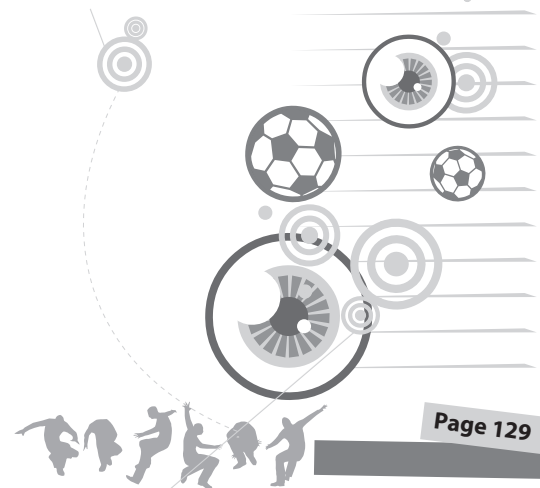
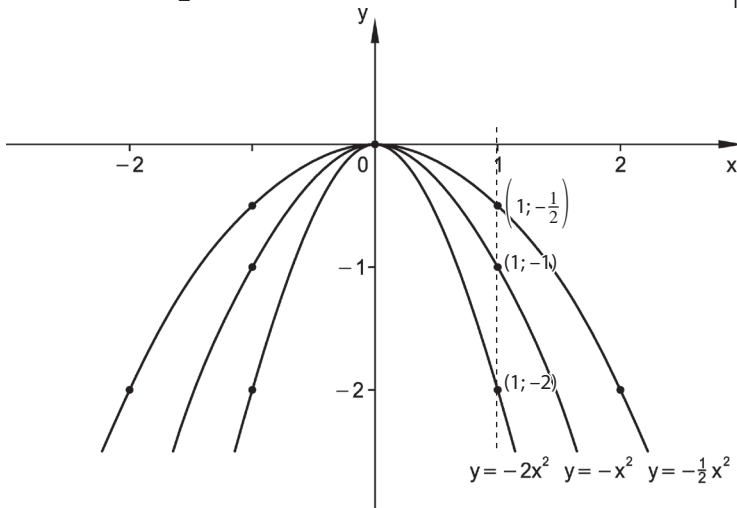
Conclusion: $y = ax^2$

The graph is stretched vertically by a factor of a .



Let us look at:

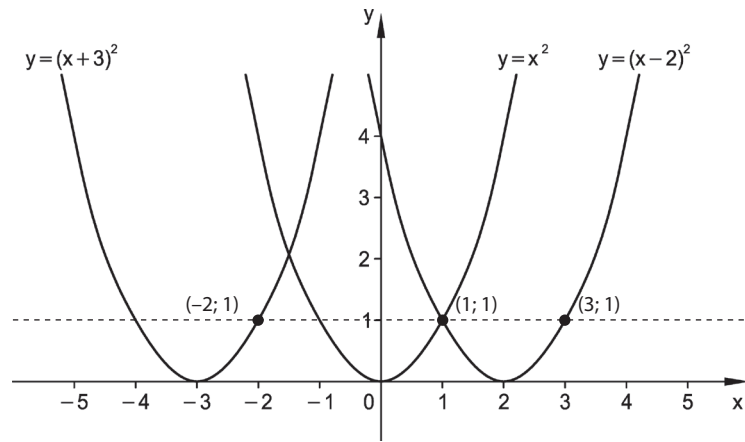
$y = -x^2$; $y = -\frac{1}{2}x^2$ and $y = -2x^2$



The graph is stretched vertically by a factor of a and reflected about the x -axis.

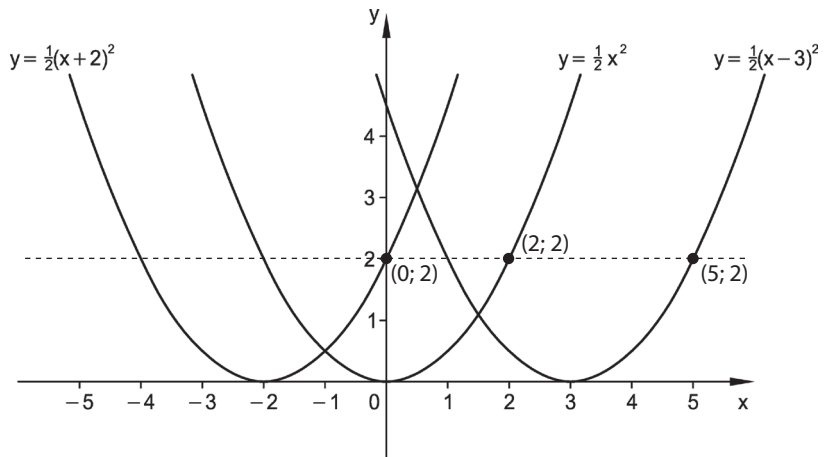
focus: $y = (x + p)^2$

We will keep playing with the parabola but we will translate it along the x -axis and see what happens.



focus: $y = (x + p)^2$

Let's translate the others



Now what will you do if you are asked to draw $y = -2(x + 2)^2$

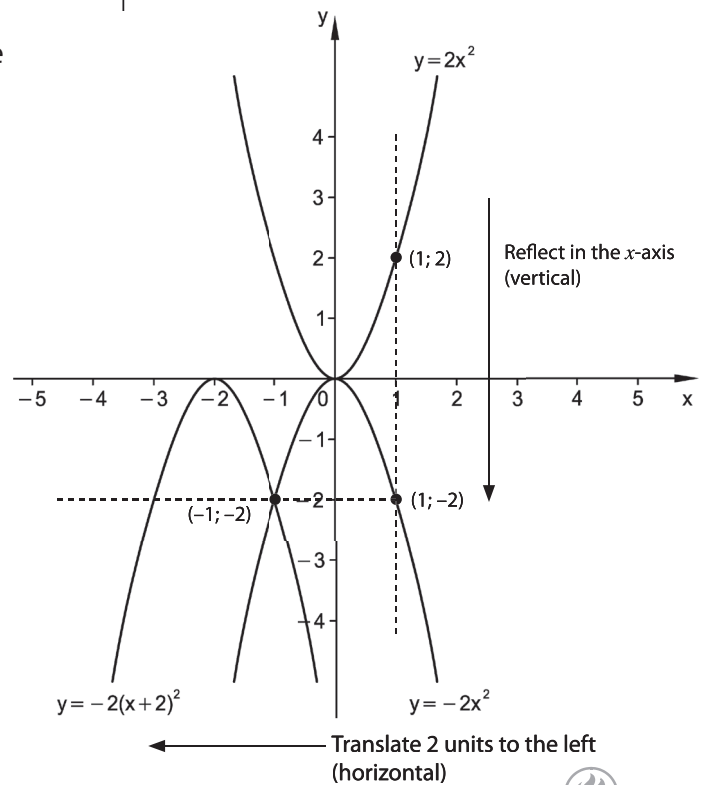
Step 1:

Draw $y = 2x^2$ and reflect it

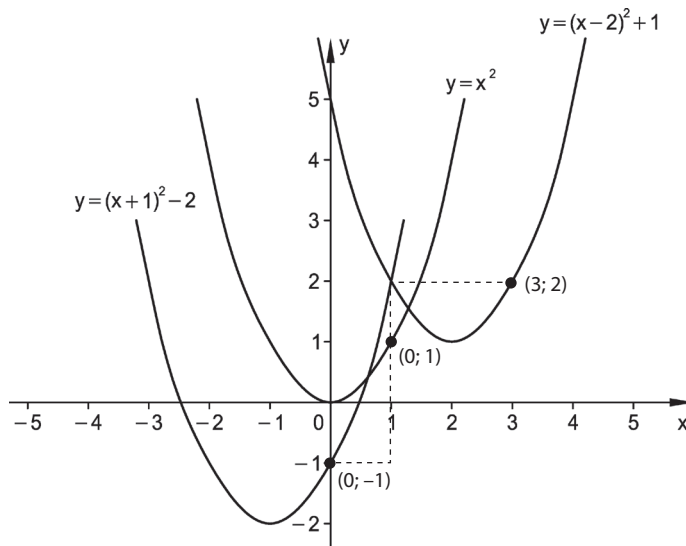
To be $y = -2x^2$

Step 2:

Translate it 2 units to the left.



How about translating a parabola horizontally and vertically?

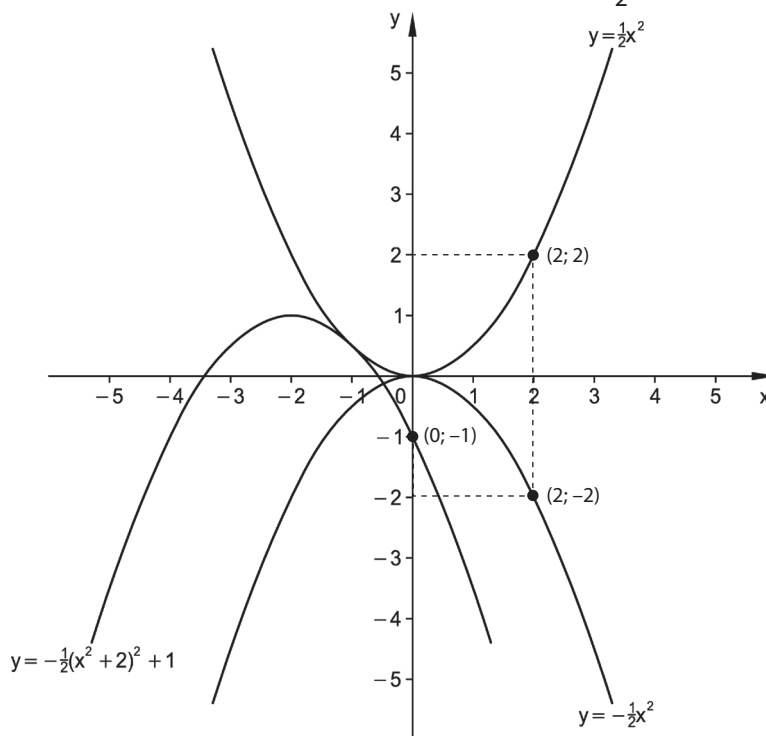


We will translate $y = x^2$ to the right 2 and up 1. What will the equation be?

We will translate $y = x^2$ to the left 1 and down 2. What will the equation be?

How will you draw $y = -\frac{1}{2}(x+2)^2 + 1$ by translating the graph of $y = x^2$?

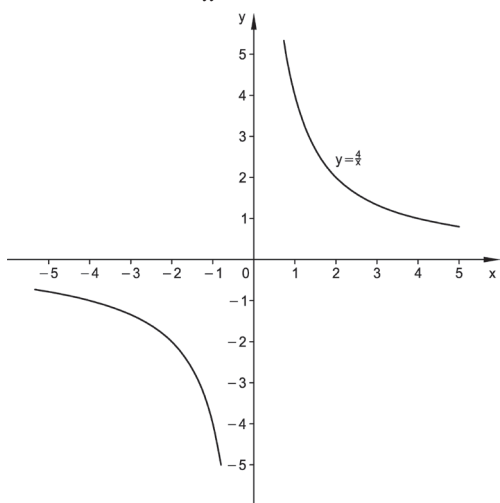
Draw $y = \frac{1}{2}x^2$ and reflect it across the x -axis to become $y = -\frac{1}{2}x^2$



Now translate it 2 units left and one unit up.

How about the other graphs you learnt last year?

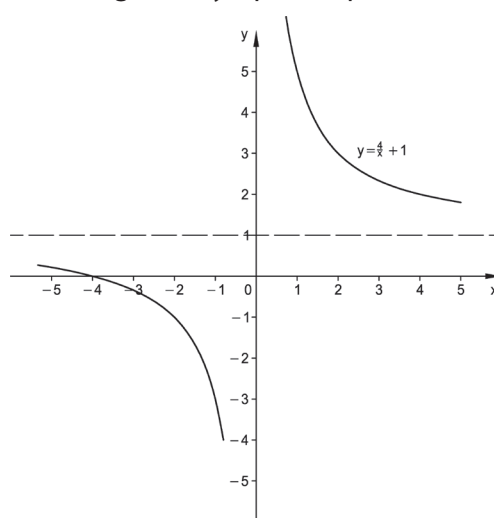
Hyperbola: $y = \frac{4}{x}$



Asymptotes at $x = 0$ and $y = 0$

What about $y = \frac{4}{x} + 1$?

(We translate the whole graph, including the asymptote up 1).

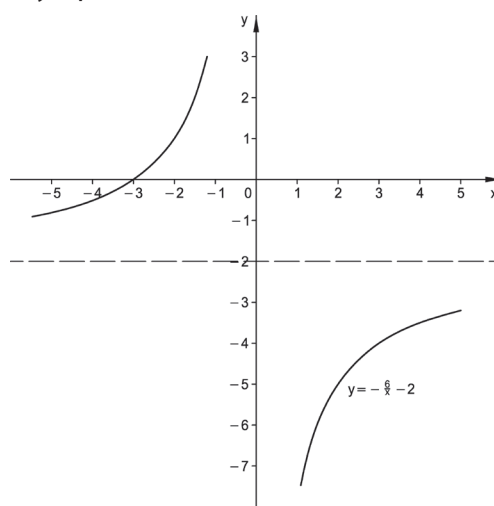
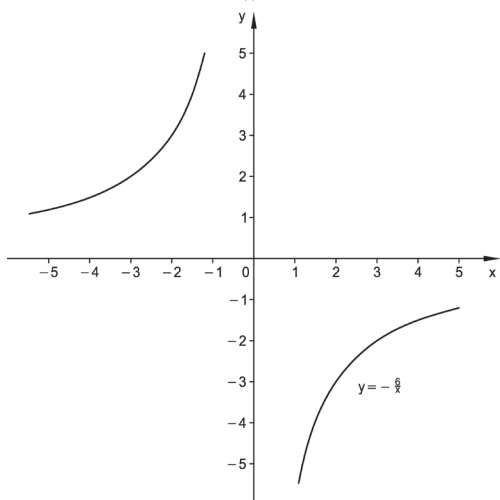


Remember this one

$y = -\frac{6}{x} - 2$

Now translate the graph (including the asymptote) down 2.

First draw $y = -\frac{6}{x}$



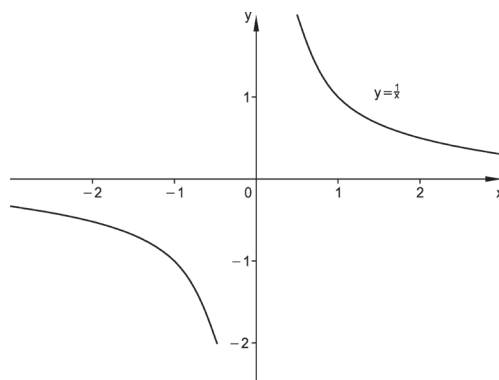
Now we will learn to translate the hyperbola horizontally.

$y = \frac{1}{x-1}$

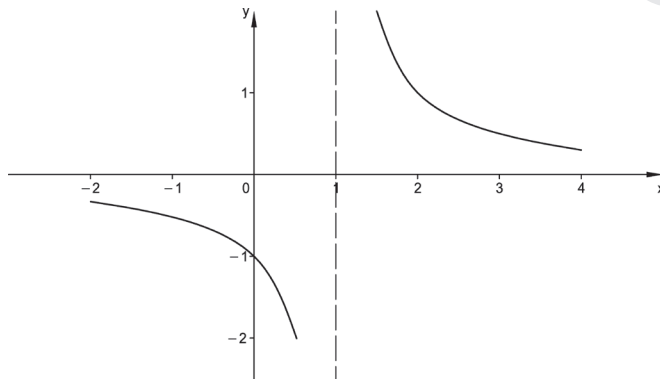
What do you think we will do?

We translate $y = \frac{1}{x}$ one unit to the right.

Let's first draw $y = \frac{1}{x}$



Now translate the whole graph (including the asymptote) 1 unit to the right.



Let's do another horizontal translation.

Sketch $y = \frac{-4}{x+2}$

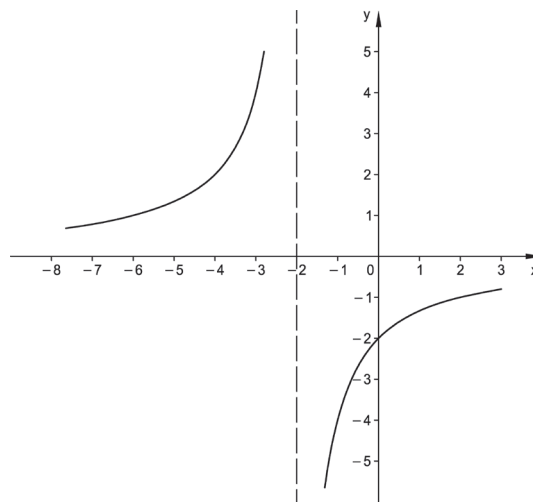
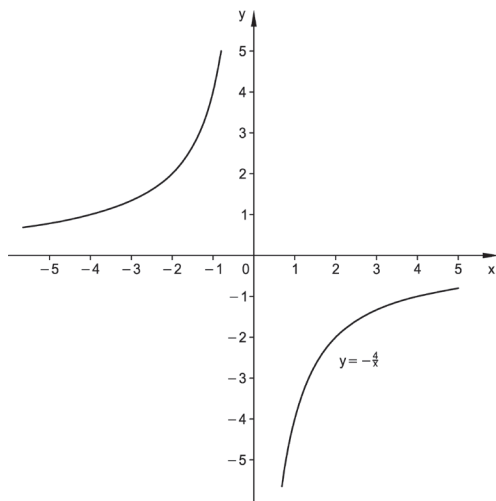
Translation plan:

Step 1:

Reflect $y = \frac{4}{x}$ across the y -axis to become $y = -\frac{4}{x}$

Step 2:

Now translate this graph 2 units to the left (asymptote first).



What about translating the hyperbola both vertically and horizontally?

Sketch: $y = \frac{2}{x+1} - 2$

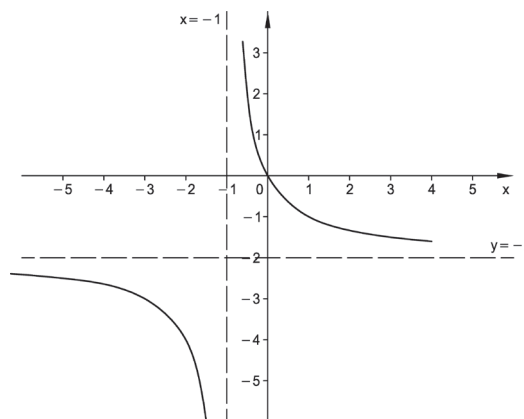
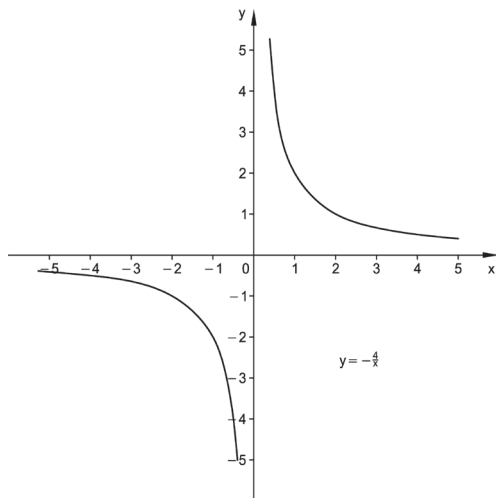
Translation plan:

Step 1: Draw $y = \frac{2}{x}$

Step 2:

Now translate the graph 1 unit to the left and 2 units down.

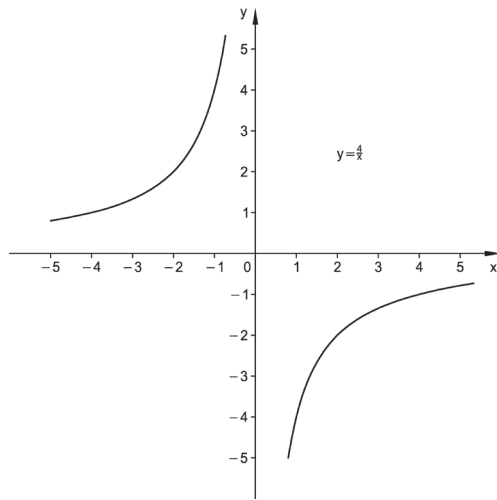
(First name the asymptotes).



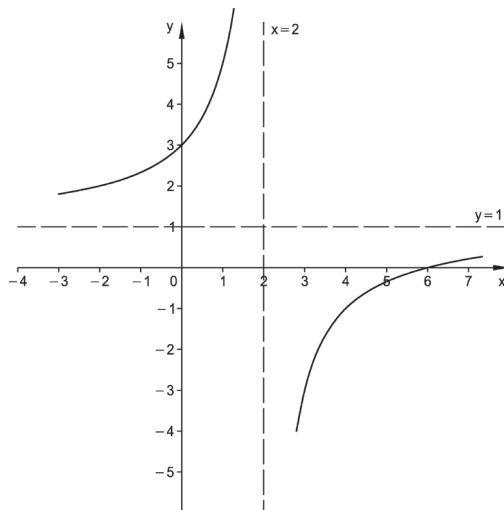
Another one: Sketch $y = \frac{-4}{x-2} + 1$

Translation plan:

Sketch $y = \frac{4}{x}$ and reflect it across the y -axis to become $y = -\frac{4}{x}$.



Now translate the graph 2 units to the right and 1 unit up (asymptotes first).



Summary

Parabola: $a(x-p)^2 + q$

Draw $y = ax^2$

Move the turning point to $(p; q)$

Keep the shape the same as $y = ax^2$

Hyperbola: $y = \frac{k}{x-p} + q$

Draw: $y = \frac{k}{x}$

Move the asymptotes $x = p$

$y = q$

Find any x -intercepts ($y = 0$)

Find any y -intercepts ($x = 0$)

Draw the arms of the hyperbola

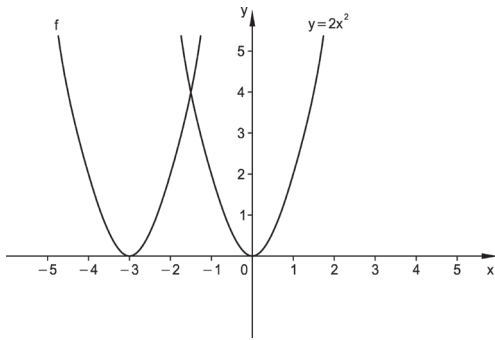


Activity 1

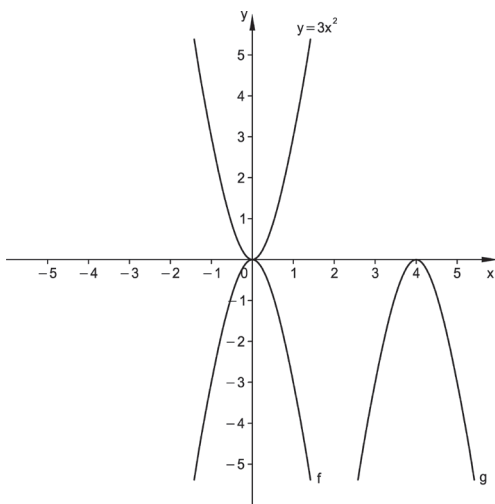
Activity

1. Identify the following graphs and explain the translation.

a)



b)



2. Draw the following graphs, each on a separate set of axes by translating $y = ax^2$ where $a > 0$.

Explain the translation you used.

a) $y = -(x + 1)^2$

b) $y = \frac{1}{3}(x - 2)^2$

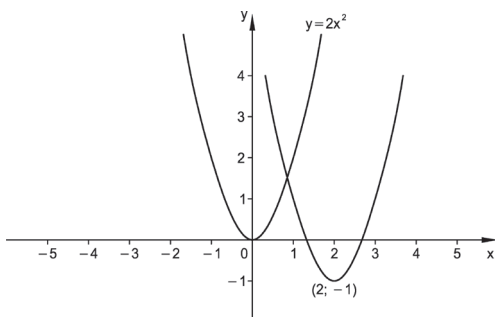
c) $y = -2(x + 3)^2$

Activity 2

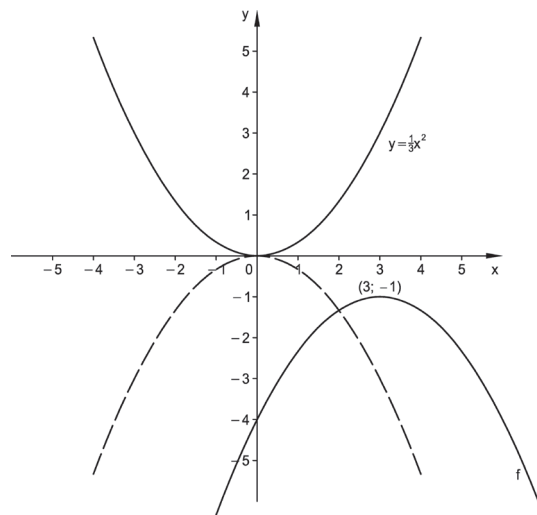
Activity

1. Identify the following graphs and explain the translation.

a)



b)



2. Use translation to draw the following graphs on separate sets of axes and explain the translation you need.

a) $y = -(x + 3)^2 - 1$

b) $y = \frac{1}{2}(x + 1)^2 - 2$

c) $y = -2(x - 3)^2 + 2$

Activity 

Activity 3

1. Sketch the following graphs. In each case write down the translation plan. Show on the graph the asymptotes and any x - and y -intercepts.

a) $y = \frac{3}{x-2} + 1$

b) $y = \frac{-6}{x+1} - 2$

c) $y = \frac{2}{x+3} - 1$

d) $y = \frac{-4}{x-1} + 2$

2. If $y = \frac{6}{x}$ is translated 1 unit up and 3 units to the left, draw the graph and give the equation.

3. If the graph $y = \frac{12}{x}$ is reflected across the y -axis, translated 1 unit down vertically and two units to the left horizontally, write down the equation and draw the graph.