

ANALYTICAL GEOMETRY (3)

Learning Outcomes and Assessment Standards

Learning Outcome 3: Space, shape and measurement

Assessment Standard As 3(c) and AS 3(a)

- The gradient and inclination of a straight line.
- The equation of a straight line.

Overview

In this lesson you will:

- Use a formula to find the equation of a straight line
- Find equations of parallel and perpendicular lines
- Find the co-ordinates of the point of intersection of two lines.

Lesson

Equations of straight lines (used to find the equations of lines)

Here we have four cases:

1. If the gradient m_{AB} and the **y-intercept** c of the line are given:
 \Rightarrow We use $y = m_{AB}x + c$ (**the gradient/y-int formula**)
2. If the gradient m_{AB} and another point $C(x_c; y_c)$ on the line are given:
 \Rightarrow We use $(y - y_c) = m_{AB}(x - x_c)$ (**the point gradient formula**)
3. If **two points** $A(x_A; y_A)$ and $B(x_B; y_B)$ on the line are given:
 \Rightarrow We use $\frac{y - y_B}{x - x_B} = \frac{y_A - y_B}{x_A - x_B}$ (**the two point formula**)
4. If the **x-intercept** and **y-intercept** are given:
 \Rightarrow We use $\frac{x}{x - \text{int}} + \frac{y}{y - \text{int}} = 1$ (**the dual intercept formula**)

Examples

1. Find the equation of the line passing through $A(-1; -4)$ with a gradient of -2 .

Solution

Here we have the gradient and a point, so we use our point gradient formula:

Now simply substitute

$$y + 4 = -2(x + 1)$$

$$y + 4 = -2x - 2$$

$$\frac{y - y_A}{x - x_A} = m$$

$$\therefore y + 4 = -2(x + 1)$$

$$y = -2x - 6$$

2. Find the equation of the line AB if A(6; -2) and B(-3; 1)

Solution



Solution

Here we have two points:

$$\begin{aligned}\text{So } \frac{y - y_A}{x - x_A} &= \frac{y_B - y_A}{x_B - x_A} \\ \therefore \frac{y + 2}{x - 6} &= \frac{-2 - 1}{6 - (-3)} = -\frac{1}{3} \\ \therefore 3(y + 2) &= -(x - 6) \\ \therefore 3y + 6 &= -x + 6 \\ \therefore 3y &= -x\end{aligned}$$

3. Find the equation of the line passing through A(2; -4) parallel to the line passing through B(1; 1) and C(-3; -2)

Solution



Solution

$$m_{BC} = \frac{-2 - 1}{-3 - 1} = \frac{3}{4}$$

Now we have a gradient and a point, so

$$\begin{aligned}\frac{y - y_A}{x - x_A} &= m_{BC} \\ y - (-4) &= \frac{3}{4}(x - 2) \\ y + 4 &= \frac{3}{4}(x - 2) \\ 4y + 16 &= 3x - 6 \\ 4y &= 3x - 22\end{aligned}$$

Remember: if we write an equation in the form $y = mx + c$ then m is the gradient.

4. Find the equation of the line parallel to $2y + 4x = 5$ passing through the point P(-1; -6)

Solution



Solution

We need a gradient and a point P(-1; -6)

$$\begin{aligned}\text{Gradient } 2y &= -4x + 5 \\ y &= -2x + \frac{5}{2} \\ m &= -2\end{aligned}$$

We now have a gradient, and a point, so

$$\frac{y - y_P}{x - x_P} = m$$

$$\begin{aligned}\text{Now substitute: } y + 6 &= -2(x + 1) \\ y + 6 &= -2x - 2 \\ y &= -2x - 8\end{aligned}$$

Remember: If $AB \perp DE$ then $m_{AB} \times m_{DE} = -1$

5. Find the equations of the line perpendicular to $4y + x = -8$ through the point Q(-1; -3)



**Solution**

We need the gradient

$$4y = -x - 8$$

$$y = -\frac{1}{4}x - 2$$

$$m = -\frac{1}{4} \quad \text{So the gradient we want is } 4$$

$$m = 4 \text{ and } Q(-1; -3)$$

$$\text{So } \frac{y - y_Q}{x - x_Q} = m$$

$$\text{Equation } y + 3 = 4(x + 1)$$

$$y + 3 = 4x + 4$$

$$y = 4x + 1$$

6. Find the equation of the line perpendicular to the line joining $A(-3; -4)$ and $B(2; 6)$ and passing through $C(-5; 1)$

Solution

$$\text{Gradient } AB = \frac{10}{5} = 2$$

Our new line is perpendicular to AB so $m = -\frac{1}{2}$

$$\text{So } \frac{y - y_C}{x - x_C} = m$$

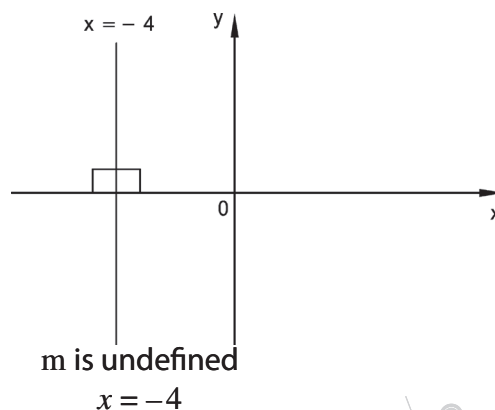
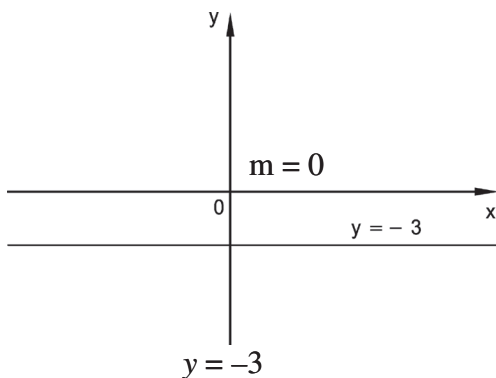
$$C(-5; 1)$$

$$\text{Equation } y - 1 = -\frac{1}{2}(x + 5)$$

$$2y - 2 = -(x + 5)$$

$$2y - 2 = -x - 5$$

$$2y = -x - 3$$

**REMEMBER THOSE IMPORTANT LINES**

7. (a) Find the equation of the line parallel to the x -axis through the point $(4; 2)$

$$\text{Solution } m = 0 \quad \text{so } y = 2$$

- (b) Find the equation of the line through the points $(3; 1)$ and $(3; -4)$

$$\text{Solution } m = -\frac{5}{0} \text{ } m \text{ is undefined so } x = 3$$



1. The points $A(3; 1)$, $B(1; -2)$ and $C(2; 3)$ are points in the Cartesian plane.
 - (a) If $CD \parallel AB$ with $D(-\frac{1}{3}; t + 1)$ determine the values of t .
 - (b) If B , A and $E(r; 4)$ are collinear, determine the value of r .
2. The equation of the line PQ is $(3 - 2k)x = (k + 1)y = 12$
Calculate the numerical value of k in each case, if line PQ
 - a) is parallel to the line $y = 4x + 7$
 - b) is perpendicular to the line through points $(1; 3)$ and $(-2; 4)$
 - c) passes through the point $(-3; 4)$
 - d) is parallel to the x -axis.
 - e) is parallel to the y -axis.
3. Find the equation of the straight lines
 - a) Parallel to the line $y + 8 = 0$ and 6 units from $(2; 1)$.
 - b) Perpendicular to the line $y - 2 = 0$ and 4 units from $(-1; 7)$