ANALYTICAL GEOMETRY (2)

Learning Outcomes and Assessment Standards

Learning Outcome 3: Space, shape and measurement
Assessment Standard AS 3(c) and AS 3(a)

- The gradient and inclination of a straight line.
- The equation of a straight line.

Overview

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In this lesson you will:

- Discover what is meant by inclination
- Use trigonometry to find the inclination of a straight line
- Find the angle between two straight lines
- Use analytical methods to find the three angles of a triangle.

Lesson

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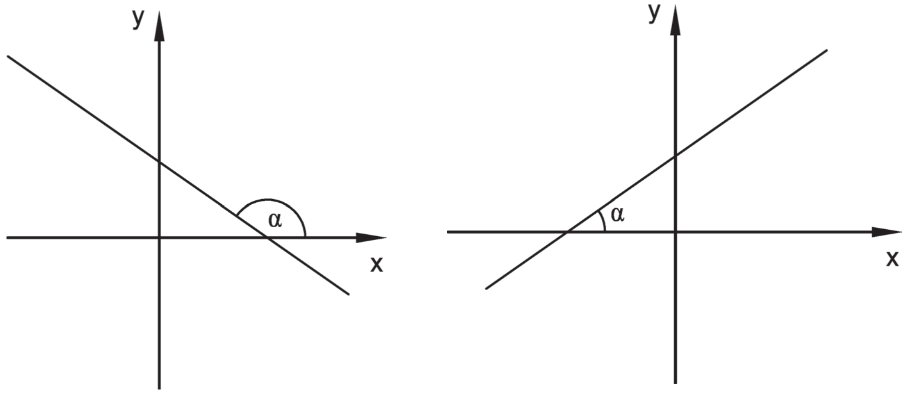
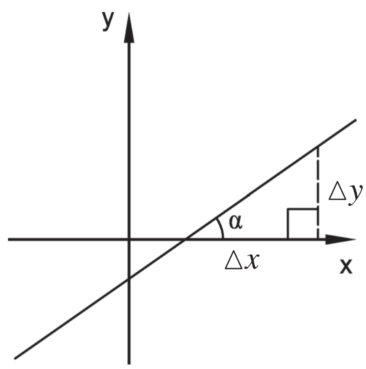
The inclination of a straight line

Definition: The angle formed by a straight line and the positive direction of the horizontal.

4 OPTIONS



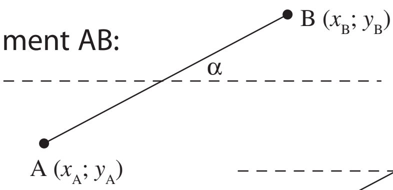
To find this angle we need the concept of a gradient and link it directly to trigonometry.



$m = \frac{\text{change in } \triangle y}{\text{change in } \triangle x}$

and $\tan \alpha = \frac{y}{x}$

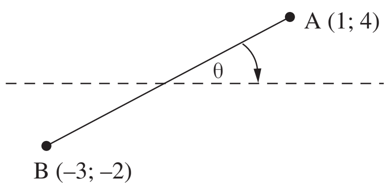
∴ For any line segment AB:



$\tan \alpha = m_{AB}$

$\therefore \tan \alpha = \frac{y_B - y_A}{x_B - x_A}$

So $\alpha = \tan^{-1}(m_{AB})$



LIBERTY LIFE

Examples

1. We introduce any horizontal line through AB:

Then $\tan\theta = m_{AB}$

$= \frac{4+2}{1+3}$

$= \frac{6}{4}$

$= \frac{3}{2}$

$\therefore \theta = \tan^{-1}\left(\frac{3}{2}\right)$

$= 56{,}3°$

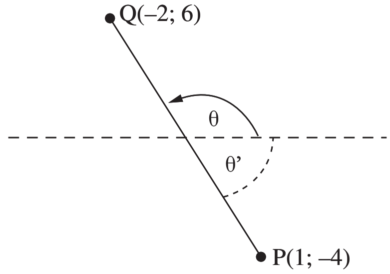
2. $\tan\theta = m_{PQ}$

$= \frac{6+4}{-2-1}$

$= -\frac{10}{3}$

$\therefore \theta = \tan^{-1}\left(-\frac{10}{3}\right)$

$= -73{,}3°$



Notice that when $\tan\theta < 0$, we are working with the negative angle θ. So to get to θ, we need to add a period of tan, which is 180°.

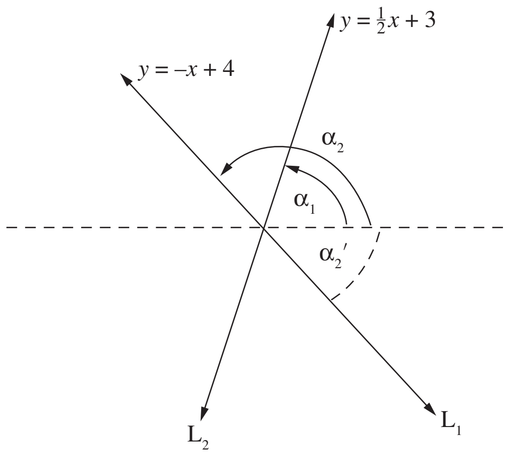
So $\theta = \theta' + 180° = -73{,}3° + 180° = 106{,}7°$.

3. Find the size of the angles between the following lines

a. $y = -x + 4$ and $y = \frac{1}{2}x + 3$

b. $y = -3x - 4$ and $y = -x$

c. $y = 8x$ and $y = 2x + 3$



a. For L_1: $\tan\alpha_2 < 0$

So $\alpha_2 = \tan^{-1}(-1) + 180°$

$= -45° + 180°$

$= 135°$

$\alpha_2' = 45°$

For L_2: $\alpha_1 \tan^{-1}\left(\frac{1}{2}\right) + 26{,}6°$

The acute angle between the lines will be: $45° + 26{,}6° = 71{,}6°$

The obtuse angle between them is: $180° - 71{,}6° = 108{,}4°$

b. For L_1: $m = -1$

$\theta_1 = \tan^{-1}(-1) + 180°$

$= -45° + 180°$

$\therefore \theta_1 = 135°$

and $\theta_1' = 45°$

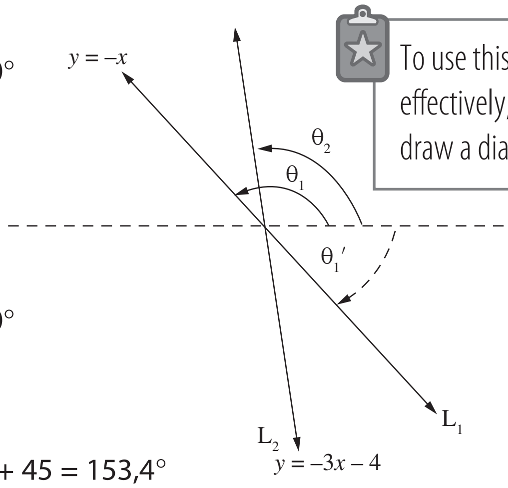
For L_2: $m = -3$

$\theta_2 = \tan^{-1}(-3) + 180°$

$= -71{,}6° + 180°$

$\theta_2 = 108{,}4°$

Thus: obtuse angle: $108{,}4° + 45 = 153{,}4°$



To use this concept effectively, always draw a diagram.

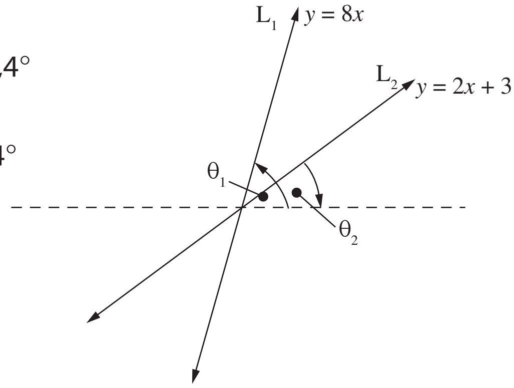
Acute angle: 26,6°

c. L_1: m = 8: – θ_1 = $\tan^{-1}(8)$ = 82,8°

L_2: m = 2: – θ_2 = $\tan^{-1}(2)$ = 63,4°

∴ ∠ between lines (acute) = 82,8 – 63,4°
= 19,4°

The obtuse angle will be = 180° – 19,4°
= 160,6°



4. A(–1; 5) B(2; 3) and C(–6; –1) are co-ordinates of the vertices of △ABC

Find the size of the angles.

Draw a picture

Tip: Find α, β and θ the use geometry.

$m_{AC} = \frac{6}{5}$	$m_{BC} = \frac{4}{8}$	$m_{AB} = \frac{2}{-3}$
$\tan\alpha = \frac{6}{5}$	$\tan\beta = \frac{1}{2}$	$\tan\theta = -\frac{2}{3}$
$\alpha = 50{,}2°$	$\beta = 26{,}6°$	$\theta = 146{,}3°$

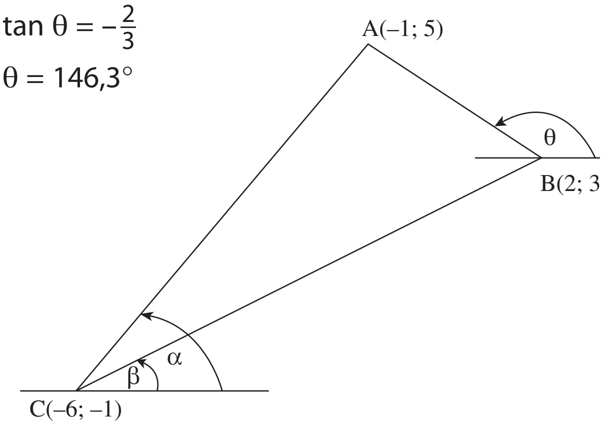
By geometry $\hat{C} = \alpha - \beta$

$\hat{C} = 23{,}6°$

$\hat{A} = \theta - \alpha$

$\hat{A} = 96{,}1°$

$\hat{B}_1 = 60{,}3$ (<'s in △)



Let's find the inclination of each side

First:

AC: $m = \frac{5+1}{-1+6} = \frac{6}{5}$

∴ $\theta = \tan^{-1}\left(\frac{6}{5}\right) = 50{,}2°$

AB: $m = \frac{5-3}{-1-2} = -\frac{2}{3}$

∴ $\beta = \tan^{-1}\left(-\frac{2}{3}\right) + 180°$

$= 146{,}3°$

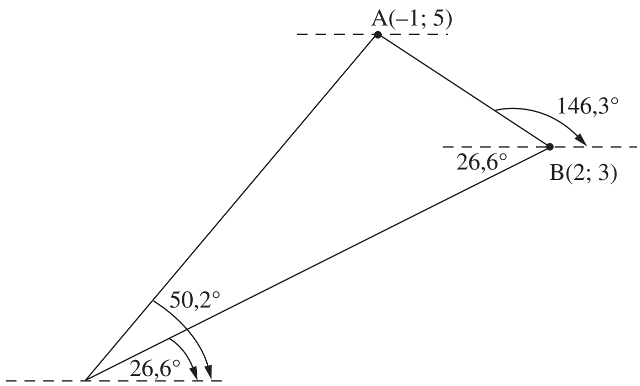
BC: $m = \frac{3+1}{2+6} = \frac{1}{2}$

$\alpha = \tan^{-1}\left(-\frac{1}{2}\right)$

$= 26{,}6°$

$\hat{C} = \theta - \alpha = 50{,}2°\ 26{,}6°$

$\hat{C} = 23{,}6°$



$\hat{B} = (180° - 146{,}3°) + 26{,}6° = 60{,}3°$

$\hat{A} = 180° - 23{,}6° - 60{,}3°$ Sum angles in triangle

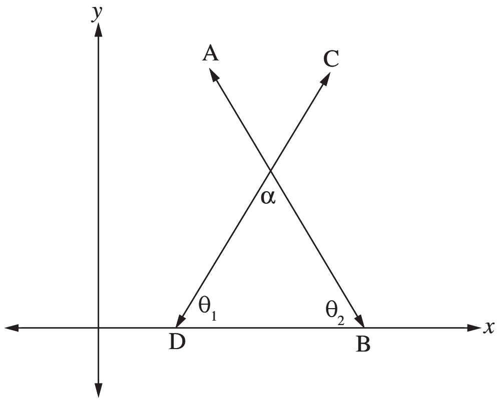
$= 96{,}1°$

The angle between two lines AB and BC

$\alpha = \theta_2 - \theta_1$

The acute angle between AB and CD:

$\tan \ = \dfrac{m_{AB} - m_{CD}}{1 + m_{AB} \cdot m_{CD}}$



Applying this to:

$m_{AC} = \frac{6}{5}$

$m_{AB} = -\frac{2}{3}$

$m_{BC} = \frac{1}{2}$

$\tan \hat{A} = \dfrac{m_{AB} - m_{AC}}{1 + m_{AB} \cdot m_{AC}}$

$= \dfrac{-\frac{2}{3} - \frac{6}{5}}{1 + \left(-\frac{2}{3}\right)\left(\frac{6}{5}\right)}$

$= -\frac{28}{3}$

$\therefore \hat{A} = \tan^{-1}\left(-\frac{28}{3}\right) + 180°$

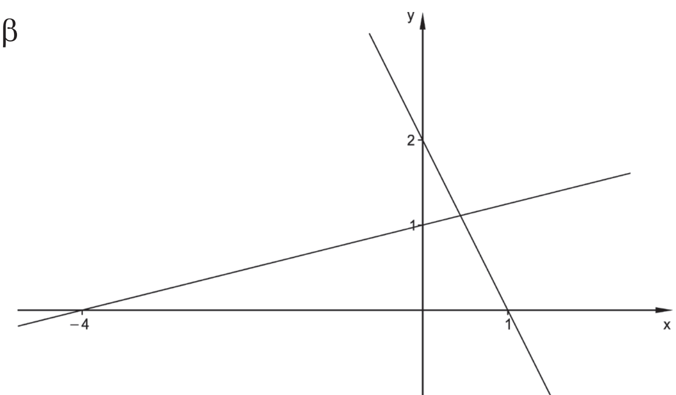
$= 96{,}1°$

Activity

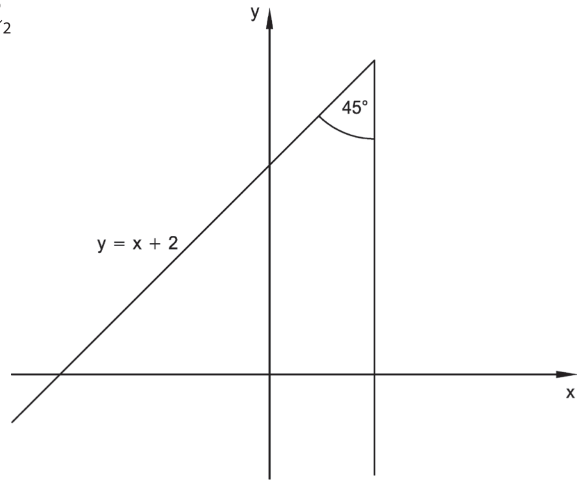
1. Calculate the inclination of AB if
   - a) A(–3; 2) and B(–5; 0)
   - b) A($\sqrt{3}$; 1) and B($2\sqrt{3}$; –2)
   - c) A(–1; 2) and B(1; –1)
   - d) A(3; 4) and B(3; –6)
   - e) A(1; –1) and B(6; –1)
2. Calculate the inclination of the following straight lines
   - a) $3y + x = 4$
   - b) $2y - x = 3$
   - c) $4x + y = 1$
   - d) $x = \frac{1}{2}y$
3. A(1; 2), B(–3; –2) and C(3; 6). Calculate $B\hat{A}C$
4. $y = -\frac{1}{2}x$ and $y = 3x + 3$

   Intersect at one place.

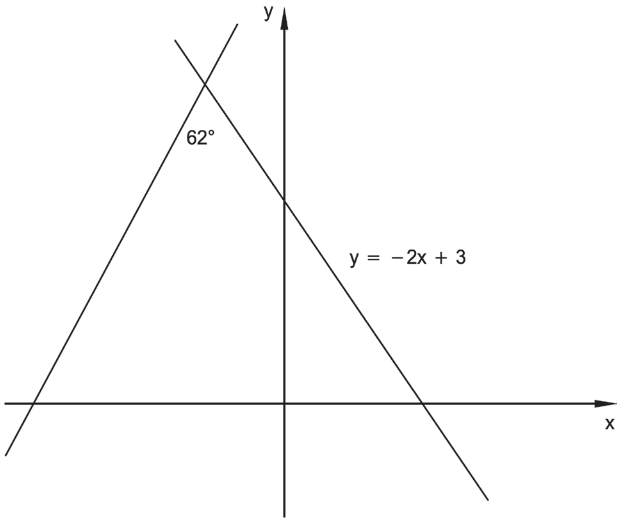
   Calculate the possible sizes of the angles at the point of intersection.
5. Calculate α and β



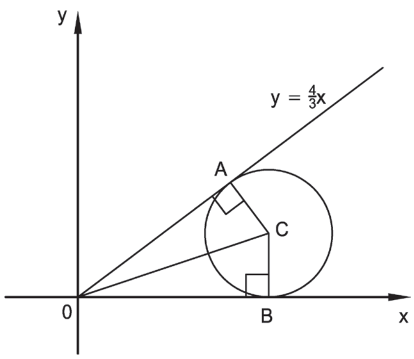
6. $\hat{A} = 45°$ Find the gradient of ℓ_2



7. Find the gradient of AB



8. Calculate the gradient of OC



9. P(−1; 4) Q(2; 2) and
R(−6; −1) are co-ordinates of △PQR, find the angles of the triangle.