LESSON 26

# ANALYTICAL GEOMETRY (1)

**Learning Outcomes and Assessment Standards**

**Learning Outcome 3: Space, shape and measurement**
**Assessment Standard AS 3(c) and AS 3(a)**

- The gradient and inclination of a straight line.
- The equation of a straight line.

Overview

## Overview

In this lesson you will:

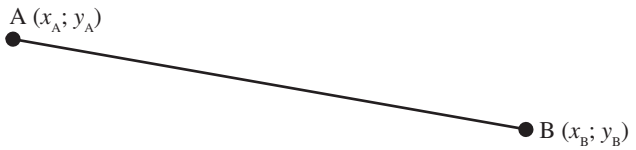
- Review the distance formula, the mid-point and the gradient covered in Grade 10 in order to cope with the progression needed for this section of the curriculum
- Use analytical geometry and properties of quadrilaterals to solve various problems
- Use correct formulae, interpret questions and make the necessary equations.

Lesson

*Lesson*

## The distance formula:

To find the distance between two points A and B, we use the formula:




$$AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$$

Example

**Examples**

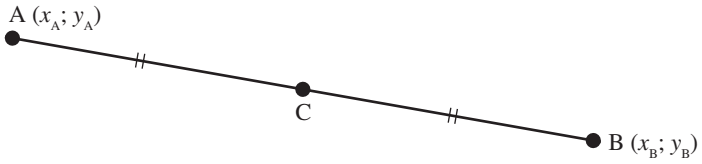
1.The distance between $A(-7; y)$ and $B(-3; 4)$ is $4\sqrt{5}$, find $y$

$$AB = 4\sqrt{5} \quad \text{(given)}$$
$$\therefore AB^2 = (4\sqrt{5})^2$$
$$= 16 \cdot 5$$
$$= 80$$
$$\therefore (x_A - x_B)^2 + (y_A - y_B)^2 = 80$$
$$\therefore (-7 + 3)^2 + (y - 4)^2 = 80$$
$$\therefore (y - 4)^2 = 80 - 16$$
$$= 64$$
$$\therefore y - 4 = 8$$
$$\therefore y = 4 \pm 8$$
$$y = 12 \quad \text{or} \quad y = -4$$

## The mid-point of a line

To determine the co-ordinates of the midpoint of a line segment AB:

$$x_C = \frac{x_A + x_B}{2} \qquad y_C = \frac{y_A + y_B}{2}$$

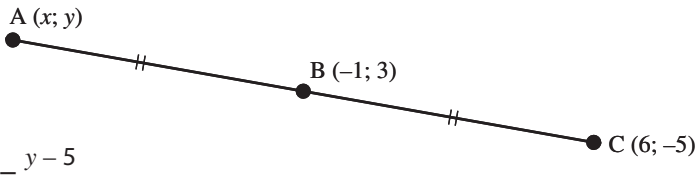



**Example**

B(–1; 3) is the mid-point of AC. Find the co-ordinates of A if

A$(x; y)$ and C(6; -5)

Draw a picture




Form equations

$$-1 = \frac{x+6}{2} \quad \text{and} \quad 3 = \frac{y-5}{2}$$

$$-2 = x + 6 \qquad 6 = y - 5$$

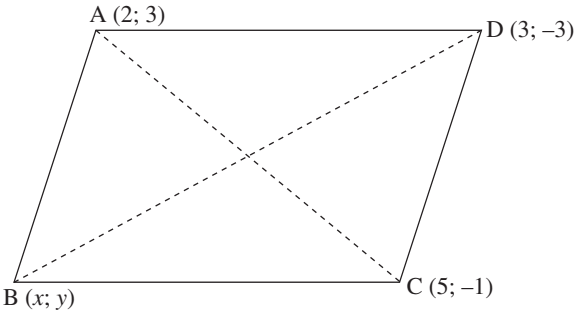
$$x = -8 \qquad y = 11$$

A(–8; 11)

**Example**

A(2; 3), C(5; –1), B$(x; y)$ and D(3; –3) are the co-ordinates of the parallelogram ACBD, find the co-ordinates of B.

**Gradient**




Since ABCD is a parallelogram, we can use the properties of parallelogram to help us find the co-ordinates of point B.

**Using the fact that the diagonals bisect one another:**

Midpoint of AC:

$$\left.\begin{aligned} x_E &= \frac{x_A + x_C}{2} = \frac{5+2}{2} = \frac{7}{2} \\ y_E &= \frac{y_A + y_C}{2} = \frac{3-1}{2} = 1 \end{aligned}\right\} \quad E\left(\frac{7}{2}; 1\right)$$

Now midpoint BD:

$$x_E = \frac{x_B + x_D}{2} \text{ and} \qquad y_E = \frac{y_B + y_D}{2}$$

$$\frac{7}{2} = \frac{x+3}{2} \qquad 1 = \frac{y-3}{2}$$

$$\therefore x = 7 - 3 \qquad \therefore y - 3 = 2$$

$$x = 4 \qquad y = 5$$

$$\therefore E = (4; 5)$$

**Using the fact that the gradients of sides are equal:**

$m_{AB} = m_{DC}$

$\therefore \triangle y_{AB} = \triangle y_{DC}$ and $\triangle x_{AB} = \triangle x_{DC}$

$\therefore y - 3 = -1 - (-3)$ $\therefore x - 2 = 5 - 3$

$\therefore y - 3 = 2)$ $\therefore x = 2 + 2$

$y = 5$ $x = 4$

$\therefore E = (4; 5)$

Example

**We can use BC = AD, and the equation of BC, to find the co-ordinates of B.**

We could also use $m_{BC} = m_{AD}$ in the same way as above.

**The gradient of a line segment**

To determine the gradient (slope) of a line PQ, we use

$$m_{PQ} = \frac{\triangle y}{\triangle x} = \frac{y_Q - y_P}{x_Q - x_P}$$


If two lines are parallel, then they have equal gradients

Example

If two lines are perpendicular, then the product of their gradients is –1

If a line is horizontal, then $\triangle y = 0$ : So $m = 0$

If a line is vertical, then $\triangle x = 0$ : So $m = \alpha$

3 points A, B and C are said to be collinear if $m_{AB} = m_{BC}$

Example

**Examples**

If A(1; 4), B(–3; 2), C(–1; –1) and D($x$; 0) are four points in the Cartesian plane, find the value of $x$ if:

(a) AB $\parallel$ CD

(b) AB $\perp$ BD

(c) BC and D are collinear.

1. a) AB $\parallel$ CD Make the equation

$m_{AB} = m_{CD}$

$\frac{2-4}{-3-1} = \frac{0+1}{x+1}$

$\frac{-2}{-4} = \frac{1}{x+1}$

$\frac{1}{2} = \frac{1}{x+1}$ cross multiply

$x + 1 = 2$

$x = 1$

b) AB $\perp$ BD

$m_{AB} \times m_{BD} = -1$

$\therefore \frac{1}{2} \times \frac{2-0}{-3-x} = -1$

$\frac{1}{2} \times \frac{2}{-3-x} = -1$

$\therefore \frac{1}{x+3} = 1$

$\therefore 1 = 3 + x$

$x = -2$

c) B; C and D are collinear

$m_{BC} = m_{CD}$

$\frac{2+1}{-3+1} = \frac{0+1}{x+1}$

$-\frac{3}{2} = \frac{1}{x+1}$

$3(x + 1) = -2$

$3x + 3 = -2$

$3x = -5$

$x = \frac{-5}{3}$

## Activity

*Activity 1*

1. In each case determine the value of $x$ and $y$.
   a) The distance of $(x; -3)$ from the origin is 5 units.
   b) $(-1; -1)$ is equidistant from $(0; 2)$ and $(x; -2)$.
   c) $(x; y)$ is the mid-point of the line segment joining $(-1; 3)$ and $(7; 1)$.
   d) (1;-2) is the centre of the circle passing through (5; 1) and $(-2; y)$.
2. If A(3; 1), B–5; 7), C(11; –5) and D$(x; y)$ are the co-ordinates of parallelogram ABCD, find the co-ordinates of D.
3. A(–2; 3), B(1; 4), C(– 4; 1) and D$(x; 4)$ are points in the Cartesian plane.
   In each case find the value of $x$.
   a) AB $\parallel$ CD
   b) AB $\perp$ CD
   c) B; C and D are collinear
4. Show that the points A(–5;–6) B(–2; 0), and C(–1; 2) are collinear.
5. Prove that P(6; 2), Q(3; 5), R(–3;–1) and S(0; – 4) are the vertices of a rectangle.
6. Prove that A(–3; –1), B(–2; 2), C(1; 3) and D(0; 0) are the vertices of a rhombus.
7. Show that A(6; – 4) B(5; 3,) C(–2; 2) and D(–1; –5) are the vertices of a square.
8. If A(–5; 3), B(4; 0), C(6; 6) and D are the co-ordinates of parallelogram ABCD.
   a) Find the co-ordinates of D.
   b) Show that ABCD is a rectangle.