SOLUTIONS OF TRIANGLES

Learning Outcomes and Assessment Standards

Learning Outcome 3: Shape, space and measurement
Assessment Standard
Solve problems in two dimensions by using the sine, cosine and area rules, and by constructing and interpreting geometric and trigonometric models.

Overview

In this lesson you will:

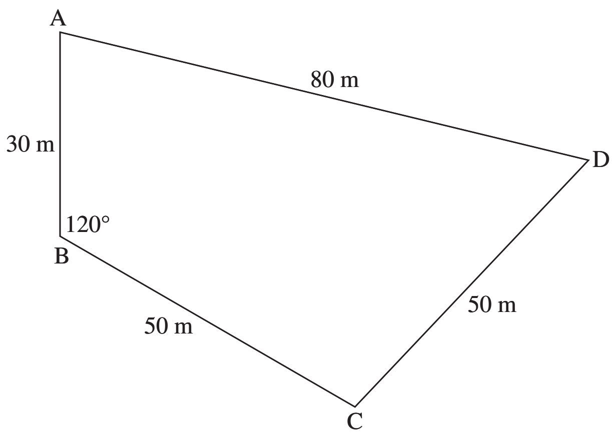
- Look at geometric and trigonometric models in two dimensions and solve real life problems.
- Be required to use your calculator correctly.
- Need to apply the sine, cosine and the area rule

Lesson

Example 1

Look at the diagram.

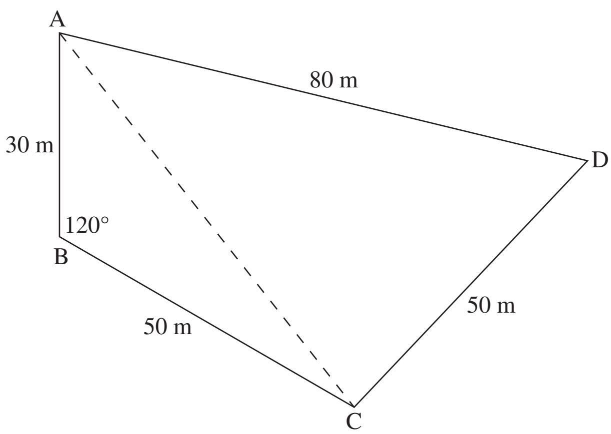
A, B, C and D are 4 beacons on a farm. Find the area of the land enclosed by these four beacons if $\hat{B} = 120°$.



Solution

To find the area of the quadrilateral ABCD, it will be wise to divide it into two triangles. That way we can use our method to determine the area of a non-right angled triangle.

So we join A and C. (If we join B and D, we will split up the angle B, and we do not know how this split happens). So now:



As we have two sides and the included angle, we can find the area of ΔABC.

So: Area $\Delta ABC = \frac{1}{2}$ AB.BC sin B

$= 649{,}5\ m^2$.

We do not have enough information in ΔADC, so we need to find AC first and use the cosine rule for this (since we have 3 sides), and find $\hat{D}$. After that, we can find the area.

So in ΔABC: $AC^2 = AB^2 + BC^2 - 2AB.BC.\cos \hat{B}$

$\therefore AC^2 = 30^2 + 50^2 - 2(30)(50) \cos 120°$

$= 4\,900$

$\therefore AC = 70$

In ΔADC: $AC^2 = AD^2 + DC^2 - 2AD.DC.\cos \hat{D}$

$4\,900 = 6\,400 + 2\,500 - 2(80)(50)\cos \hat{D}$

$\therefore \cos \hat{D} = \frac{1}{2}$

$\therefore \hat{D} = 60°$

$\therefore$ Area $\Delta ADC = \frac{1}{2} AD.DC.\sin \hat{D}$

$= \frac{1}{2} 80.50.\sin 60°$

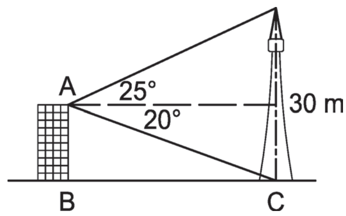
$= 1\,732 \text{ m}^2$

$\therefore$ Area of quadrilateral ABCD $= 649{,}5 + 1\,732 = 2\,381{,}6 \text{ m}^2$

Example 2

Look at this diagram

From a point A on top of a building, the angle of elevation to the top of a tower is 25° and the angle of depression to the foot of the tower is 20°. If the height of the tower is 30m, how far is the building from the tower, if they lie in the same horizontal plane.



Solution

Let's put in the information.

We need to find BC.

First we must find AC.

In ΔADC

$\frac{AC}{\sin 65°} = \frac{30}{\sin 45°}$

$AC = \frac{30 \sin 65°}{\sin 45°}$

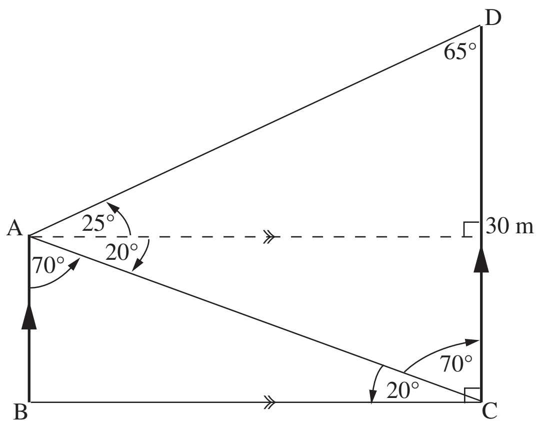
$AC = 38{,}5\text{m}$

Now in $\triangle ABC$

$\hat{B} = 90°$:

So $\cos 20° = \frac{BC}{AC}$

$\therefore BC = \cos 20 \times 38{,}5 \text{ m} = 36{,}1 \text{ m}$



Activities 1–5

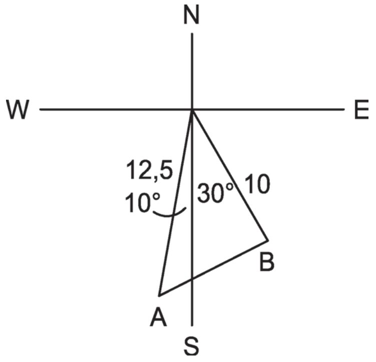
Example 3

Now you need to draw your own diagram

Two pleasure cruisers leave the same pier simultaneously, one travelling at 20 km/h in a direction S30°E and the other at 25 km/h in a direction S10°W.

How far apart will they be after 30 minutes?

Solution



Start south and move 30° East.

Then start south and move

West 10°

Distance = speed × time

1st Cruiser $= 20 \times \frac{1}{2}$ $= 10$ km

2nd Cruiser $= 25 \times \frac{1}{2}$ $= 12{,}5$ km

We need to find AB

$AB^2 = (10)^2 + (12{,}5)^2 - 2(10)(12{,}5)\cos 40°$

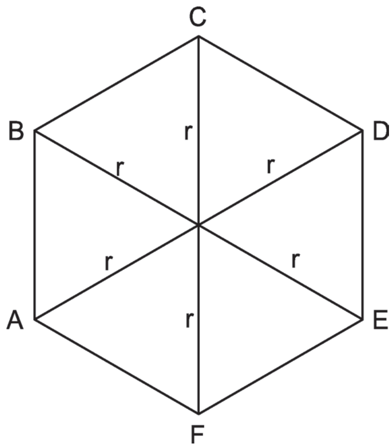
$AB = 8$ km

Example 4 **(**Formula's and problem solving.)

Look at this regular hexagon.

r is the radius of the circumscribed circle.

Find a formula for the area of the hexagon.



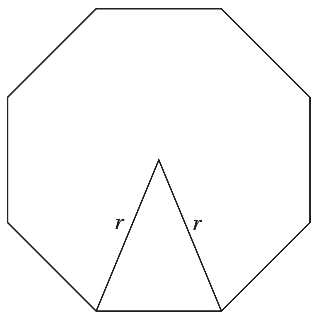
Solution

The central angle is $\frac{360°}{6} = 60°$

$\therefore$ area $= 6 \times \frac{1}{2} r \cdot r \sin 60°$

$= \frac{3r^2 \cdot \sqrt{3}}{2}$

Find the area of a regular octagon with radius of the circumscribed circle being *r*.



centre angle: $\frac{360°}{8} = 45°$

There are 8 iscoceles triangles in this regular octagon.

So area $= 8 \times \frac{1}{2} r.r. \sin 45°$

$= 4r^2 \frac{1}{\sqrt{2}}$

$= \frac{4r^2}{\sqrt{2}}$

Hence, find a formula for the area of a regular polygon with n sides

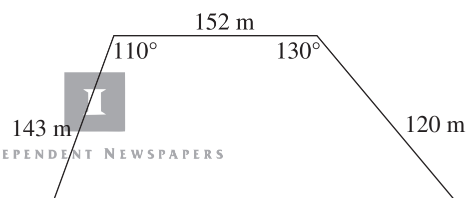
Solution

The central angle is $\frac{360°}{n}$

Area $= n \times \frac{1}{2} r^2 \cdot \sin \frac{360°}{n}$

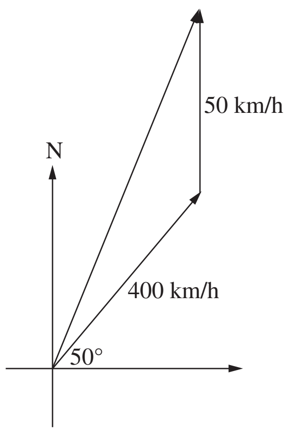
Activity 1

1.



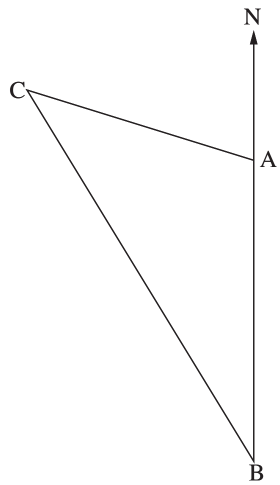
a) Find $\hat{A}$.

b) Determine the length of AB

2. 

A plane travelling at 400 km/h is flying with a bearing of 40°. There is a wind of 50 km/h from the South. If no correction is made for the wind, what are the final bearing and ground speed of the plane?

3. A reconnaissance plane leaves an aircraft carrier at A and flies due south at a steady speed of 500 km/h. The carrier meanwhile proceeds at 30 km/h on a coarse N 60° W (or a bearing of 300°). The plane has fuel for only four hours flying and the intention is to use all the fuel. This means that after flying to a certain point B, it must turn and fly in the direction BC to intercept the carrier at C before its fuel is exhausted.



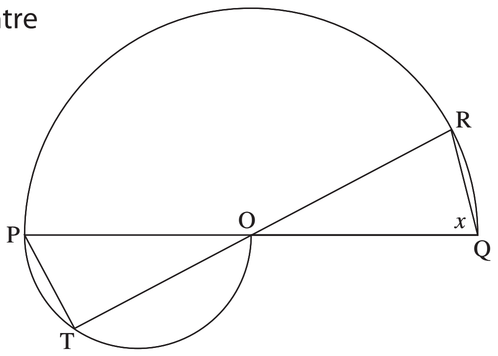
3.1 What is the distance of AC

3.2 What total distance does the plane fly

3.3 Use the rule of cosines to write down an equation in x and hence find, to the nearest kilometre, the maximum distance South that the plane can fly before turning to rejoin the ship.

3.4 Calculate the angle B and hence give the direction BC in which the plane must fly in order to meet the carrier at C.

4. In the figure alongside, O is the centre of the semi-circle PRQ with radius r. PO is a diameter of the semicircle PTO. Angle Q = x.



Determine:

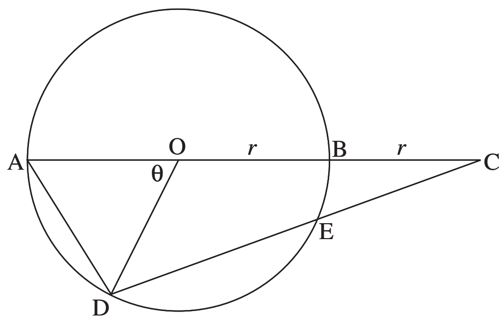
4.1 RQ in terms of r and x by using the sine rule, and simplify the expression if $\sin 2x = 2 \sin x \cos x$

4.2 The area of ΔROQ in terms of r and x

4.3 The area of ΔPOT in terms of r and x

4.4 The ratio of $\frac{\text{Area } \Delta ROQ}{\text{Area } \Delta POT}$ if $x = 75°$, leaving your answer in surd form.

5. In the figure, AB is a diameter of circle O with radius equal to r. AB is produced to C such that BC = r. CED is a secant such that CE = ED = DA and angle AOD = θ.



5.1 Prove that: $\cos\theta = \frac{1}{4}$

5.2 Hence prove that the area of ΔAOD = $\frac{r^2\sqrt{15}}{8}$

6. A tower PQ stands on top of a hill QR. At a point A in the same horizontal as R, the angle of elevation of P, the top of the tower is 2θ and the angle of the elevation of Q, the base of the tower, is θ. The height of the tower is a.

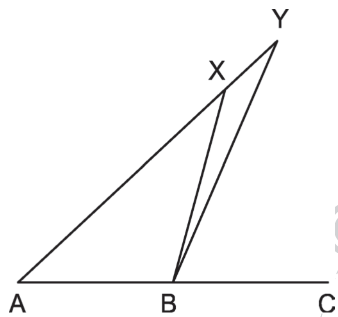
6.1 Show that AP = $\frac{a}{\tan\theta}$.

6.2 Hence show that the height of P above the horizontal plane, is $2a \cos^2\theta$ if $\sin 2\theta = 2 \sin\theta = 2 \sin\theta \cos\theta$.

6.3 If θ = 30°, calculate PQ/PR.

7. A shadow, cast on a lake by a stationary balloon, moves by 12 m as the angle of elevation of the sun increases form 62° to 70°. Determine the height of the balloon.

8. A man walking down a straight road ABC notice two trees X and Y. he also notice that A, X and Y lie in a straight line and $X\hat{A}C = 43°$. He walks 800 m to B, and observes that $X\hat{B}C = 74°$ and $Y\hat{B}C = 66°$.



Calculate the distance between the two trees.