DIFFERENT SEQUENCES

Learning Outcomes and Assessment Standards

Learning Outcome 1: Number and number relationships
Assessment Standards AS 3
Investigate number patterns (including but not limited to those where there is a constant second difference between consecutive terms in a number pattern, and the general term is therefore quadratic) and hence:
a) Make conjectures and generalisations
b) Provide explanations and justifications and attempt to prove conjectures.

Overview

Overview

In this lesson you will:

- Look at unusual sequences where there is no first or second difference
- If possible find the nth term
- Prove conjectures.

Lesson

Lesson

Example

Example 1

What will the next three terms be in the sequence 2; 6; 18; 54; …

	T_1	T_2	T_3	T_4		
	2 ;	6 ;	18 ;	54		
	$\frac{1}{3} \times 6$;	1×6 ;	3×6 ;	9×6		
=	$3^{-1} \times 6$;	$3^0 \times 6$;	$3^1 \times 6$;	$3^2 \times 6$;	$3^3 \times 6$;	$3^4 \times 6$

So the next 3 terms will be: 162; 486; 1458

and $T_n = 3^{n-2} \times 6 = 3^n.3^{-2}.3.2 = 2.3^{n-1}$

Alternatively:

	T_1	T_2	T_3	T_4		
	2 ;	6 ;	18 ;	54		
=	2×1 ;	2×3 ;	2×9 ;	2×27		
=	2×3^0 ;	2×3^1 ;	2×3^2 ;	2×3^3 ;	2×3^4 ;	2×3^5 ;

…

So the next 3 terms will be: $2 \times 3^4 = 162$
$2 \times 3^5 = 486$
$2 \times 3^6 = 1458$

Example

Example 2

Again they all are multiples of 6:

	T_1	T_2	T_3			
	24 ;	12 ;	6			
=	4×6 ;	2×6 ;	1×6 ;	$\frac{1}{2} \times 6$;	$\frac{1}{4} \times 6$;	$\frac{1}{8} \times 6$
=	$2^2 \times 6$;	$2^1 \times 6$;	$2^0 \times 6$;	$2^{-1} \times 6$;	$2^{-2} \times 6$;	$2^{-3} \times 6$

$= \quad 2^3 \times 6\ ; \quad 2^2 \times 3\ ; \quad 2^1 \times 3\ ; \quad 2^0 \times 3\ ; \quad 2^{-1} \times 6; \quad 2^{-2} \times 3$

Next 3 terms will be: $3; \frac{3}{2}; \frac{3}{4}$

$T_n = 2^{4-n}.3$

Alternatively

24; 12; 6; …

$24; \frac{24}{2}; \frac{24}{4}; \frac{24}{8}; \frac{14}{16}; \frac{24}{32}; \ldots$

$Tn = 24. \frac{1}{2}^{n-1}$

Both these general terms are the same:

$T_n = 2^{4-n}.3$

and $T_n = 24. (\frac{1}{2})^{n-1}$

$= 8.3.2^{1-n}$

$= 2^3.3.2^{1-n}$

$= 3.2^{4-n}$

Example

Example 3 (Problem solving)

A rubber ball is dropped from a height of 30m. After each bounce, it returns to a height that is 4/5 of the previous height.

a) express the first three heights as a sequence

Solution

Solution

$30\left(\frac{4}{5}\right); 30\left(\frac{4}{5}\right)^2, 30\left(\frac{4}{5}\right)^3$

b) how high will the ball be after 21 bounces?

Solution

Solution

$T_{21} = 30\left(\frac{4}{5}\right)^{21}$

Example

Example 4 (The Fibonacci sequence)

Write down the next three terms in the sequence 1; 1; 2; 3; 5; 8;

Solution

Solution

13; 21; 34

These are fascinating numbers because they appear all over in our world.

Activity

Activity 1

1. Find the nth term of the following sequences.

   a) $4; -2, 1; -\frac{1}{2}; \frac{1}{4}; \ldots$ b) $-\frac{1}{8}; -\frac{1}{2}; -2; -8; \ldots$

   c) 32; 16; 8; 4; 2; … d) $3a; 6a^2; 9a^3; 18a^4 \ldots$

2. a) Find T_n b) Find the 8th term 9; 3;1 …

3. Which term of the sequence

   $1; \frac{3}{2}; \frac{9}{4} \ldots$ is equal to $\frac{243}{32}$? (Hint; Find T_n first)

4. A tree grows 120 cm during the first year. Each year it grows $\frac{9}{10}$ of the previous year's growth. How much in the 58th year?

Activity

Activity 2

1. a) Show why the 5th triangular number is $T_5 = \frac{1}{2} \times 5 \times 6$

   b) Find an expression in n for the nth triangular number.

2. a) Use the diagrams of the 3rd and 4th triangular numbers, T_3 and T_4 to show that their sum that their sum is the 4th square number, i.e. $S_4 = T_3 + T_4$

   b) Generalise with a formula the connection between triangular and square numbers.

3. a) Write down a table of square numbers from the 1st to the tenth.

   b) Find two square numbers which add to give a square number.

   c) Repeat part (b) for at least three other pairs of square numbers.

4. Without using a calculator explain whether:

   a) 441 b) 2001

   c) 1007 d) 4096 is a square number

5. Show that the difference between any two consecutive square numbers is an odd number.

6. Show that the difference between

   a) The 7th square number and the 4th square number is a multiple of 3.

   b) The 8th square number and the 5th square number is a multiple of 3.

   c) The 11th square number and the 7th square number is a multiple of 4.

   d) Generalise the statement implied in parts (a) (b) and (c).

   e) 64 is equal to the 8th square number $S_8 = 8^2$

   64 is equal to the 4th cube number $C_4 = 4^3$

   Find other cube numbers which are also square numbers.

   If you can, make a general comment about such cube numbers.

7. One way of making the number 5 by adding ones and threes is:

   $5 = 3 + 1 + 1$ … and another different way is: $5 = 1 + 3 + 1$

   Investigate the number of different ways of making any number by adding ones and threes.

8.

$1^3 + 2^3$	$1^3 + 2^3 + 3^3 + 4^3$	$1^3 + 2^3 + 3^3$
$= 1 + 8$	$= 1 + 8 + 27 + 64$	$= 1 + 8 + 27$
$= 9$	$= 100$	$= 36$
$= 3^2$	$= 10^2$	$= 6^2$
$= (1 + 2)^3$	$= (1 + 2 + 3 + 4)^2$	$= (1 + 2 + 3)^2$

   Investigate this situation further. Try other powers.