

QUADRATIC SEQUENCES (2)

Learning Outcomes and Assessment Standards

Learning Outcome 1: Number and Number relationships Assessment Standards

Investigate number patterns (including but not limited to those where there is a constant second difference between consecutive terms in a number pattern, and the general term is therefore quadratic) and hence:

- (a) Make conjectures and generalization.
- (b) Provide explanations and justifications and attempt to prove conjectures.

Overview

In this lesson you will:

- Revise second order difference patterns..
- Study recursive number patterns.

Lesson

Revision of quadratic number patterns

Quadratic number patterns have the general term $T_n = an^2 + bn + c$.

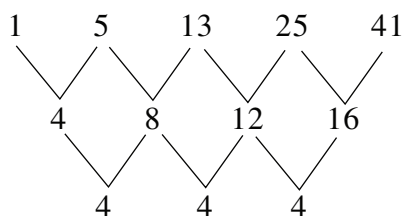
$a + b + c =$ first term

$3a + b =$ first term of first difference row

$2a =$ second difference

Example

Determine the general term of the number pattern 1; 5; 13; 25; 41;



It is clearly a quadratic number pattern because it has a constant second difference.

You can now proceed as follows:

$$\begin{array}{lll} 2a = 4 & 3a + b = 4 & a + b + c = 1 \\ \therefore a = 2 & \therefore 3(2) + b = 4 & \therefore 2 - 2 + c = 1 \\ & \therefore b = -2 & \therefore c = 1 \end{array}$$

Therefore the general term is $T_n = 2n^2 - 2n + 1$

Recursive linear number patterns

Consider the number pattern: 3; 5; 7; 9; 11;

$$T_1 = 3$$

$$T_2 = 3 + 2 = T_1 + 2$$

$$T_3 = 5 + 2 = T_2 + 2$$



$$T_4 = 7 + 2 = T_3 + 2$$

$$T_5 = 9 + 2 = T_4 + 2$$

$$T_6 = 11 + 2 = T_5 + 2$$

$$\therefore T_{n+1} = T_n + 2$$

This formula represents the recursive form of the number pattern.

Further examples

(a) Express in recursive form:

4; 10; 16; 22; 28; 34;

(b) Determine the number pattern if:

$$T_1 = 2 \quad \text{and} \quad T_n = T_{n-1} + 5$$

(a) 4; 10; 16; 22; 28; 34;

$$T_1 = 4$$

$$T_2 = 4 + 6 = T_1 + 6$$

$$T_3 = 10 + 6 = T_2 + 6$$

$$T_4 = 16 + 6 = T_3 + 6$$

$$\therefore T_n = T_{n-1} + 6$$

(b) $T_1 = 2$

$$T_2 = T_1 + 5 = 2 + 5 = 7$$

$$T_3 = T_2 + 5 = 7 + 5 = 12$$

$$T_4 = T_3 + 5 = 12 + 5 = 17$$

Therefore the number pattern is:

2; 7; 12; 17; 22;

Recursive quadratic number patterns

Example

Express the following number patterns in recursive form:

(a) 2; 5; 10; 17; 26;

(b) 9; 16; 28; 45; 67;

(a) $T_1 = 2$

$$\therefore T_2 = 2 + 3 = T_1 + 3$$

$$\therefore T_3 = 5 + 5 = T_2 + 5$$

$$\therefore T_4 = 10 + 7 = T_3 + 7$$

$$\therefore T_5 = 17 + 9 = T_4 + 9$$

$$\therefore T_n = T_{n-1} + \text{?????}$$

Now consider the pattern 2; 3; 5; 7; 9;

The first term 2 is the odd man out in the pattern because 3; 5; 7; 9; ... is linear with a constant difference of 2 between the terms. With the first term, 2, you have to add 1 to get to the second term, 3.

What you can do is work out the general term for the pattern 3; 5; 7; 9; ... and then subtract the odd man out.

For the linear pattern 3; 5; 7; 9;

$$T_n = 3 + (n - 1)(2)$$

$$\therefore T_n = 3 + 2n - 2$$

$$\therefore T_n = 2n + 1$$

Now let's remove the odd man out by subtracting 1 from n :

$$T_n = 2(n - 1) + 1$$

$$\therefore T_n = 2n - 2 + 1$$

$$\therefore T_n = 2n - 1$$

If we now add in $(2n - 1)$ to the recursive formula, we will get:

$$T_n = T_{n-1} + (2n - 1)$$

We can now verify this recursive formula as follows:

$$T_1 = 2$$

$$T_2 = T_1 + (2(2) - 1) = 2 + 3 = 5$$

$$T_3 = T_2 + (2(3) - 1) = 5 + 5 = 10$$

$$T_4 = T_3 + (2(4) - 1) = 10 + 7 = 17$$

Clearly, the recursive formula generates the terms of the number pattern.

(b) 9; 16; 28; 45; 67;

$$T_1 = 9$$

$$\therefore T_2 = 9 + 7 = T_1 + 7$$

$$\therefore T_3 = 16 + 12 = T_2 + 12$$

$$\therefore T_4 = 28 + 17 = T_3 + 17$$

$$\therefore T_5 = 45 + 22 = T_4 + 22$$

$$\therefore T_n = T_{n-1} + \text{??????}$$

Now consider the pattern 9; 7; 12; 17; 22;

The first term 9 is the odd man out in the pattern because it is linear with a constant difference of 5 between the terms. With the first term, 9, you have to subtract 2 to get to the second term, 7.

What you can do is work out the general term for the pattern 7; 12; 17; 22;

and then subtract the odd man out.

For the linear pattern 7; 12; 17; 22;

$$T_n = 7 + (n - 1)(5)$$

$$\therefore T_n = 7 + 5n - 5$$

$$\therefore T_n = 5n + 2$$

Now let's remove the odd man out by subtracting 1 from n :

$$T_n = 5(n - 1) + 2$$

$$\therefore T_n = 5n - 5 + 2$$

$$\therefore T_n = 5n - 3$$



If we now add in $(5n - 3)$ to the recursive formula, we will get:

$$T_n = T_{n-1} + (5n - 3)$$

We can now verify this recursive formula as follows:

$$T_1 = 9$$

$$T_2 = T_1 + (5(2) - 3) = 9 + 7 = 16$$

$$T_3 = T_2 + (5(3) - 3) = 16 + 12 = 28$$

$$T_4 = T_3 + (5(4) - 3) = 28 + 17 = 45$$

Clearly, the recursive formula generates the terms of the number pattern.

Activity 1

- Write each of the following number patterns in recursive form.
 - 4; 7; 10; 13;
 - 0; 4; 8; 12;
 - 10; 6; 2;
- Determine the number pattern in each of the following cases:
 - $T_1 = 3$ and $T_n = T_{n-1} + 3$
 - $T_1 = 2$ and $T_n = 3T_{n-1}$

Activity 2

Write each of the following number patterns in recursive form.

- 7; 24; 51; 88;
- 3; 12; 27; 48;
- 6; 17; 34; 57;