QUADRATIC SEQUENCES (1)

Learning Outcomes and Assessment Standards

Learning Outcome 1: Number and Number relationships
Assessment Standards
Investigate number patterns (including but not limited to those where there is a constant second difference between consecutive terms in a number pattern, and the general term is therefore quadratic) and hence:
(a) Make conjectures and generalization.
(b) Provide explanations and justifications and attempt to prove conjectures.

Overview

In this lesson you will:

- Revise grade 10 first order difference patterns.
- Study second order difference patterns

Lesson

Revision of linear number patterns

Consider the linear number pattern 3; 5; 7; 9; 11; …

The **first term** (T_1) is 3.

The pattern is formed by adding 2 to each new term.

We say that the **constant difference** between the terms is 2.

$T_1 = 3$

$T_2 = 3 + 2$

$T_3 = 3 + 2 + 2 = 3 + (2) \times 2$

$T_4 = 3 + 2 + 2 + 2 = 3 + (3) \times 2$

$T_6 = 3 + 2 + 2 + 2 + 2 + 2 = 3 + (5) \times 2$

$T_7 = 3 + 2 + 2 + 2 + 2 + 2 + 2 = 3 + (6) \times 2$

$T_{50} = 3 + (49) \times 2 = 101$

$\therefore T_n = 3 + (n - 1)2$

You will probably notice that the first term in this formula is 3 and the constant difference is 2.

In general, these linear number patterns have the general form:

$\therefore T_n = a + (n - 1)d$

where a represents the first term and d represents the constant difference.

In the previous example, the general term will be

$\therefore T_n = 3 + (n - 1)2$

$\therefore T_n = 3 + 2n - 2$

$\therefore T_n = 2n + 1$

Further example

Consider the number pattern: 4; 9; 14; 19; ……..

INDEPENDENT NEWSPAPERS

Determine the nth term and hence the 100^{th} term.

You will probably notice that the first term in this formula is 4 and the constant difference is 5.

$\therefore T_n = 4 + (n - 1)(5)$

$\therefore T_n = 4 + 5n - 5$

$\therefore T_n = 5n - 1$

$\therefore T_{100} = 5(100) - 1$

$\therefore T_{100} = 499$

Quadratic number patterns

We will now focus on **quadratic number patterns** with general terms of the form

$T_n = an^2 + bn + c$.

Consider the pattern 2; 5; 10; 17; 26; ………

The general term of the pattern is $T_n = n^2 + 1$

This general term works since:

$T_1 = (1)^2 + 1 = 2$

$T_2 = (2)^2 + 1 = 5$

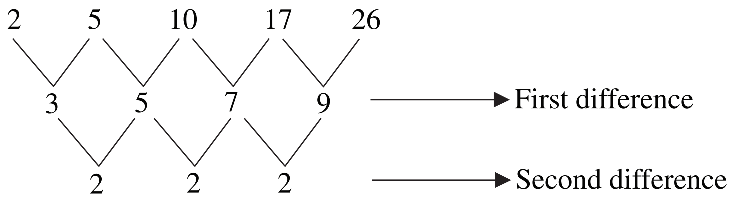
$T_3 = (3)^2 + 1 = 10$

$T_4 = (4)^2 + 1 = 17$

$T_5 = (5)^2 + 1 = 26$

But the question arises as to how one actually gets the general term.

The following method with assist you in this regard.



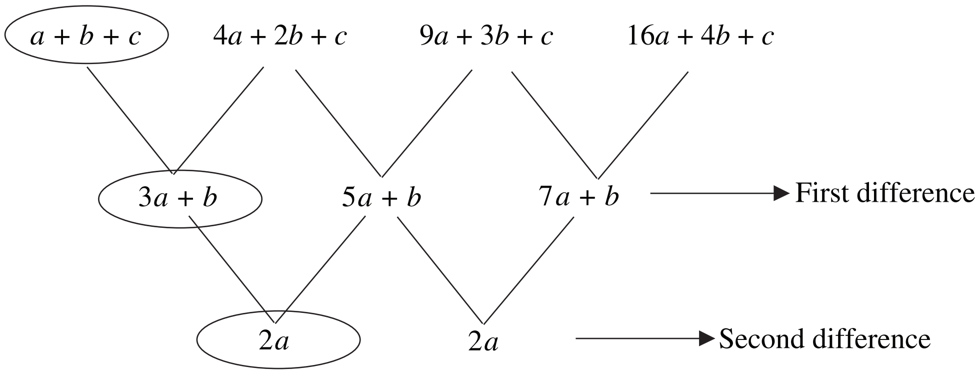
It is clear that this number pattern does not have a constant first difference. However, there is a constant second difference.

Suppose that the general term of a particular quadratic number pattern is given by $T_n = an^2 + bn + c$.

The terms of the number pattern would then be:

$T_1 = a(1)^2 + b(1) + c = a + b + c$ $T_2 = a(2)^2 + b(2) + c = 4a + 2b + c$

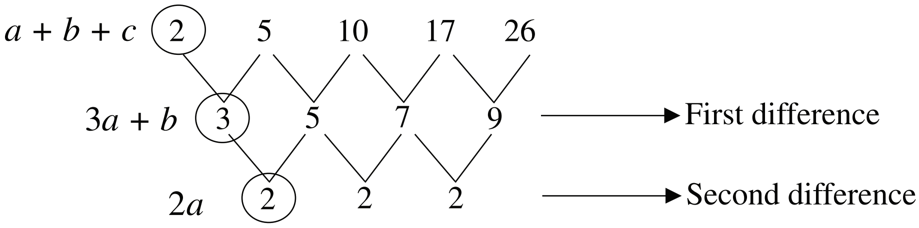
$T_3 = a(3)^2 + b(3) + c = 9a + 3b + c$ $T_4 = a(4)^2 + b(4) + c = 16a + 4b + c$.



You will notice that the constant second difference is given by the expression $2a$.

The first term in the first difference row is given by $3a + b$ and the first term is given by $a + b + c$.

So, consider the previous number pattern: 2; 5; 10; 17; 26; ……..



$2a = 2$	$3a + b = 3$	$a + b + c = 2$
$\therefore a = 1$	$\therefore 3(1) + b = 3$	$\therefore 1 + 0 + c = 2$
	$\therefore b = 0$	$\therefore c = 1$

Therefore the general term is $T_n = 1n^2 + 0n + 1 = n^2 + 1$

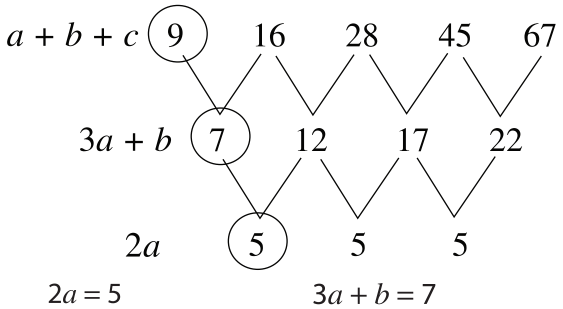
Further example

Consider the number pattern: 9; 16; 28; 45; 67; ……..

(a) Determine the general term.

(b) Determine the 40th term.

(a)



$2a = 5$	$3a + b = 7$	$a + b + c = 9$
$\therefore a = \frac{5}{2}$	$\therefore 3\left(\frac{5}{2}\right) + b = 7$	$\therefore \frac{5}{2} - \frac{1}{2} + c = 9$
	$\therefore b = 7 - \frac{15}{2} = -\frac{1}{2}$	$\therefore c = 7$

Therefore the general term is $T_n = \frac{5}{2}n^2 - \frac{1}{2}n + 7$

(b) $T_n = \frac{5}{2}n^2 - \frac{1}{2}n + 7$

$\therefore T_{40} = \frac{5}{2}(40)^2 - \frac{1}{2}(40) + 7$

$\therefore T_{40} = 3\ 987$

Activity 1

For each of the following number patterns determine the next two terms, the general term and hence the 150th term.

(a) 4; 7; 10; 13; ……………………………..

(b) 0; 4; 8; 12; ………………………………

(c) 10; 6; 2; ………………………………..

Activity 2

1. For each of the following number patterns determine the general term.

   (a) 3; 12; 27; 48; ……………………………..

   (b) 7; 24; 51; 88; ……………………………

   (c) 0; 3; 8; 15; ………………………………..

   (d) 6; 17; 34; 57; …………………….

3. Consider the following number pattern:

   $1 = 1$

   $1 + 2 = 3$

   $1 + 2 + 3 = 6$

   $1 + 2 + 3 + 4 = 10$

   $1 + 2 + 3 + 4 + 5 = 15$

   (a) Determine a general rule in terms of n for evaluating:

   $1 + 2 + 3 + 4 + 5 + 6 + \ldots + n$

   (b) Hence calculate the value of:

   $1 + 2 + 3 + 4 + 5 + 6 + 7 + \ldots + 200$

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