TRIGONOMETRIC FUNCTIONS

Learning Outcomes and Assessment Standards

Learning Outcome 3: Space, shape and measurement

The learner is able to describe, represent, analyse and explain properties of shapes in 2- and 3- dimensional space with justification.

Assessment Standard AS 5

Derive and use the values of trigonometric functions.

Overview

In this lesson you will:

- Look at angles bigger than 90° and smaller than zero
- Define the sine, cosine and tangent ratios in terms of the coordinates of a point and the radius of a circle
- Establish the sign of the three trigonometric ratios in different quadrants
- Use Pythagoras' theorem to find the values of the three different sides
- Consider angle as a revolution instead of just in a shape.

Lesson

Prior knowledge

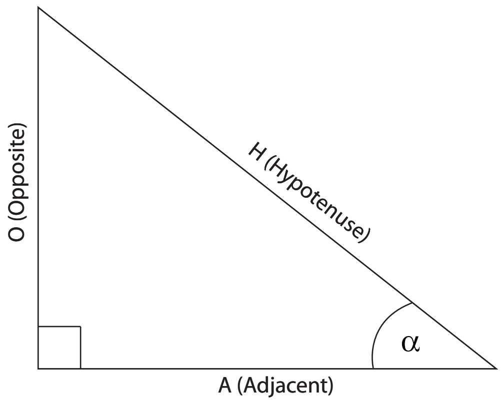
In Grade 10 we learnt:

$\sin \alpha = \frac{O}{H}$

↓ angle ↓ ratio

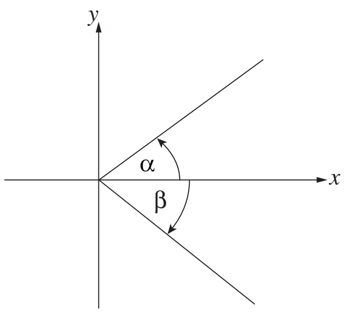
$\cos \alpha = \frac{A}{H}$

$\tan \alpha = \frac{O}{A}$



Part One

Different angle sizes



If an angle is measured anti-clockwise it is positive.

If an angle is measured clockwise it is negative.

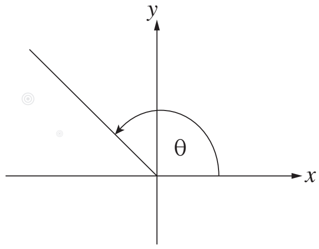
so $\alpha = 30°$

or $\alpha = 390°$

or $\alpha = -330°$

or $\alpha = -690°$

If $\beta = -40°$ or $\beta = 320°$ or $\beta = 680°$
we call these co-terminal angles.

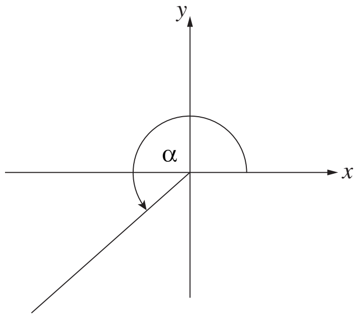


θ is in the second quadrant

If $\theta = 120°$ it could be

$\theta = -240°$

or $\theta = 480°$



If $\alpha = 200°$ it could be

$\alpha = -160°$

or $\alpha = 560°$

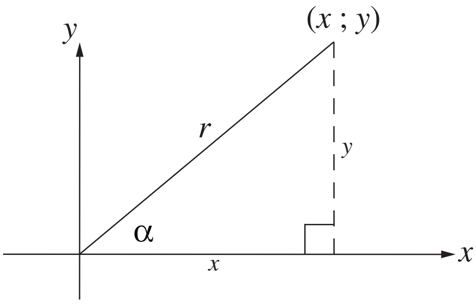
In which quadrant do you find:

1.	300°	4th	2.	(–150°)	3rd
3.	472°	2nd	4.	884°	2nd
5.	1 740	4th	6.	(–1 010)	1st

Now we will put the triangle into the Cartesian plane.

The first quadrant

Here $x > 0$ and $y > 0$. Remember that r will always be positive.



The angles are in the first quadrant

$\sin \alpha = \frac{y}{r} > 0$

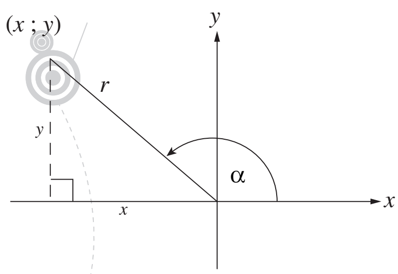
$\cos \alpha = \frac{x}{r} > 0$

$\tan \alpha = \frac{y}{x} > 0$

In the 1st quadrant, all ratios are positive.

The second quadrant

Here $x < 0$ and $y > 0$; $r > 0$



$\sin \alpha = \frac{y}{r} > 0$

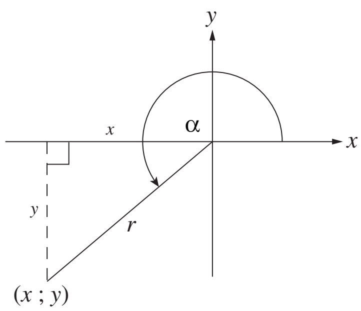
$\cos \alpha = \frac{x}{r} < 0$

$\tan \alpha = \frac{y}{x} > 0$

In the 2nd quadrant, sin α is positive but cos α and tan α are negative.

The third quadrant

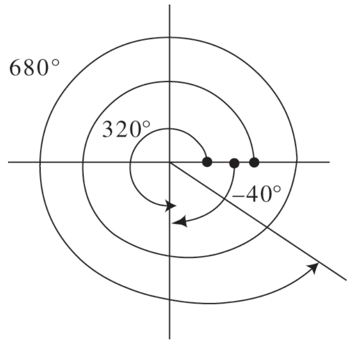
Here $x < 0$ and $y < 0$; $r > 0$



$\sin\alpha = \frac{y}{r} < 0$

$\cos\alpha = \frac{x}{r} < 0$

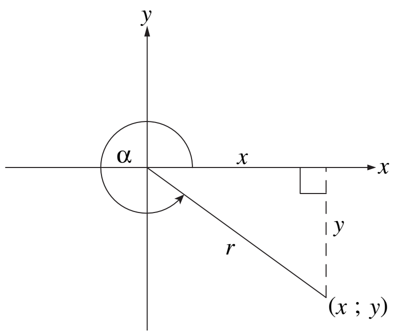
$\tan\alpha = \frac{y}{x} > 0$



In the 3rd quadrant tan α is positive but sin α and cos α are negative.

The fourth quadrant

Here $x > 0$ and $y < 0$; $r > 0$



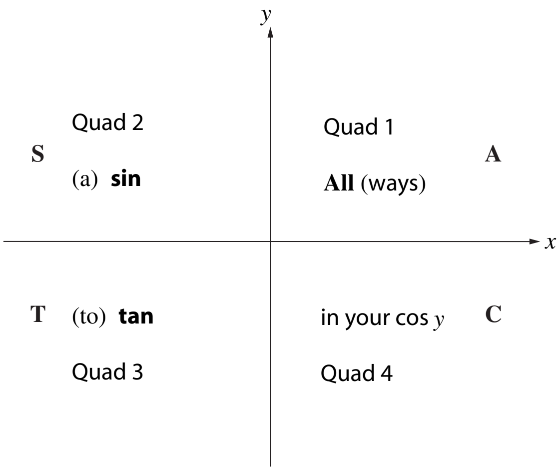
$\sin\alpha = \frac{y}{r} < 0$

$\cos\alpha = \frac{x}{r} > 0$

$\tan\alpha = \frac{y}{x} > 0$

In the 4th quadrant cos α is positive but sin α and tan α are negative.

Summary



In the 1st quadrant, **all** ratios are positive.

In the 2nd quadrant, **sin** α is positive but cos α and tan α are negative.

In the 3rd quadrant **tan** α is positive but sin α and cos α are negative.

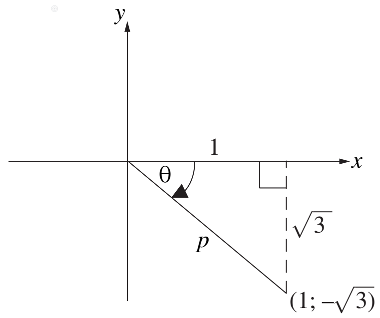
In the 4th quadrant **cos** α is positive but sin α and tan α are negative.

Part Two:

Example

Example 1

Calculate the value of sin θ, cos θ and tan θ if the terminal ray of θ is in standard position and passes through $(1; -\sqrt{3})$



$p^2 = (-\sqrt{3})^2 + (1)^2 = 3 + 1 = 4$ (Pythagoras)

$\therefore p = 2$ $(p > 0)$.

So $\sin\theta = \frac{-\sqrt{3}}{2}$; $\tan\theta = -\sqrt{3}$

$\cos\theta = \frac{1}{2}$

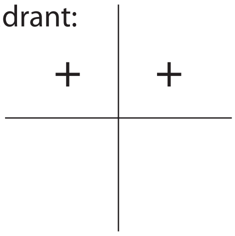
Example

Example 2

If 13 sin θ = 12 and θ ε [90° ; 270°] find cos θ tan θ.

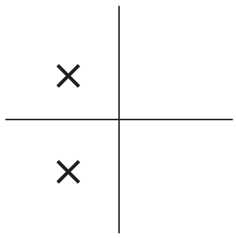
Locate the quadrant:

$\sin\theta = \frac{12}{13} > 0$

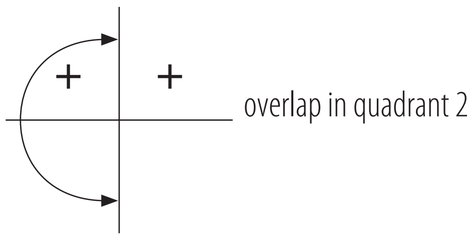


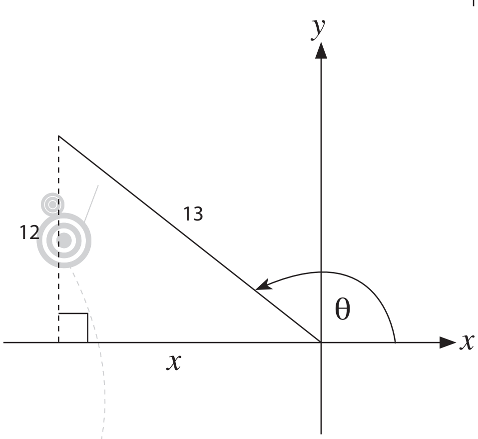
Since θ ∈ [90°; 270°]

The overlap is in quadrant 2.



Place both on one sketch:





Pythagoras:

$x^2 = 13^2 - 12^2$

$= 169 - 144$

$= 25$

$\therefore x = -5$ $(x < 0)$

Now:

$\cos\theta \times \tan\theta = \left(\frac{-5}{13}\right)\left(\frac{12}{-5}\right) = \frac{12}{13}$

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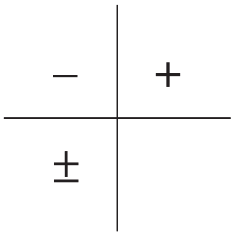
Example

Example 3

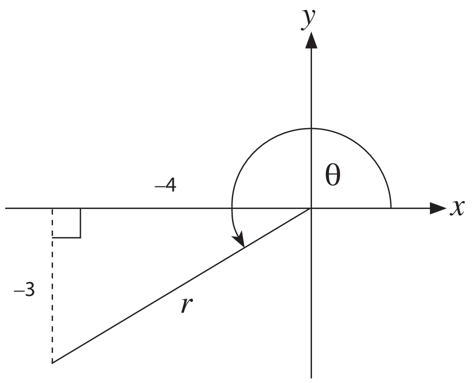
If 4 tan $\theta = 3$ and cos $\theta < 0$ find 2 sin θ - cos θ

tan $\theta = \frac{3}{4} > 0$

cos $\theta < 0$



Overlap in 3rd quadrant



$r = 5$ (Pythagorean triplet)

So 2sin θ – cos θ

$= 2\left(-\frac{3}{5}\right) - \left(-\frac{4}{5}\right)$

$= -\frac{6}{5} + \frac{4}{5}$

$= -\frac{2}{5}$

Activity 1

Activity

1. Locate the quadrant where
   - a) tan $\theta < 0$ and sin $\theta > 0$
   - b) cos $\theta < 0$ and sin $\theta > 0$
   - c) $\theta \in [180°; 360°]$ and cos $\theta < 0$
   - d) sin $\theta < 0$ and $\theta \in [0°; 270°]$
   - e) tan $\theta > 0$ and cos $\theta > 0$
   - f) tan $\theta > 0$ and $\theta \in [90°; 270°]$?
2. If P(–3 ; 1) is a point in the Cartesian plane and $X\hat{O}P = \theta$. Find
   - a) sin θ + cos θ
   - b) 1 + $\tan^2 \theta$
   - c) $\sin^2 \theta + \cos^2 \theta$

   (If necessary leave your answer in surd form)
3. If 12 tan θ = -5 find the value of (a) sin θ (b) cos θ
4. If 25 cos x = –24 and x ε [0° ; 180°], find the value of $\frac{2 \tan x}{\sin x}$
5. If 41 cos θ = -40 and sin $\theta > 0$, find the value of
   - a) 1 + $\tan^2 \theta$
   - b) 2 sin θ – cos θ
6. If sin $\theta = p$ and $p < 0$ and θ ε [90° ‘; 270°], find the value of cos θ.
7. If tan $\theta = mm > 0$ and θ ε [0° ; 90°], find the value of $\sin^2 \theta - \cos^2 \theta$
8. In $\triangle ABC$ $\hat{B}$ is an obtuse angle. sin $A = \frac{3}{5}$ sin $B = \frac{4}{5}$, find $\dfrac{\cos A + \cos B}{\tan A - \frac{1}{\tan B}}$
9. If sin $x = \frac{2m}{1+m^2}$ and x ε [0° ; 90°] , find $\frac{1}{\cos x}$ + tan x.
10. If tan $\theta = \frac{2xy}{x^2 - y^2}$ and θ ε [0° ; 90°] find sin θ + cos θ
11. If cos $\theta = p$ and $p < 0$ and θ ε [180° ; 360°] find tan θ