QUADRATIC INEQUATIONS

Solving inequalities

LESSON 10

Learning Outcomes and Assessment Standards

Learning Outcome 2: Functions and algebra
Assessment Standard AS 2.5(a)
Solving linear and quadratic equations (inequations).

Overview

In this lesson you will:

- Explore the number line to decide where inequalities are negative, zero or positive
- Learn the correct notation for inequalities
- Use a number line to solve quadratic inequalities.

Lesson

Examples

1. Solve: $3x + 6 < -3$

Solution

$3x < -3 - 6$

$3x < -9$ Divide by +3

$x < -3$ Don't change the inequality sign

When we multiply or divide an inequality by a negative number, the inequality changes direction.

2. Solve: $4 - 2x < 5$

Solution

$-2x < 5 - 4$

$-2x < 1$

Divide both sides by -2 and change the inequality sign.

$x > -\frac{1}{2}$

Quadratic inequalities

- To solve a quadratic inequality, make the one side of the inequality equal to zero and x^2 have a positive coefficient
- Factorise
- Draw a number line
- Decide whether the zeroes can be nought
- Test the number line
- Write down the correct solution.

Rules

- Never multiply the denominator away.
- If you multiply or divide by a negative, change the inequality sign.
- Non-real numbers are square roots of negative numbers.

- An expression is undefined when the denominator is zero.
- Always make sure that every x has a positive coefficient, then the far right of the number line will be positive.

Look at the expression $(x - 2)(x + 4)$

Explore the number line: $+$ ● -4 $-$ ● 2 $+$

Note $(x - 2)(x + 4) = 0$

when $x = 2$ or $x = -4$

$(x - 2)(x + 4) > 0$

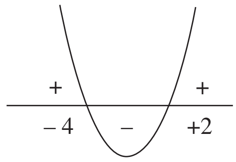
when $x < -4$ or $x > 2$

$(x - 2)(x + 4) < 0$

when $-4 < x < 2$

Why is this so?

If we look at the graph of $y = (x - 2)(x + 4)$, it looks like this:



So if $-4 < x < 2$ the y values on this graph are negative.

Always make sure that your x^2 term is positive, and that all x terms have positive coefficients. Then you will always have a + sign on the far RHS of the number line. If it is not positive, make it positive by multiplying both sides with a negative.

Rules

- Ensure that one side of the inequality is zero.
- Factorise.
- Explore the number line.
- Write down the correct solution.

Example

Examples

1. Solve for x: $x^2 < 4x$

Solution

Solution

Isolate the inequality

$x^2 - 4x < 0$ Make one side zero

$x(x - 4) < 0$ Factorise.

Number line: $+$ ○ 0 $-$ ○ 4 $+$

Write the zeroes on the number line.

We are looking for $x(x - 4) < 0$ (negatives on the number line).

$0 < x < 4$

2. Solve for x: $x^2 - x \geq 6$

Solution

Solution

$x^2 - x - 6 \geq 0$

$(x - 3)(x + 2) \geq 0$

Number line: $+$ ● -2 $-$ ● 3 $+$

We are looking for positive on the number line.

$x \leq -2$ or $x \geq 3$

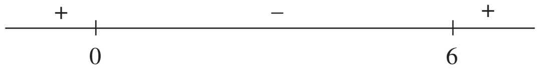
3. Solve for x: $6x \leq x^2$

Solution

$-x^2 + 6x \leq 0$

$\therefore x^2 - 6x \geq 0$ Make sure that x^2 has a positive sign

$\therefore x(x - 6x \geq 0$ Do not use $6x - x^2 \leq 0$



We are looking for negative

$x \leq 0$ or $x \geq 6$

4. Look at this one

Solve for: $\frac{2x+1}{x-4} \geq 0$

Solution

Note : You already have factors and all the x's have positive coefficients.

Note: $-\frac{1}{2}$ can be zero but 4 cannot be zero.

$x \leq -\frac{1}{2}$ or $x > 4$

5. A strange one

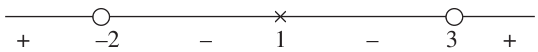
Solve for x: $\frac{(x-1)^2}{x^2 - x - 6} \geq 0$

Solution

We know that $x^2 \geq 0$ for all x so $(x-1)^2 \geq 0$ for all x.

Furthermore: $x^2 - x - 6 = (x-3)(x+2)$

but $x \neq 3$ and $x = -2$ since they are both in the denominator. Because $(x-1)^2 \geq 0$, we are not going to change sign when we pass through 1.



So we get that $x < -2$; $x = 1$; $x > 3$

6. And this one

Solve for x: $\frac{5x - x^2}{(x+2)^2} \leq 0$

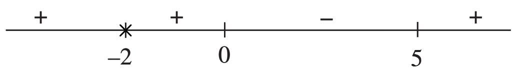
Solution

Let us change the sign so that x^2 has a positive coefficient:

$\frac{x^2 - 5x}{(x+2)^2} \geq 0$

$\therefore \frac{x(x-5)}{(x+2)^2} \geq 0$

$\therefore x < -2; -2 < x \leq 0; x \geq 5$



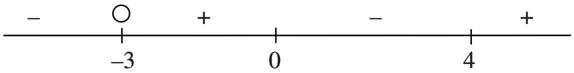
Application of inequalities

Examples

1. Find the values of m for which the following expression is real: $\sqrt{\dfrac{m^2-4m}{m+3}}$

Solution

$\dfrac{m^2-4m}{m+3} \geq 0$



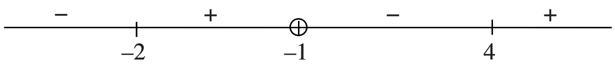
$\therefore \dfrac{m(m-4)}{m+3} \geq 0$

$-3 < m \leq 0$ or $m \geq 4$

2. Find the values of k for which the following expression is undefined or non-real: $\sqrt{\dfrac{(4-k)(2+k)}{(k+1)}}$

Solution

$\dfrac{(4-k)(2+k)}{k+1} < 0$



$\dfrac{(k-4)(k+2)}{k+1} > 0$

$-2 < k < -1$ or $k > 4$

Activity 1

1. $x^2 - x > 0$
2. $x^2 - x < 6$
3. $x(x-2) \geq 8$
4. $4x - x^2 < 0$
5. $3x \geq x^2$

Activity 2

1. $x^3 - x^2 \geq 12x$
2. $\dfrac{4+x}{5-x} \leq 0$
3. $\dfrac{x^2-x-6}{x^2-x} \geq 0$
4. $\dfrac{2x-x^2}{x^2+7x+6} \leq 0$
5. $\dfrac{-x-4}{1-x^2} \geq 0$
6. $\dfrac{7x^2-14x}{1+x} > 0$

Example

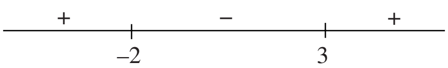
Solve $\dfrac{x^2+1}{x^2-x-6} < 0$

$x^2 \geq 0$ so $x^2 + 1 \geq 1$

So then for the outcome to be negative,

$x^2 - x - 6 < 0$

$\therefore (x-3)(x+2) < 0$



$-2 < x < 3$

Activity 3

1. For which values of k will the expression $\sqrt{9-k^2}$ be real?
2. For which values of p will the expression $\sqrt{\dfrac{3-p}{2+p}}$ be real?
3. For which values of m will the expression $\sqrt{\dfrac{6+m-m^2}{m-1}}$ be non-real or invalid?
4. For which values of t will $\sqrt{\dfrac{-(t-1)^2}{2t}}$ be real?
5. For which values of p will $\dfrac{\sqrt{4-p^2}}{p-1}$ be real?