QUADRATIC EQUATIONS

Simultaneous equations and modelling

Learning Outcomes and Assessment Standards

Learning Outcome 2: Functions and algebra
Assessment Standard
Equations in two unknowns, one of which is linear and one of which is quadratic.
Use mathematical models to investigate problems that arise in real–life contexts.

Overview

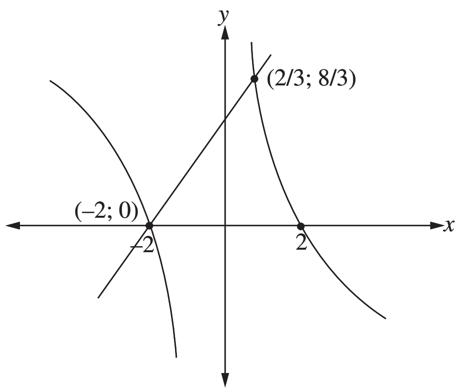
In this lesson you will:

- Solve simultaneous equations.
- Form your own equation and use the techniques of simultaneous equations to solve problems.

Lesson

Why solve simultaneous equations?

It is a method that helps us to determine where the graphs of $y - x = 2$ and $x^2 + 2xy - 4 = 0$ cut one another graphically



Example 1

Solve for x and y in

$x + 2y = 5$ and $2y^2 - xy - 4x^2 = 8$

$\Rightarrow$ We use: $x + 2y = 5$

(1) – $\therefore x = 5 - 2y$ (isolate x to avoid fraction)

Now substitute:

$2y^2 - (5 - 2y)y - 4(5 - 2y)^2 = 8$

$\therefore 2y^2 - 5y + 2y^2 - 4(25 - 20y + 4y^2) = 8$

$\therefore 4y^2 - 5y - 100 + 80y - 16y^2 - 8 = 0$

$\therefore -12y^2 + 75y - 108 = 0$

Thus

$12y^2 - 75y + 108 = 0$

$\therefore 4y^2 - 25y + 36 = 0$

$\therefore (4y - 9)(y - 4) = 0$

$\therefore y = \frac{9}{4}$ or $y = 4$

Now substitute back into (1)

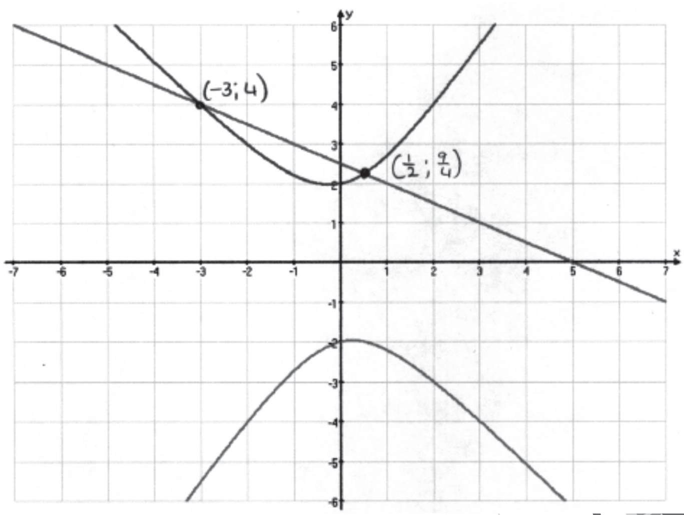
Then $x = 5 - 2\left(\frac{9}{4}\right)$ or $x = 5 - 2(4)$

$x = 5 - \frac{9}{2}$ $\qquad$ $x = 5 - 8$

$x = \frac{10 - 9}{2}$ $\qquad$ $x = -3$

$x = \frac{1}{2}$

$\therefore$ If $x = \frac{1}{2}$; $y = \frac{9}{4}$ and if $x = -3$; $y = 4$



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Example 2

Solve for x and y if

$x + y = 9$ and $x^2 - 4xy - y^2 = 8$ –(2)

(correct to two decimals)

$\Rightarrow x + y = 9$

Then $y = 9 - x$ –(1)

(2) $\mapsto$ (1) $x^2 - 4x(9 - x) - (9 - x^2) = 8$

$\therefore x^2 - 36x + 4x^2 - (81 - 18x + x^2) - 8 = 0$

$\therefore 5x^2 - 36x - x^2 + 18x - 81 - 8 = 0$

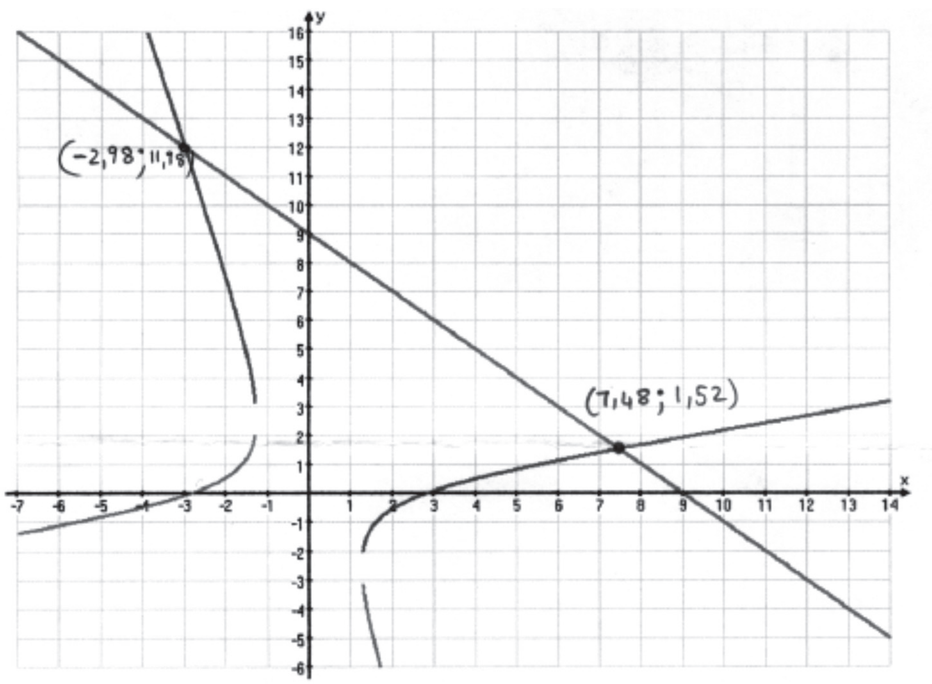
$\therefore 4x^2 - 18x - 89 = 0$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{18 \pm \sqrt{(18)^2 - 4(4)(-89)}}{2(4)}$

$x = \frac{18 \pm \sqrt{1\,748}}{8}$

$\therefore x = 7{,}48$ or $x = -2{,}98$



Then $y = 9 - (7{,}48)$ $y = 9 - (-2{,}98)$

$\therefore y = 1{,}52$ $y = 9 + 2{,}98$

$y = 11{,}98$

Thus: If $x = 7{,}48$; $y = 1{,}52$ and if $x = -2{,}98$; $y = 11{,}98$

Example 3

3. Solve for x and y: $3x - 4y = 3$ and $xy = 15$

$\Rightarrow$ We cannot avoid fractions:

$xy = 15$

Thus $x = \frac{15}{y}$ … (1) and $3x - 4y = 3$ … (2)

(1) $\rightarrow$ (2): $3\left(\frac{15}{y}\right) - 4y = 3$

$$\therefore \frac{45}{y} - 4y = 3$$

$$\therefore 45 - 4y^2 = 3y$$

$$\therefore 4y^2 + 3y - 45 = 0$$

$$\therefore (4y + 15)(y - 3) = 0$$

$$\therefore y = -\frac{15}{4} \text{ or } y = 3$$

Then

$$x = \frac{15}{-\frac{15}{4}} = 15.-\frac{4}{15}$$

$$\therefore x = -4 \text{ or } x = \frac{15}{3}$$

$$\therefore x = 5$$

Example 4

Matthew Matic and Allen Algebra attended the polynomial pop concert in Trigoville.

"Do you know" said Matthew, "that if there were 60 people less attending this concert, then the audience could be arranged in a perfect square".

"Yes" replied Allen, "and if there were 41 more people attending, then the length of the square would be increased by 1".

How many people were attending this concert?

Solution

Let x be the number of people attending the concert.

$x - 60 = y^2$ (1)

$x + 41 = (y + 1)^2$ (2)

Note: It is of utmost importance that you define your variable

Make x the subject in equation 1

$x = y^2 + 60$

Substitute into equation 2

$y^2 + 60 + 41 = (y + 1)^2$

Solve

$y^2 + 101 = y^2 + 2y + 1$

$2y = 100$

$y = 50$

$x = y^2 + 60$

$x = 2\,500 + 60$

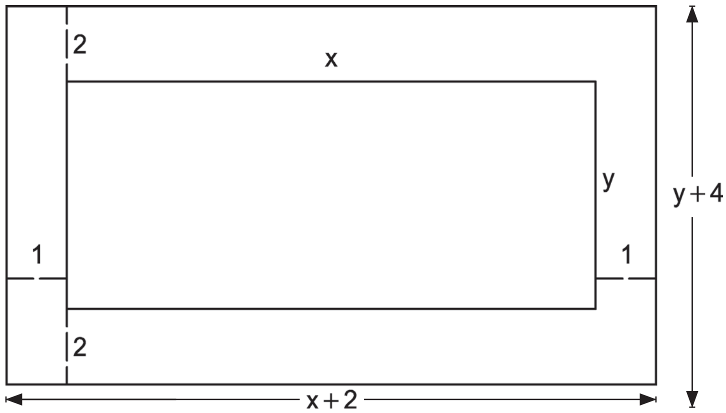
$x = 2\,560$

Always answer the question

Thus 2 560 people attended the concert.

Example 5

A card has 10 cm^2 of printing. The margins at the top and the bottom are 2 cm and along the sides are 1 cm. If the area of the whole card is 39 cm^2, find possible dimensions for the card.



Solution

$xy = 10$

$(x + 2)(x + 4) = 39$

$xy + 2y + 4x + 8 = 39$

Substitute $y = \frac{10}{x}$ into the above equation.

$x\left(\frac{10}{x}\right) + 2\left(\frac{10}{x}\right) + 4x + 8 = 39$

$10 + \frac{20}{x} + 4x + 8 = 39$

$10x + 20 + 4x^2 + 8x = 39$

$4x^2 - 21x + 20 = 0$

$(4x - 5)(x - 4) = 0$

$x = \frac{5}{4}$ or $x = 4$

$y = \frac{10}{\frac{5}{4}}$ $y = \frac{10}{4}$

$y = 8$ $y = \frac{5}{2}$

Dimensions

$x = 6$ $y = 6,5$ or $x = 1,23$ $y = 12$

Now do Activity 2.

Activity 1

In the following, solve for x and y

1. $x + y = 9$ and $x^2 + xy + y^2 = 61$
2. $x - 2y = 3$ and $4x^2 - 5xy = 3 - 6y$
3. $(x - 1)^2 + (y - 2)^2 = 5$ and $2x + y + 1 = 0$
4. $(x - 2)^2 + (y - 3)^2 = 0$
5. $(y - 3)(x^2 + 2) = 0$
6. $2x - 3y = 1$ and $x(x - 6) - y = (1 - y)(1 + y)$

Activity 2

1. A number of two digits is seven times the sum of its digits. The product of the digits is diminished by the tens digit, leaves twelve times the ten digits divided by the units digit. Find the number.
2. The product of two consecutive integers is 90. Find the integers.