LESSON 7

QUADRATIC FORMULA

Using the formula to solve quadratic equations

Learning Outcomes and Assessment Standards

Learning Outcome 2: Functions and algebra
Assessment Standard
The solution of quadratic equations by using the formula.

Overview

In this lesson you will:

- Use the quadratic equation formula to solve quadratic equations.
- Read the questions carefully so as to decide whether to leave the answer in surd form or to use your calculator and write the answer in surd form.
- Learn to write the equation in the simplified surd form.
- Look at the formula and decide when you have solutions and when you do not.
- Look at the formula and decide whether you could have factorised or whether you had to use the formula to solve the equation.

Lesson

Using the formula to solve quadratic equations. Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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We look at $b^2 - 4ac$

Why is $b^2 - 4ac$ so important?

It is under a root, and we know that square roots of negative numbers do not exist in the real domain.

So if $b^2 - 4ac < 0$ there will be no real solution

$2x^2 + 4x + 3 = 0$

$b^2 - 4ac = 16 - 4(6)$

$b^2 - 4ac$ is Negative – so x is non real.

If $b^2 - 4ac$ is a perfect square we don't need to use a calculator. **Why? Because a perfect square can be square rooted** e.g. $\sqrt{25} = \sqrt{5^2} = 5$. So we can then factorise.

$2x^2 - 5x - 3 = 0$

$b^2 - 4ac = 25 - 4(2)(-3)$

$= 49$ Perfect square, so

$(2x + 1)(x - 3) = 0$

$x = \frac{-1}{2}$ or $x = 3$

So when do we need to apply the formula?

– When $b^2 - 4ac$ is a positive, non-perfect square

Like: $\sqrt{12}$ or $\sqrt{17}$ etc.

If $b^2 - 4ac$ is a non-perfect square, x is an irrational number.

Example 1

Solve for x and write your answer correct to two decimal places.

$2x^2 - 5x + 1 = 0$

Solution

$a = 2 \quad b = -5 \quad c = 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{5 \pm \sqrt{17}}{2(2)}$

$\therefore x = 5 \pm \frac{\sqrt{17}}{4}$

$x = 2{,}28 \quad \text{or} \quad x = 0{,}22$

Check $b^2 - 4ac$: $b^2 - 4ac = 25 - 4(2)(1)$

$= 25 - 8$

$= 17$

17 is not a perfect square, so we know that we need to use the formula.

Check $b^2 - 4ac$: $b^2 - 4ac = (-2)^2 - 4(1)(-1)$

$= 4 + 4$

$= 8$

8 is not a perfect square

Example 2

Solve for x and write your answer in the simplest surd form.

$x^2 - 2x - 1 = 0$

Solution

$\therefore x = \frac{-(-2) \pm \sqrt{8}}{2}$

$= \frac{2 \pm \sqrt{2}}{2}$

$x = 1 \pm \sqrt{2}$

Example 3

Solve for x: $-3x^2 + 2x - 5 = 0$: $b^2 - 4ac = 4 - (4)(-3)(-5)$

$= 4 - 60$

$= -56$

$b = 4ac$ is negative

Thus no real solution

Example 4

Solve for x: $6x^2 + 5x - 6 = 0$

$b^2 - 4ac = 25 - 4(6)(-6)$

$= 169$

169 is not a perfect square, so we can factor

Solutions

Method 1: We can factorise

$(2x + 3)(3x - 2) = 0$

$x = \frac{-3}{2}$ or $x = \frac{2}{3}$

Method 2: Maybe we did not want to factorise, so

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore x = \frac{-5 \pm \sqrt{169}}{12}$$

$$\therefore x = \frac{-5 + 13}{12} \quad \text{or} \quad x = \frac{-5 - 13}{12}$$

$$\therefore x = \frac{8}{12} \quad \text{or} \quad x = \frac{-18}{12}$$

$$\therefore x = \frac{2}{3} \quad \text{or} \quad x = \frac{-3}{2}$$

So, when are there solutions and when are there no solutions, and when could you actually have factorised instead of using the formula?

Strange example

If the roots of $x^2 - mx + n = 0$ are consecutive integers, find m in terms of n.

$a = 1 \qquad b = -m \qquad c = n$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{m \pm \sqrt{m^2 - 4(n)}}{2}$$

If $R_1 = \frac{m + \sqrt{m^2 - 4n}}{2}$ and $R_2 = \frac{m - \sqrt{m^2 - 4n}}{2}$ then $R_1 \geq R_2$

So: $R_1 = R_2 + 1$ Consecutive numbers

$$\therefore \frac{m + \sqrt{m^2 - 4n}}{2} = \frac{m - \sqrt{m^2 - 4n}}{2} + 1$$

$$\therefore m + \sqrt{m^2 - 4n} - m + \sqrt{m^2 - 4n} = 2$$

$$2\sqrt{m^2 - 4n} = 2$$

$$\sqrt{m^2 - 4n} = 1$$

$$m^2 - 4n = 1$$

$$m^2 = 1 + 4n$$

$$m = \pm\sqrt{1 + 4n}$$

Activity 1

Solve for *x*:

1. $9x(x - 1) = -2$
2. $2x^2 + 4x + 3 = 0$
3. $2x^2 + x - 2 = 0$
4. $\frac{1}{4}x^2 + \frac{1}{3}x - 7 = 0$
5. $24x^2b^2 + 2xb - 1 = 0$
6. $36x^4 - 25x^2 + 4 = 0$
7. $x^2(x + m) - 2x(x + m) + x + m = 0$
8. $\frac{1}{2}x^2 - 3x + 1{,}3 = 0$ (correct to 2 dec)
9. $(x + 3)(2 - x) = 7x$ (correct to 2 dec)