LESSON 5

QUADRATIC EQUATIONS

Involving Surds

Learning Outcomes and Assesments Standards

Learning Outcomes 2: Functions and algebra
Assessment Standard
Solutions of a quadratic equation with a single surd.

Overview

In this lesson you will:

- Examine when a surd expression is real and solve the surd equation.
- Examine when a surd equation is defined.

Lesson

Quadratic equations

If $x^2 = 4$

$x = \pm\sqrt{4}$

$x = \pm 2$

Remember:
$\sqrt{4} = 2$ (positive)
and $-\sqrt{4} = -2$
(negative) and $\sqrt{-4}$
is non-real

Why are roots (surds) so tricky?

Can we determine $\sqrt{-4}$? The answer is yes, but if we allow non $\mathbb{R}$ numbers. At school we work with Real ($\mathbb{R}$) numbers only, so $\sqrt{-4}$ is not possible!

So when we see a root sign, we must make sure that what is underneath that root sign, is positive in value.

So if $y = \sqrt{x}$, then $x \geq 0$

Note that we have $y = +\sqrt{x}$, so $y \geq 0$
if $x \geq 0$

If we have $y = -\sqrt{x}$, then $y \leq 0$
if $x \geq 0$

In both cases, if x is not positive, we cannot determine y.

This forms the conceptual basis that builds sufficiently for flexible application!

So for $y = \sqrt{x+1}$; $x \geq -1$ and $y \geq 0$
$y = 2\sqrt{x+1} - 4$; $x \geq -1$ and $y \geq -4$

Method of solving a surd equation

- Isolate the surds
- State restrictions on x
- Square both sides
- Get into standard form
- Solve for x
- Check your solution against the restriction.

Examples

1. Solve for x.

$\sqrt{x+6} - x = 4$ Isolate the surd

$\sqrt{x+6} = 4 + x$

Remember:

$(a+b)^2 = a^2 + 2ab + b^2$

Now state restrictions on x.

$x + 6 \geq 0 \ \therefore x \geq -6$

$4 + x \geq 0 \ \therefore x \geq -4$

Thus $x \geq -4$

$x + 6 = (4 + x)^2$ Square both sides, not each term

$x + 6 = 16 + 8x + x^2$

$0 = x^2 + 7x + 10$ Get into standard form

$0 = (x + 5)(x + 2)$ Factorise

$x = -5$ or $x = -2$ Solve

But $4 + x \geq 0 \ \therefore x \geq -4$ Check answer with restriction. Omit the solution $\therefore x \neq -5$ that does not satisfy the restriction

$\therefore x = -2$

2. $\sqrt{2x+1} + 7 = x$

$\therefore \sqrt{2x+1} = x - 7$ (Isolate the surd)

Restriction: $2x + 1 \geq 0$ and $x - 7 \geq 0$

$x \geq -\frac{1}{2}$ $x \geq 7$

$\therefore x \geq 7$

Now: $2x + 1 = (x - 7)^2$ (square both sides)

$\therefore 2x + 1 = x^2 - 14x + 49$

$\therefore x^2 - 16x + 48 = 0$

$\therefore (x - 12)(x - 4) = 0$

$\therefore x = 12$ or $x = 4$

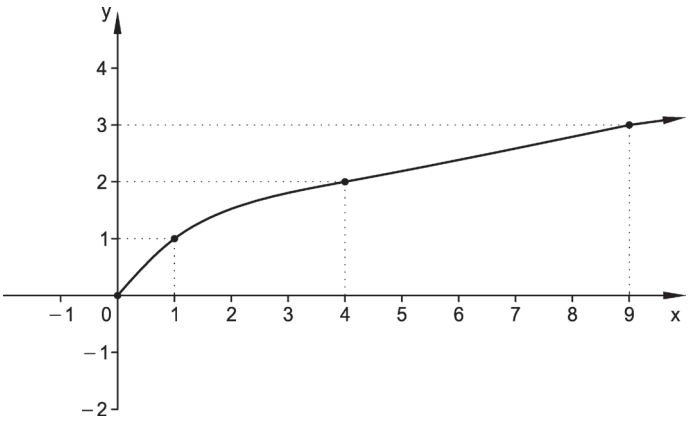
But $x \geq 7$

Thus $x = 12$ only.

Linking to graphs

1. Let's sketch a graph $y = \sqrt{x}$.

x	0	1	4	9
y	0	1	2	3



Remember that $x \geq 0$, so we only use positive values of x in our table.

Domain $(D_f) = \{x : x \geq 0\}$

Range $(R_f) = \{y : y \geq 0\}$

Alternatively, use interval notation:

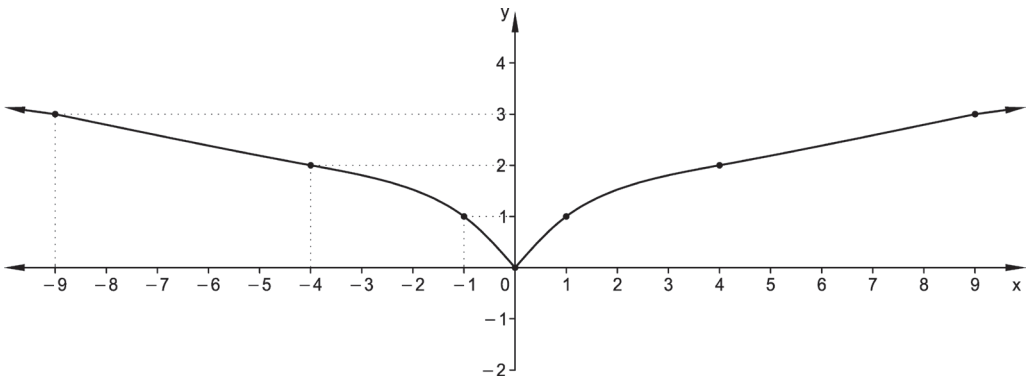
$D_f = [0; \infty)$

$R_f = [0; \infty)$

2. What happens if you reflect the graph in the y-axis?

x changes sign.

New graph:



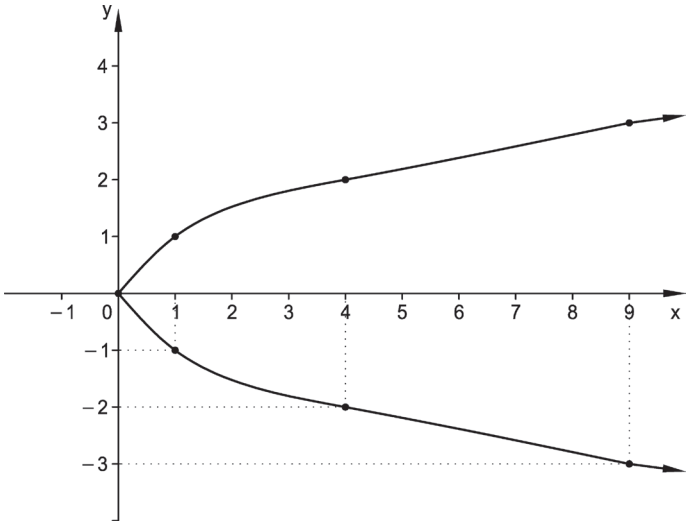
For $y = \sqrt{-x}$

Domain $= \{x : x \leq 0\} x \in \mathbb{R}$

Range $= \{y : y \geq 0\}\ y \in \mathbb{R}$

3. What happens is you reflect the graph in the x–axis?

y changes sign



For $y = -\sqrt{x}$

Domain = $\{\frac{x}{x} \geq 0\}\ x \in \mathbb{R}$

Range = $\{\frac{y}{y} \leq 0\}\ y \in \mathbb{R}$

Alternatively, use interval notation:

Domain = $\{0;\infty\}$

Range = $\{-\infty;0\}$

4. Sketch graph of $f\ (x) = \sqrt{x-2}$

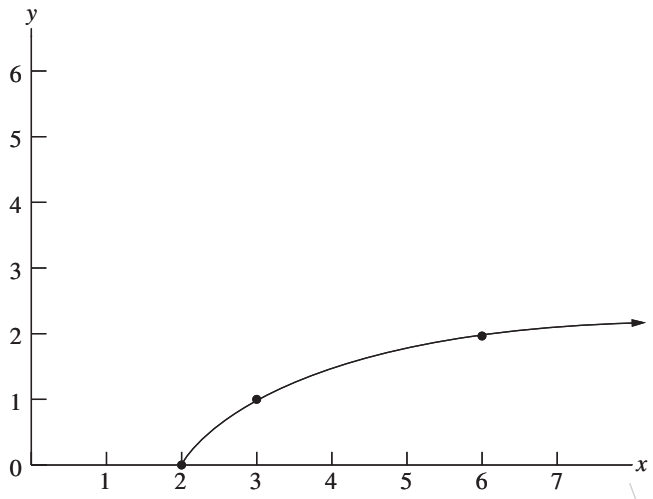
Restriction: $x - 2 \geq 0$

$x \geq 2$

x	2	3	6	11
y	0	1	2	3

D_f: $x \geq 2; x \in \mathbb{R}$

R_f: $y \geq 0; y \in \mathbb{R}$



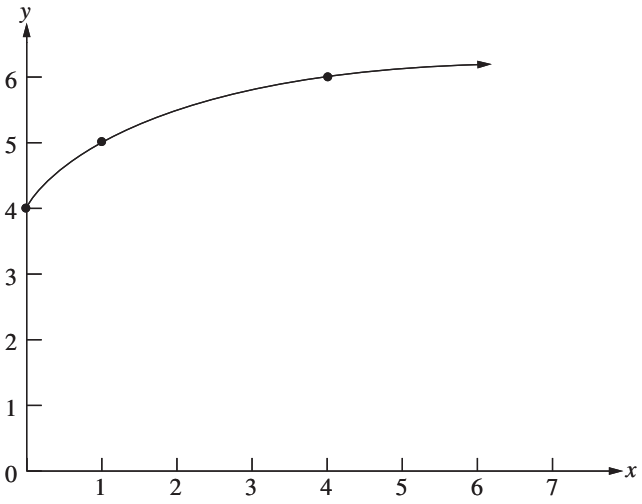
5. Sketch graph of $f(x) = \sqrt{x+4}$

Restriction: $x \geq 0$

x	0	1	4
y	4	5	6

D_f: $x \geq 0; x \in \mathbb{R}$

R_f: $y \geq 4; y \in \mathbb{R}$



Now do Activity 2.

In summary:

If we reflect in the y-axis; $x \leftrightarrow -x$

If we reflect in the x-axis; $y \leftrightarrow -y$

If we reflect in the line $y = x$; $x \leftrightarrow y$

Activity 1

1. Solve for x:
   (a) $2 = x - \sqrt{2x-1}$
   $\sqrt{x+6} - x = 4$
   $x - 2\sqrt{x-3} = 3$
   (b) $\sqrt{11-x} - x = 1$
   (c) $\sqrt{5x-1} + 1 - 2x = 0$
   (d) $x + \sqrt{x-2} = 4$
   (e) $\sqrt{10-3x} = x - 2$
   (f) $x\sqrt{x-2} - 5x - 3\sqrt{x-2} + 15 = 0$
2. If given $\sqrt{8-2x} = \frac{x}{2} + 1$
   (a) Show that $-2 \leq x \leq 4$
   (b) Solve for x
   (c) Without any further calculation, solve the equation
   $-\sqrt{8-2x} = \frac{x}{2} + 1$
3. Solve for x. $\sqrt{x-5} - 4 = \frac{12}{\sqrt{x-5}}$
4. Solve for x: $\sqrt{\frac{12x+8}{x-1}} = 3 - \sqrt{\frac{x-1}{3x+2}}$

Activity 2

(Investigation)

1. Sketch the graph of $y = \sqrt{-x}$ and write down the domain and range.
2. If $m = \sqrt{n}$ give all possible real values of m and all possible real values of n.
3. If $a = -\sqrt{-c}$ give all possible real values of a and all possible real values of c.