ALGEBRA

Simplifying algebraic expressions

Learning Outcomes and Assessment Standards

Assessment standards
(LO2 AS 11.2.4 (b))

Overview

In this lesson you will:

- Revise Grade 10 Algebra.
- Multiply and divide algebraic fractions.
- Add and subtract algebraic fractions.

A. Simplification of fractions

Rule

- Factorise the denominator
- Factorise the numerator
- Cancel the common fractions

Prior knowledge examples

Simplify the following:

1. $\frac{x^2 - x - 6}{x^2 - 9}$

$= \frac{(x-3)(x+2)}{(x-3)(x+3)}$ Factorise numerator and denominator

$= \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x+3)}$ Cancel

$= \frac{x+2}{x-3}$ This is the simplified form

Pitfall

$\frac{\cancel{x^2} - x - 6}{\cancel{x^2} - 9}$

$= \frac{-(x+6)}{-9}$

$= \frac{x + \cancel{6}^{2}}{\cancel{9}_{3}}$

$= \frac{x+2}{3}$

One cannot cross cancel over terms!

$\frac{x+2}{x-3}$ does not cancel to give $\frac{\cancel{x}+2}{\cancel{x}-3} = -\frac{2}{3}$ since one cannot cancel over a term.

That is why we factorise the expression first and then cancel common factors.

2. $\frac{2(x-2)(x+1) + 3(x-2)}{6x^2 + 13x - 5}$ Take out the common factor $(x - 2)$

$= \frac{(x-2)[2(x+1)+3]}{(3x-1)(2x+5)}$ Distribute and simplify in the square brackets

$= \frac{(x-2)(2x+5)}{(3x-1)(2x+5)}$

$= \frac{(x-2)\cancel{(2x+5)}}{(3x-1)\cancel{(2x+5)}}$ Cancel

$= \frac{x-2}{3x-1}$

Example 1

$$\frac{2x^2 + 2ax - 3xy - 3ay}{2x^2 + 3xy - 9y^2}$$

$$= \frac{2x(x + a) - 3y(x + a)}{(2x - 3y)(x + 3y)}$$ (Factorise numerator by grouping in pairs)

$$= \frac{(x + a)\cancel{(2x - 3y)}}{(2x - 3y)\cancel{(x + 3y)}}$$ (Take out the common factor $(x + a)$ in numerator and cancel)

$$= \frac{x + a}{x + 3y}$$

$$\frac{2x^2 + 2ax - 3xy - 3ay}{2x^2 + 3xy - 9y^2}$$

$$= \frac{x(2x - 3y) + a\,(2x - 3y)}{(2x - 3y)(x - 3y)}$$

$$= \frac{\cancel{(2x - 3y)}(x + a)}{\cancel{(2x - 3y)}/(x + 3y)}$$

$$= \frac{x + a}{x + 3y}$$

Example 2 (Additional practise for you)

$$\frac{2x^2 - x - 3}{x^2 + x} \times \frac{3x^2 - x - 4}{6x^2 - 17x + 12} \div \frac{2x + 2}{x^2}$$

$$= \frac{(2x - 3)(x + 1)}{x(x + 1)} \times \frac{(x + 1)(3x - 4)}{(2x - 3)(3x - 4)} \times \boxed{\frac{x^2}{2(x + 1)}}$$ Factorise

$$= \frac{\cancel{(2x - 3)}\cancel{(x + 1)}}{\cancel{x}\cancel{(x + 1)}} \times \frac{\cancel{(x + 1)}\cancel{(3x - 4)}}{\cancel{(2x - 3)}\cancel{(3x - 4)}} \times \frac{x^2}{2\cancel{(x + 1)}}$$ (Cancel)

$= x$

To turn the ÷ sign into a × sign, we reciprocate the fraction.

Important facts to remember:

since $(y - x)^2 = (x - y)^2$

and $x + y = y + x$,

so we see that $\frac{x + y}{y + x} = 1$

and $\frac{(x - y)^2}{(y - x)^2} = 1$

but $\frac{x - y}{y - x} = -1$

To change the sign of a fraction, we simply remove the negative, so

$$\frac{p - q}{q - p} = \frac{-(q - p)}{(q - p)} = 1$$

Example 1

$$\frac{x^2 - 6x + 8}{3x + 9} \times \frac{3 + x}{2 - x}$$

$$= \frac{(x - 4)\cancel{(x - 2)}}{3\cancel{(x + 3)}} \times \frac{\cancel{x + 3}}{\cancel{-(x - 2)}}$$ } $x + 3 = 3 + x$ and $2 - x = -(x - 2)$

$$= \frac{x - 4}{3}$$

Example 2 (For you to practise)

$$\frac{4 - 9x^2}{6x^2 - x - 2}$$

$$= \frac{(2 - 3x)(2 + 3x)}{(3x - 2)(2x + 1)}$$

$$= \frac{^{(-1)}\cancel{(2 - 3x)}(2 + 3x)}{\cancel{(3x - 2)}(2x + 1)}$$ [Since $\frac{(2 - 3x)}{(3x - 2)} = \frac{-\cancel{(3x - 2)}}{\cancel{(3x - 2)}} = -1$]

$$= \frac{(-1)(2 + 3x)}{(2x + 1)}$$

$$= \frac{-2 - 3x}{2x + 1}$$

Worksheet 1

Simplify:

1. $\dfrac{x^2 - x - 6}{18 - 6x}$

2. $\dfrac{9 - 12x + 4x^2}{2x^2 - x - 3} \times \dfrac{4 + x - 3x^2}{6x^2 - 17x + 12}$

3. $\dfrac{(x - 1) - 2(1 - x)^2}{2x^2 + x - 6}$

4. $\dfrac{p^2 - 2p + 15}{6p - 12} \times \dfrac{2 - p}{5 - p}$

5. $\dfrac{xy - 3y + 4x - 12}{3 - x} \times \dfrac{2y}{4y + y^2}$

B. Addition and subtraction of fractions

Rule

- Find the LCM of the denominators (LCD).

Example 1

Write the following as a fraction of the form $\frac{p}{q}$:

$\dfrac{3}{a} + \dfrac{a}{a - 3}$

$= \dfrac{3(a - 3) + a^2}{a(a - 3)}$ (LCD $= a(a - 3)$)

$= \dfrac{3a - 9 + a^2}{a(a - 3)}$

$= \dfrac{a^2 + 3a - 9}{a(a - 3)}$

Example 2

$\dfrac{3}{x + 2} - \dfrac{4}{1 - x} + 2$

$= \dfrac{3}{x + 2} + \dfrac{4}{x - 1} + 2$ Since $1 - x = -(x - 1)$

$= \dfrac{3(x - 1) + 4(x + 2) + 2(x - 1)(x + 2)}{(x + 2)(x - 1)}$ LCD $= (x + 2)\cdot(x + 1)$

$= \dfrac{3x - 3 + 4x + 8 + 2(x^2 + x - 2)}{(x^2 + x - 2)}$

$= \dfrac{7x + 5 + 2x^2 + 2x - 4}{x^2 + x - 2}$

$= \dfrac{2x^2 + 9x + 1}{x^2 + x - 2}$

Example 3

Sometimes we need to factorise denominators first before finding the LCD.

The next example deals with this.

$\dfrac{4}{x^2 - 9} + \dfrac{3}{(x+ 3)^2} - \dfrac{1}{3 - x}$

$= \dfrac{4}{(x - 3)(x + 3)} + \dfrac{3}{(x + 3)^2} + \dfrac{1}{x - 3}$ Remember: $3 - x = -(x - 3)$

$= \dfrac{4(x + 3) + 3(x - 3) + (x + 3)^2}{(x - 3)(x + 3)^2}$ LCD $= (x + 3)^2(x - 3)$

$= \dfrac{4x + 12 + 3x - 9 + x^2 + 6x + 9}{(x - 3)(x + 3)^2}$

$= \dfrac{x^2 + 13x + 12}{(x - 3)(x + 3)^2}$

Now do worksheet 2.

Worksheet 2

Write as a single fraction:

1. $\frac{1}{y-2}+\frac{3}{3-y}-\frac{4}{(3-y)^2}$
2. $\frac{3x}{x+2}-\frac{2}{5-2x}+\frac{4x}{2}$
3. $\frac{6}{9-x^2}-\frac{3}{(x+3)^2}+\frac{1}{x-3}$
4. $\frac{3y+2x}{y+x}-\frac{3}{3y^2+5xy+2x^2}$
5. $x+\frac{1}{x-1}+\frac{2}{2-x}$
6. $\frac{2}{m^2-2m+1}+\frac{3}{1-m}$

Write as a single fraction:

$\frac{3}{x-1}+\frac{2}{x}-\frac{4}{3x}$

$=\frac{9x+6(x-1)-4(x-1)}{3x(x-1)}$

$=\frac{9x+6x-6-4x+4}{3x(x-1)}$

$=\frac{11x-2}{3x(x-1)}$

We keep the denominators when we work with an expression

But

Solve for x: (Multiply each term by the LCD)

$\frac{3}{x-1}+\frac{2}{x}=\frac{4}{3x}$

LCD $= 3x(x-1)$

So LCD $3x(x-1)\neq 0$

When we solve an equation we can get rid of the denominator, ***provided that we state*** our restrictions!!

$\therefore\ x\neq 0$ or $x\neq 1$

$\therefore\ \frac{3}{x-1}\times 3x(x-1)+\frac{2}{x}\times 3x(x-1)=\frac{4}{3x}\times 3x(x-1)$

$\therefore\ 9x+6(x-1)=4(x-1)$

$\therefore\ 9x+6x-6=4x-4$

$\therefore\ 11x=2$

$\therefore\ x=\frac{2}{11}$ (which is not 0 nor 1)

Worksheet 3

Practise

1. Simplify: $\frac{2}{2x-3}+\frac{4}{x}$
2. Solve for x: $\frac{2}{2x-3}=\frac{4}{x}$
3. Simplify: $2+\frac{5}{x-1}$
4. Solve for x: $2=\frac{5}{x-1}$
5. Simplify: $\frac{3}{2x}+\frac{4}{3x}-\frac{1}{x+3}$
6. Solve for x: $\frac{3}{2x}+\frac{4}{3x}=\frac{1}{x+3}$
7. Simplify: $\frac{2}{x+1}+\frac{3}{1-x}$
8. Solve for x: $\frac{2}{x+1}+\frac{3}{1-x}=0$