STUDY Mate

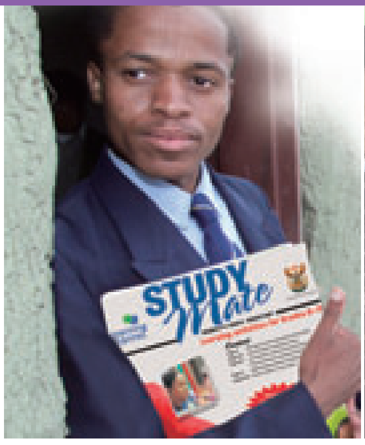
GRADE 12

ISSUE 12 2008

Mathematics · Mathematical Literacy · English FAL · Afrikaans FAL · Physical Science · Life Sciences · Accounting · Business Studies · History · Geography

learning channel

150 DAYS before final exams!



Dear Learners

I am glad to once again have the opportunity to speak to you through STUDYMATE. This is the 12th edition of Study Mate and as you know time is flying. This means you should be very advanced in your preparations for the final examinations.

In this edition we have a contribution from well known role model Mr Mark Shuttleworth. He will share with you his thoughts on why it is so important to invest time and effort in your education today.

If you have not yet done so, get hold of the first 11 issues of Study Mate and start studying today. The hard work and effort you put in today will assist you to succeed at the end of the year.

Study Mate will continue to provide you with tests and examination questions so that you can track your progress in your preparation for the end of year examination. It will also provide tips and hints on how to study. It also has a countdown so you can see exactly how many days you have before the examinations begin.

Make sure you have a study timetable, somewhere to study, textbooks and resources as well as stationery. Maintain a healthy diet and find time to relax in between all the studying. Talk to your classmates, friends and family for tips and encouragement.

Remember, this is your Study Mate so write to us and let us know what you would like to see in Study Mate and send us your test answers.

I will write to you again in the near future. In the meantime, work hard and keep focused on the road ahead. I think of you all everyday.

My warmest best wishes,

Mrs Naledi Pandor, MP
Minister of Education

education

Department:
Education
REPUBLIC OF SOUTH AFRICA

READ ...what **Mark Shuttleworth** has to say to our readers!

"I'd like to inspire you to spend a little extra time on Maths and Science this year. Studies have shown, in several different countries, that the amount of time we spend on Maths and Science at school directly correlates to income levels later on in life. It's not that calculus will make you rich – it's that studying Maths and Science will help you develop analytical skills that will make you more effective whatever your choice of career later on in life.

So whether you want to be an artist or a scientist, whether you want to go into business, government or the academic life, whether you want to be a rock star or a geologist –some extra time on Maths and science today will give you a better chance of success. And if you don't yet know exactly what you want to do in life, Maths and Science will keep the most doors open for you to choose from later. Mark"

Miss F Ally a Grade 11 student sent her answers to the Department of Education and we sent her a diary and two work books. She responded with this letter:

" I was overjoyed on receiving the letter, diary and work books. I make it my duty every Tuesday to get the Study Mate as I really enjoy working with them. I will continue using them till the end – I am working hard and wish to achieve excellent results. I will also try to attempt the Grade 12 papers to extend my knowledge."

HIGHLIGHTS FOR MAY 2008

DID YOU KNOW?

1. On the 23rd of May 2008, 83 schools received cash rewards from Standard Bank for increasing the number of learners who passed Mathematics on the Higher Grade in 2007 as compared to 2006. Over R1 million was paid out to the schools.

   Study Mate would like to congratulate all these schools on their effort and hard work. Mbilwi Secondary School in Limpopo increased their Mathematics Higher Grade passes by 65 passes from 52 in 2006 to 117 in 2007. Greenbury Secondary in KwaZulu Natal had 41 more passes from 27 in 2006 to 68 in 2007. Star College also in KwaZulu Natal, increased their passes by 28 from 6 in 2006 to 34 in 2007.
2. On 27th of May 2008, 650 schools received 120 000 Mathematics workbooks from Liberty Life and Independent Newspapers
3. On the 27th of May 2008, 9 schools received awards at the National Science and Technology Forum Awards (NSTF).

MATHEMATICS

SAMPLE EXAMINATION PAPER – PAPER 1

TIME: 3 HOURS
[150 MARKS]

QUESTION 1

1.1 The first term of a geometric sequence is $a^{10}b^{10}$ and the third term is b^{-10}. Write all your answers with positive exponents.

1.1.1 Write down the nth term of this sequence. (3)

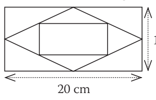
1.1.2 If this sequence converges, find the sum to infinity of this sequence. (3)

1.1.3 Write down, in terms of a and b, the condition for this sequence to be convergent. (2)

1.2 Calculate the value of $\sum_{i=1}^{31} 31i - 31$. (3)

1.3 The first three terms of an arithmetic sequence form a Pythagorean triplet. Show that $d = \frac{a}{3}$. (7)

1.4 The midpoints of a rectangle with a length of 20 cm and a breadth of 10 cm are joined to form a rhombus. The midpoints of the sides of the rhombus are joined to form a rectangle, as shown in the diagram on the right. This process is repeated indefinitely.



1.4.1 Calculate the sum of the areas of the rectangles. (5)

1.4.2 Hence calculate the sum of the areas of the rhombuses. (2)

[25]

QUESTION 2

2.1 Solve for x (correct to three decimal places, where appropriate) in each of the following:

2.1.1 $x^1 = \log 1$ (1)

2.1.2 $x^{10} = \log 10$ (2)

2.1.3 $x^{100} = \log 100$ (2)

2.1.4 $9^{27x} \times 27^{9x} = 927$ (2)

2.2 If $\log 3 = p$ and $\log 5 = q$, express the following in terms of p and q:

2.2.1 $\log 2$ (2)

2.2.2 $\log 30$ (2)

2.2.3 $\log_5 45$ (2)

2.2.4 1,6 (2)

2.3 If $f(x) = \log \frac{x}{(x-1)}$, simplify the following without the use of a calculator:

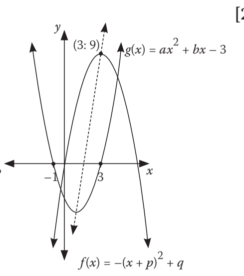
$f(11) + f(12) + f(13) + \ldots + f(98) + f(99) + f(100)$ (2)

2.4 Calculate the future value of a 15-year annuity with a monthly payment of R150 and an interest rate of 15% p.a., compounded monthly. (3)

[20]

QUESTION 3

3.1 The graphs on the right are those of the functions $f(x) = -(x+p)^2 + q$ and $g(x) = ax^2 + bx - 3$.



3.1.1 Find the values of p and q and hence the exact rule for f. (3)

3.1.2 Find the values of a and b and hence the exact rule for g. (3)

3.1.3 Find the coordinates of the turning point of $g(x)$. (2)

3.1.4 For what value(s) of k will $-(x+p)^2 + k = 0$ have equal roots? (1)

3.1.5 Determine the equation of the straight line through the turning points of the two parabolas. (2)

3.2.1 Copy and complete this table: (3)

x	–3	–2	–1	0	1	2	3
$y = (\frac{3}{2})^x$							

3.2.2 Sketch the graph of $f(x) = (\frac{3}{2})^x$. Use a scale of 1 cm per unit on both axes. (3)

3.2.3 Write down the rule for f^{-1}, the inverse of $f(x)$, and draw its graph on the same set of axes as in **3.2.2**. (3)

3.2.4 On the same set of axes draw the graph of $g(x) = -2(\frac{3}{2})^x + 1$. Indicate its asymptote. (3)

3.2.5 Write down the equation of an exponential function of which the graph is the result of shifting the graph of $g(x)$ three units to the right. (1)

3.3 The number of mosquitoes in a certain region of Africa depends on the rainfall in January of a given year. The function $N(x) = 250x - x^2$ gives the approximate number, $N(x)$, of thousands of mosquitoes when the rainfall is x mm in January.

3.3.1 Determine the rainfall that will produce the maximum number of mosquitoes, as well as the corresponding maximum number of mosquitoes. (3)

3.3.2 It is estimated that the number of cases of malaria caused by the maximum number of mosquitoes in that region is given by the formula $M(t) = 3\log_2 t + 5$, where t is the number of months following the January rain season. Calculate the number of cases of malaria after:
i 1 month **i** 2 months **iii** 4 months. (3)

[30]

QUESTION 4

4.1 Solve for x:

4.1.1 $x + \sqrt{x-2} = 2$ (3)

4.1.2 $(2x-3)^2 < 4$ (3)

4.1.3 $x^2 + 2 + \frac{12}{x^2+2} = 7$ (3)

4.1.4 $\frac{2x+1}{x-1} \leq 1$ (2)

4.1.5 $3x^3 - 17x^2 + 18x + 8 = 0$ (3)

4.1.6 $x^2 - 5 < \frac{x^2-3}{x}$ (3)

4.2 Determine the values of a and b so that $x^2 - x - 6$ is a factor of $h(x) = ax^3 - 3x^2 - bx + 6$. (3)

[20]

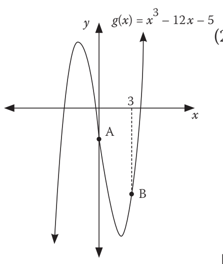
QUESTION 5

5.1 It is given that $f(x) = -3x + \frac{1}{x}$.

5.1.1 Determine $f'(x)$ from first principles. (3)

5.1.2 Calculate $f(-1) + f'(-1)$. (1)

5.2 Determine:



5.2.1 $\lim_{s \to 2} \frac{s^4 - 16}{s - 2}$ (2)

5.2.2 $\frac{dy}{dx}$ if $f(x) = -3x^3 - \frac{\sqrt[3]{x}}{x\sqrt{x}} - 4\pi^3$ (2)

5.2.3 $D^x(x^2 - \sqrt{x})^2$ (2)

5.3 The sketch graph represents the function $g(x) = x^3 - 12x - 5$.

5.3.1 Determine the equation of the tangent to the curve at A. (1)

5.3.2 Determine the equation of the tangent to the curve at B where $x = 3$. (2)

5.3.3 Determine the coordinates of the points of contact of tangents of the form $y = -9x + c$ to the curve. (2)

[15]

QUESTION 6

6.1 Given the function $f(x) = x^3 - x^2 - x + 1$.

6.1.1 Prove that $x + 1$ is a factor of $f(x)$ and find the other factors of $f(x)$. (3)

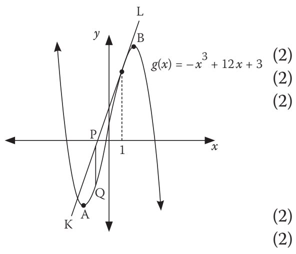
6.1.2 Write down the x- and y-intercepts of the graph of $y = f(x)$. (2)

6.1.3 Determine the stationary points of $f(x)$. (2)

6.1.4 Show that the sign of $f'(x)$ changes at each stationary point and say what kind of turning point each is. (2)

6.1.5 Sketch the graph of $y = f(x)$. (2)

6.2 On the right is the sketch graph of $g(x) = -x^3 + 12x + 3$.



Determine:

6.2.1 the co–ordinates of A and B. (2)

6.2.2 the equation of the tangent KL. (2)

6.2.3 the maximum length of PQ between A and B. (2)

6.3 A sports shop calculates that they will sell 1 000 mountain bikes if they charge R600 per bike. For each R20 decrease in the price, an additional 100 bikes can be sold.

6.3.1 Find an expression for the total income if there are x R20-decreases in the price. (2)

6.3.2 Find the value of x that maximises the total income. (2)

6.3.3 Find the maximum income. (1)

[22]

QUESTION 7

7.1 Beach Leisure specialises in hand-made beach articles. It makes beach umbrellas and deck chairs. A beach umbrella uses 2 m of canvas and takes 2 hours to make, whereas a deck chair uses 1,5 m of canvas and takes 3 hours to make. Every week, Beach Leisure has a total of 600 m of canvas and 900 man-hours of labour available. Canvas costs R20/m. The other raw materials for an umbrella and a deck chair cost R35 and R45 respectively. A beach umbrella sells for R150 and a deck chair sells for R135. Beach Leisure has a standing order for 40 deck chairs and 24 beach umbrellas per week. Let x and y be the number of beach umbrellas and deck chairs respectively that Beach Leisure makes per week.

7.1.1 Write down the constraints of this situation in terms of x and y. (3)

7.1.2 Draw a neat graph showing these constraints and shade the feasible region. (5)

7.1.3 Find the coordinates of the vertices of the feasible region. (4)

7.1.4 Write down the equation to calculate the weekly profit in terms of x and y. (1)

7.1.5 What is Beach Leisure's maximum weekly profit? (4)

7.1.6 How many beach umbrellas and deck chairs must Beach Leisure make per week to make a maximum profit? (1)

[18]

TOTAL: [150 marks]

MEMORANDUM

QUESTION 1

1.1.1 $a = a^{10}b^{10}$ and $ar^2 = b^{-10}$

$\therefore r^2 = \frac{b^{-10}}{a^{10}b^{10}} = a^{-10}b^{-20}$

$\therefore r = a^{-5}b^{-10}$

$\therefore T_n = ar^{n-1} = a^{10}b^{10}(a^{-5}b^{-10})^{n-1} = a^{10}b^{10}a^{-5n}a^5b^{-10n}b^{10} = \frac{a^{15}b^{20}}{a^{5n}b^{10n}}$

1.1.2 $S_\infty = \frac{a}{1-r} = \frac{a^{10}b^{10}}{1-a^{-5}b^{-10}} = \frac{a^{10}b^{10}}{1-\frac{1}{a^5b^{10}}} = \frac{a^{15}b^{20}}{a^5b^{10}-1}$

1.1.3 $-1 < \frac{1}{a^5b^{10}} < 1$

1.2 $a = 31 - 31 = 0$

$d = 31$

$S_{31} = \frac{31}{2}[2(0) + (31-1)31] = 15{,}5(30)(31) = 14\,415$

1.3 The numbers will be a; $a + d$; a + $2d$.

The numbers form a Pythagorean triplet, so:

$(a+2d)^2 = a^2 + (a+d)^2$

therefore $a^2 + 4ad + 4d^2 = a^2 + a^2 + 2ad + d^2$

therefore $-a^2 + 2ad + 3d^2 = 0$

therefore $a^2 - 2ad - 3d^2 = 0$

therefore $(a - 3d)(a + d) = 0$

therefore $d = \frac{a}{3}$ or $d = -a$

If $d = \frac{a}{3}$, the numbers are: a; $a + \frac{a}{3}$; $a + \frac{2a}{3}$.

If $d = -a$, the numbers are: a; 0, $-a$, which is not a Pythagorean triplet, so $d = \frac{a}{3}$.

1.4.1 The area of the first rectangle = 20 cm × 10 cm = 200 cm².
The area of the second rectangle = 10 cm × 5 cm = 50 cm².
The areas of the rectangles form a geometric sequence with $a = 200$ and $r = \frac{1}{4}$.
$S = \frac{a}{1-r} = \frac{200}{1-\frac{1}{4}} = \frac{200}{\frac{3}{4}} = 266{,}6$ cm².

1.4.2 The area of the first rhombus = $\frac{1}{2}$ (product of the diagonals) = $\frac{1}{2}(20 \times 10)$ cm²
= $\frac{1}{2}$ (area of the first rectangle). Continuing this logic, the sum of the areas of the rhombuses
= $\frac{1}{2}$ (the sum of the areas of the rectangles) = 133,3 cm².

QUESTION 2

2.1.1 $x = 0$

2.1.2 $x^{10} = \log 10$

$\therefore x^{10} = 1$

$\therefore x = \pm 1$

2.1.3 $x^{100} = \log 100 = 2$

$\therefore 100 \log x = \log 2$

$\therefore \log x = \frac{\log 2}{100}$

$\therefore x = \pm 1{,}007$

2.1.4 $9^{27x} \times 27^{9x} = 927$

$\therefore 3^{54x} \times 3^{27x} = 927$

$\therefore 3^{81x} = 927$

$\therefore 81x \log 3 = \log 927$

$\therefore x = \frac{\log 927}{81 \log 3}$

$\therefore x = 0{,}077$

2.2.1 $\log 2 = \log \frac{10}{5} = \log 10 - \log 5 = 1 - q$

2.2.2 $\log 30 = \log(3 \times 10) = \log 3 + \log 10 = p + 1$

2.2.3 $\log_5 45 = \log_5(9 \times 5) = \log_5 9 + \log_5 5 = \log_5 3^2 + 1 = 2\log_5 3 + 1 = 2\left(\frac{\log 3}{\log 5}\right) + 1 = \;+ 1 = 2\frac{p}{q} + 1$

2.2.4 $\log 1{,}6 = \log \frac{5}{3} = \log 5 - \log 3 = q - p$

2.3 $f(11) + f(12) + f(13) + \ldots + f(99) + f(100)$
$= \log 11 - \log 10 + \log 12 - \log 11 + \log 13 - \log 12 + \ldots + \log 99 - \log 98 + \log 100 - \log 99$
$= \log 100 - \log 10 = 2 - 1 = 1$

2.4 $FV = \frac{150[(1 + \frac{0{,}15}{12})^{180} - 1]}{\frac{0{,}15}{12}} = R100\,276{,}01$

QUESTION 3

3.1.1 T.P. of $f(x) = -(x+p)^2 + q$ is (3; 9) $\therefore p = -3$ and $q = 9$.
Thus $f(x) = -(x-3)^2 + 9$.

3.1.2 $g(x) = ax^2 + bx - 3$.
The x–intercepts (–1; 0) and (3; 0) come from the equation
$(x+1)(x-3) = 0 \therefore x^2 - 2x - 3 = 0$
Compare with $ax^2 + bx - 3 = 0$
Thus $a = 1$ and $b = -2$ and $g(x) = x^2 - 2x - 3$.

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MATHEMATICS

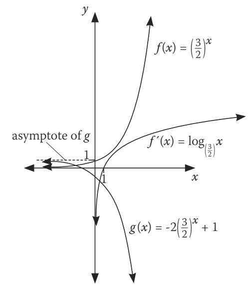
3.1.3 T.P.: $x = \frac{-(-2)}{2} = 1$ and $g(1) = 1 - 2 - 3 = -4$
Thus the T.P. is $(1; -4)$.

3.1.4 $-(x + p)^2 + k = 0$ will have equal roots if $k = 0$.

3.1.5 Let $y = mx + c$ be the equation of the line through $(1; -4)$ and $(3; 9)$.
$m = \frac{[9 - (-4)]}{[3 - 1]} = \frac{[13]}{2} = 6{,}5$
$\therefore y = 6{,}5x + c$ which must be satisfied by $(1; -4)$,
$\therefore 6{,}5(1) + c = -4$
$\therefore c = -10{,}5$
The equation is $y = 6{,}5x - 10{,}5$

3.2.1

x	-3	-2	-1	0	1	2	3
$y = (\frac{3}{2})^x$	$\frac{8}{27} \approx 0{,}3$	$\frac{4}{9} \approx 0{,}4$	$\frac{2}{3} \approx 0{,}7$	1	$\frac{3}{2} = 1{,}5$	$\frac{9}{4} = 2{,}25$	$\frac{27}{8} \approx 3{,}4$



3.2.2 $f(x) = (\frac{3}{2})^x$. See graph on the right.

3.2.3 $f^{-1}(x) = \log_{\frac{3}{2}} x$. See graph on the right.

3.2.4 $g(x) = -2(\frac{3}{2})^x + 1$. See graph on the right.

3.2.5 $h(x) = -2(\frac{3}{2})^{x-3} + 1$

3.3.1 $N(x) = 250x - x^2$ is a quadratic function with a maximum value at its T.P.
$x = \frac{-b}{2a} = \frac{-250}{2(-1)} = 125$
$N(150) = 250(125) - (125)^2 = 31\,250 - 15\,625 = 15\,625$.
Thus a maximum number of 15 625 mosquitoes are produced if the rainfall is 125 mm.

3.3.2 $M(t) = 3 \log_2 t + 5$, the number of cases of malaria after t months.
i $M(1) = 3 \log_2(1) + 5 = 3 \times 0 + 5 = 5$ cases
ii $M(2) = 3 \log_2(2) + 5 = 3 \times 1 + 5 = 8$ cases
iii $M(4) = 3 \log_2(4) + 5 = 3 \times 2 + 5 = 11$ cases

QUESTION 4

4.1.1 $x + \sqrt{x - 2} = 2 \therefore \sqrt{x - 2} = 2 - x$
$\therefore x - 2 = (2 - x)^2$
$\therefore x - 2 = 4 - 4x + x^2$
$\therefore x^2 - 5x + 6 = 0$
$\therefore (x - 2)(x - 3) = 0$
$\therefore x = 2$ or $x = 3$
Check: $x = 2$: LHS $= 2 + \sqrt{2 - 2} = 2 =$ RHS
$x = 3$: LHS $= 2 + \sqrt{3 - 2} \neq$ RHS
$\therefore x = 2$

4.1.2 $(2x - 3)^2 < 4$
$\therefore 4x^2 - 12x + 9 < 4$
$\therefore 4x^2 - 12x + 5 < 0$
$\therefore (2x - 5)(2x - 1) < 0$
$\therefore \frac{1}{2} < x < \frac{5}{2}$

$(2x - 5)$	$-$		$-$		$+$
$(2x - 1)$	$-$		$+$		$+$
$(2x - 5)(2x - 1)$	$+$	$\frac{1}{2}$	$-$	$\frac{5}{2}$	$+$

4.1.3 $x^2 + 2 + \frac{12}{x^2 + 2} = 7$
Let $k = x^2 + 2$. Then:
$x^2 + 2 + \frac{12}{[x^2 + 2]} = 7$
$\therefore k + \frac{12}{k} = 7$
$\therefore k^2 + 12 = 7k$
$\therefore k^2 + 12 - 7k = 0$
$\therefore (k - 3)(k - 4) = 0$
$\therefore k = 3$ or $k = 4$
$\therefore x^2 + 2 = 3$ or $x^2 + 2 = 4$
$\therefore x^2 = 1$ or $x^2 = 2$
$\therefore x = \pm 1$ or $x = \pm\sqrt{2}$

4.1.4 $\frac{2x + 1}{x - 1} \leq 1 \therefore \frac{2x + 1}{x - 1} - 1 \leq 0$
$\therefore \frac{2x + 1 - x + 1}{x - 1} \leq 0$
$\therefore \frac{x + 2}{x - 1} \leq 0$
$\therefore x \geq -2$ and $x < 1$
$\therefore -2 \leq x < 1$

$(x + 2)$	$-$		$-$		$+$
$(x - 1)$	$-$		$+$		$+$
$\frac{(x + 2)}{(x - 1)}$	$+$	-2	$-$	1	$+$

4.1.5 Let $f(x) = 3x^3 - 17x^2 + 18x + 8$
Now $f(2) = 3(2)^3 - 17(2)^2 + 18(2) + 8 = 0$
$\therefore (x - 2)$ is a factor.
$3x^3 - 17x^2 + 8x + 8 = (x - 2)(3x^2 - 11x - 4)$
$= (x - 2)(x - 4)(3x + 1)$
$\therefore (x - 2)(x - 4)(3x + 1) = 0$
$\therefore x = 2$ or $x = 4$ or $x = -\frac{1}{3}$

4.1.6 $x^2 - 5 < \frac{x^2 + 3}{x}$
$\therefore x^2 - 5 - \frac{x^2 + 3}{x} < 0$
$\therefore \frac{x^3 - 5x - x^2 - 3}{x} < 0$
Let $f(x) = x^3 - 5x - x^2 - 3$.
Now $f(3) = (3)^3 - 5(3) - (3)^2 - 3 = 0$
and $x^3 - x^2 - 5x + 3 = (x - 3)(x^2 + 2x + 1) = (x - 3)(x + 1)^2$
$\therefore \frac{(x - 3)(x + 1)^2}{x} < 0$
Now $(x + 1)^2 > 0$ for all x.
From the sign graph of $\frac{x - 3}{x} < 0$, the solution is $0 < x < 3$.

4.2 $x^2 - x - 6 = (x - 3)(x + 2)$.
$(x + 2)$ is a factor of $h(x) = ax^3 - 3x^2 - bx + 6$, so $h(-2) = 0$,
i.e. $a(-2)^3 - 3(-2)^2 - b(-2) + 6 = 0$
$\therefore -8a + 2b = 6$
$\therefore -4a + b = 3$ (1)
$(x - 3)$ is a factor of $h(x)$, so $h(3) = 0$
$\therefore a(3)^3 - 3(3)^2 - b(3) + 6 = 0$
$\therefore 27a - 3b = 21 \therefore 9a - b = 7$ (2).
Add (1) and (2).
Then $5a = 10$
$\therefore a = 2$
Substitute $a = 2$ into (1):
$-4(2) + b = 3$
$\therefore b = 11$

QUESTION 5

5.1.1 $f'(x) = \lim_{h \to 0} \frac{-3(x + h) + \frac{1}{x + h} + 3x - \frac{1}{x}}{h}$
$\lim_{h \to 0} \frac{-3h + \frac{1}{x + h} - \frac{1}{x}}{h}$
$= -3 + \lim_{h \to 0} \frac{x - (x + h)}{(x + h)(x)(h)}$
$= -3 + \lim_{h \to 0} \frac{-h}{(x + h)(x)(h)}$
$= -3 + \lim_{h \to 0} \frac{-1}{(x + h)x}$
$= -3 + \frac{-1}{x^2}$

5.1.2 $f(-1) + f'(-1) = -3(-1) + \frac{1}{-1} - 3 - \frac{1}{(1)^2}$
$= 3 - 1 - 3 - 1 = -2$

5.2.1 $\lim_{s \to 2} \frac{s^4 - 16}{s - 2}$
$= \lim_{s \to 2} \frac{(s^2 + 4)(s - 2)(s + 2)}{s - 2}$
$= \lim_{s \to 2} (s^2 + 4)(s + 2)$
$= 32$

5.2.2 $\frac{\sqrt[3]{x}}{x\sqrt{x}} = x^{\frac{1}{3} - 1 - \frac{1}{2}} = x^{-\frac{7}{6}}$
$f'(x) = -9x^2 - (-\frac{7}{6}x^{-\frac{13}{6}})$
$= -9x^2 + \frac{7}{6x^{\frac{13}{6}}}$

5.2.3 $(x^2 - \sqrt{x})^2 = x^4 - 2x^{\frac{5}{2}} + x$
$D_x(x^2 - \sqrt{x})^2 = 4x^3 - 5x^{\frac{3}{2}} + 1$

5.3 $g(x) = x^3 - 12x - 5 \therefore g'(x) = 3x^2 - 12$

5.3.1 Tangent at A: Let equation be $y = mx - 5$.
$m = g'(0) = -12$
Thus the equation is $y = -12x - 5$.

5.3.2 Tangent at B $(3; y)$: Let equation be $y = mx + c$.
$m = g'(3) = 3(3)^2 - 12 = 27 - 12 = 15$
$\therefore y = 15x + c$
$g(3) = 3^3 - 12(3) - 5 = 27 - 36 - 5 = -14$
$(3; -14)$ must satisfy $y = 15x + c$
$\therefore -14 = 15(3) + c$
$\therefore c = -14 - 45 = -59$.
Thus the equation is $y = 15x - 59$.

5.3.3 Tangents of the form $y = -9x + c$ have gradient $m = -9$
$\therefore g'(x) = 3x^2 - 12 = -9$
$\therefore 3x^2 = 3 \therefore x^2 = 1 \therefore x = \pm 1$
$g(1) = 1^3 - 12 - 5 = -16$
$g(-1) = (-1)^3 - 12(-1) - 5 = 6$
Thus the points of contact are $(-1; 6)$ and $(1; -16)$.

QUESTION 6

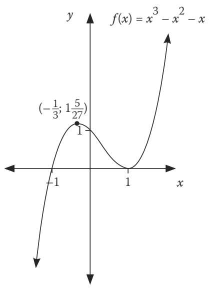
6.1 $f(x) = x^3 - x^2 - x + 1$

6.1.1 $f(1) = 1^3 - 1^2 - 1 + 1 = 0 \therefore x - 1$ is a factor of $f(x)$.
$f(x) \div (x - 1) = x^2 - 1 = (x + 1)(x - 1)$
$\therefore f(x) = (x + 1)(x - 1)^2$

6.1.2 x-intercepts: $(-1; 0)$ and $(1; 0)$; y-intercept is $(0; 1)$.

6.1.3 $f'(x) = 3x^2 - 2x - 1$
Solve $3x^2 - 2x - 1 = 0 \therefore (3x + 1)(x - 1) \therefore x = -\frac{1}{3}$ or $x = 1$
$f(-\frac{1}{3}) = (-\frac{1}{3})^3 - (-\frac{1}{3})^2 - (-\frac{1}{3}) + 1 = -\frac{1}{27} - \frac{1}{9} + \frac{1}{3} + 1 = 1\frac{5}{27}$ and $f(1) = 0$
Thus the stationary points are $(-\frac{1}{3}; 1\frac{5}{27})$ and $(1; 0)$.

6.1.4 Choose test points at $x = -1$, 0 and 2:
$f'(-1) = 3(-1)^2 - 2(-1) - 1 = 4 > 0$
$\therefore f(x)$ increases if $x < -\frac{1}{3}$,
$f'(0) = -1 < 0 \therefore f(x)$ decreases if $-\frac{1}{3} < x < 1$, and
$f'(2) = 3(2)^2 - 2(2) - 1 = 7 > 0$
$\therefore f(x)$ increases if $x > 1$.
Thus $\left(-\frac{1}{3}; 1\frac{5}{27}\right)$ is a local maximum T.P. and $(1; 0)$ is a local minimum T.P.



6.1.5 See graph on the right.

6.2 $g(x) = -x^3 + 12x + 3$

6.2.1 $g'(x) = -3x^2 + 12$
T.P.s where $g'(x) = -3x^2 + 12 = 0$
$\therefore x^2 = 4$
$\therefore x = \pm 2$
$g(-2) = -(-2)^3 + 12(-2) + 3 = 8 - 24 + 3 = -13$, and
$g(2) = -(2)^3 + 12(2) + 3 = -8 + 24 + 3 = 19$
Thus A is $(-2; -13)$ and B is $(2; 19)$.

6.2.2 Let the equation of the tangent be $y = mx + c$.
At the point of contact $g'(1) = -3(1)^2 + 12 = 9$
$\therefore m = 9$
$\therefore y = 9x + c$ which must be satisfied by $(1; 14)$:
$\therefore 14 = 9(1) + c$
$\therefore c = 5$
Thus the equation of the tangent is $y = 9x + 5$.

6.2.3 $PQ(x) = (9x + 5) - (-x^3 + 12x + 3) = x^3 - 3x + 2$
$\frac{d}{dx} PQ(x) = 3x^2 - 3$
$3x^2 - 3 = 0 \therefore x^2 = 1$
$\therefore x = \pm 1$ give stationary points.
$PQ(1) = (1)^3 - 3(1) + 2 = 1 - 3 + 2 = 0$
$PQ(-1) = (-1)^3 - 3(-1) + 2 = -1 + 3 + 2 = 4$
Thus the maximum length of PQ is 4 units when $x = -1$.

6.3.1 The price charged per bike will be $600 - 20x$.
The number of bikes sold will be $1\,000 + 100x$.
The total income will be $t(x) = (600 - 20x)(1\,000 + 100x)$
$= 600\,000 + 40\,000x - 2\,000x^2$

6.3.2 $t'(x) = 40\,000 - 4\,000x = 0$
$\therefore 40\,000 = 4\,000x \therefore x = 10$.
Let $x = 1$: $t'(1) = 36\,000 > 0$
Let $x = 20$: $t'(20) = 40\,000 - 80\,000 < 0$
Therefore $x = 10$ gives a maximum total income.

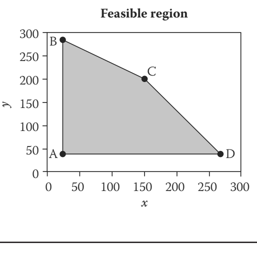
6.3.3 The maximum income will be:
$t(10) = 600\,000 + 40\,000(10) - 2\,000(10)^2 = 800\,000$
Therefore the maximum income is R800 000.

QUESTION 7

7.1.1 $x \geq 24$; $y \geq 40$; $2x + 1{,}5y \leq 600$; $2x + 3y \leq 900$.

7.1.2 See graph on the right.

Feasible region



7.1.3 A: $(24; 40)$
B: $x = 24$; $2x + 3y = 900$
$\therefore 2(24) + 3y = 900$
$\therefore 48 + 3y = 900$
$\therefore 3y = 900 - 48 = 852$

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MATHEMATICS

$\therefore y = \frac{852}{3} = 284$
$\therefore$ B: (24; 284)
C: $2x + 3y = 900$; $2x + 1{,}5y = 600$
$\therefore 1{,}5y = 300$
$\therefore y = 200$
$\therefore 2x + 3(200) = 900$
$\therefore 2x = 900 - 600 = 300$
$\therefore x = 150$
$\therefore$ C: (150; 200)
D: $y = 40$; $2x + 1{,}5y = 600$
$\therefore 2x + 1{,}5(40) = 600$
$\therefore 2x + 60 = 600$
$\therefore 2x = 600 - 60 = 540$
$\therefore x = 270$
$\therefore$ D: (270; 40)

7.1.4 $P = 150x - 2(20)x - 35x + 135y - 1{,}5(20)y - 45y = 75x + 60y$

7.1.5 $P_A = 75(24) + 60(40) = 1\,800 + 2\,400 = 4\,400$
$P_B = 75(24) + 60(284) = 1\,800 + 17\,040 = 18\,840$
$P_C = 75(150) + 60(200) = 11\,250 + 12\,000 = 23\,250$
$P_D = 75(270) + 60(40) = 20\,250 + 2\,400 = 22\,650$
Maximum profit = 23 250.

7.1.6 Beach Leisure must make 150 beach umbrellas and 200 deck chairs per week.

All Grade 11 & 12 learners, try the questions on these pages. 150 days left before the exams start. We hope you are working hard.

PHYSICAL SCIENCE

EXAMPLE 1:

During the contact process for the preparation of sulfuric acid, the following reaction occurs:

$2SO_{2(g)} + O_{2(g)} \rightleftharpoons 2SO_{3(g)}$ $\Delta H < 0$.

1. 0,6 mole of SO_2 and 0,5 mole of O_2 react in a 1,0 dm^3 closed container. When equilibrium is established at 620 K, it is found that 0,4 mole of SO_3 are present in the container. Determine the equilibrium constant for this reaction at 620 K.
2. How would the number of moles of SO_3 at equilibrium, and the equilibrium constant (K_c), be affected by each of the following changes to the system after equilibrium was initially reached (see table below)?

Hint: Write only increases, decreases or remains the same next to each of the numbers, for example: (vii) – increases.

Change in system	Number of moles of $SO_{3(g)}$	K_c
Only temperature increases	–	(i)
Only pressure increases	(ii)	(iii)
Increase in SO_2 concentration	–	(iv)
Addition of a suitable catalyst	(v)	(vi)

ANSWERS:

1.

	$2SO_2$ +	O_2 $\rightleftharpoons$	$2SO_3$
Number moles at start	0,6	0,5	0
Moles used/ produced	–0,4	–0,2	+0,4
Moles at equilibrium	0,2	0,3	0,4
Concentration $c = \frac{n}{V}$	$\frac{0{,}2}{1}$ = 0,2 mol·dm^{-3}	$\frac{0{,}3}{1}$ = 0,3 mol·dm^{-3}	$\frac{0{,}4}{1}$ = 0,4 mol·dm^{-3}

$$K_c = \frac{[SO_3]^2}{[SO_2]^2[O_2]} = \frac{(0{,}4)^2}{(0{,}2)^2(0{,}3)} = 13{,}33$$

2. i) decreases
ii) increases
iii) remains the same
iv) remains the same
v) remains the same
vi) remains the same

Note:

Filling in the table:
1. Fill in moles at the start as given in the problem.
2. Fill in moles of SO_3 at equilibrium.
3. Using the ratio, fill in moles SO_2 and O_2 used in the reaction.
$2SO_2$:$2SO_3$
$\therefore$ 0,4 mole SO_3 produced uses 0,4 mole SO_2.
Do the same for O_2.
4. Subtract the moles of SO_3 used from the SO_2 at the start. Do the same for O_2.
5. Calculate the concentration of all products and reactants at equilibrium.

PHYSICAL SCIENCE

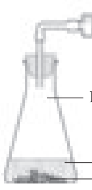
Self-assessment activity (Unit 1)

1. Michael is decalcifying an electric kettle. He uses a dilute solution of hydrochloric acid. The balanced equation for the decalcification is:
$CaCO_{3(s)} + 2HC\ell_{(aq)} \rightarrow CaC\ell_{2(aq)} + H_2O$.

To increase the rate of reaction Michael should:
A. use more acid of the same concentration
B. use another bottle of acid with a higher concentration
C. increase the pressure
D. replace the kettle's lid. [3]

2. Investigate the factors affecting the rate of the reaction represented by the following equation:
$CaCO_{3(s)} + 2HC\ell_{(aq)} \rightarrow CaC\ell_{2(aq)} + H_2O_{(\ell)} + CO_{2(g)}$.

2.1 How many moles of $HC\ell$ are there in 100 cm^3 of a solution of 0,5 mol·dm^{-3} concentration? [4]
2.2 According to the equation, how many moles of $HC\ell$ are required to react with 2 g (0,02 mole) $CaCO_{3(s)}$? [2]
2.3 The reaction is repeated six times using the reagents shown in the table, and the apparatus shown in the sketch.
Use the first table to complete last table, comparing the expected rate of reaction and yield of $CO_{2(g)}$, under the various conditions for the experiment. (*Hint*: Use only the words 'decreases', 'increases' and 'same' to complete the table.) [10]



Experiment	Mass $CaCO_{3(s)}$	State of $CaCO_3$	Concentration $HC\ell_{(aq)}$ (mol·dm^{-3})	Volume $HC\ell_{(aq)}$ (cm^3)	Temperature (°C)
1	2	Lumps	0,5	100	50
2	2	Lumps	1,0	100	50
3	2	Powder	0,5	100	50
4	2	Lumps	0,5	120	50
5	2	Lumps	0,5	100	50
6	2	Lumps	0,5	100	25

Experiment	Rate	Yield of $CO_{2(g)}$
2		
3		
4		
5		
6		

Total = 19

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7 NCS SUBJECTS	MINIMUM REQUIREMENTS
4 COMPULSORY SUBJECTS	
• 2 Languages (*one Language at Home Language and other Language at least at 1st Additional level*)	Obtain at least 40% in three subjects one of which must be an official language on Home Language level.
• Mathematics OR Mathematical Literacy	Obtain at least 30% for three subjects
• Life Orientation	
+	
3 CHOICE SUBJECTS	
• Any 3 other NCS subjects	

NCS RATING CODES

RATING CODE	RATING	MARKS
7	Outstanding achievement	80 – 100
6	Meritorious achievement	70 –79
5	Substantial achievement	60 – 69
4	Adequate achievement	50 – 59
3	Moderate achievement	40 – 49
2	Elementary achievement	30 – 39
1	Not achieved	0 – 29

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