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GRADE 12
Mathematics
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National Senior Certificate Grade 12
Mathematics
Exemplar 2008 – Paper 2

Watch out for mathematics formulas Paper 2 in Edition 10.

MARKS: 150

TIME: 3 hours

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions. Answer ALL the questions.
2. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.
3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
5. Number the answers correctly according to the numbering system used in this question paper.
6. Diagrams are NOT necessarily drawn to scale.
7. It is in your own interest to write legibly and to present the work neatly.
8. TWO diagram sheets for answering QUESTION 3.2.1, QUESTION 9.3, QUESTION 10.1 and QUESTION 11.1 are attached at the end of this question paper. Write your name/examination number in the spaces provided and hand them in together with your ANSWER BOOK.

QUESTION 1

ABCD is a quadrilateral with vertices A(2 ; 5), B(– 3 ; 10), C(– 4 ; 3) and D(1 ; – 2).

1.1 Calculate the length of AC. (Leave the answer in simplest surd form.) (2)

1.2 Determine the coordinates of M, the midpoint of AC. (2)

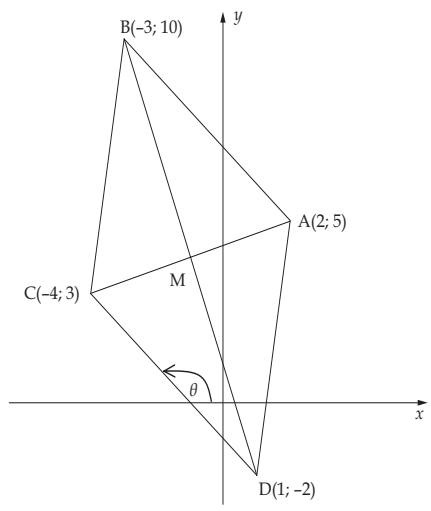
1.3 Show that BD and AC bisect each other at right angles at M. (5)

1.4 Calculate the area of ΔABC. (4)

1.5 Determine the equation of DC. (3)

1.6 Determine θ, the angle of inclination of DC. (2)

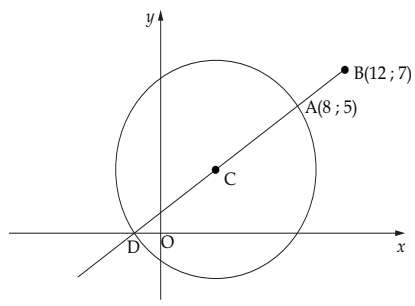
1.7 Calculate the size of $A\hat{D}C$. (4)



[22]

QUESTION 2

A(8 ; 5) and B(12 ; 7) are two points in a Cartesian plane. BA produced intersects the x-axis at D. AD is a diameter of the circle centred at C.



2.1 Show that the equation of the line through A and B can be given as $x - 2y + 2 = 0$. (3)

2.2 Determine the coordinates of D. (2)

2.3 Determine the coordinates of C. (2)

2.4 Determine the equation of the circle. (5)

2.5 Determine the equation of the tangent passing through A(8 ; 5). (3)

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URGENT MESSAGE TO ALL GRADE 11 AND 12 TEACHERS

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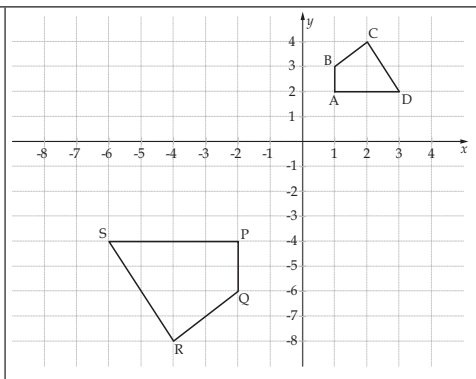
Regards
Your Study Mate team

3.2.1		✓ coordinates P ✓ coordinates Q ✓ coordinates R ✓ coordinates S (4)
3.2.2	$(x\,;y) \rightarrow (-x\,;-y)$ $(-x\,;-y) \rightarrow (-2x\,;-2y)$ $\therefore (x\,;y) \rightarrow (-2x\,;-2y)$	✓ $(x\,;y) \rightarrow (-x\,;-y)$ ✓ $(-x\,;-y) \rightarrow (-2x\,;-2y)$ ✓✓ $\therefore (x\,;y) \rightarrow (-2x\,;-2y)$ (4)
3.2.3	Area ABCD : area PQRS = 1 : 4	✓ answer (1)
3.3.1	Let r = OP = OP′ The x co-ordinate of P′ = $r\cos(\alpha + 30°)$ $x' = r\cos(\alpha + 30°)$ $= r(\cos\alpha.\cos 30° - \sin\alpha.\sin 30°)$ $= r\cos\alpha.\cos 30° - r\sin\alpha.\sin 30°$ $= r.\frac{x}{r}\cos 30° - r.\frac{y}{r}\sin 30°$ $= x.\frac{\sqrt{3}}{2} - y.\frac{1}{2}$ The y co-ordinate of P' is $r\sin(\alpha + 30°)$ Similarly $y' = y\cos 30° + x\sin 30°$ $= y.\frac{\sqrt{3}}{2} + x.\frac{1}{2}$	✓ formula ✓ expansion ✓ simplification ✓ substitution ✓ formula ✓ expansion ✓ simplification ✓ substitution (8)
3.4	$K' = \left(x.\frac{\sqrt{3}}{2} - y.\frac{1}{2}\,;\, y.\frac{\sqrt{3}}{2} + x.\frac{1}{2}\right)$ $= \left(4.\frac{\sqrt{3}}{2} - 3.\frac{1}{2}\,;\, 3.\frac{\sqrt{3}}{2} + 4\frac{1}{2}\right)$ $= (1.96\,;\,4.60)$ $L' = \left(x.\frac{\sqrt{3}}{2} - y.\frac{1}{2}\,;\, y.\frac{\sqrt{3}}{2} + x.\frac{1}{2}\right)$ $= \left(3.\frac{\sqrt{3}}{2} - 6.\frac{1}{2}\,;\, 6.\frac{\sqrt{3}}{2} + 3.\frac{1}{2}\right)$ $= (-0.40\,;\,6.70)$	✓ x-coordinate of K′ ✓ y-coordinate of K′ ✓ x-coordinate of L′ ✓ y-coordinate of L′ (4) **[25]**

QUESTION 4

4.1	$\frac{\sin 140°.\tan(-315°)}{\cos 230°.\sin 420°}$ $= \frac{\sin 40°.(\tan 315°)}{(\cos 50°).\sin 60°}$ $= \frac{-1}{\frac{\sqrt{3}}{2}}$ $= \frac{-2}{\sqrt{3}}$	✓ $\sin 40°$ ✓ $-\cos 50°$ ✓ $\tan 45°$ ✓ $\sin 60°$ ✓ $\sin 40°$ ✓ answer (6)
4.2	$\tan(180° + x).\cos(540° + x)\left(\frac{\sin(-x) + \sin^2(90° - x)}{\cos(90° + x)}\right)$ $= \tan x.(-\cos x)\left(-\sin x + \frac{\cos^2 x}{-\sin x}\right)$ $= \frac{\sin x}{\cos x}.(-\cos x)\left(\sin^2 x + \frac{\cos^2 x}{-\sin x}\right)$ $= \sin^2 x + \cos^2 x$ $= 1$	✓ $\tan x$ ✓ $-\cos x$ ✓ $-\sin x$ ✓ $\cos^2 x$ ✓ $-\sin x$
4.3	$\sin 15°$ $= \sin(45° - 30°)$ $= \sin 45°.\cos 30° - \cos 45°.\sin 30°$ $= \frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}.\frac{1}{2}$ $= \frac{\sqrt{2}(\sqrt{3} - 1)}{4}$	✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ $\sin^2 x + \cos^2 x$ ✓ answer (8) ✓ $\sin 15°$ ✓ $\sin(45° - 30°)$ ✓ expansion ✓✓ substitution (5) (note using 60° and 45° will also give same answer)
4.4.1	$\cos 2\theta + 3\cos\theta - 1$ $= 2\cos^2\theta - 1 + 3\cos\theta - 1$ $= 2\cos^2\theta + 3\cos\theta - 2$	✓ $\cos 2\theta = 2\cos^2\theta - 1$ ✓ simplification (2)
4.4.2	$\cos 2\theta + 3\cos\theta - 1 = 0$ $2\cos^2\theta + 3\cos\theta - 2 = 0$ $(2\cos\theta - 1)(\cos\theta + 2) = 0$ $\cos\theta = \frac{1}{2}$ or $\cos\theta = -2$ invalid $\theta = \pm 60° + k.360° \quad k \in Z$	✓ factors ✓ $\cos\theta = -2$ invalid ✓ answers ✓ $k \in \mathbb{Z}$ (4) **[25]**

QUESTION 5

5.1	Joyce did not use the correct expansion for sin(A + B).	✓ answer (1)
5.2	$\sin(45° + 21°)$ $= \sin 45°.\cos 21° + \sin 21°.\cos 45°$ $= \frac{\sqrt{2}}{2}(t) + \sqrt{1 - t^2}\left(\frac{\sqrt{2}}{2}\right)$ $= \frac{\sqrt{2}}{2}(t + \sqrt{1 - t^2})$	✓ expansion ✓ substitution ✓ $\sin 21 = \sqrt{1 - t^2}$ ✓ answer (4) **[5]**

QUESTION 6

6.1	$\frac{BC}{OB} = \cos\theta$ $BC = 2a.\cos\theta$	✓ $\frac{BC}{OB} = \cos\theta$ (1)
6.2	area $\Delta OBC = \frac{1}{2}.OB.BC.\sin\theta$ area $\Delta OBC = \frac{1}{2}.(2a).(2a.\cos\theta).\sin\theta$ area $\Delta OBC = a^2.2\cos\theta.\sin\theta$ area $\Delta OBC = a^2.\sin 2\theta$	✓ area rule ✓ substitution ✓ double angle (3)
6.3	Area will be a maximum when $\sin 2\theta = 1$. $\theta = 45°$ ΔOBC will be isosceles C(a ; a)	✓ answer (1) **[5]**

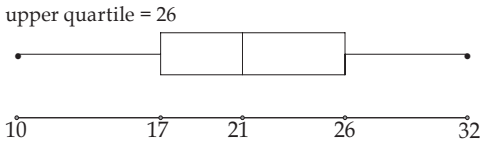
QUESTION 7

7.1	$\frac{20,7}{BD} = \cos 43,6°$ $BD = \frac{20,7}{\cos 43,6°}$ $BD = 28,6$ m	✓ ratio ✓ solving for BD ✓ answer (3)
7.2	$BE^2 = 28,58)^2 + (28,1)^2 - 2(28,58)(28,1)\cos 35,7°$ $= 302,0610874\ldots$ $BE = 17,4$ m	✓✓ cos rule ✓ substitution ✓ answer (4)
7.3	Area $\Delta BEC = \frac{1}{2}(20,7)(17,4)\sin 63°$ $= 160,4$ square metres	✓ substitution ✓ answer (2) **[9]**

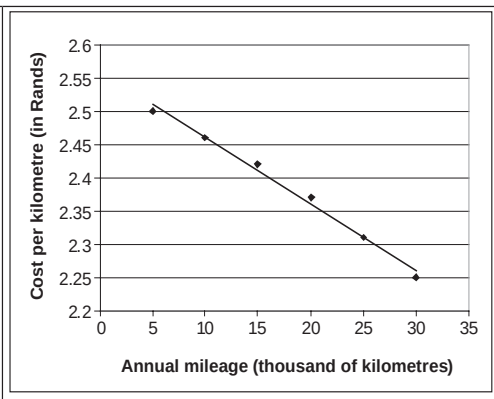
QUESTION 8

8.1	$\cos\frac{x}{2} = \sin(x - 30°)$ $\cos\frac{x}{2} = \cos(90° - x + 30°)$ $\frac{x}{2} = 120° - x + k.360°$ or $\frac{x}{2} = -120° + x + k.360° \; k \in \mathbb{Z}$ $\frac{3x}{2} = 120° + k.360°$ $\quad \frac{-x}{2} = -120° + k.360°$ $x = 80° + k.240°$ $\quad x = 240° - k.720°$ $x = 80°;\ -160°$	✓cos(90° − x + 30°) ✓✓ answer ✓✓ answers ✓✓ answer (7)
8.2	A(−160° ; 0,174) and B(80° ; 0,766) $-160° < x < 80°$	✓ critical values ✓ statement (2) **[9]**

QUESTION 9

9.1	10, 13, 13, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 30, 32. Median = 21	✓ ordered data ✓ answer (2)
9.2	lower quartile = 17 upper quartile = 26	✓ Q_1 ✓ Q_3 (2)
9.3		✓✓quartiles ✓ ✓box ✓ whiskers (5) ✓ range the same
9.4	The range of distances that Geoff and Thabo's travelled is the same. Thabo's summary is skewed to the right while Geoff's summary is more evenly distributed. This suggest that Thabo had covered greater distances than Geoff. Also Thabo's median (viz. 25) is larger than Geoff's median (21).	✓ explanation of skewed data. (2) **[11]**

QUESTION 10

10.1 & 10.2		✓✓ plotting points ✓ labels (3) ✓✓ line of best fit (2)
10.3	R 2,47 or R 2,48	✓ answer in this range (1) **[6]**

QUESTION 11

11.1		✓midpoints ✓✓✓ (total) one mark for every two numbers correct in the last column)

Percentages	Midpoint of interval (x)	Frequency (f)	Total ($f(x)$)
10 – 19	14,5	6	87
20 – 29	24,5	14	343
30 – 39	34,5	16	552
40 – 49	44,5	11	489,5
50 – 59	54,5	3	163,5
Sum			1635

	Mean $= \frac{1\,635}{50} = 32,7$	✓ answer (5)
11.2	$\bar{x} = \frac{\sum x}{n} = \frac{7+4+9+4+9+5+4+6}{8} = 6$	✓✓ mean
	$\delta = \sqrt{\frac{\sum(x - \bar{x})^2}{n}}$	✓ formula
	$= \frac{\sqrt{(7-6)^2 + (4-6)^2 + (9-6)^2 + (4-6)^2 + (9-6)^2 + (5-6)^2 + (4-6)^2 + (6-6)^2}}{8}$	✓✓ calculation
	$= \frac{1+4+9+4+9+1+4+0}{8} = \sqrt{4} = 2$	✓ simplification ✓ answer (7) **[12]**

QUESTION 12

12.1	Supplier B	✓ answer (1)
12.2	I would select supplier A. The graph for supplier A shows a fairly consistent lifetime of their bulbs. Whilst the graph for supplier B shows that their bulbs last longer than supplier A, we also have a situation where their bulbs have a shorter lifetime than supplier A.	✓ supplier A ✓ explanation (2) **[3]**

2.6 Determine A′, the image of A reflected about the straight line through C, perpendicular to the x-axis. (3)

[18]

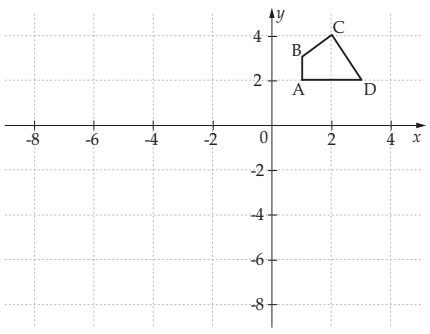
QUESTION 3

3.1 The point P(2 ; $\sqrt{3}$) lies in a Cartesian plane. Determine the coordinates of the image of P if:

3.1.1 P is reflected across the x-axis (2)

3.1.2 P has been rotated about the origin through 90° in an anticlockwise direction (2)

3.2 A transformation T of the Cartesian plane is described as follows:
A point is first rotated about the origin through 180° in the anticlockwise direction. Thereafter it is enlarged through the origin by a factor of 2. In the diagram below quadrilateral ABCD is given with A(1 ; 2), B(1 ; 3), C(2 ; 4) and D(3 ; 2).

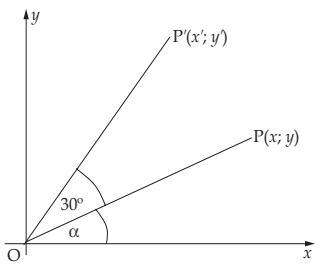


3.2.1 Use the grid on the attached DIAGRAM SHEET 1 to sketch and label PQRS, the image of ABCD under the transformation T. (4)

3.2.2 Write down the image of (x ; y) in terms of x and y. (4)

3.2.3 Write down the ratio of area ABCD : area PQRS. (1)

3.3 Show that the coordinates of P′, the image of P(x ; y) rotated about the origin through an angle of 30° in the anticlockwise direction, are given by $\left(\frac{\sqrt{3}}{2}x - \frac{y}{2} ; \frac{\sqrt{3}}{2}y + \frac{x}{2}\right)$



(8)

3.4 K′ and L′ are the images of K(4 ; 3) and L(3 ; 6) under a rotation of 30°, in the anticlockwise direction, about the origin.

Using the results in QUESTION 3.3, determine the coordinates of K′ and L′. (4)

[25]

QUESTION 4

4.1 Simplify, without the use of a calculator:

$$\frac{\sin 140°.\tan(-315°)}{\cos 230°.\sin 420°}$$ (6)

4.2 Simplify: $\tan(180° + x).\cos(540° + x)\left(\sin(-x) + \frac{\sin^2(90° - x)}{\cos(90° + x)}\right)$ (8)

4.3 Show, without the use of a calculator:

$\sin 15° = \frac{\sqrt{2}(\sqrt{3} - 1)}{4}$ (5)

4.4.1 Show that $\cos 2\theta + 3\cos\theta - 1 = 2\cos^2\theta + 3\cos\theta - 2$. (2)

4.4.2 Hence determine the general solution for: $\cos 2\theta + 3\cos\theta - 1 = 0$ (4)

[25]

QUESTION 5

The following question was given to candidates to answer:

If $\cos 21° = t$, determine, without the use of a calculator, the value of $\sin 66°$ in terms of t.
Joyce gave the following solution:

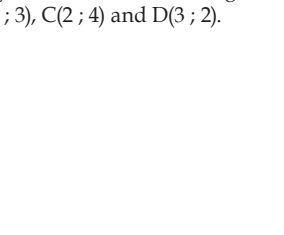
$\sin 66°$

$= \sin(45° + 21°)$ line 1

$= \sin 45° + \sin 21°$ line 2

$= \frac{\sqrt{2}}{2} + \sqrt{1 - t^2}$ line 3

$= \frac{\sqrt{2} + 2\sqrt{1 - t^2}}{2}$ line 4



Unfortunately, Joyce's answer is incorrect.

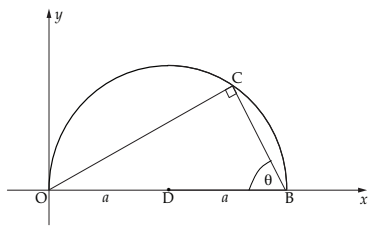
5.1 Explain why Joyce's answer is incorrect. (1)

5.2 Give a correct solution to Joyce's problem. (4)

[5]

QUESTION 6

OCB is a semicircle with centre D and radius a. $\hat{OCB} = 90°$ and $D\hat{B}C = \theta$



6.1 Show that BC = $2a\cdot\cos\theta$ (1)

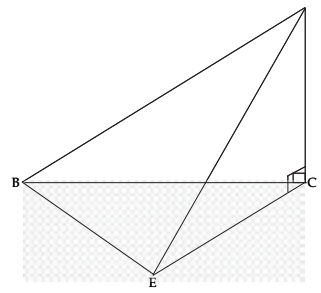
6.2 Show that the area $\Delta OCB = a^2.\sin 2\theta$ (3)

6.3 Determine the coordinates of C such that the area of ΔOCB is a maximum. (1)

[5]

QUESTION 7

CD is a vertical mast. The points B, C and E are in the same horizontal plane. BD and ED are cables joining the top of the mast to pegs on the ground. DE = 28,1 m and BC = 20,7 m. The angle of elevation of D from B is 43,6°. $C\hat{B}E = 63°$; $B\hat{D}E = 35,7°$.



Give your answers correct to ONE decimal place in each of the following questions:

7.1 Calculate the length of BD. (3)

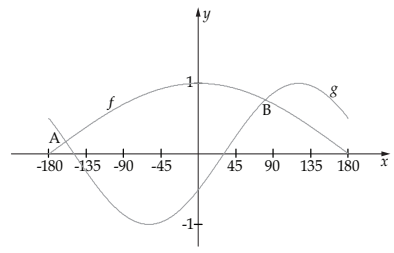
7.2 Show that the length of BE rounds to 17,4 m. (4)

7.3 Calculate the area of ΔBEC. (2)

[9]

QUESTION 8

Sketched below are the graphs of the functions $f(x) = \cos\frac{x}{2}$ and $g(x) = \sin(x - 30°)$ for $x \in [-180° ; 180°]$. The curves intersect at points A and B.



8.1 Determine the coordinates of the points A and B. (7)

8.2 For which values of x is $f(x) > g(x)$? (2)

[9]

QUESTION 9

Geoff, a driver of a courier motorcycle, recorded the distances he travelled (in kilometres) on 15 trips. The data is given below:

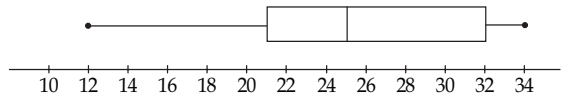
24 19 21 27 20 17 32 22
26 18 13 23 30 10 13

9.1 What is the median for the above-mentioned data? (2)

9.2 Write down the upper and lower quartiles. (2)

9.3 Draw a box and whisker diagram for the data of Geoff's travels, in the space provided on the attached diagram sheet 1. (5)

9.4 Another driver, Thabo, in the same company had also travelled and recorded (in kilometres) the distance he travelled on 15 trips. The five number summary of his data is (12 ; 21 ; 25 ; 32 ; 34). The box and whisker diagram is shown below.



Carefully analyse the box and whisker diagrams, on the diagram sheet, of Geoff and Thabo's travels and comment on the differences or similarities, if any, between the distances covered by each on the 15 trips. (2)

[11]

QUESTION 10

A company that rents out cars calculated the average cost per kilometre of maintaining a new car for different distances covered during the first year. The data gathered is given in the table below.

Annual mileage (thousands of kilometres)	5	10	15	20	25	30
Cost per kilometre (in rands)	2,50	2,46	2,42	2,37	2,31	2,25

10.1 Draw a scatter diagram to represent the above data on DIAGRAM SHEET 2. (3)

10.2 Decide which of the following graphs fit the above-mentioned data best: straight line, parabola or exponential. (2)

10.3 Estimate, by using a suitable graph, the average cost per kilometre of operating a new car if it is driven 8 000 kilometres during the first year. (1)

[6]

QUESTION 11

11.1 Fifty shoppers were asked what percentage of their income they spend on groceries. Six answered that they spend between 10% and 19%, inclusive. The full set of responses is given in the table below.

Percentage	**Frequency (f)**
10 – 19	6
20 – 29	14
30 – 39	16
40 – 49	11
50 – 59	3

By using the table on DIAGRAM SHEET 2, calculate the mean percentage of family income allocated to groceries. (5)

11.2 The marks of 8 learners in a test for which the maximum mark is 10, were:

7, 4, 9, 4, 9, 5, 4, 6.

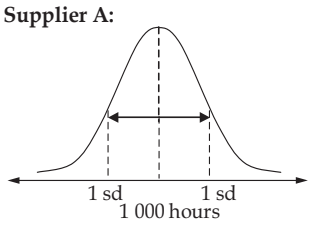
Calculate the standard deviation of this data. (7)

[12]

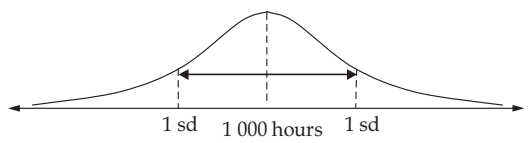
QUESTION 12

Your school makes use of two suppliers of light bulbs. Both companies claim that on average their light bulbs last 1 000 hours. The graphs below show the distribution of time taken in hours before a light bulb from each supplier burnt out. The horizontal and vertical scales of the two graphs are the same.

Supplier A:



Supplier B:



12.1 Compare the graphs above. Which company supplies bulbs that have a higher deviation from the mean? (1)

12.2 The clerk asks for your assistance in selecting the supplier from which he will purchase 100 light bulbs for the school. Which supplier would you select? Explain the reason for your choice. (2)

[3]

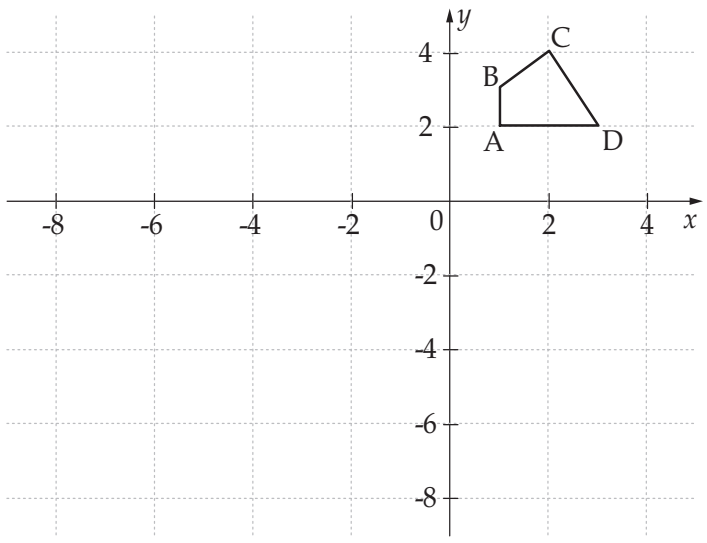
TOTAL: 150

NAME/EXAMINATION NUMBER:

DIAGRAM SHEET 1

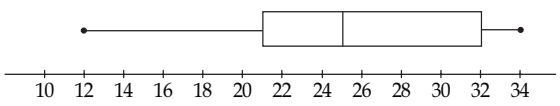
QUESTION 3

3.2.1



QUESTION 9

9.3

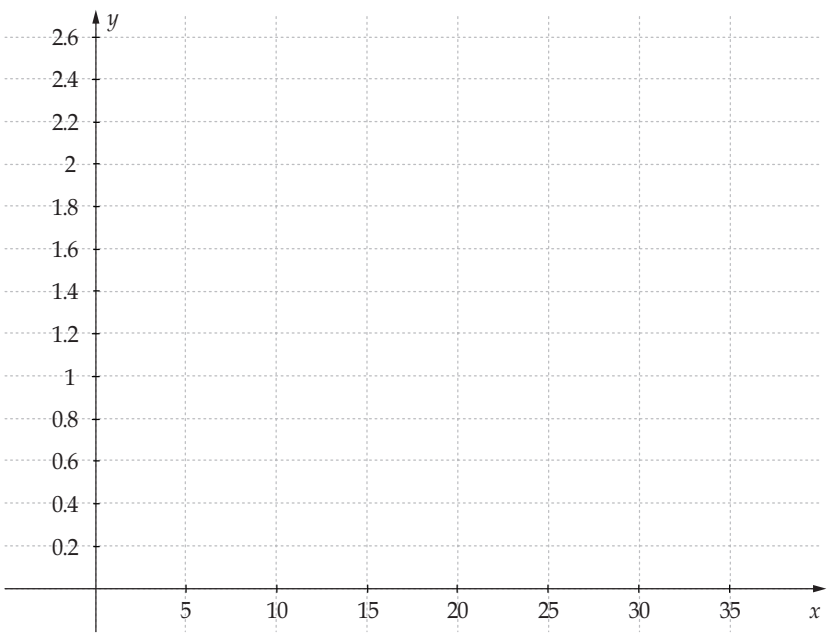


NAME/EXAMINATION NUMBER:

DIAGRAM SHEET 2

QUESTION 10

10.1



QUESTION 11

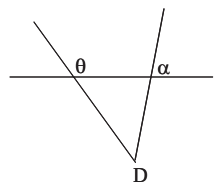
11.1

Percentages	Frequency (f)		
10 – 19	6		
20 – 29	14		
30 – 39	16		
40 – 49	11		
50 – 59	3		

Mean =

Memorandum

QUESTION 1

1.1	$AC = \sqrt{(2+4)^2 + (5-3)^2}$ $AC = \sqrt{40}$ $AC = 2\sqrt{10}$	✓ substitution ✓ answer (2)
1.2	$M\left(\frac{-4+2}{2}; \frac{3+5}{2}\right)$ $\therefore$ M(–1 ; 4)	✓ substitution ✓ answer (2)
1.3	$m_{BD} = \frac{10-(-2)}{-3-1} = \frac{12}{-4} = -3$ $m_{AC} = \frac{5-3}{2-(-4)} = \frac{2}{6} = \frac{1}{3}$ $\therefore m_{BD} \times m_{AC} = -3 \times \frac{1}{3} = -1$ $\therefore$ BD⊥AC *Midpoint BD* $\left(\frac{-3+1}{2}; \frac{10-2}{2}\right)$ = *Midpoint of AC* = (–1 ; 4) bisect at 90°	✓ answer ✓ answer ✓ –1 ✓ coordinates ✓ = Midpoint AC (5)
1.4	Area ΔABC $= \frac{1}{2}.AC.MB$ $= \frac{1}{2}.\sqrt{40}.\sqrt{(10-4)^2 + (-3+1)^2}$ $= \frac{1}{2}\sqrt{40}.\sqrt{40}$ = 20	 ✓ formula ✓ substitution ✓ MB = $\sqrt{40}$ ✓answer (4)
1.5	$m_{DC} = \frac{3+2}{-4-1} = -1$ $y + 2 = -1(x - 1)$ $y = -x - 1$	✓ substitution ✓ answer ✓ answer (3)
1.6	$m_{DC} = -1$ $\tan\theta = -1$ $\theta = 135°$	 ✓ substitution ✓ answer (2)
1.7	$m_{AD} = \frac{5+2}{2-1} = 7$ $\therefore \tan\alpha = 7$ $\alpha = 18{,}9°$ $A\hat{D}C = \theta - \alpha$ $A\hat{D}C = 135° - 81{,}9° = 53{,}1°$ OR ADC = 180° – (45° + 81,9°) ADC = 53,1° OR Use Cosine Rule $AC^2 = DC^2 + AD^2 - 2DC.AD\cos D$ $\therefore 40 = 50 + 50 - 2 \times 50\cos D$ $\therefore \cos D = 0.6$ $\therefore \hat{D} = 53{,}13°$	 ✓ 7 ✓ 81,9° ✓ $A\hat{D}C = \theta - \alpha$ ✓ 53,1° (4) ✓ Use Cosine Rule ✓✓ Substitution ✓ answer **[22]**

QUESTION 2

2.1	$m_{AB} = \frac{7-5}{12-8}$ $m_{AB} = \frac{1}{2}$ equation of AB is $y - 5 = \frac{1}{2}(x - 8)$ $y = \frac{1}{2}x + 1$ $2y - x = 2$ $x - 2y + 2 = 0$	 ✓ $m_{AB} = \frac{1}{2}$ ✓ substitution ✓ answer (3)
2.2	At D: $x - 2(0) + 2 = 0$ $x = -2$ D(– 2 ; 0)	✓ substitution of $y = 0$ ✓ x answer (2)
2.3	$C\left(\frac{8-2}{2}; \frac{5+0}{2}\right)$ $C\left(3; \frac{5}{2}\right)$	✓ x – value ✓ y –value (2)
2.4	$AC^2 = (8-3)^2 + \left(5 - \frac{5}{2}\right)^2$ $AC^2 = 25 + \frac{25}{4}$ $AC^2 = \frac{125}{4}$ equation of the circle is $(x-3)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{125}{4}$	✓ substitution ✓ AC^2 ✓ $(x-3)$ ✓ $\left(y - \frac{5}{2}\right)$ ✓ answer (5)
2.5	gradient of tangent = – 2 (tangent ⊥ radius) equation of tangent is $y - 5 = -2(x - 8)$ $y - 5 = -2x + 16$ $y = -2x + 12$	✓ gradient ✓ substitution ✓ answer (3)
2.6	Axis of symmetry $x = 3$ A′ (– 2 ; 5)	✓ axis of symmetry ✓ x-answer ✓ y-answer (3) **[18]**

QUESTION 3

3.1.1	$P(2 ; -\sqrt{3})$	✓✓ coordinates (2)
3.1.2	$P(-\sqrt{3} ; 2)$	✓✓ coordinates (2)

NEXT WEEK:
History Grade 12