OUTLINE OF PRESENTATION AT THE

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THE MATHEMATICS COMPREHENSION TEST

1. The Origin of the Mathematics Comprehension Test

A teaching concept, known as Mathematics Comprehension, was developed by academics in the Mathematics Department at the University of Cape Town. This is a method of teaching new topics by means of graded test questions.

This concept is being used by the Alternative Admissions Research Project (AARP) as a basis for a test that attempts to reduce the advantage of Mathematical knowledge in assessing the writer. The test is an attempt to gauge the "potential" of the writer to assimilate new Mathematical knowledge while assuming very little Mathematical proficiency on the part of the writer.

A major assumption is that if the writer is able to assimilate new Mathematical knowledge in this context, he will be able to use the same technique at university.

2. Background to the problem that the AARP wishes to address

- At UCT it has been found that for the majority of pupils from an ex-Department of Education and Training (DET) background, Mathematics marks are clumped in a very narrow band. This makes it very difficult to assess the potential of these students to learn Mathematics at university level.
- Vast inequalities in preparedness make standard achievement tests ineffective in gauging the Mathematical abilities of the writers.
- Very few DET writers have Mathematics on the HG, which is a prerequisite for many courses.

These pupils are thus effectively barred from entrance to the university because of their school background. A different form of assessing their "potential" to learn Mathematics

therefore needed to be developed for them to be considered to study Mathematics at university level.

3. Assumptions made about Mathematical Proficiency of the Writer:

- It is assumed that the writer has a Mathematical background of at least Grade 9 level i.e.
- The basic Mathematical operations of addition, subtraction, multiplication, division, multiplication as well as exponentiation are assumed.
- The ability to use the aforementioned operations in the context of integers and rational numbers is assumed.
- Knowledge of irrational numbers is assumed. (writers are not expected to apply the operations to them but only to realize certain properties such as $\sqrt{3}+\sqrt{3}\neq\sqrt{6}$ etc
- The ability to do point-by-point plotting of bar graphs and line graphs is assumed.
- The ability to read information from a graph is assumed.
- Working knowledge of the formulas for the area of rectangles, squares and triangles is assumed.
- Knowledge of basic Euclidean geometry including angles on a straight line, intersecting lines, parallel lines, triangles and quadrilaterals is assumed, as is knowledge that the radii of circle are equal is expected as well as the nomenclature with respect to the circle.
- The ability to solve linear equations and basic quadratic equations in the form $x^2 =$ perfect square is assumed.
- Basic Algebra is assumed viz. simplifying polynomials and multiplication of at most binomials with binomial is assumed.
- The ability to solve simple linear equations is assumed.

    - If these are not assumed as a basic starting point, too many “trivial” concepts will have to be included to teach the desired concepts.
    - A prospective student who does not have these basic Mathematical skills will require extensive support to bring the applicant up to par with the

rigours required at tertiary education in any of the numerate fields of study. Setting these minimum requirements does not detract from AARP's aim to identify those with the potential to succeed in Mathematics at tertiary level.

- If higher Mathematical proficiency is required, this might disadvantage those students who have not had access to this level of proficiency and those that have not been in contact with Mathematics for a length of time.

4. Conceptual Basis of the Test

- Writers are taught a new concept or theme by means of leading questions.
- The initial questions are based on the writer's present knowledge.
- Scaffolding questions are used to teach or introduce the new concept.
- Later questions test whether the concepts have been understood and at what level.
- Some questions require the writer to verbalise what he has learnt.
- There are higher-level questions that require the writer to make his own assumptions to arrive at the answers.
- The theme must be of sufficient complexity that the writer will not be able to grapple with it without adequate intervention.
- The aim of the test is to identify those learners that have the potential to learn new Mathematical concepts.

5. Practicalities with regard to the Set Up of the Test

- The concept tested has to be something that the learner will not have encountered before in any depth
- The language of the text should be sufficiently simple so that a second language speaker will be able to follow the teaching.
- Two different themes are written to reduce the chance that a particular writer may have done a project in one of the areas.
- The topics are different in that one is usually number orientated and the other visually orientated.

- Initial questions are set that should be accessible to all learners and these guide them into learning the topic. (Scaffolding)
- Subsequent questions test writers' knowledge of the newly acquired work with graded questions that eventually sees if the writer is able to make practical usage of the topic.
- Certain higher order questions are included which require the writer to make an intuitive leap. Clues are normally provided in previous questions but no scaffolding is provided to lead writers to the answers.

6. Analysis of Results

Test results are reported in three ways:

- The overall mark achieved on the Test.
- A general Mathematical Taxonomy, based on Bloom's taxonomy.
- Reporting based on the types of questions in the Mathematics Comprehension Test.

- The general Mathematical taxonomy is divided into 5 categories
    - Basic Mathematics: Writers' knowledge of the school Mathematics syllabus up to the Grade 9 Level.
    - Translating Information: Writers' ability to translate word problems into Mathematical expressions, to represent tables of values by graphs or Mathematical expressions, to convert from one graphical representation to another and to represent the information of a complex diagram in a simplified sketch.
    - Comprehending concepts: Writers' ability to distinguish between examples and non-examples of concepts according to given criteria, to compare and contrast similar concepts and to identify and represent critical elements of a concept in various forms.
    - Analysing principles: Writers' ability to identify key concepts involved in a principle and to recognize relationships between those concepts.

- Synthesis of principles: Writers' ability to formulate new relationships or relationships in different contexts based on a knowledge of existing relationships.

- Types of questions in the Test are divided into four categories
  - Scaffolding Questions: Writers' ability to respond to the teaching that is given and show an ability to comprehend the new concepts in the light of this teaching.
  - Testing Questions: Writers' ability to apply the new concepts and show that the writer is able to use these concepts in similar circumstances.
  - Discussion Questions: Writers' ability to explain verbally or diagrammatically the method he employs to arrive at an answer or to justify the writer's choice of answer.
  - Intuitive Questions: Writers' ability to apply the knowledge gained to a more complex situation without any scaffolding.

7. Examples of questions set in the Test:

Writer will have been introduced to the concept of complex numbers with addition, subtraction and multiplication in previous questions including a motivation as to why the concept is formulated.
The test will continue as follows

We know that $3 \times 4 = 12$

Therefore we can say $3 = \frac{12}{4}$.

Similarly if ab = c then b = $\frac{c}{a}$

Question 1

If $x(3 + 4\boldsymbol{i}) = 2 - 3\boldsymbol{i}$ then write down what x will be equal to. You do not have to work out (simplify) the expression. .....

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This is an example of a scaffolding question that takes their knowledge of rational numbers and expects them to transfer this knowledge to complex numbers.

Now $(5 + 2\boldsymbol{i})(5 - 2\boldsymbol{i})$ $= 5(5 - 2\boldsymbol{i}) + 2\boldsymbol{i}\,(5 - 2\boldsymbol{i})$

$= 25 - 10\boldsymbol{i} + 10\boldsymbol{i} - 4\boldsymbol{i}^2$

$= 25 + 0 - 4(-1)$

$= 25 + 4 = 29$

Question 2 Calculate the following expression

2.1 $(2 + 3\boldsymbol{i})(2 - 3\boldsymbol{i})$

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2.2 $(1 + 2\boldsymbol{i})(1 - 2\boldsymbol{i})$

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This question scaffolds the concept of what a conjugate is. The answers would be $2^2 + 3^2 = 4 + 9 = 13$ *and* $1^2 + 2^2 = 1 + 4 = 5$

Question 3

Without multiplying out, what would be the imaginary part of $(a + b\boldsymbol{i})(a - b\boldsymbol{i})$?

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This is an intuitive type question, which determines whether they realise that there would be no imaginary part, whatever the numbers.

Question 4

4.1 Calculate the value of $(2 - \boldsymbol{i})(2 + \boldsymbol{i})$

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4.2 What would be the value of $\frac{5}{2-i}$ in the form $a+b\boldsymbol{i}$.

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4.3 What would be the value of $\frac{1}{2-i}$ in the form $a + b\boldsymbol{i}$.

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These questions lead them to a method to write the expression $\frac{1}{p+qi}$ *in the form* $a + b\boldsymbol{i}$

4.4 Write $\frac{3+2i}{2-i}$ in the form $a+b\boldsymbol{i}$.

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Writers are required to see that he has to rewrite the denominator in the form $a + b\boldsymbol{i}$ *before multiplying*

4.5 Write $\frac{2+5i}{3+2i}$ in the form $a + b\boldsymbol{i}$.

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Writers are now required to perform division with complex numbers from scratch.

Question 5

Explain the steps you would follow to write the expression $\frac{p+qi}{u+vi}$ in the form a + b$\boldsymbol{i}$.

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The writer now has to verbalise the method he has used.

Question 6 In the following sum "x" is a complex number in the form a + b$\boldsymbol{i}$ 1.

6.1 Solve for *x* if $x + 2\boldsymbol{i} = 2 + 5\boldsymbol{i}$

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This is a scaffolding question to introduce solving of equations with complex numbers.

6.2 Solve for *x* if $\frac{1}{x} = 1 + 3i$

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This is a testing question to ascertain whether writer is able to see that inverse has to be found.

6.3 Solve for x if $\frac{1}{x^2} = 1 + 3i$

This is an Intuitive question that expects the writer to derive a method for solving a quadratic equation in complex numbers.

8. A brief explanation of statistical terms used in the following sections:

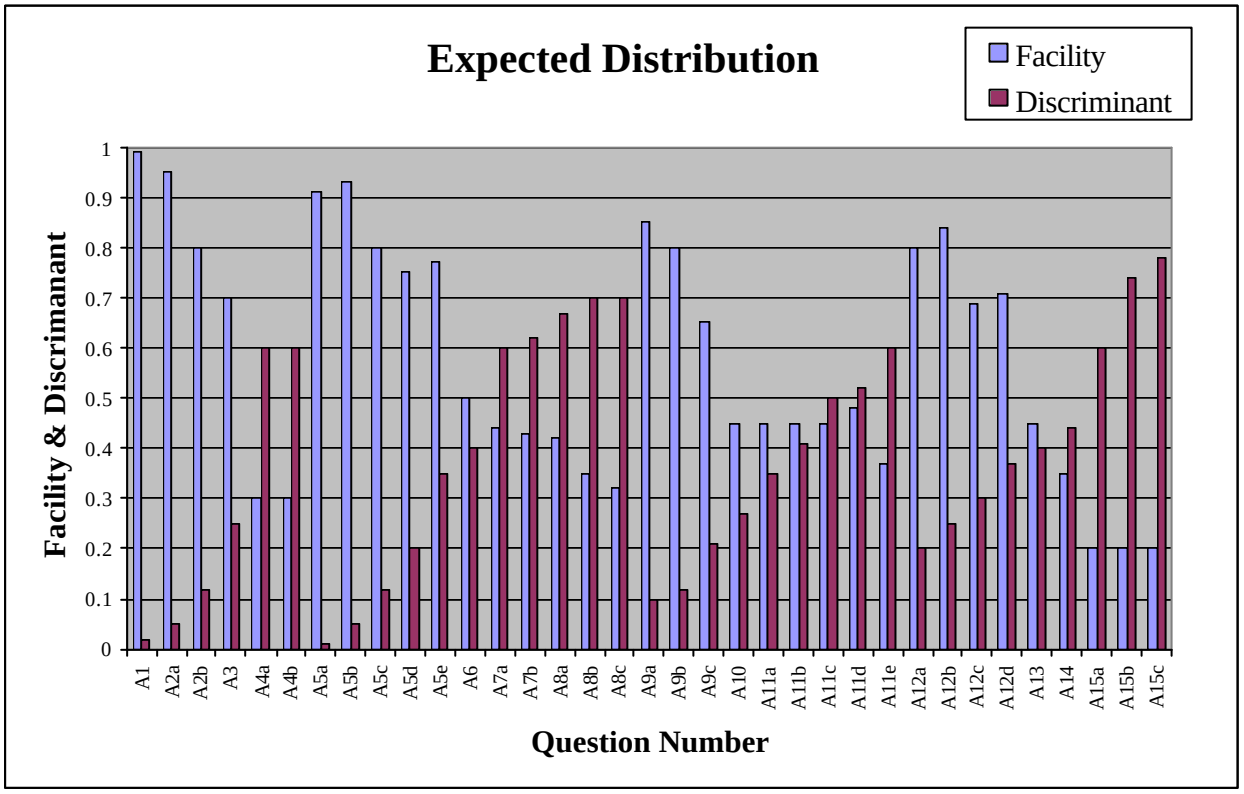
Facility Value： A measure of how easy the item was found by the writers. A facility of 1 indicates all the writers obtained the correct answer. A facility of 0 indicates none of the writers obtained the correct answer. A facility of more than

0.6 indicates the item is too easy and a facility of less than 0.4 indicates the item is too difficult.

Discriminant: A measure of how well an item separates the weak students from the stronger ones. A discriminant of less than 0.3 is said not to discriminate well enough. The weaker students perform as well on the item as the stronger students.

The graphs that follow plot the facility and the discriminant for each question in the test.

9. Overview of methodology used to revise the test



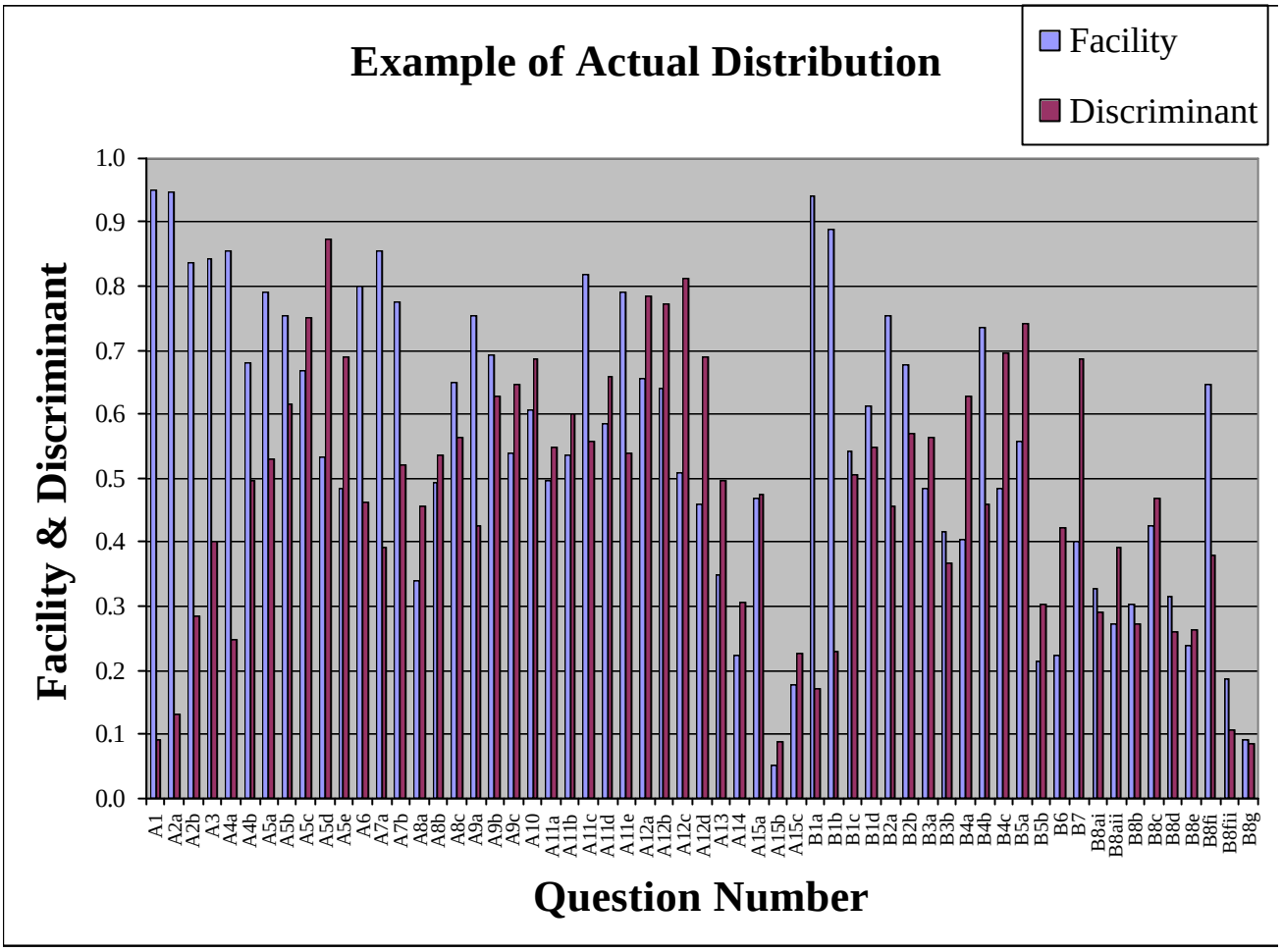
The above graph shows what the desired facility and discriminants should be.

- We would aim to achieve a distribution like the above where in the scaffolding question we have a high facility value, as all the writers should be able to accomplish this task. The discriminant in the scaffolding questions should be very low, as we do not want the better writers to be advantaged at this stage. We would prefer if all started at an equal level and everybody were able to accomplish the scaffolding type question.

- When we leave the scaffolding type question and start engaging the writer with testing or discussion type questions the facility values should start to drop, as more writers are unable to engage with the concepts. We would want the discriminant to increase correspondingly as only the weaker student is unable to score marks in these questions.

- There would follow peaks in the facility values as scaffolding questions introduce new concepts. These peaks will have decreasing facility values, as the concepts will increasingly rely on knowledge learnt in the test.

- Towards the end of the test (and sometimes within the test) higher level or intuitive questions will be asked which should have a low facility value. We would aim at these identifying the top 20% of the pool. The discriminant should be very high for these intuitive questions.



The above graph is results from an actual test, consisting of two sections, run by AARP. Approximately 2000 writers took the test. Once again the facility and discriminant is plotted for each item in the test.

Section A has the desired distribution in that the initial questions in each section, "scaffolding" has high facility values and low discriminant.
The intuitive type questions are from question 13 onwards. A13 and A14 perform satisfacto rily but A15b and A15c have an unacceptably low discriminant. We would have to introduce more scaffolding to A15b to enable the stronger students to do the question.

In Section B B5b was intended as a scaffolding question but has a very low facility value as well as a reasonably high discriminant and would therefore have to be reworded.

Question B8f(ii) and B8g have unacceptably low discriminants and will have to have some scaffolding added to make it more accessible to the stronger writers.

Use of the MCOM as a selection tool

This test is suitable for use as a selection tool, which is complementary to school-leaving results, in the following respects:

- It does not require the writer to have a strong background in Mathematics as per the usual achievement type tests and thus, to a certain extent, levels the playing field for those from disadvantaged scholastic backgrounds.
- The concepts tested are different to those tested in the matriculation examination.
- There is little emphasis given on prior studying – this allows us to differentiate those who "swot" the work to those who have a more innate understanding of the Mathematics.

It tests whether the writer has the ability to learn concepts that they would be expected to encounter in most courses with a Mathematical base