basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

Mechanics

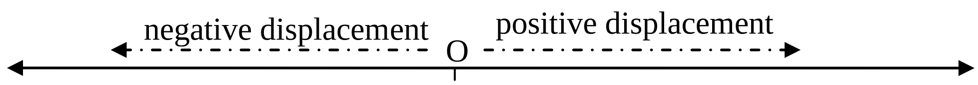
Self Study Guide

Book 2

Content

1.1.1 Frames of reference

Let us firstly consider the motion of a particle along a straight line. We will assume that we are working with SI units which means that length is measured in *metres*, and time in *seconds*. We choose as our origin a point *O* on the line as well as a positive direction. If we consider East as positive, then the displacement of the particle from the origin in an easterly direction will be positive and on the opposite side of *O*, its displacement will be negative. In general, if the particle is moving, its position along the line will be a function of time.



Objects in the real world generally require three spatial dimensions to adequately describe their motion through space. For example, a tumbling satellite in space may follow a complicated path which requires three coordinates to fix its position in space.

In this course we will simplify our investigations of motion by considering objects moving along a straight line, ie., we restrict our motion to one dimension. To describe the position of a particle as a function of time we require an origin (starting point). Motion will be described with respect to this origin which may be at rest or moving with constant velocity. For example a train leaves a station and travels along a straight track for some time t and covers a distance $\mathbf{\Delta x}$. We take the train station to be the origin and we assign to the station the position x = 0. If the train covers a distance x in time $\mathbf{\Delta t}$, then it's displacement with respect to the origin in time Δt is $\Delta x = x_2 - x_1 = x - 0 = x$, east (say). Here we assumed that the displacement of the train away from the station is positive. Should the train return to the station, it's displacement on its return journey will be negative. Upon arrival at the station, it's displacement will be zero. It's distance however, will not be zero. Why?

RELATIVE VELOCITY

The velocity of an object, just like its position, must be described relative to a reference frame. For objects moving on the Earth's surface, we take the Earth to be a fixed frame. We measure position, velocity and acceleration of an object relative to this fixed frame. We know that the Earth is orbiting the sun, and the sun together with planets move through space.

ACTIVITY 1

An aircraft carrier moves smoothly across the water at a speed of V_{AW}, relative to the still water. A US marine walks along the flight deck to the front of the carrier with velocity V_{MA}, relative to the flight deck. The velocity of the marine relative to the water is

$$V_{MW} = V_{MA} + V_{AW}$$

We note that the resultant velocity of the marine relative to the water is the sum of her velocity relative to the aircraft carrier and the velocity of the aircraft carrier relative to the water. If the marine was walking in the opposite direction to the motion of the aircraft carrier, then we subtract the velocities concerned. For example, if the aircraft is moving at 50 $km.hr^{-1}$ and the marine is walking at 7 $km.hr^{-1}$ in the same direction as the carrier, then her velocity will be $V_{MW} = 50 + 7 =$ 57 $km.hr^{-1}$ relative to the water. If the marine was walking in the opposite direction then her velocity relative to the water will be

$$V_{MW} = 50 - 7 = 43 \text{ km.hr}^{-1}.$$

Here we have taken the positive direction in the initial direction of the aircraft carrier.

ACTIVITY 2

Another illustrative example is the following: You are driving along in your car which is travelling at 100 $km.hr^{-1}$. The speed has to be relative to something. This is usually taken to be the Earth. The speed of the car is relative to the road. You the passenger, is at rest relative to the car, but you are travelling at 100 $km.hr^{-1}$ relative to the ground. Now suppose that while you are travelling at 100 $km.hr^{-1}$, another car pulls up alongside you, and that car is also travelling at 100 km.hr^{-1}. How fast are you travelling relative to the other car? The answer is 0 km.hr^{-1}. And then you may ask, what about a car travelling at 80km.hr^{-1} behind you? Relative to the car behind you, you are travelling at 20km.hr^{-1}, in the forward direction.

EVERYDAY EXAMPLES OF RELATIVE VELOCITIES

1. Safe following distance while driving.
2. Navigation – air traffic control and sea-faring.
3. Use of conveyor belts in manufacturing, eg. bottling and canning of goods.
4. Escalators and treadmills.
5. Sport – skiing, biking, F1 racing, etc.

ACTIVITY 3

A man is driving a truck at 50 km.hr^{-1} along a straight flat road. He is spotted by a motorcycle officer, who gives chase. By the time the truck driver sees the police officer on his bike, the bike is travelling at 75 km.hr^{-1}. What is the bike's velocity relative to the truck?

Let the velocity of the bike relative to the earth be V_{BE} and the velocity of the truck relative to the earth be V_{TE}. We may write:

$$V_{BE} = V_{BT} + V_{TE}$$

where V_{BT} is the velocity of the bike relative to the truck.

$$75 = V_{BT} + 50$$
$$V_{BT} = 25 \text{ km.hr}^{-1}$$

Discuss the direction of V_{BT}.

ACTIVITY 4

An escalator in a shopping mall moves at 1 m.s^{-1} and is 150 m long. If a girl steps on at one end and walks 2 m.s^{-1} relative to the escalator, how much time does she require to reach the opposite end if she walks

4.1 in the same direction as the escalator's motion?

4.2 in the opposite direction?

4.1 $$V_{gE} = V_{ge} + V_{eE}$$
$$= 2 + 1 = 3\text{m.s}^{-1}$$

Time taken to get across = D/V_{gE} =150/3 = 50s.

4.2 Similarly, we obtain t = 150s.

OPTIONAL – 2-D RELATIVE VELOCITY

Here is a realistic problem involving relative velocities. A boat capable of travelling at 4 m.s^{-1} in still water is used to cross a river which is flowing at 3 m.s^{-1}. The river is 1 km wide.

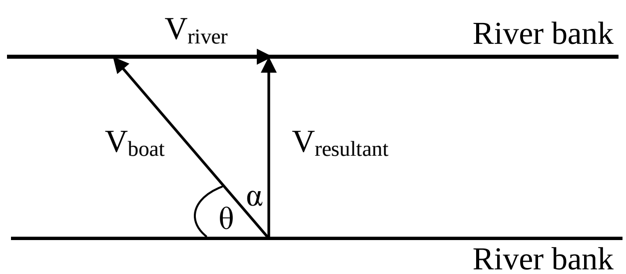
[a] If the captain of the boat wishes to travel directly across the river, at what angle relative to the river bank should he head the boat?

[b] Should he head in the direction determined in [a] above, calculate the time taken for the boat to reach the opposite bank.

[c] If the captain decides to head directly across the river, how far downstream would he find himself, upon reaching the opposite bank?

SOLUTIONS

[a]

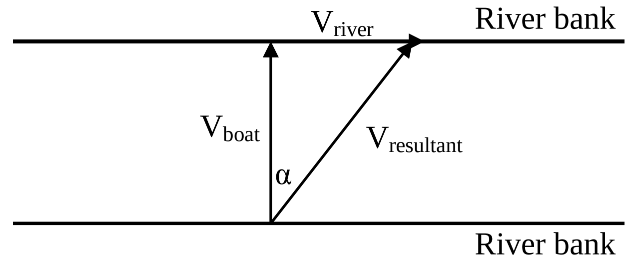


$$\sin\alpha = \frac{V_{river}}{V_{boat}} = \frac{3}{4}$$
$$\Rightarrow \alpha = \arcsin\left(\frac{3}{4}\right)$$
$$\theta = 90^0 - \alpha$$

[b]

$$V_{resul\tan t} = \sqrt{3^2 + 4^2} = 5m/s$$
$$t = \frac{D}{V} = \frac{1000}{5} = 200s$$

[c] $\tan\alpha = \frac{3}{4} = \frac{X}{1} \Rightarrow X = \frac{3}{4}km$



WORK BOOK ONE – PRACTICE PROBLEMS

1. Neo follows an exhaustive training program in preparation for the Comrades marathon. One of her cardio-workouts requires her spending twenty minutes on the treadmill. The treadmill conveyor belt travels at 2 $m.s^{-1}$. How fast must Neo run on the treadmill for her to be stationary relative to the ground?

2. In the latest James Bond movie, agent 007 stands on the top of a train which is travelling at a constant speed of 90$km.hr^{-1}$ relative to the ground. A Russian agent is standing a distance of 25 m away from agent 007, on the roof of the train. If 007 fires a dart at 25 $m.s^{-1}$ towards the Russian agent, calculate the time taken for the dart to strike the Russian agent. Assume that the dart is fired in the opposite direction to the train's motion. Air friction is negligible. How will your answer compare if the dart was fired in the same direction as the motion of the train?

3. Nathi and Selvan decide to play a game of ping-pong aboard a boat which is travelling at 30 $m.s^{-1}$, relative to the water. They stand 10 m apart. If Selvan hits the ball at 5 $m.s^{-1}$ towards Nathi (in the direction of motion of the boat), calculate the time taken for the ball to reach Nathi. Will this time be the same for spectators aboard the boat? What about for someone watching the match from the pier?

4. A boat which is 10km from shore travels at a constant velocity of 5$m.s^{-1}$ towards the beach. A seagull aboard the boat starts to fly at a constant speed of 10$m.s^{-1}$ and heads for the shore. Upon reaching the shore the seagull flies back to the boat and repeats the journey to the shore. What distance does the seagull cover when the boat finally reaches the beach?

WORK BOOK ONE – SOLUTIONS

1. Neo will have to run 2 $m.s^{-1}$ relative to the treadmill.

2. We note here that 007 and the Russian agent are at rest relative to each other. The time taken for the dart to reach the Russian agent is independent of the train's velocity. The time taken for the dart to travel 25 m = D/V = 25/25 = 1s. The time will also be the same if the dart were fired in the same direction as the train's velocity. To understand we note that everything happens on the train.

3. Similar to question 2. The time taken for the ball to travel between players will be the same for the everyone aboard the boat, ie., time = D/V = 10/5 = 2s.

   The situation is different for someone on the pier. If the ball is moving in the same direction as the boat, then the velocity of the ball is added to the velocity of the boat. If the velocities are in opposite directions, then the resultant velocity is the difference between the two.

4. This is a classic question. A slight variation of this question was posed to Richard Feynman, the renowned American physicist, when he was pursuing a PhD. The answer is quite simple. If we know how the seagull is in the air, then we can easily calculate the total distance covered by the seagull. The boat approaches the shore at constant velocity. The time taken for the boat to reach the shore must equal to the total flight time of the seagull = D/V_{boat} =10000/5 = 2000s. Distance covered by the seagull = V_{gull} x t = 10 x 2000 = 20km.

1.1.2 Projectile motion

Consider an object being projected (thrown) vertically upwards with some initial velocity v_i. If we choose the upward direction as positive, then the acceleration of the object is $g = -9.8$ m.s^{-2}, since the acceleration due to gravity always points towards the center of the Earth. It is also important to note that the object is travelling in a straight line. If we ignore air resistance, then the equations required to describe the object's position, velocity and flight time are:

$$\boxed{\begin{aligned} v_f &= v_i + gt \\ v_f^2 &= v_i^2 + 2g\Delta y \\ \Delta y &= v_i t + \tfrac{1}{2} g t^2 \\ \Delta y &= \left(\frac{v_i + v_f}{2}\right) t \end{aligned}}$$

where Δy is the object's vertical displacement, v_i, its initial velocity, v_f, the final velocity, g, the acceleration due to gravity and t, the time in seconds.

NOTES

1. At the highest point of its motion, the object's final velocity is zero.

2. At the highest point, the object's acceleration is g = 9.8m.s^{-2}, pointing downwards. We cannot switch off gravity!

3. The time taken to rise is equal to the time taken to fall back to the point of projection.

In grade 11, you would have derived the following relationship:

$$\boxed{g = \frac{GM}{R^2}},$$

where $G = 6.7 \times 10^{-11}$ N.kg^{-2}.m^2, M is the mass of the gravitating body (for example earth) and R is the distance between the centres of the gravitating body and the falling body.

ACTIVITY 1

Let us work out the acceleration due to gravity at the Earth's surface.
Take the radius of the Earth to be R = 6000 km and M = 6 x 10^{24}kg.

$$g_E = \frac{Gm_E}{R^2_{E22}}$$
$$= \frac{(6.7 \times 10^{-11})(6 \times 10^{24})}{(6000 \times 10^3)^2} \approx 9.81 m/s$$

In our derivation, we have made the following simplifications:

1. We ignored the rotation of the earth.
2. We have assumed that the mass is uniformly distributed over the Earth.
3. We have also taken the Earth to be perfectly spherical.

A COMMON MISCONCEPTION

Most people believe that there is no gravity on the moon. This cannot be true since the moon has mass. Also, gravity cannot be switched off.

ACTIVITY 2

Using the fact that the moon's mass is about 0.012 times the Earth's mass and the diameter of the moon is 3476 km, calculate the acceleration due to gravity on the moon.

The acceleration due to gravity on the moon is given by

$$g_{moon} = \frac{Gm_{moon}}{R^2_{moon}}$$
$$= \frac{G(0.012 m_E)}{\left(\frac{3476 \times 10^3}{2}\right)^2} \approx \frac{g_{earth}}{6}$$

ACTIVITY 3

A ball is projected vertically upwards from the ground with an initial velocity of 10 m.s^{-1}. Calculate

3.1 the maximum height reached by the ball above level ground.
3.2 the time taken for the ball to reach maximum height.
3.3 the speed of the ball after 1.5 s.

3.1 Take up as positive. Given v_i = 10m.s^{-1}, g = - 9.8 m.s^{-2}.
At maximum height v_f = 0m.s^{-1}. We obtain

$$v_f^2 = v_i^2 + 2g\Delta y$$

$$\Delta y = \frac{v_f^2 - v_i^2}{2g}$$

$$= \frac{0^2 - 10^2}{-9.8} = 10.2m$$

3.2

$$v_f = v_i + gt$$

$$t = \frac{v_f - v_i}{g} = \frac{0-10}{9.8} = 1.02s$$

3.3

$$v_f = v_i + gt$$

$$= -10 + (9.8)(1.5) = 4.7m.s^{-1}$$

(downwards)

ACTIVITY 4

A stone is projected vertically upwards from a 50m high cliff, with an initial velocity of 35 $m.s^{-1}$. Ignore air resistance.

4.1 Calculate the time taken for the stone to reach its maximum height.
4.2 How fast is the stone travelling at t = 7 seconds?
4.3 How far is the stone above ground, after 8 seconds?

4.1 Given $v_i = 35\ m.s^{-1}$, taking up as positive $g = -9.8 m.s^{-2}$. At the highest point $v_f = 0\ m.s^{-1}$.

$$v_f = v_i + gt$$
$$t = \frac{v_f - v_i}{g} = \frac{0-35}{-9.8} = 3.57s$$

4.2 $v_f = 35\ m.s^{-1}$, downwards. Can you see why?

4.3

$$\Delta y = v_i t + \tfrac{1}{2} g t^2$$
$$= -35(8) + \tfrac{1}{2}(9.8)(8)^2 = 33.6m$$

Height above ground = 50 + 33.6 = 83.6 m.

ACTIVITY 5

A cricket ball passes a window sill 15 m above a street with an instantaneous velocity of 5$m.s^{-1}$ upward. The ball was hit from the street below.

5.1 Define the acceleration of the body.
5.2 Determine the initial velocity of the ball immediately after it was hit.
5.3 What maximum height above the street level does the ball reach?

5.1 Acceleration is equal to the rate of change of velocity.

5.2 Given $\Delta y = 15m$, $v_f = 5m.s^{-1}$, $v_i = ?$

$$v_f^2 = v_i^2 + 2g\Delta y$$

$$v_i = \sqrt{v_f^2 - 2g\Delta y}$$

$$= \sqrt{5^2 - 2(-9.8)(15)} = 17.9m.s^{-1}$$

5.3 At maximum height, $v_f = 0m.s^{-1}$

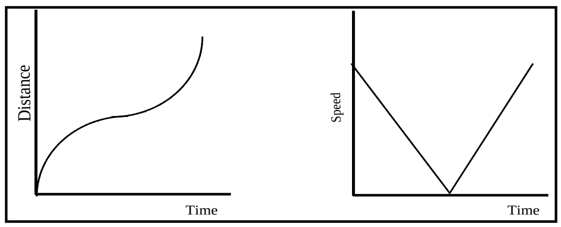
$$v_f^2 = v_i^2 + 2g\Delta y$$

$$\Delta y = \frac{v_f^2 - v_i^2}{2g} = \frac{0^2 - (17.9)^2}{2(-9.8)} = 16.4m.s^{-1}$$

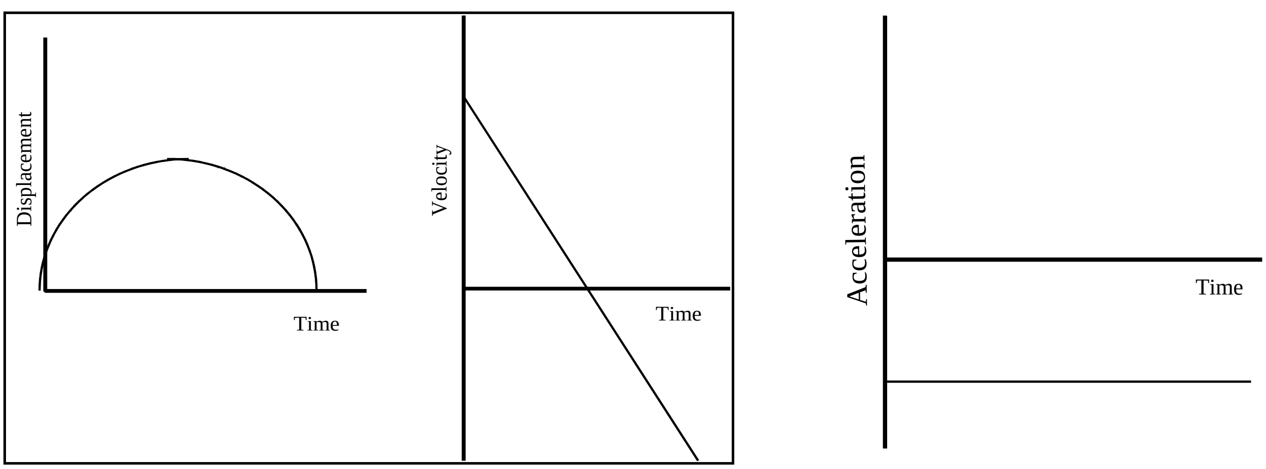
1.1.3 GRAPHS OF MOTION

Let us consider the example of a ball thrown straight up into the air which returns to the thrower's hand after some time *t*.

Can we represent the scalar quantities such as distance and speed graphically for the above situation? The answer is yes and below are the graphs of distance vs time as well as speed vs time. Before plotting our graphs, we will *choose a direction* which we will stick to. Since the ball is thrown vertically upwards, we will choose up as positive. This means that the acceleration of the ball on its way up as well as on its way down is negative. Do you agree?



We have got to be careful when we are plotting vector quantities, such as displacement, velocity and acceleration, since we need to take direction into account.



If we compare the graphs (and have a good eye for detail) we notice the following relationships:

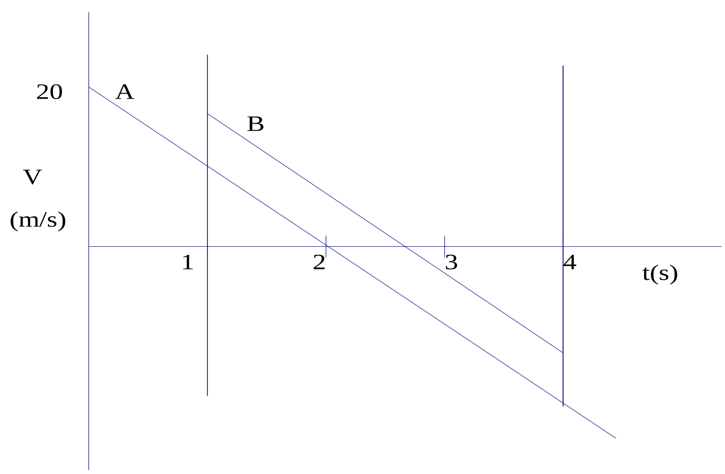
- For a displacement vs time graph the gradient gives the velocity.
- The gradient of a velocity vs time graph gives us the magnitude of the acceleration.
- The area under a velocity vs time graph gives us displacement.
- The area under an acceleration vs time graph gives the velocity.

ACTIVITY 1

An object *A*, is thrown vertically upwards with a velocity of 20 $m.s^{-1}$ and subsequently falls back to ground, landing at 20$m.s^{-1}$.

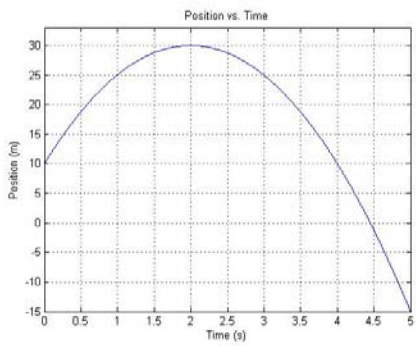
1.1 Sketch a velocity versus time graph representing the motion of object *A*.

1.2 Another object *B*, was thrown upwards 1s after object *A* was thrown, but reached the ground at the SAME TIME as *A*. On the same graph, sketch the motion of object *B*.



ACTIVITY 2

2. Study the x–t graph and answer the questions that follow.



2.1 Calculate the time taken to reach maximum height.

2.2 How long after launch does the object pass its launch point?

2.3 Calculate the initial velocity of the object.

2.4 Determine v (t) for the object.

2.5 Calculate the position of the object (with respect to ground) at t=3s.

2.1. 2s

2.2. 4 s

2.3.

$$\Delta y = v_i t + \tfrac{1}{2} g t^2$$

$$30 - 10 = v_i(2) + \tfrac{1}{2}(-9.8)(2)^2$$

$$v_i = 19.8 m.s^{-1}$$

2.4 v(t) = 19.8 + gt

2.5. 25 m

1.1.4 Air resistance and terminal velocity

In the presence of air resistance there are three forces acting on the object as depicted below:

F_D = drag force
F_B = buoyancy force
W = weight of object

If the sphere is moving slowly, the drag force will be small, as will the buoyancy force, so that the sphere will be accelerated downwards by its weight. As the velocity of the sphere increases, so does the drag force, so that at some point, the buoyancy force and the drag force will balance the weight, and the sphere will cease to accelerate downwards, i.e. it will reach a terminal velocity, v_T. At this point,

$$\boxed{W = F_D + F_B}$$

This only applies to slow moving objects in very viscous fluids. It is found that for slow moving spheres through a viscous medium, the drag force is proportional to the square of the velocity of the sphere.

DID YOU KNOW?

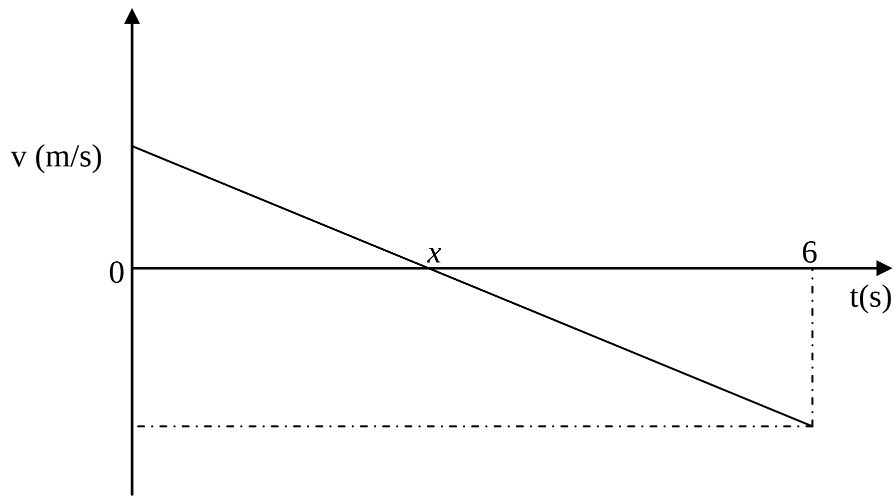
A raindrop about the size of a fly falls at a rate of 9 m/s while a drizzle, about the size of a grain of salt, falls at a rate of 2 m/s.

A human being tumbling through the air, without a parachute falls at a rate of 56 m/s. That’s fast!

WORK BOOK TWO – PRACTICE PROBLEMS

1. A hot air balloon is rising vertically upwards with a constant velocity of $8\ m.s^{-1}$ when one of the passengers accidently lets go of a cell phone while leaning over the edge. The cell phone reaches the ground after 6 s.

   Refer to the graph and answer the questions that follow.



1.1	What is the initial velocity of the cell phone?	
1.2	What is the value of the slope of the graph? Explain.	
1.3	Calculate the time x on the time-axis.	
1.4	Which point on the path of the cell phone corresponds to time x?	
1.5	Calculate the magnitude of the velocity with which the cell phone reaches the ground.	
1.6	Calculate the height of the balloon above the ground at the moment the passenger let's go of the cell phone.	

2. Consider an experiment which resembles an experiment Galileo performed from the leaning tower of Pisa.
   Ball A is dropped from rest from a height of 80m.
   Ignore air resistance.

21. Calculate the time taken ball for A to reach the ground.

22. Ball B is thrown down from the same height 1.5 s after ball A is released. Calculate the magnitude of the velocity with which B must be thrown downwards in order to reach the ground at the same instant as A.

3. A ball is dropped into a dark well; the sound of the ball hitting the bottom of the well comes back after 2 s. Taking the speed of sound c = 340 m.s^{-1}, determine the depth of the well.

4. Ball A is thrown vertically upwards with an initial velocity of 10 m.s^{-1}. A second ball, B is thrown vertically upwards, 5 seconds later, with an initial velocity of 20 m.s^{-1}.

4.1 Calculate the height above the ground at which the two balls will meet?

4.2 When the two balls meet, their displacements will be the same, however, they cover different distances. Explain.

WORK BOOK TWO – SOLUTIONS

1.1 Take up as positive.

8 m.s^{-1}, upwards

1.2 slope = -9.8m.s^{-2}

1.3 $v_f = v_i + gt$ which gives t = 0.8 s

1.4 Point x corresponds to the maximum height reached by the cell phone from where its starts falling towards earth.

1.5 v_f = 8 + gt

$$v_f = v_i + gt$$
$$= 8 + (-9.8)(6)$$
$$= -50.8 m.s^{-1}$$

Or v_f = 50.8 m.s^{-1} downwards.

1.6 Time taken to fall the distance *h*, above the ground = 6 – 2(0.8) = 4.4s.

$$\Delta y = v_i t + \tfrac{1}{2} g t^2$$
$$= (8)(4.4) + \tfrac{1}{2}(9.8)(4.4)^2 = 130m$$

2.1

$$\Delta y = v_i t + \tfrac{1}{2} g t^2$$
$$= 0 + \tfrac{1}{2} g t^2$$
$$t = \sqrt{\frac{2\Delta y}{g}} = \sqrt{\frac{2(80)}{9.8}} = 4.04s$$

2.2 For ball B, the flight time will be t = 4.04 – 1.5 = 2.54 s

$$\Delta y = v_i t + \tfrac{1}{2} g t^2$$
$$v_i = \frac{\Delta y - \frac{1}{2} g t^2}{t} = \frac{80 - \frac{1}{2}(9.8)(2.54)^2}{2.54}$$
$$= 19.05 m.s^{-1}$$

3. Think carefully here. The time t = 2s is made up of two parts – (i) the time taken for the ball to travel down the well and (ii) the time taken for the sound signal to travel up the well.

$$2 = \sqrt{\frac{2h}{g}} + \frac{h}{c} \text{ ----- (1)}$$

Note that we have a quadratic in *h*. Let h = k^2, and so we can write (1) as

$$2c = \omega k + k^2$$
$$k^2 + \omega k - 2c = 0$$

where we have defined $\omega = c\sqrt{\frac{2}{g}}$. Substitute the values for c and ω and solve for k (h).

5. Taking the ground as our zero level and up as positive, we write the displacement equations for both stones:

$S_A = 10t + \frac{1}{2}gt^2$

$S_B = 20(t-5) + \frac{1}{2}g(t-5)^2$

At point of collision

$S_A = S_B$

$\Rightarrow 10t - 400 - 20gt + 200g = 0$

$t = 8.39s$

$S = 10(3.77) + \frac{1}{2}(-9.8)(3.77)^2 = -31.94m$

ie., the balls will collide 31.94 m above the ground.

4.2 Class discussion.