Mrs Angie Motshekga,
Minister of Basic Education

Mr Enver Surty,
Deputy Minister of Basic Education

These workbooks have been developed for the children of South Africa under the leadership of the Minister of Basic Education, Mrs Angie Motshekga, and the Deputy Minister of Basic Education, Mr Enver Surty.

The Rainbow Workbooks form part of the Department of Basic Education's range of interventions aimed at improving the performance of South African learners in the first six grades. As one of the priorities of the Government's Plan of Action, this project has been made possible by the generous funding of the National Treasury. This has enabled the Department to make these workbooks, in all the official languages, available at no cost.

We hope that teachers will find these workbooks useful in their everyday teaching and in ensuring that their learners cover the curriculum. We have taken care to guide the teacher through each of the activities by the inclusion of icons that indicate what it is that the learner should do.

We sincerely hope that children will enjoy working through the book as they grow and learn, and that you, the teacher, will share their pleasure.

We wish you and your learners every success in using these workbooks.

Rainbow WORKBOOKS

MATHEMATICS IN ENGLISH
GRADE 9 – BOOK 1
TERMS 1 & 2
ISBN 978-1-4315-0226-4

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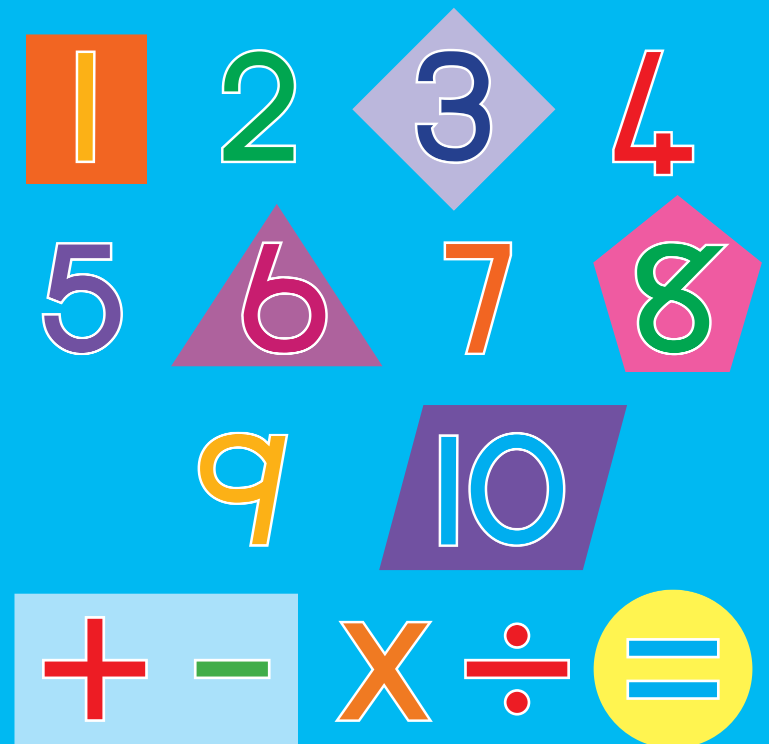

MATHEMATICS IN ENGLISH – Grade 9 Book 1

ISBN 978-1-4315-0226-4

Grade 9

Name: | Class:

basic education
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REPUBLIC OF SOUTH AFRICA

MATHEMATICS IN ENGLISH

Book 1
Terms 1 & 2

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2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
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19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
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Grade 9

Rainbow WORKBOOKS

Mathematics

in ENGLISH

ENGLISH

Book 1

Name:

0 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35

1 Whole numbers and properties of numbers

What does 'arithmetic' mean? Why is it important?

Arithmetic is the oldest and most elementary branch of mathematics and deals with the properties and handling of numbers. It is used by almost everyone for everyday tasks of counting and calculating through to complicated science and business calculations. It involves the study of quantity, especially as the result of combining numbers. Basic arithmetic uses the four operations of addition, subtraction, multiplication and division with integers, rational and real numbers and includes measurement and geometry.

Activities 1–16 are not just revision activities. They also summarise important concepts you need in grade 9.

1. Calculate: round off your answers to the nearest tenth, hundredth and thousandth.

a. 78 438 + 19 469	b. 83 408 – 46 753	c. 37 489 x 128	d. 39) 87 652

2. Use a calculator to check your answers.

3. Draw a flow diagram using the words natural numbers, whole numbers and integers.

4. **Complete the following:**

a. The **commutative** property of addition and multiplication:

i. a + b = ______ ii. a x b = ______

b. The **associative** property of addition and multiplication:

i. (a + b) + c = ______ ii. (a x b) x c = ______

c. The **distributive** property of multiplication over addition and subtraction:

i. a(b + c) = ______ ii. a(b – c) = ______

d. 0 (zero) as the **identity** element for addition: ______ = ______

e. 1 (one) is the **identity** element of multiplication: ______ = ______

5. **Calculate the following by illustrating the properties of whole numbers:**

Example: 33 + 99 = 99 + 33 = 132

a. 51 + (19 + 46) =

b. 4 (12 + 9) =

c. (9 x 64) + (9 x 36) =

d. If 33 + 99 = 132, then 132 =

e. If 20 x 5 = 100, then 100 =

Problem solving

Create a problem using all four basic operations. This should be an everyday example.

2 Multiples and factors

Multiples: The result of multiplying by an integer, e.g. 3 x 4 = 12. The multiples of 3 are: 3, 6, 9, ...

Prime factors of a number are prime numbers that divide that number exactly.

Factors: Factors are the numbers you multiply together, e.g. 3 and 4 are factors of 12. All the factors of 12 are 1, 2, 3, 4, 6, 12.

Talk about ...

LCM: Lowest common multiple

HCF: Highest common factor

1. Identify the LCM.

Example: Multiples of 3: {3, 6, 9, (12), 15, 18, ...}
Multiples of 4: {4, 8, (12), 16, 20, ...}
LCM = 12

a. Multiples of:

7: {____________________}

6: {____________________}

LCM: ________________

b. Multiples of:

8: {____________________}

2: {____________________}

LCM: ________________

c. Multiples of:

5: {____________________}

4: {____________________}

LCM: ________________

d. Multiples of:

9: {____________________}

6: {____________________}

LCM: ________________

2. **Calculate the HCF using factorisation or inspection:**

Example: Factors of 192 and 216

192	2
96	2
48	2
24	2
12	2
6	2
3	3
1	

216	2
108	2
54	2
27	3
9	3
3	3
1	

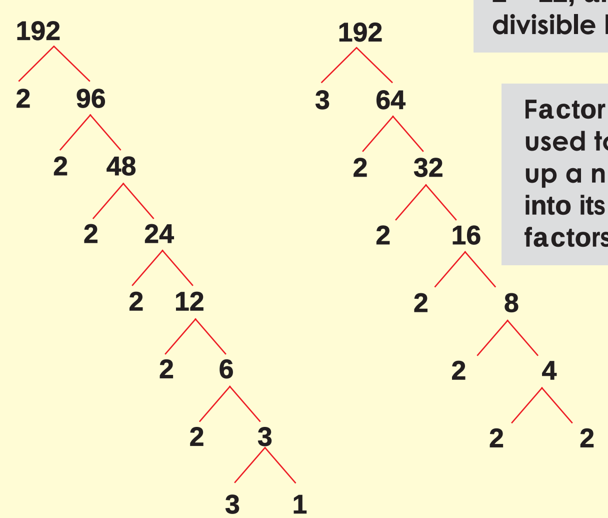
192 = 2 x 2 x 2 x 2 x 2 x 2 x 3

216 = 2 x 2 x 2 x 3 x 3 x 3

Common factors are = 2, 2, 2, 3

HCF = 2 x 2 x 2 x 3 = 24

Factor trees of 192



I know that 192 is divisable by 3 because 1 + 9 + 2 = 12, and 12 is divisible by 3.

Factor trees are used to break up a number into its prime factors.

a. Factors of 204 and 252

b. Factors of 208 and 234

c. Factors of 72 and 188

d. Factors of 275 and 350

continued

Multiples and factors continued

e. Factors of 456 and 572

f. Factors of 205 and 315

3. Calculate the LCM using factorisation or inspection.

Example: Factors of 123 and 141

123	3
41	41
1	

141	3
47	47
1	

3 x 41 x 47 = 5 781

LCM = 5 781

a. Factors of 243 and 729

b. Factors of 200 and 1 000

c. Factors of 225 and 675

d. Factors of 128 and 256

e. Factors of 162 and 486

f. Factors of 225 and 675

Problem solving

Explain calculating HCF using factorisation to a family member.

15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

3 Exponents

Revise the laws of exponents by completing the following:

$x^m x^n$ = ☐ x^1 = ☐

$x^m \div x^n$ = ☐

$(x^m)^n$ = ☐ x^0 = ☐ and $x \neq 0$

Why should you study the laws of exponents?

1. Write these numbers in exponential form:

Example: 144
= 12 x 12
= 12^2

a. 64

b. 9

2. Write these numbers in exponential form:

Example: 81
= 3 x 3 x 3 x 3
= 3^4

a. 27

b. 8

3. Write the following in exponential form:

Example: 64 + 8
= $8^2 + 2^3$

a. 125 + 25 =

b. 64 + 125 =

4. Write the following in exponential form:

Example: 50 x 50 x 50 x 50 x 50 x 50 x 50 = 50^7

a. 30 x 30 x 30 x 30 x 30 =

b. 40 x 40 x 40 x 40 x 40 x 40 x 40 x 40 x 40 x 40 x 40 =

5. Look at the examples and calculate.

Example: $3^1 = 3$, $25^1 = 25$, $m^1 = m$, $9^1 = 9$

a. x^1 =

b. a^1 =

6. Answer positive or negative without calculating.

Example: $(-15)^2$ will be positive
$(15)^2$ will be positive
$(-15)^3$ will be negative

a. $(-9)^2$

b. $(18)^2$

7. Simlify.

Example: a x b x a x b
= $a^2 \times b^2$

$b^2 \times c^2 \times c^2 \times b^2$
= $b^4 \times c^4$

a. g x g x h x h x h

b. a x a x b x b x a x a

8. Revision: calculate the square root.

Example: $\sqrt{9}$
$= \sqrt{3 \times 3}$
$= 3$

a. $\sqrt{64}$

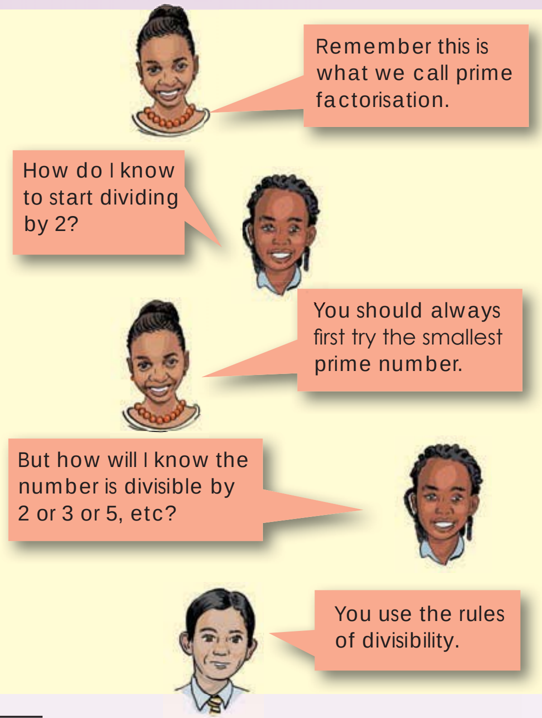
b. $\sqrt{25}$

9. Calculate the square root using the example to guide you:

Example: $\sqrt{256}$
$= \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \times 2 \cdot 2 \cdot 2 \cdot 2}$
$= 2 \cdot 2 \cdot 2 \cdot 2$
$= 16$

256	2
128	2
64	2
32	2
16	2
8	2
4	2
2	2
1	

Test your answer: 16 x 16 = 256



a. $\sqrt{324}$

b. $\sqrt{1296}$

continued

Exponents continued

10. Revise: calculate.

Example: $\sqrt{12 \cdot 12}$
$= 12$

a. $\sqrt{2 \cdot 2}$

b. $\sqrt{3 \cdot 3}$

11. Represent the square root in its simplest form.

Example: $\sqrt{2 \cdot 2 \cdot 2}$
$= 2\sqrt{2}$

a. $\sqrt{3 \cdot 3 \cdot 3}$

b. $\sqrt{6 \cdot 6 \cdot 6}$

12. Represent the square root in its simplest form:

Example: $\sqrt{8}$
$= \sqrt{2 \times 2 \times 2}$
$= 2\sqrt{2}$

a. $\sqrt{12}$

b. $\sqrt{45}$

13. Look at the example and complete the following:

Example: $3^2 = 9$ therefore $\sqrt{9} = 3$

a. 5^2

b. 9^2

14. Calculate and test your answer:

Example: $2^3 \times 2^2$
$= 2^{3+2}$
$= 2^5$
$= 32$

Test: $2^3 \times 2^2$
$= 8 \times 4$
$= 32$

$8^5 \times 8^9 =$

15. Simplify and test your answer:

Example: $x^3 \times x^4$
$= x^{3+4}$
$= x^7$

Test your answer: $x = 2$

$2^3 \times 2^4$
$= 8 \times 16$
$= 128$

2^{3+4}
$= 2^7$
$= 128$

$p^7 \times p^3 =$

16. Calculate and test your answer:

Example: $3^5 \div 3^2$
$= 3^{5-2}$
$= 3^3$
$= 27$

Test: $3^5 \div 3^2$
$= 243 \div 9$
$= 27$

$1^{10} \div 1^{10} =$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

17. Simplify and test your answer.

Example:

$x^5 \div x^3$
$= x^{5-3}$
$= x^2$

Test your answer: $x = 2$

$2^5 \div 2^3$
$= 2^{5-3}$
$= 2^2$
$= 4$

$2^5 \div 2^3$
$= 32 \div 8$
$= 4$

$g^{20} \div g^{15} =$

Test whether $g = 3$

18. Simplify and test your answer:

[you may use your calculator]

Example: $(2^3)^2$
$= 2^{3x2}$
$= 2^6$
$= 64$

Test: $(2^3)^2$
$= (8)^2$
$= 64$

$(7^9)^4 =$

19. Simplify and test your answer:

Example:

$(x^3)^2$
$= x^{3X2}$
$= x^6$

Test your answer: $x = 3$

$(3^3)^2$
$= (3)^{3x2}$
$= 3^6$
$= 729$

$(3^3)(3^3)$
$= 27 \times 27$
$= 729$

$(p^2)^6 =$

Test is $p = 2$

20. Simplify:

Example: $(3x^2)^3$
$= 3^{\cdot}x^{2x3}$
$= 27x^6$

$(23s^{10})^2 =$

21. Simplify:

Example: $(a \times t)^n$
$= a^n \times t^n$

$(b \times c)^y =$

22. Solve using both methods.

Example: $a^4 \div a^4$
$= \frac{a \cdot a \cdot a \cdot a}{a \cdot a \cdot a \cdot a}$
$= 1$

If $a \neq 0$

a^4 means $a \times a \times a \times a$ which means the same as $a.a.a.a$

$= a^{4-4}$
$= a^0$
$= 1$

If $a \neq 0$

$m^3 \div m^3 =$

Problem solving

Add the first 10 square numbers.

Represent the square root of any four–digit number using prime factorisation.

4 Integers and patterns

Integers include the counting numbers {1, 2, 3, ...}, zero {0}, and the negative of the counting numbers {–1, –2, –3, ...}

Commutative property:
a + b = b + a
a x b = b x a

Associative property:
a + (b + c) = (a + b) + c
a x (b x c) = (a x b) x c

Distributive property
a × (b + c)
= a × b + a × c or a × (b + c) × a
= a × b + a × c

1. Identify the last term in each pattern. What is the rule?

Example: –8, –7, –6, –5, –4, –3, –2. –2 is the 7th term. The rule is + 1.

–20, –18, –16, –14, –12, –10, – 8 It is the ______ term.

The rule is ______

2. Write the following in ascending order:

–5, 5, 15, 55, 10, –15, –10, –55 ______

3. Fill in <, >, or =

a. 4 ______ –4 b. –18 ______ –8 c. –2 ______ 2

4. Calculate the following:

Example: (–7) + (5)
= –7 +5
= –2

a. (–6) – (8) =

b. (–8) + (–4) =

5. Calculate the following:

Example: (–5 – 4) x (6 – 2)
= –9 x 4
= –36

a. (–2 – 3) ÷ (–4 – 1)

b. (5 – 6) x (8 – 7)

6. **Calculate the following:**

Example: (–3 + 2) + (5 – 3) x (8 – 9)
= (–1) + (2) x (–1)
= –1 + (–2)
= –1 – 2
= –3

(–7 + 5) x (–2 – 7) + (–5 + 3)=

7. **Commutative property: use the example to guide you to calculate the following:**

Example: 8 + (–3) = (–3) + 8 = 5
8 x (–3) = (–3) x 8 = –24

a. 33 + (–14) = ______ = ______

b. 7 x (–6) = ______ = ______

8. **Use subtraction to check addition or vice versa.**

Example: 8 + (–3) = 5 then
5 – 8 = –3 or
5 – (–3) = 8

a. 17 + (–8) = ______ = ______

b. 9 + (–5) = ______ = ______

9. **Associative property: use the example to guide you to calculate the following:**

Example: [(–6) + 4] + (–1) = (–6) + [4 + (–1)] = (–6) + 3 = –3

a. [(–3) + 2] + (–4) = ______ = ______

b. [(–4) + (–10)] + 5 = ______ = ______

10. **Use division to check or vice versa.**

Example: 5 x (–6) = –30 then
–30 ÷ 5 = –6 and
–30 ÷ (–6) = 5

a. 6 x (–8) =

b. 4 x (–2) =

11. **Complete the pattern.**

Example: (+5) x (+5) = 25
(–5) x (–5) = 25
(+5) x (–5) = –25
(–5) x (+5) = –25

(+12) x (+12) = ______
(–12) x (–12) = ______
(+12) x (–12) = ______
(–12) x (+12) = ______

Problem solving

If the answer is 20 and the calculation has three operations, what could the calculation be?

Common fractions

Look at these examples and give five more examples of each.

Proper fraction $\frac{3}{4}$

Improper fraction $\frac{8}{3}$

Mixed number $1\frac{1}{2}$

Improper fraction to mixed number $\frac{8}{3} = 2\frac{2}{3}$

Mixed number to improper fraction $1\frac{1}{4} = \frac{5}{4}$

1. Add and simplify if necessary.

Example:

$\frac{6}{8} + \frac{4}{8}$

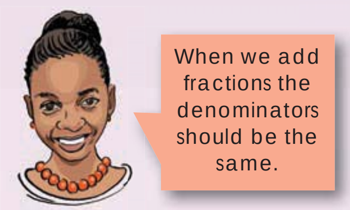
$= \frac{10}{8}$

$= 1\frac{2}{8}$

$= 1\frac{1}{4}$

a. $\frac{6}{12} + \frac{8}{12} =$

b. $\frac{3}{15} + \frac{7}{15} =$



2. Calculate and simplify the answer if necessary.

Example:

$\frac{2 \times 2}{3 \times 2} + \frac{3}{6}$

$= \frac{4}{6} + \frac{3}{6}$

$= \frac{7}{6}$

$= 1\frac{1}{6}$

a. $\frac{1}{4} - \frac{3}{8} =$

b. $\frac{3}{6} + \frac{7}{18} =$

3. Calculate and simplify the answer if necessary.

Example:

$\frac{2 \times 4}{3 \times 4} + \frac{3 \times 3}{4 \times 3}$

$= \frac{8}{12} + \frac{9}{12}$

$= \frac{17}{12}$

$= 1\frac{5}{12}$

a. $\frac{6}{5} + \frac{5}{6} =$

b. $\frac{3}{7} + \frac{7}{9} =$

4. **Calculate and simplify the answer if necessary.**

Example: $\frac{2}{x} + \frac{3}{x}$

$= \frac{2+3}{x}$

$= \frac{5}{x}$

a. $\frac{6}{x} - \frac{5}{x} =$

b. $\frac{1}{x^2} + \frac{4}{x^2} =$

5. **Calculate and simplify.**

Example: $\frac{3}{4} \times \frac{2}{3}$

$= \frac{6}{12}$

$= \frac{1}{2}$

a. $\frac{5}{6} \times \frac{4}{7} =$

b. $\frac{6}{12} \times \frac{4}{5} =$

6. **Simplify:**

Example: $\frac{3}{x} \times \frac{x}{4}$

$= \frac{3x}{4x}$

$= \frac{3}{4}$

a. $\frac{3}{x} \times \frac{x}{12} =$

b. $\frac{x}{21} \times \frac{14}{x} =$

7. **Calculate and simplify the answer:**

Example: $\frac{3}{4} \div \frac{2}{3}$

$= \frac{3}{4} \times \frac{3}{2}$

$= \frac{9}{8}$

$= 1\frac{1}{8}$

a. $\frac{4}{7} \div \frac{4}{6} =$

b. $\frac{9}{12} \div \frac{3}{4} =$

Problem solving

Name five fractions that are between two tenths and three tenths.

What is $\frac{5}{8} + \frac{8}{5}$ in its simplest form?

Can two unit fractions give you a unit fraction if you:
- add it?
- multiply it?

What is $\frac{3}{12} \times \frac{12}{4}$ in its simplest form?

If the answer is $\frac{33}{99}$, what are the two fractions that have been multiplied? Is there only one answer.

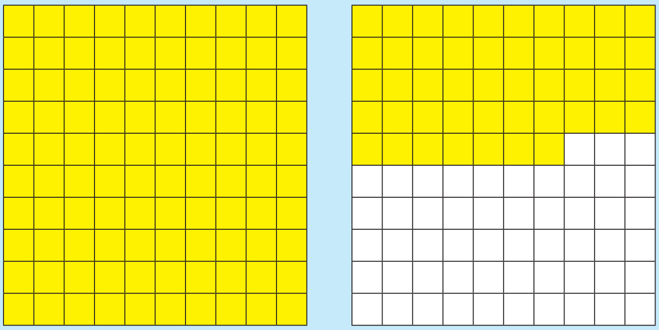
If ____ (whole number) x ____ (fraction) = $\frac{32}{40}$, how many possible solutions are there for this sum?

Multiply any two improper fractions and simplify your answer if necessary.

15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

6 Percentages and decimal fractions

Look at the following. What does it mean?



$$\frac{147}{100} = 1{,}47 = 147\%$$

When in everyday life do we use:

- Decimal fractions?
- Percentages?

1. Write each of the following percentages as a fraction and a decimal fraction:

Example: 18% or $\frac{18}{100}$ or 0,18
$= \frac{9}{50}$

a. 42%

b. 65,5%

2. Calculate.

Example: 25% of R60
$= \frac{25}{100} \times \frac{R60}{1}$
$= \frac{R1\,500}{100}$
= R15,00

a. 30% of R150

b. 65% of R125

3. Calculate the percentage *increase.*

Example:
Calculate the percentage increase if the price of a bus ticket of R60 is increased to R72.

$\frac{12}{60} \times \frac{100}{1}$
$= \frac{1200}{60}$
= 20

∴ **20% increase**

We first need to say by how much the price of the bus ticket was increased.

Then to work out the **percentage increase** we need to multiply $\frac{12}{60}$ by 100.

It was increased by R12 (R72 – R60 = R60).

The price is increased by $\frac{12}{60}$ or by 20%.

R95 to R125

Price increase: ______

4. Calculate the percentage *decrease.*

Example:
Calculate the percentage decrease if the price of petrol goes down from 25 cents to 17 cents a litre. Amount decreased is 8 cents.

$\frac{8}{25} \times \frac{100}{1}$

$= \frac{800}{25}$

= 32

∴ **32% increase**

We first need to say by how much the price of petrol was decreased by.

It was decreased by 8c because 17c + 8c gives you 25c.

Then to work out the **percentage increase** we need to multiply $\frac{8}{25}$ by 100.

R52 of R46

Price decrease: ______

5. Write the following in expanded notation:

Example: 30,405 = 30 + 0,4 + 0,005

a. 39,482

b. 458,917

c. 873,002

d. 903,9301

6. Calculate using both methods. Check your answer.

Example 1: 2,37 + 4,53

= (2 + 4) + (0,3 + 0,5) + (0,07 + 0,03)

= 6 + 0,8 + 0,1

= 6,9

Example 2:

```
  2,37
+ 4,53
  6,90
```

a. 89,879 – 39,999 =

b. 802,897 + 78,873 =

continued

15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

6b Percentages and decimal fractions continued

7. **Calculate. Check your answers using a calculator.**

Example:

0,4 x 0,3 = 0,12 | 0,04 x 0,3 = 0,012 | 0,04 x 0,03 = 0,0012

a. 0,4 x 0,5 =

b. 0,04 x 0,5 =

c. 0,04 x 0,05 =

d. 0,6 x 0,3 =

e. 0,06 x 0,3 =

f. 0,06 x 0,03 =

g. 0,8 x 0,7 =

h. 0,08 x 0,7 =

i. 0,08 x 0,07 =

8. **Calculate. Check your answers using a calculator.**

Example 1: 0,3 x 0,5 x 100
= 0,15 x 100
= 15

Example 2: 0,7 x 0,4 x10
= 0,28 x 10
= 2,8

a. 0,9 x 0,4 x 10 =

b. 0,7 x 0,06 x 10 =

9. **Calculate. Check your answers using a calculator. Round off your answers.**

Example: 4,387 x 30
= (4 x 30) + (0,3 x 30) + (0,08 x 30) + (0,007 x 30)
= 120 + 9 + 2,4 + 0,21
= 120 + 9 + 2 + 0,4 + 0,2 + 0,01
= 131,421

Round off your answers to the:
Nearest unit: 131
Nearest tenth: 131,4
Nearest hundredth: 131,42

a. 16,467 x 40 =

b. 298,999 x 60 =

10. **Calculate the following:**

Example: 9,81 ÷ 9
= 1,09
1,09 rounded off to the nearest tenth is 1,1.

a. 5,25 ÷ 5 =

b. 72,08 ÷ 8 =

c. 48,48 ÷ 6 =

d. 39,97 ÷ 7 =

Problem solving

Multiply the number that is exactly between 2,71 and 2,72 by the number that equals ten times three.

You need twelve equal pieces from 144,12 m of rope. How long will each piece be?

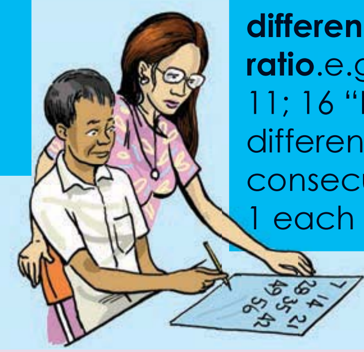
My mother bought 77,12 m of rope. She has to divide it into eight pieces. How long will each piece be?

7 Input and output

What does each statement tell you? Give two more examples of each.

Constant difference
e.g. –3; –7; –11; –15 "Add –4 " or "Count in –4s" or "Add –4 to the previous pattern".

Constant ratio
e.g. –2; –4; –8; –16; –32 "Multiply the previous term by 2."



Not a constant difference or a ratio.e.g. 1; 2; 4; 7; 11; 16 "Increase the difference between consecutive terms by 1 each time."

1. What is the constant difference between the consecutive terms?

a. 8, 12, 16, 20. ______   b. 7, 14, 21, 28. ______

2. What is the constant ratio between the consecutive terms?

a. 3, 9, 27, 81 ______   b. 9, –27, 81, –243 ______

3. Does this pattern have a constant difference or ratio?

a. 1, 4, 10, 19 ______   b. 2, 4, 8, 16 ______

4. What is the constant difference or ratio between the consecutive terms?

a. 5, –15, 45, –135 ______   b. 7, 14, 21, 28, ______

5. Complete the table and then state the rule.

Example:

Position	1	2	3	4	5	n
Value of the term	5	10	15	20	25	n x 5

Rule?
The term x 5.

a. Complete the table

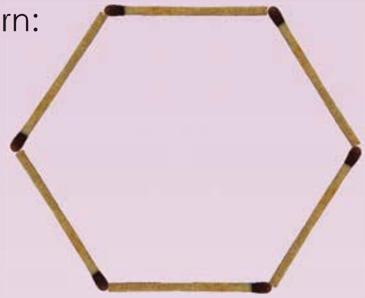
Position	2	4	6	8	n
Value of the term	4	8		16	

b. State the rule. ______

c. What will the 20th term value be? ______

6. Make a drawing to find the next pattern. Complete the questions.

Hexagon number pattern:



(1 x 6)

a. What will the next pattern be? The rule: Increase the length of each side by one match.

b. Record your results in this table.

Hexagon	1	2	3	4	5	6		10	n
Number of matches									

7. Complete the following table. Describe it.

Example: 8, 15, 22, 29…

Term	**1**	**2**	**3**	**4**		**18**	**n**
Value of the term	8	15	22	29		127	7(n) + 1

- Add 7 to the previous position.
- 7 x the position of the term + 1.
- 7(n) + 1, where “n” is the position of the term.
- 7(n) + 1, where “n” is a natural number.

5, 7 , 9, 11,

Term	**1**	**2**	**3**	**4**		**17**	**n**
Value of the term							

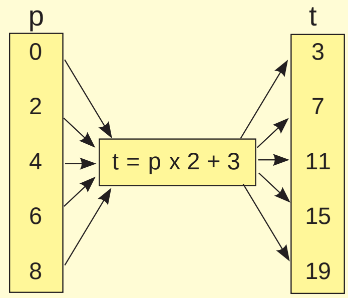
continued

15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

7b Input and output continued

8. Complete the following:

Example:



t = p x 2 + 3 (rule)

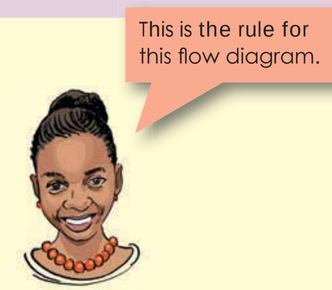
0 x 2 + 3 = 3 (t = 3)

2 x 2 + 3 = 7 (t = 7)

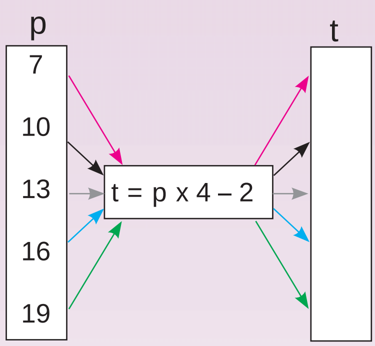
4 x 2 + 3 = 11 (t = 11)

6 x 2 + 3 = 15 (t = 15)

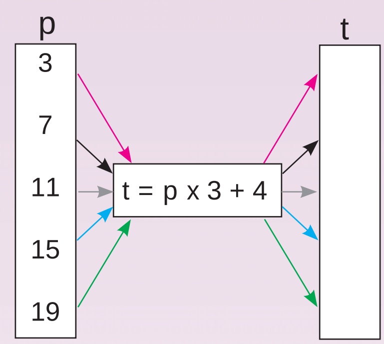
8 x 2 + 3 = 19 (t = 19)



a.

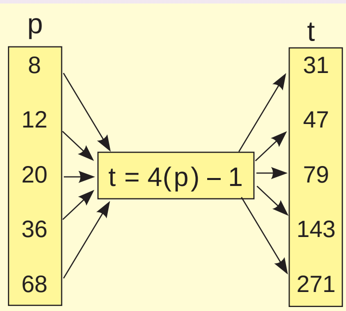


b.



9. What is the rule?

Example:



31 = 4(8) – 1

47 = 4(12) – 1

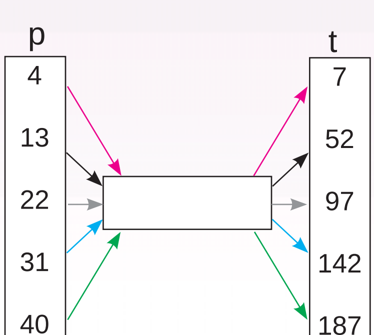
79 = 4(20) – 1

143 = 4(36) – 1

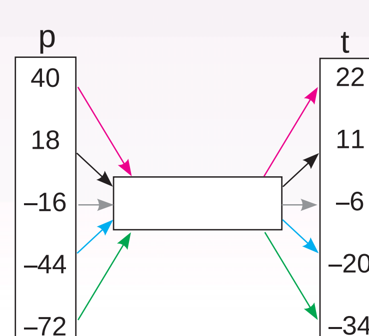
271 = 4(68) – 1

The rule is: t = 4(p) – 1

a.



b.



10. **Describe the relationship between the numbers in the top row and the numbers in the bottom row of the table.**

Example:

x	0	1	2	20	50	100
y	5	7	9	45	105	205

Rule is $y = 2x + 5$

x	–2	–1	0	1	2	3
y	10	8	6	4	2	0

11. **Describe the relationship between the numbers in the top row and those in the bottom row of the table. Write down the values of m and n.**

Example:

x	–2	–1	0	m	2	3
y	30	27	n	21	18	15

m = 1

n = 24

Rule is $y = -3x + 24$

x	–3	–2	m	0	1	2
y	–1	0	1	2	3	n

m =

n =

Rule is y =

Problem solving

a. If the constant ratio is – 7, what could a sequence be?

b. If t = g x 4 – 9, where g = –8, what is t?

c. y = – x + (– 2) is the rule. Show this in a table with x = –3, –2, –1, 0, 1, 2.

8 Algebra

Revise the following:

variable → $x + 18 = 52$ ← constants

operation → $+$; equal sign → $=$

Say whether if the following is an:
- expression, or an
- equation,

and why.

a) $x + 18 = 52$

b) $x + 18$

1. Calculate the following and also underline the variable in red and the constants in blue:

Example 1: $3a + 4a$
$= 7a$

Note: $3a^2 + 5a^2$ is not $8a^4$

Example2: $3a^2 + 4a^2$
$= 8a^2$

Example 3: $5x^2 + 4x^2 = 9x^2$

Example 4: $5x + 4x^2 = 5x + 4x^2$

Example 5: $3a^2 \times 4a^2$
$= (3a^2)(4a^2)$
$= 12a^4$

Example 6: $3a^2 \div 4a^2$
$= \frac{3a^2}{4a^2}$
$= \frac{3}{4}$

a. $5a + 3a =$

b. $6m - 2m =$

c. $1a^2 + 2a^2 =$

d. $8r^2 + 5r^2 =$

e. $4x^2 + 2x^2 =$

f. $5x^2 + 5x =$

g. $2a \times 3a =$

h. $2c^2 \times 5c^2 =$

i. $1a \div 7a =$

j. $3f \div 5f =$

2. Complete.

Example: $4 \times \_\_\_ = 1$

$4 \times \frac{1}{4} = 1$

a. $5 \times \_\_\_ = 1$

b. $7 \times \_\_\_ = 1$

3. Solve for x:

Example 1: $2x = 16$

$\frac{2x}{2} = \frac{16}{2}$

$x = 8$

Example 2: $x - 2 + 3 = -5$

$x + 1 = -5$

$x + 1 - 1 = -5 - 1$

$x = -6$

Example 3: $\frac{2x}{3} = 12$

$\frac{2x}{3} \times 3 = 12 \times 3$

$\frac{2x}{2} = \frac{36}{2}$

$x = 18$

a. $3x = 27$

b. $5x + x = 18$

c. $x + 3 + 2 = 4$

d. $x + 8 + 7 = -8$

e. $\frac{4x}{6} = 12$

f. $\frac{x}{5} = 15$

4. Calculate, if $x = 2$, then:

Example: $2x + 5$
$= 2(2) + 5$
$= 4 + 5$
$= 9$

Example: $x^2 + 5$
$= (2)^2 + 5$
$= 4 + 5$
$= 9$

a. $4x + 8 =$

b. $6 + 3x =$

c. $x^2 + 2 =$

d. $x^2 + 11 =$

e. $x^2 - x =$

f. $3x - x^2 =$

continued ☛

15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

Algebra continued

5. Solve for x.

Example 1: $-5x = 10$

$$\frac{-5x}{-5} = \frac{10}{-5}$$

$x = -2$

Example 2: $2x - 6x = 16$

$-4x = 16$

$$\frac{-4x}{-4} = \frac{16}{-4}$$

$x = -4$

a. $-2x = 10$

b. $-6x = -12$

c. $4x - 5x = 8$

d. $8x + 4x = 4$

6. Calculate:

Example 1: $\frac{x^4}{x^2}$

$= \frac{x.x.x.x}{x.x}$

$= x.x$

$= x^2$

This is a monomial – it has only one term.

Example 2: $\frac{x^4 - x^2}{x^2}$

$= \frac{x^2(x^2 - 1)}{x^2}$

$= x^2 - 1$

This is a binomial – it has two terms connected by a plus or minus sign.

a. $\frac{x^2}{x}$

b. $\frac{x^3}{x^2}$

c. $\frac{x^6 - x^2}{x^2} =$

d. $\frac{x^9 - x^3}{x^3} =$

Example 3: $\frac{x^4 - 6x^2 - 1}{x^2}$

$= \frac{x^4}{x^2} - \frac{6x^2}{x^2} - \frac{1}{x^2}$

$= x^2 - 6 - \frac{1}{x^2}$

e. $\frac{x^4 - 2x^2 - 3}{x^2} =$

f. $\frac{x^6 - 2x^3 - 1}{x^3} =$

7. Revision: Simplify the following using the distributive law:

Example 1: 2 (3 + 4)

= 2 x 3 + 2 x 4

= (2 x 3) + (2 x 4)

= 6 + 8

= 14

2	3	4
	6	+ 8

a. 2 (3 + 6) =

b. 4 (8 + 1) =

Example 2: 2 (x + 5)

= (2 x x) + (2 x 5)

= $2x$ + 10

2	x	5
	$2x$	+ 10

c. 2 (x + 4) =

d. 4 (x + 7) =

Example 3: 2 (x^2 + x + 3)

= (2 x x^2) + (2 x x) + (2 x 3)

= $2x^2$ + $2x$ + 6

2 (x^2 + x + 3)

= $2x^2$ + $2x$ + 6

2	x^2	x	3
	$2x^2$	+ $2x$	+ 6

e. 2 (x^2 + x + 4) =

f. 4 (3 + x + x^2) =

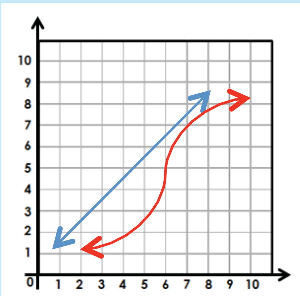
Problem solving

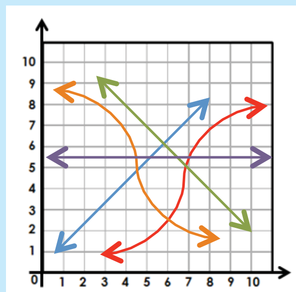
Betty has 8n marbles and Peter has 3n. How many do they have altogether? Write a number sentence.

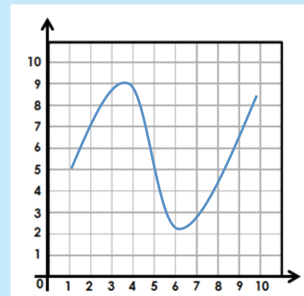
9 Graphs

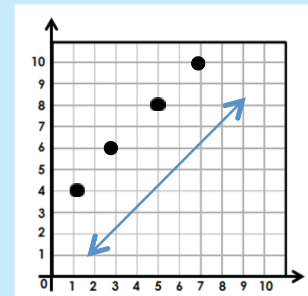
What do the graphs or words tell us about the concept?

Linear and non–linear	Constant, increasing and decreasing	Maximum or minimum	Discrete or continuous

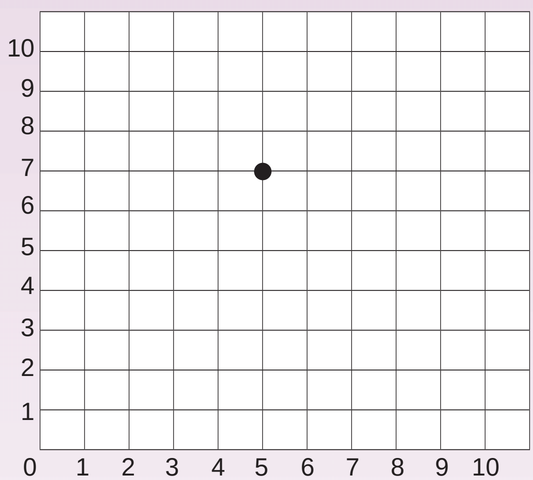








1. Plot the following and write it in words.



Example: The point **(5,7)** is 5 units along, and 7 units up.

a. (3,7) is ☐ units along, and ☐ units up.

b. (4,8) is ☐ units along, and ☐ units up.

c. (5,9) is ☐ units along, and ☐ units up.

d. (10,2) is ☐ units along, and ☐ units up.

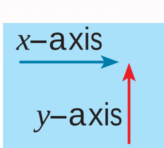
e. (0,6) is ☐ units along, and 2 ☐ units up.

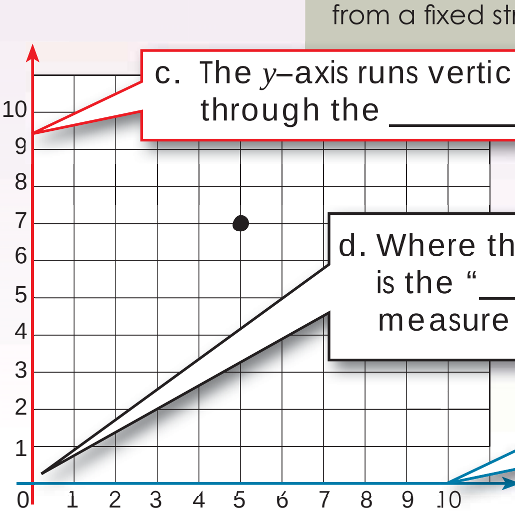
A parabola is the line made by a point travelling so that it is always at the same distance from a fixed point (the focus) as it is from a fixed straight line (the directrix).

2. Complete the following:

a. The left–right (____________) direction is called the *x*–axis.

b. The ____________ (vertical) direction is called the ____________.





c. The *y*–axis runs vertically through the ____________.

d. Where the *x*–axis crosses the *y*–axis is the "__________" point. You measure everything from here.

e. The *x*–axis runs horizontally through the ____________.

3. **Complete the ordered pairs for the equations $y = x^2 + 4$ and $y = -x^2 + 4$ and the plot them on the set of axis on the graph.**

x	–4	–3	–2	–1	0	1	2	3	4
y	20								

$y = (-4)^2 + 4$
$= 16 + 4$
$= 20$

$y = x^2 + 4$
=
=

$y = x^2 + 4$
=
=

$y = x^2 + 4$
=
=

$y = x^2 + 4$
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$y = x^2 + 4$
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$y = x^2 + 4$
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=

$y = x^2 + 4$
=
=

$y = x^2 + 4$
$= 16 + 4$
$= 20$

x	–4	–3	–2	–1	0	1	2	3	4
y	–16								

$y = (-4)^2 + 4$
$= -20 + 4$
$= -16$

$y = -x^2 + 4$
=
=

$y = -x^2 + 4$
=
=

$y = -x^2 + 4$
=
=

$y = -x^2 + 4$
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$y = -x^2 + 4$
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$y = -x^2 + 4$
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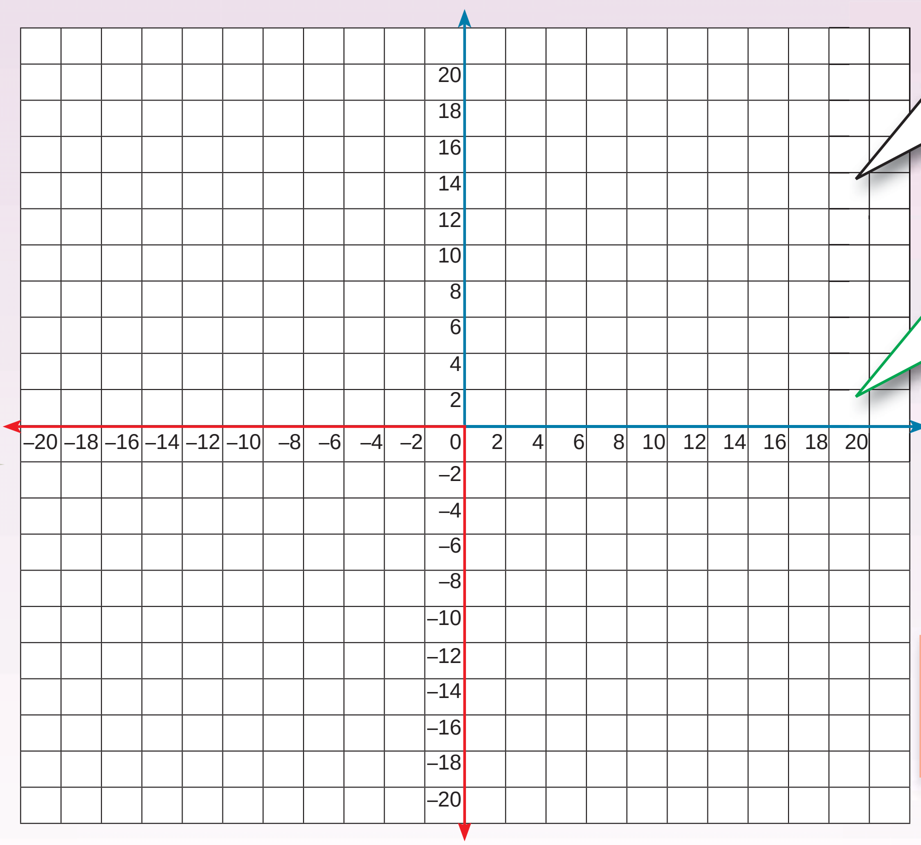


The first parabola has a minimum point (___,___) and it opens upwards (u–shaped).

The second parabola has a maximum point (___,____) and it opens downwards (n–shaped).

What happens if you throw a ball into the air?

It will arc up into the air and come down again. The ball follows the path of a parabola

Problem solving

Describe the graph $y = x^2 + 10$

10 Financial mathematics

Can you remember the meaning of the following?

DEFINITION

Profit is the surplus remaining after total costs are deducted from total revenue.

Loss is the excess of expenditure over income.

Discount is the amount deducted from the asking price before payment.

Budget is the estimate of cost and revenues over a specified period.

A **loan** is a sum of money that an individual or a company lends to an individual or company with the objective of gaining profits when the money is paid back.

Hire purchase is a system by which a buyer pays for an asset in regular installments, while enjoying the use of it.

During the repayment period, ownership of the item does not pass to the buyer. Upon the full payment of the loan, the ownership passes to the buyer.

Interest is the fee charged by a lender to a borrower for the use of borrowed money, usually expressed as an annual percentage of the amount borrowed, (also called the interest rate).

VAT (Value Added Tax) is the tax payable on all goods and services in South Africa. The current VAT rate is 14%. Some essential foods are exempt – that means they have a 0% VAT rate.

An **exchange rate** is the current market price for which one currency can be exchanged for another.

1. Solve the following financial problems:

a. Kabelo receives R120 per week pocket money. He goes ten pin bowling twice (cost R20,00 per session excluding VAT). He has coffee for R5,00 and buys R30,00 of airtime, both with VAT included. How much pocket money can he carry over to the next week?

b. Your receive R400 pocket money per month for chores you do around the house. Draw up a budget in the budget column. You had the following expenses last month: Movie R60,00; Taxi R90,00; Ice Cream R5,75; New shirt R65,00; Donation to welfare R50,00; Stationery R45,00; Repairs to your bicycle R150,00. Enter these expenses in the actual amount column. You have saved R375,00. Did you save anything or will you need to use some of your savings?

	Budget	Actual amount	Difference
Income (Pocket money)			
Expenses			
Taxi			
Movies			
Sweets			
Clothes			
Donations			
Savings			
Stationery			
Totals			
Net Income			

continued ☛

c. A total of R36 000 was invested in two accounts. One account earned 7% annual interest and the other earned 9%. The total annual interest earned was R2 920. How much was invested in each account?

d. David buys a new car on hire purchase. The car costs R75 000 (excluding VAT) and he trades in his old car (that is fully paid for) for R9 500. The car registration, documentation and licence fees are R2 000. What will his instalment be if he pays 7% p.a. in simple interest and repays the money he borrows over a period of 54 months?

e. Lindy has €45. She wants to buy jeans for $15 CAD and a T–shirt for $10 CAD. After her purchases, how much money will she have left in ZAR?

Use the exchange rates in the table below to help you solve the word problems. Show your work in the space provided.

	ZAR (R)	USD ($)	GBP (£)	CAD ($)	EUR (€)	AUD ($)
ZAR (R)	1,00	**6,76**	11,06	6,89	9,88	7,17
USD ($)	**0,15**	1,00	1,60	0,92	1,46	0,87
GBP (£)	0,09	1,09	1,00	0,58	0,91	0,55
CAD ($)	0,15	1,09	1,74	1,00	1,59	0,95
EUR (€)	0,10	0,69	1,10	0,63	1,00	0,60
AUD ($)	0,14	1,15	1,83	1,05	1,67	1,00

Example: **1 ZAR (R) = 0,15 USD ($)**

1 USD ($) = 6,76 ZAR (R)

Problem solving

Make notes of the important financial tips you have learned, and share them with a family member.

11 Geometric figures

Symbols you need to revise or learn.

Triangle	Angle	Perpendicular	Parallel	Degrees	Right angles
$\triangle$	$\angle$	$\perp$	$\parallel$	$^{\circ}$	∟
Line segments	**Line**	**Ray**	**Congruent**	**Similar**	**Therefore**
$\overline{AB}$	$\overleftrightarrow{AB}$	$\overrightarrow{AB}$	$\cong$	$\sim$	$\therefore$

Geometric figures to remember.

Geometric figures

Triangles	Quadrilaterals	More polygons
Equilateral triangle	Parallelogram	Pentagon
Isosceles triangle	Rectangle	Hexagon
Scalene triangle	Square	Heptagon
	Rhombus	Octagon
	Trapezium	Nonagon
	Kite	Decagon, etc.

These are also polygons

Angles to remember.

Acute angle: an angle that is less than 90°

Right angle: an angle that is 90°

Obtuse angle: an angle that is greater than 90° but less than 180°

Straight angle: an angle that is exactly 180°

Reflex angle: an angle that is greater than 180°

Complementary angle: an angle in a pair of angles which add together to make 90°

DEFINITION: polygon is a geometric figure with three or more straight sides.

How would you calculate the total sum of the interior angles of a polygon?

Similar and congruent triangles

Similar triangles have the same shape but are not the same size. Each pair of corresponding angles is equal and the ratio of any pair of corresponding sides is the same.

Congruent triangles are triangles that have the same size and shape. This means that the corresponding sides are equal and the corresponding angles are equal.

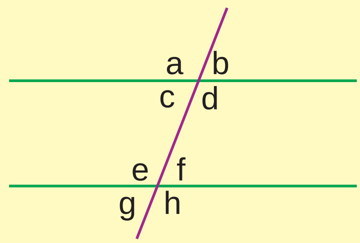
1. **Construct using a protractor.**

a. An angle smaller than 90º.	b. A polygon with more than four sides.	c. A triangle.
i. Name the angle.	i. Calculate the interior angles of the polygon.	i. Draw a traingle that is congruent to the triangle above. Label it.
ii. Construct another angle such that this angle and the angle above, when added together, total 90º. What do you call such a pair of angles?	ii. Where in everyday life will we find such a shape?	ii. Draw a triangle similar to the triangle above. Label it.

continued

Geometric figures continued

2. Describe the constructions using the words below.



Parallel lines

Transversal

Vertical opposite angles

a = d; b = c;

e = h; f = g

Corresponding angles

a = e; b = f;

c = g; d = h

Alternate interior angles

c = f; d = e

Alternate exterior angles

a = h; b = g

Consecutive interior angles

$c + e = 180^{o}$

$d + f = 180^{o}$

a.

b.

A **diagonal** is a straight line inside a shape that joins one vertex to another but is not an edge of that shape.

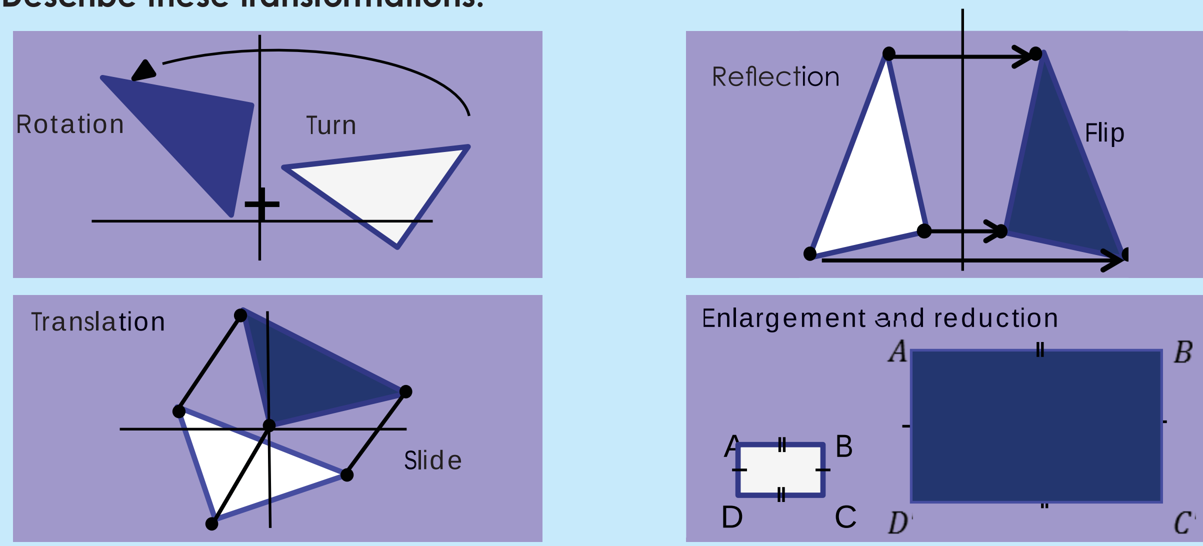
3. Can you identify any diagonals? If not draw a few.

Problem solving

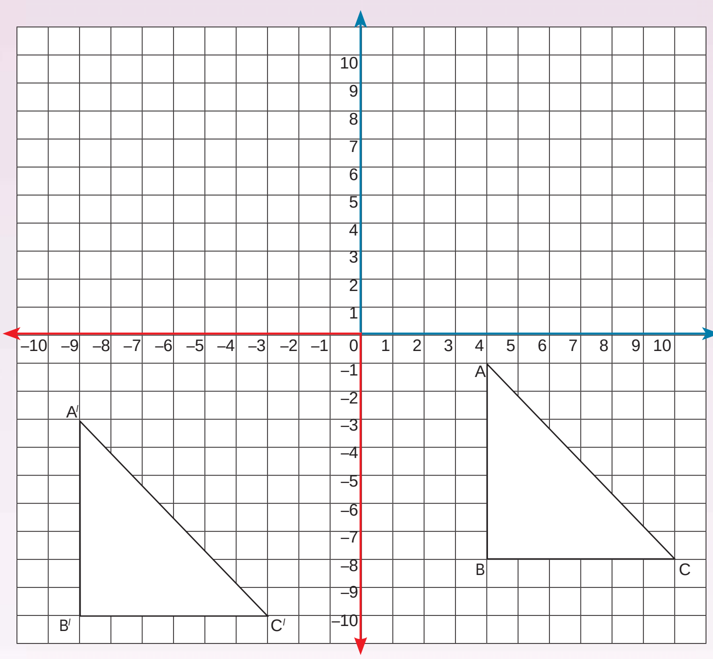
In which job, other than that of an engineer, will people need to calculate angles. Give an example of such a person and say why the person is calculating angles.

12 Transformations

Describe these transformations.



1. Answer the following questions:



a. The coordinates of ABC are:

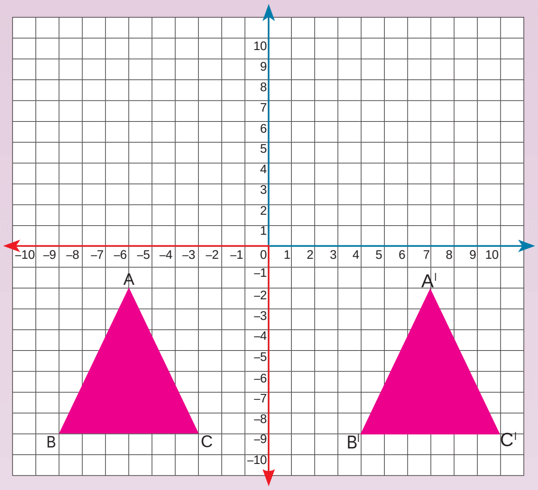
b. The coordinates of A' B' C' are:

c. The translation vector is:

(x – 13, y

d. Explain the translation vector in words.

2. **Answer the following questions:**



a. The coordinates of ABC are:

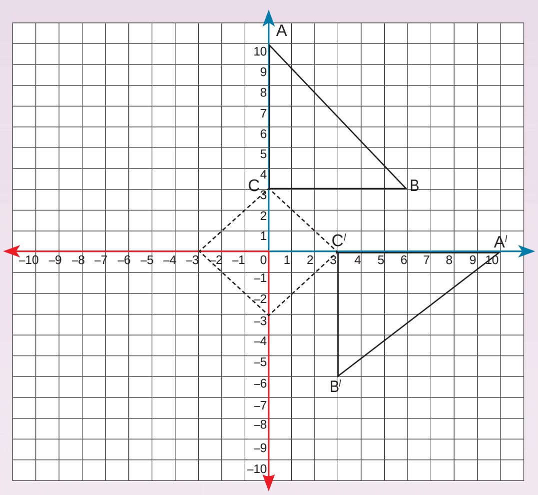
b. The coordinates of A' B' C' are:

c. ABC is reflected over the ______ .

d. Which coordinates remain the same?

e. Which coordinates differ?

3. **Answer the following questions:**

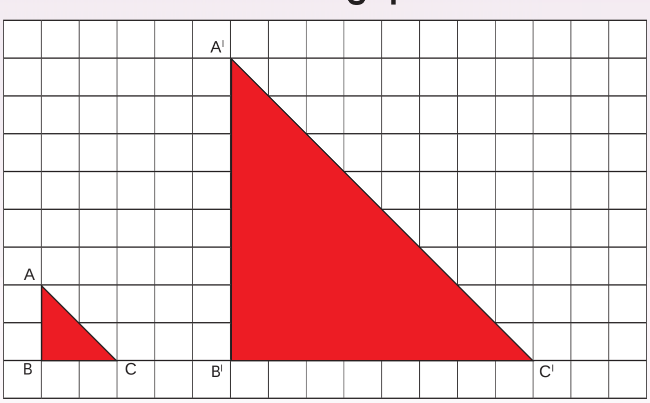


a. The coordinates of ABC are:

b. The coordinates of A' B' C' are:

c. Compare the corresponding vertices.

4. **Answer the following questions:**



a. A'B' = ______ × AB

b. B'C' = ______ × BC

c. A'C' = ______ × AC

d. Therefore, we say that the transformation is an **enlargement** with **scale factor**.

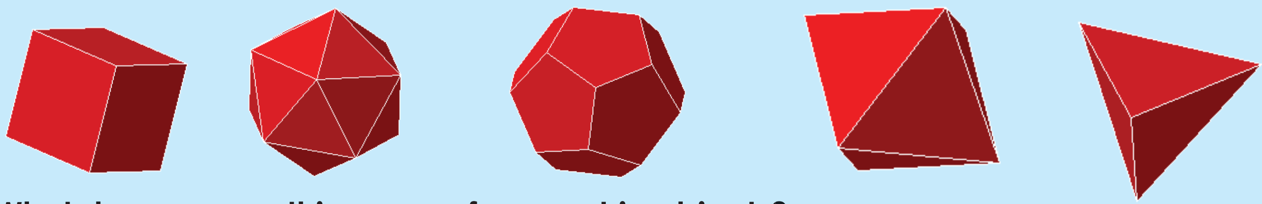
Problem solving

Design a house on grid paper (top view).
Enlarge your plan by a scale factor of 2.
Reflect the house, rotate it by 90 degrees and translate it two units up and three down.

13 Geometric objects

What do all these geometric objects have in common?

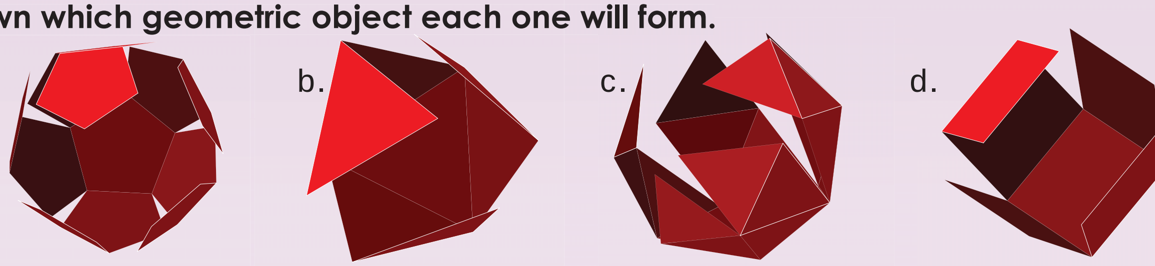
cube icosahedron dodecahedron octahedron tetrahedron



What do we name this group of geometric objects?

1. Label the following using the words: surface (face), edge and vertex. Also write down which geometric object each one will form.

a. b. c. d.



a.	b.	c.	d.
dodecahedron			

Describe each.

e.	f.	g.	h.
30 edges 20 vertices 12 faces			

2. Complete the following:

a. If the sides of a geometric figure are equal in length and the interior angles are equal, the geometric figure is ______________.

If the sides are not equal it is ______________.

b. What do you notice if you look at a platonic solid's surfaces?

____

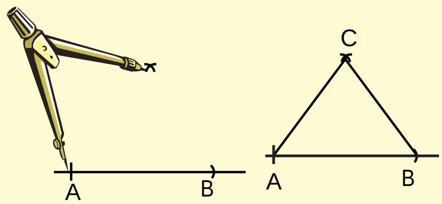
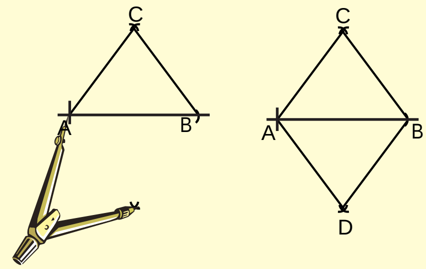
c. What do we name geometric solids if all the surfaces are congruent?

____

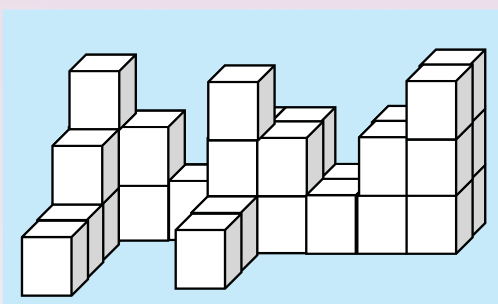
d. Name three geometric solids that are irregular.

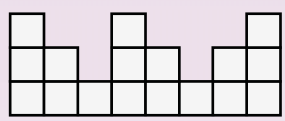
____

3. Construct the net for a tetrahedron. We have given you the first two steps.

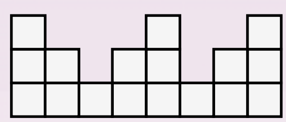
Step 1: Construct an equilateral triangle. Label it ABC. 	**Step 2:** Construct another equilateral triangle with one base joined to base AB of the first triangle. 

4. Describe the different views of the building using the drawings below.

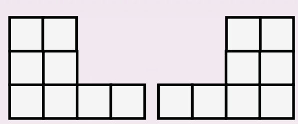




a. ______________________

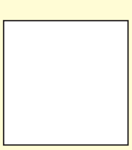
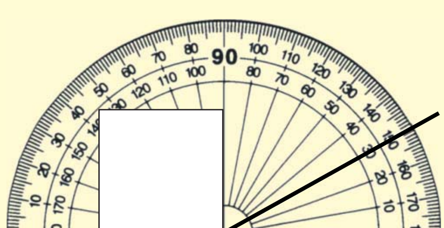
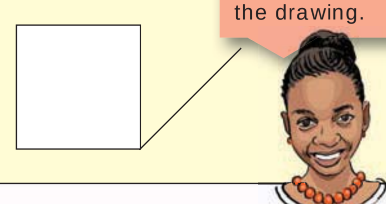


b. ______________________



c. ______________________

5. Draw a cube using a 30º oblique drawing.

Step 1: Draw a square.	**Step 2** Draw a 30º line from the bottom right vertex.	**Step 3** Draw the rest of the cube.
		Remember that the lines that are parallel in the real three–dimensional object remain parallel in the drawing. 

Problem solving

Make skeletons (outlines) of the platonic solids using recycled materials.

14 Perimeter and area

Revise these formulae:

Perimeter of a rectangle $2l + 2b$ Area of a rectangle: $l \times b$	Circumference of a circle $C = \pi d$ or $2 \pi r$
Perimeter of a square: $4l$ Area of a square: $l \times l$	Area of a circle $A = \pi r^2$
The area of a triangle is: $\frac{1}{2}\ b \times h$	

Look at these conversions:

1 cm = 10 mm   1 cm^2 (1 cm x 1 cm)   = 100 mm^2 (10 mm x 10 mm)
1m = 1 000 mm   1 m^2 (1 m x 1 m)   = 1 000 000 mm^2 (1 000 mm x 1 000 mm)
1 km = 1 000 m   1 km^2 (1 km x 1 km)   = 1 000 000 m^2 (1 000 m x 1 000 m

1. Calculate the perimeter and area of a square. Write your answer in mm.

Example: side 4,5 cm

Perimeter
P = 4 x s
= 4 (4,5 cm)
= 18 cm

Area
$A = s^2$
= 4,5 cm x 4,5 cm
= 20,25 cm^2

Write your answer in mm.

= 4 (45 mm)
= 180 mm

= 45 mm x 45 mm
= 2 025 mm^2

If the area is 2 025 mm^2 what is the answer in cm^2?

1 cm = 10 mm
1 cm^2 = 1 cm x 1 cm
1 cm^2 = 10 mm x 10 mm
1 cm^2 = 100 mm^2

$\therefore \left(\frac{2\ 025\ mm^2}{100}\right) cm^2$
= 20,25 cm^2

Side 3,5 cm

2. Calculate the area and perimeter of a rectangle. Write your answer in mm.

Example: length 3,8 cm, breadth 2,1 cm

Perimeter
P = 2 (l + b)
= 2 (3,8 cm + 2,1 cm)
= 2 (5,9 cm)
= 11,8 cm

Area
A = l x b
= 3,8 cm x 2,1 cm
= 7,98 cm^2

1 m = 100 cm
1 m^2 = 100 cm x 100 cm
1 m^2 = 10 000 cm^2

Write the area answer in mm² and m².

mm^2
= 7,98 cm^2
= 7,98 cm^2 x 100
= 798 mm^2

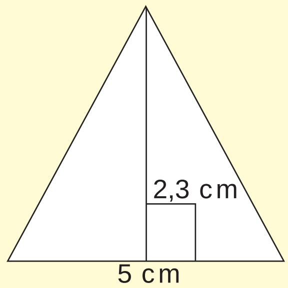
1 cm = 10 mm
1 cm^2 = 1 cm x 1 cm
1 cm^2 = 10 mm x 10 mm
1 cm^2 = 100 mm^2

m^2
$= \frac{7,98\ cm^2}{10\ 000}$
= 0,00798 m^2

Length 9,3 cm and breadth 7,2 cm

3. **Calculate the area of a triangle. Write your answer in mm.**

Example:



Area

$A = \frac{1}{2} b \times h$

$\frac{1}{2}$ (5 cm) x 2,3 cm

= 2,5 cm x 2,3 cm

= 5,75 cm^2

Write your answer in mm^2.

5,75 cm^2

(5,75 cm^2 x 100) mm^2

= 575 mm^2

Write your answer in m^2.

$\left(\frac{5,75 \text{ cm}^2}{10\ 000}\right) m^2$

= 0,000575 m^2

Base = 8 cm Height = 2,6 cm

4. **Calculate the area of the circles.**

Example: Radius is 3 cm.

A = πr^2

= (3,14159) $(3 \text{ cm})^2$

= 28,27 cm^2

a. Radius is 4 cm

b. Radius is 2,5 cm

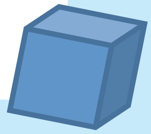
Problem solving

If the area of the circle is 314,159 cm^2. What is the radius?

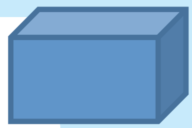
15 Volume and surface area

Revise the following formulae:

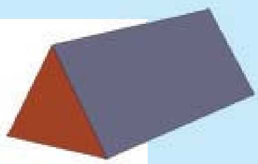
The volume of a cube
$v = l^3$



The volume of a rectangular prism
$v = l \times b \times h$



The volume of a triangular prism
$v = \frac{1}{2} b \times h \times l$



Surface area of a prism

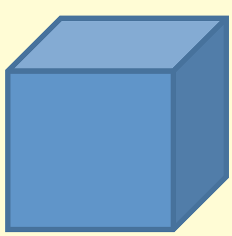
A = the sum of the area of all the faces

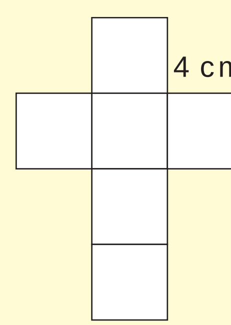
Revise the following:
- if 1 cm = 10 mm then 1 cm^3 = 1 000 mm^3
- if 1 m = 100 cm then 1 m^3 = 1 000 000 cm^3.
- An object with a volume of 1 cm^3 will displace exactly 1 ml of water.
- An object with a volume of 1 m^3 will displace exactly 1 kl of water.

1. Calculate the volume, capacity and surface area of a cube.

Example:

Volume	Capacity	Surface area
$v = l^3$ $v = (4\text{ cm})^3$ $v = 64\text{ cm}^3$	Note: An object with a volume of 1 cm^3 will displace 1 ml of water. Therefore an object that is 64 cm^3 will displace 64 ml water or 0,064 *l*.	Net of the cube. How many faces (surfaces) are there? Surface area = sum of the area of all the faces. = 6 (area of a face) = $6a^2$ = $6\ (4\text{ cm})^2$ = $6 \times 16\text{ cm}^2$ = 96 cm^2





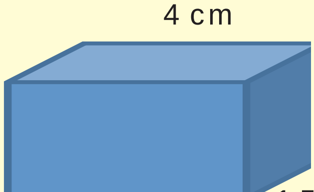
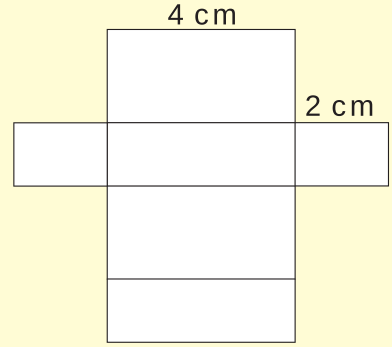
Cubic mm	Cubic cm	Cubic m	Litre
1 000 000 000	1 000 000	1	1 000
1 000 000	1 000	0,001	1
1 000	1	0,000001	0,001

The side (length) of the cube is 2,5 cm.

Volume	Capacity	Surface area

2. Calculate the volume, capacity and surface area of a rectangular prism.

Example:

Volume	Capacity	Surface area
 v = l x b x h v = 4 cm x 1,5 cm x 2 cm v = 12 cm^3	Note: An object with a volume of 1 cm^3 will displace 1 ml of water. ∴ an object that is 12 cm^3 will displace 12 ml.	Net of the rectangle. How many faces (surfaces) are there?  Surface area A = 2 lb + 2lb + 2bh = 2 (4 cm x 1,5 cm) + 2(4 cm x 2 cm) + 2(1,5 cm x 2 cm) = 12 cm^2 + 16 cm^2 + 6 cm^2 = 34 cm^2

Cubic mm	Cubic cm	Cubic m	Litre
1 000 000 000	1 000 000	1	1 000
1 000 000	1 000	0,001	1
1 000	1	0,000001	0,001

continued ☛

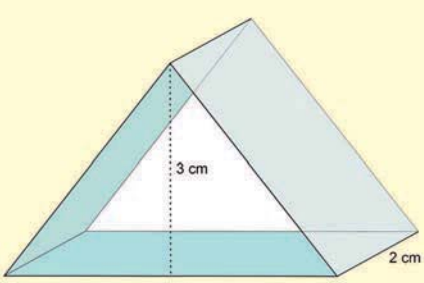
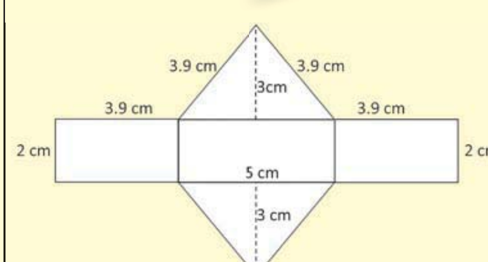
15b Volume and surface area continued

The rectangular prism's dimensions are: length = 4,5 cm; breadth = 3,5 cm and height 4 cm.

Volume	Capacity	Surface area

3. Calculate the volume, capacity and surface area of a triangular prism.

Example:

Volume	Capacity	Surface area
 v = $\frac{1}{2}$ *b* x *h* x *l* v = $\frac{1}{2}$(5cm) x 3 cm x 2 cm v = 2,5 cm x 3 cm x 2 cm v = 15 cm^3	Note: An object with a volume of 1 cm^3 will displace 1 ml of water. ∴ an object that is 15 cm^3 will displace 15 ml of water.	Net of the triangular prism. How many faces (surfaces) are there? Use Pythagoras to calculate this.  Surface area A = 2 (area of triangle) + area of 3 rectangles = 2 ($\frac{1}{2}$ (5 cm) x 3 cm) + 2(3,9 cm x 2 cm) +1(5 cm x 2 cm) = 15 cm^2 + 15,6 cm^2 + 10 cm^2 = 40,6 cm^2

The triangular prism's dimensions are: base of triangle 4 cm, height of triangle 2,5 cm, length of prism 3,2 cm, height of sides 3,2 cm.

Volume	Capacity	Surface area

Problem solving

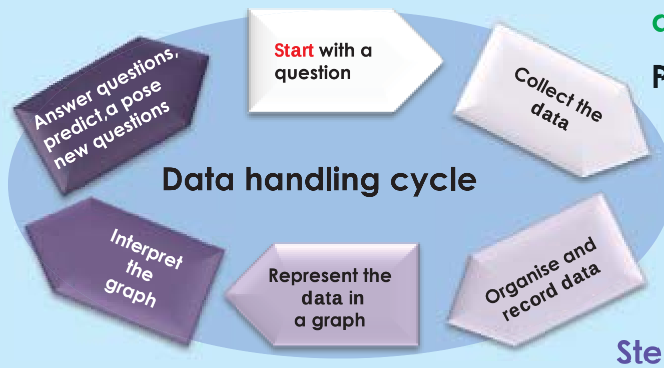
a. If the volume of a cube is 106 cm^3, what are its dimensions in mm and m?

b. Give everyday examples of where we will use the volume, capacity and the surface area of:
- cubes
- rectangular prisms
- triangular prisms

15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

16 Data

Revise: Look at the data handling cycle and describe it.



Primary data
Secondary data
Population
Survey
Questionnaire
Tables
Tallies
Stem and leaf table
Range
Mean
Median
Mode
Bar graph
Histogram
Pie charts
Pictograph

Hypothesis: grade 9 girls do better in mathematics and science than urade 9 boys.

A **hypothesis** is a statement or prediction for which sound evidence of its truth has to be found.

Here are some examples of hypotheses:
- Everybody in grade 9 owns a cell phone.
- All grade 9s like junk food.

1. **Form your research team.**

Names of your research team:

____
____
____
____

2. **What is the aim of your research?**

____
____
____

3. **What is your hypothesis?**

Primary data	Population
Secondary data	Survey
Sample	Questionnaire

4. **Questions that might help you to plan:**

a. What data do you need?

b. Who will you get it from?

c. How will you collect it?

d. How will you record it?

e. How will you make sure the data is reliable?

f. Why? Give reasons for the choices you made.

continued ☛

16b Data continued

Tables
Tallies
Stem and leaf tables
Range
Mean
Median
Mode

7. Use the data you collected and recorded to:

a. Organise your data in a frequency table.

b. Calculate the mean, median and mode.

c. Calculate the data range.

d. Draw a stem–and–leaf display.

e. Represent your data in a graph. You may use more than one type of graph.

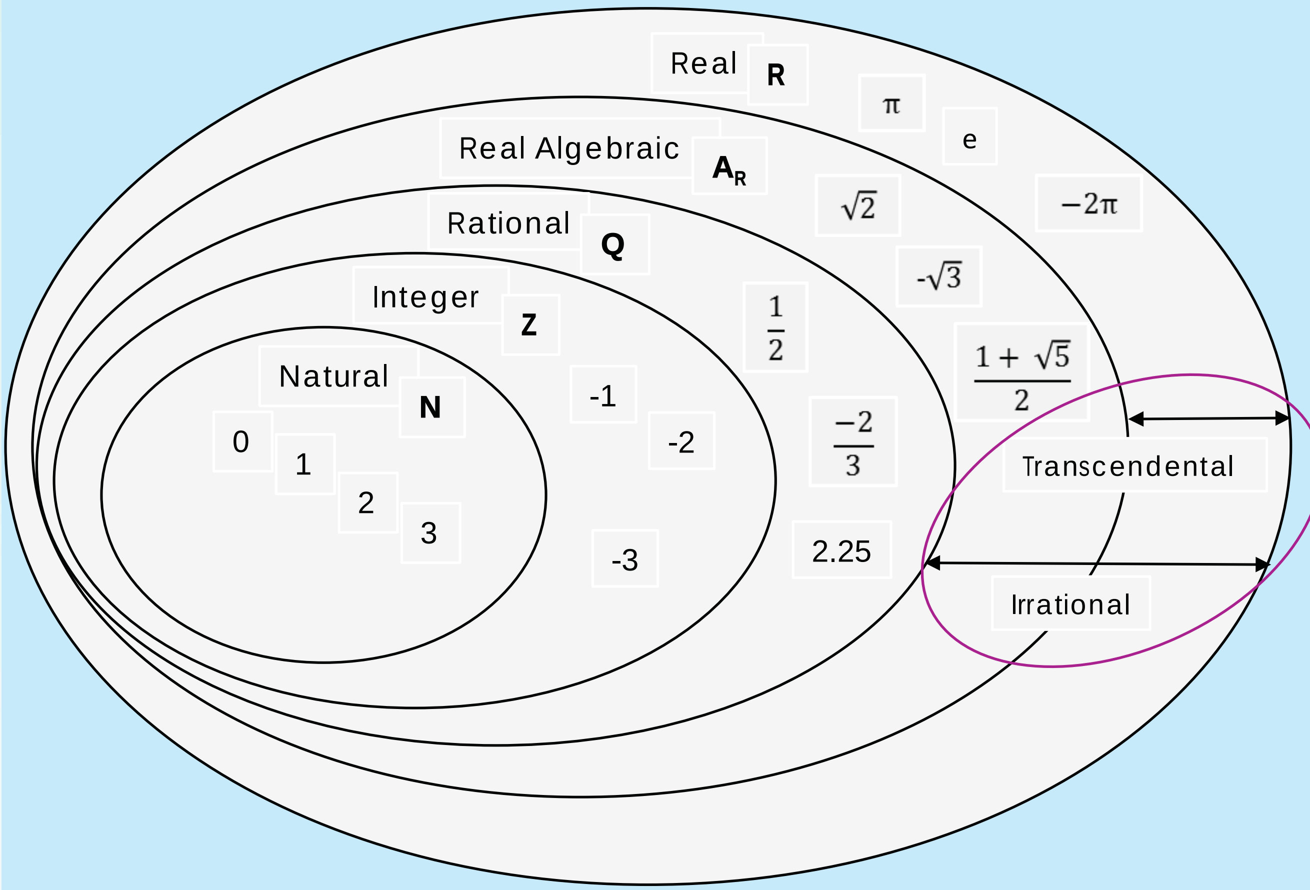
Problem solving

Interpret your graphs and tables and write a report, using the following headings:

1. Aim
2. Hypothesis
3. Plan
4. Data collection
5. Analysis
6. Conclusions
7. Appendices
8. References

17 Real numbers, rational numbers and irrational numbers

Real number Venn diagram – a diagramatic illustration of the real number system



$N \subset Z \subset Q \subset A_R \subset R$

($\subset$ = subset of)

1. **Study these definitions:**

Natural N for **Natural**

Natural numbers are counting numbers (1, 2, 3, ...), the positive integers of the whole numbers (0, 1, 2, 3, ...), the non–negative integers. Mathematicians use the term “natural” in both cases.

Integer (**Z** for **Zahlen** (‘numbers’ in German))

Integers are the natural or whole numbers and their nagatives (...–3, –2, –1, 0, 1, 2, 3, ...).

Rational (**Q** for **Quotient**)

Rational numbers are numbers that can be expressed as a fraction of an integer (that is as ratio a of an integer). Rational numbers can be added, subtracted, multiplied and divided. Eg. $\frac{1}{2}$ = 0,5 or $\frac{1}{3}$ = 0,333 ... Rational decimal expansions end or repeat.

Real Algebraic (**A_R** for **$\text{Algebraic}_{\text{Real}}$**)

A real algebraic number is defined as a number that is the root of a polynomial with rational coefficients. Real algebraic numbers may be rational or irrational. The number $\sqrt{2}$ = 1.41421 ... is a Real algebraic number that is irrational.

Real (**R** for **Real**)

Real numbers are all the numbers (all the points) on the continuous, infinitely long number line with no gaps. It is a collection of every possible infinite decimal expansion. Real numbers may be rational or **Irrational**, and algebraic or non–algebraic **(transcendental)**. The numbers π = 3.14159 ... and e = 2.71828 ... are transcendental. A transcendental number can never be written as an exact fraction of a whole number, it is definitely by an infinite series.

Irrational

These numbers cannot be written as fractions of whole numbers. Irrational decimal expansions neither end nor repeat.

Transcendental

These are irrational numbers that cannot be constituted back as an integer through an arithmetical operation.

continued

Real numbers, rational numbers and irrational numbers continued

2. **Match these descriptions with the correct number line. Start at 'Integers'.**

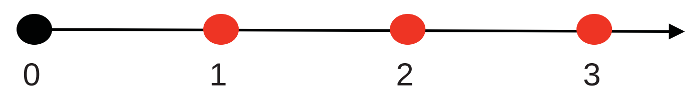
A number that can be expressed as a fraction of an integer.

All the numbers.

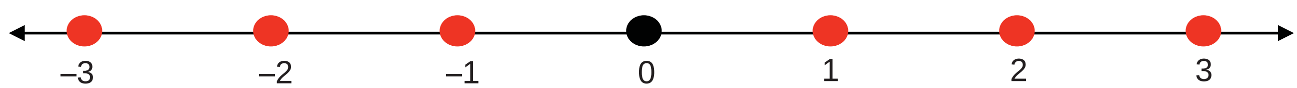
Natural numbers and their negatives.

rational or irrational numbers

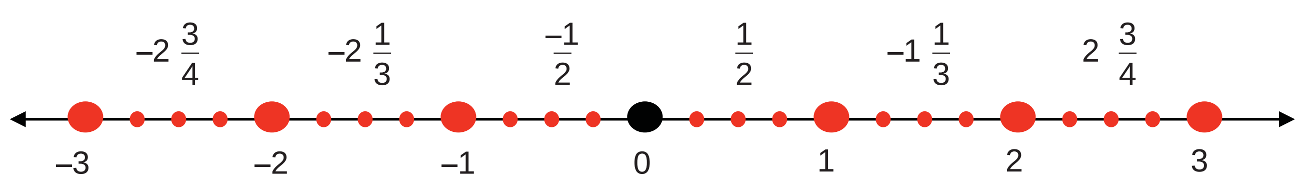
Natural, N



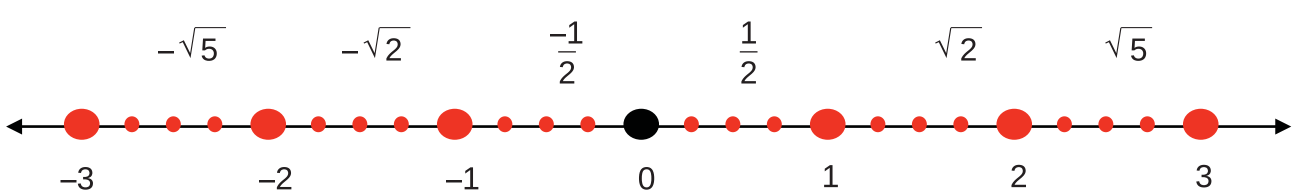
Integer, Z ______________________



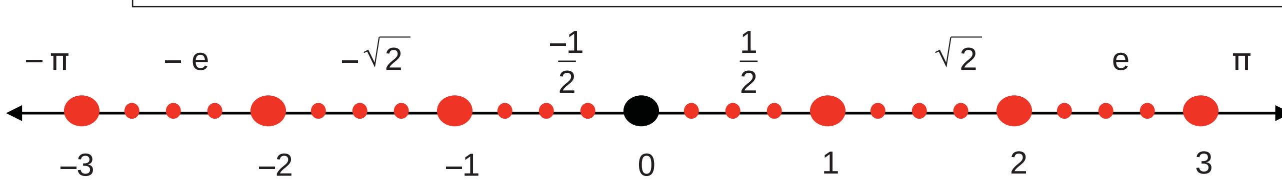
Rational, Q ______________________



Real algebraic, A_R ______________________



Real, R ______________________



3. What do the intervals between the integers on the following number lines mean:

I. Rational ______________________________

ii. Real algebraic ______________________________

iii. Real ______________________________

4. Complete the table.

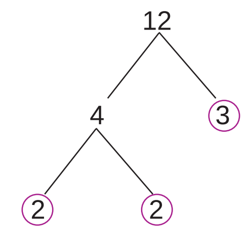
		Whole numbers	Natural numbers	Integers	Rational numbers	Irrational numbers	Real numbers
a	200	✔	✔	✔	✔		✔
b	–29						
c	0						
d	1						
e	$\frac{12}{50}$						
f	0,987						
g	$\sqrt{81}$						
h	$\sqrt{5}$						
i	π						
j	124,54						
k	$\frac{22}{7}$						
l	$\sqrt{25 + 9}$						

Problem solving

The number ***e*** (Euler's Number) is another famous irrational number. Why?

18 Factorisation

Study these methods of factorisation:

Method 1:	**Method 2:**	**Method 3:**
Ladder method. 12 | 2 6 | 2 3 | 3 1 Every factor is a prime number. We can write it as: 2 x 2 x 3 = 12 or 2^2 x 3=12	What are the prime factors of 60? Break 60 into 6 x 10. The prime factors of 6 are 2 and 3. The prime factors of 10 are 2 and 5. So the prime factors of 60 are 2, 2, 3, 5. We can write it as 2 x 2 x 3 x 5 = 60 or 2^2 x 3 x 5 = 60	

Remember it is important to know your divisibility rules when working with prime numbers.

1. a. **Factorise 15.**

Method 1:	**Method 2:**	**Method 3:**

b. **Factorise 72.**

Method 1:	**Method 2:**	**Method 3:**

Before carrying on with questions c. to e. say which method you like the most and why.

c. **Factorise 95.**

Method 1:	Method 2:	Method 3:

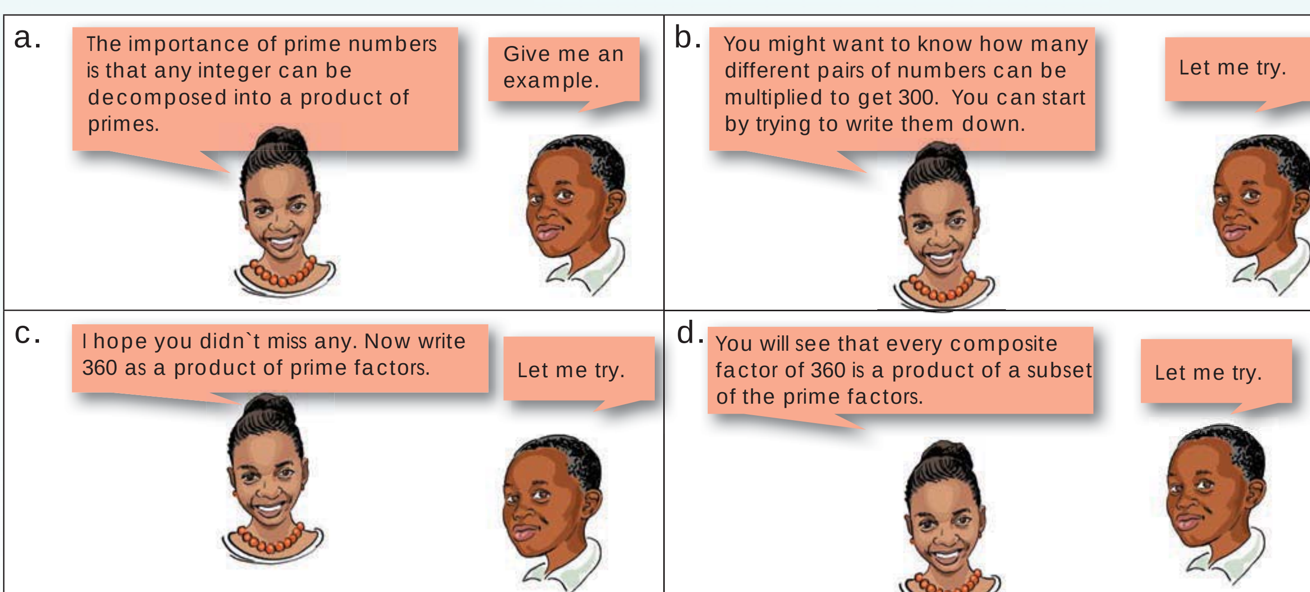
d. **Factorise 100.**

Method 1:	Method 2:	Method 3:

e. **Factorise 198.**

Method 1:	Method 2:	Method 3:

2. **Prime factorisation is finding which prime numbers multiply together to make the original number. Knowing prime factorisation will help you a lot as you carry on with maths. Why? Read the comic strip. Each time a character says 'let me try', as you carry on with, try and do it yourself.**



Problem solving

Prime numbers are numbers that can be divided only by one and themselves. Show this with all the numbers between 100 and 200.

19 Ratio, proportion and speed

Problems about the distance travelled in a given time can be solved using formulae.

To find distance:	To find time:	To find rate (speed):
Distance = Speed x Time	Time = $\frac{\text{Distance}}{\text{Speed}}$	Rate = $\frac{\text{Distance}}{\text{Time}}$
d = s x t	t = $\frac{d}{s}$	s = $\frac{d}{t}$

When we solve problems using these formulae use ratio and proportion.

A ratio is a way of comparing the sizes of two or more quantities. So 4:7 and 8:14 are ratios.

A proportion is a statement that two ratios are equal. So 4:7 is proportional to 8:14 (meaning that 4 is to 7 as 8 is to 14).

A proportion can be written in two ways:

- as two equal fractions: $\frac{4}{7} = \frac{8}{14}$ or
- like this: 4:7::8:14

When two ratios are equal, the cross-products of the ratios are equal. So for the proportion a:b::c:d, a x d = b x c, as in this example:

$\frac{4}{7} = \frac{8}{14}$ so 4 x 14 = 56 and 7 x 8 = 56

Example: My family travelled 300 km at 60 km per hour. For how long did they travel?

We can use a formula or work with ratios and proportion.

Formula to find time:

Time = $\frac{\text{Distance}}{\text{speed}}$

Time = $\frac{300}{60}$ = 5 (hours)

Working with ratio and proportion.

$\frac{60 \text{ km}}{1 \text{ h}} = \frac{300 \text{ km}}{t}$

60 x t = 300 x 1

60t = 300

$\frac{60\,t}{60} = \frac{300}{60}$

t = 5

1. **Complete the table.**

	Rate	Time	Distance	Formula	Proportion
a.	90 km/h	?	11 700 km		
b.	50 km/h	8 hours	?		
c.	120 km/h	?	61 200 km		
d.	500 km/h		?		
e.	1 000 km/h	?	20 000 m		

2. **A car travels 60 km in 18 minutes. At the same average speed, how far will it travel in 1 hour 12 minutes?**

3. **A car travelling at an average speed of 100 km/h covers a certain distance in 3 hours 36 minutes. At what average speed must the car travel to cover the same distance in 2 hours 30 minutes?**

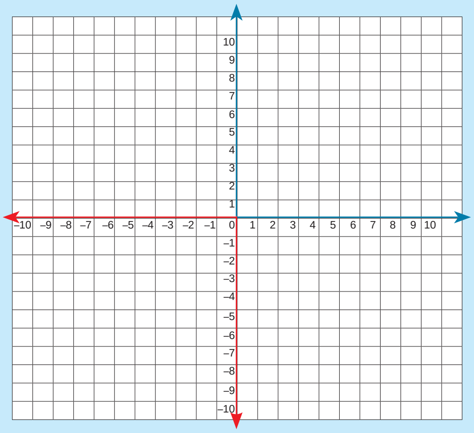
Problem solving

Write a problem using a example from you day–to–day life on speed, distance and time. Ask a family member to help you.

20 What is direct proportion?

Direct proportion

As one value increases, so does the other. How do you think this will look on a graph?



While your are busy with this worksheet think about what **inverse proportion** could mean. We will deal with it in the next worksheet.

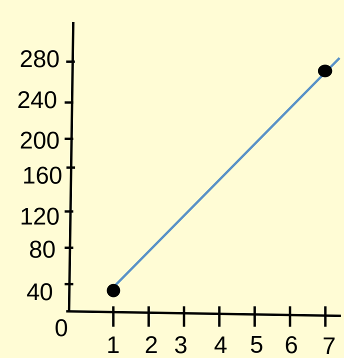
Term 1 – Week 4

Using different methods to solve proportion problems

Example: 4 books cost R150. How much do 7 books cost?

Method 1: Unitary Find the value of 1 unit and multiply to find the value of the required number of units	**Method 2:** Cross–multiply	**Method 3: Rule of three** Align terms in correct columns; multiply 3rd term by 2nd; then divide by 1st.
Books **Rands** **4** R150 **1** $\frac{R150}{4}$ = R37,50 **7** 7 x R37,50 = R262,50	**Books** **Rand** 4 R150 7 x 4 :150 :: 7 : x (1^{st} : 2^{nd} :: 3^{rd} : 4^{th}) 4 x x = 7 x R150 (1^{st} x 4^{th} = 2^{nd} x 3^{rd}) $\frac{4}{7} = \frac{R150}{x}$ $\frac{4x}{4} = \frac{R1\ 050}{4}$ x = R262,50	**Books** **Rand** 4 R150 7 x 4 :150 :: 7 : x (1^{st} : 2^{nd} :: 3^{rd} : 4^{th}) X = 7 x R150 ÷ 4 (x = 3^{rd} x 2^{nd} ÷ 1^{st}) $x = \frac{7 \times 150}{4}$ x = R262,50

Draw a graph.



How does this graph show direct proportion?

1. Use the 3 methods to solve this problem and draw a graph.
   5 T–shirts cost R120. How much will 9 cost?

Method 1:	Method 2:	Method 3:

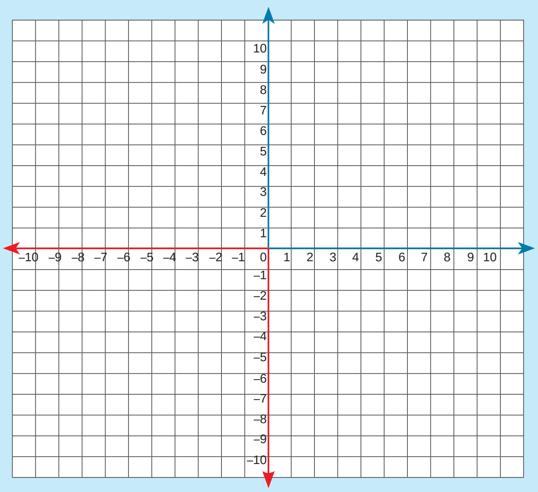
Draw a graph to show this.

Problem solving

Where in your day to day life will you use direct proportion. Draw it on a graph?

21 Inverse proportion

Inverse proportion
As one value increases, the other value shows a matching decrease.



1. Solve using all the methods and draw a graph.

Example: Ten people take 4 days to dig a hole, how long will it take 8 men?

Method 1: Unitary Find the value of 1 unit and multiply to find the value of the required number of units.	**Method 2: Vedic** Align terms in correct columns; multiply 1st term by 2nd and 3rd by 4th.	**Method 3: Rule of three** Align terms in correct columns; multiply 1st term by 2nd term then divide by 3rd.
People **Days** 10 take 4 1 takes 10 x 4 = 40 8 take $\frac{40}{8}$ = 5 8 people will take 5 days Note: Fewer people more time 	**People** **Days** 10 4 8 x 10 : 8 :: 4 : x (1st : 2nd :: 3rd : 4th) 10 x 8 = 4 x x (1st x 2nd = 3rd x 4th) 40 = 8x $\frac{8x}{8} = \frac{40}{8}$ $x = 5$	**People** **Days** 10 4 8 x 10 : 8 :: 4 : x (1st : 2nd :: 3rd : 4th) x = 10 x 4 ÷ 8 (x = 1st x 2nd ÷ 3rd) $x = \frac{10 \times 4}{8}$ $x = 5$

a. If it takes 3 people to make 21 T–shirts per day, how long will it take 12 people?

Method 1:	Method 2:	Method 3:

b. Draw a graph.

c. How does this graph show inverse proportion?

Problem solving

When in your day–to–day life would you use inverse proportion? Draw this on a graph.

22 Properties of numbers

Revise: give an example of each property. Write a rule for each.

Commutative

Associative

Distributive

Zero as a property of addition

One as a property of multiplication

1. Use the commutative property to make the equations equal.

Example:

$a + b = b + a$
$a^2 + b^2 = b^2 + a^2$
$a \times b^2 = b^2 \times a$
$2a + b = b + 2a$
$2a \times 2b = 2b \times 2a$

But:

$a \div b \neq b \div a$
and
$a - b \neq b - a$

a. $y^2 + x =$ $x + y^2$

b. $3x + y^2 =$

c. $3x^2 + 5y^2 =$

d. $2x + y =$

e. $5y + x^2 =$

If $x = 2$ and $y = -3$, solve each equation.

f.

$y^2 + x$	and	$x + y^2$
$= (-3)^2 + 2$		$= 2 + (-3)^2$
$= 9 + 2$		$= 2 + 9$
$= 11$		$= 11$

g.

h.

i.

j.

2. Use the associative property to make the equations equal.

Example:

$(a + b) + c = a + (b + c)$
$(a^2 + b^2) + c^2 = a^2 + (b^2 + c^2)$
$(a \times b) \times c = a \times (b \times c)$
$(a^2 \times b) \times c = a^2 \times (b \times c)$

But:

$(a - b) - c \neq a - (b - c)$
and
$(a \div b) \div c \neq a \div (b \div c)$

a. $(3m + n) + p^2 =$

b. $(n^2 + p^3) + 4m^2 =$

c. $(m \times p) \times n^3 =$

d. $(p^2 \times n^3) \times m^3 =$

e. $(n \times p^2) \times m^3 =$

Test both sides of your equation. If $m = -4$ and $n = 6$.

f.

g.

h.

i.

j.

continued ☛

22b Properties of numbers continued

3. Use the distributive property to make the equations equal. Test both sides of each equation If b = 1, c = 3 and d = 4.

Examples:

$a(b + c) = a \times b + a \times c$

$a(b^2 + c^2) = a \times b^2 + a \times c^2$

$a(b - c) = a \times b - a \times c$

$a(b^2 - c^2) = a \times b^2 - a \times c^2$

a. $(b^2 + c^3)d =$

b. $(d^2 \times b^3) + (d^2 \times c^3) =$

c. $d \times (c + b^2) =$

d. $c\,(b + d^2) =$

e. $(b^2 + d^2) \times c^3 =$

Test both sides of each equation If b = 1, c = 3 and d = 4.

f.

g.

h.

i.

j.

4. Use the identity property of addition or multiplication to make the equations true.

Example: a ___ = a

∴ a + 0 = a or a x 1 = a

a. b ______ = b

or b ______ = b

b. c^2 ______ = c^2

or c^2 ______ = c^2

c. p^3 ______ = p^3

or p^3 ______ = p^3

d. m^3p^2 ______ = m^3p^2

or m^3p^2 ______ = m^3p^2

e. xxx ______ = x^2

or xxx ______ = x^2

Problem solving

Use values a, b and c as well as the distributive property to write an equation. Test both sides using a = 2, b = 3 and c = –1 and then solve it using the following: a = 2, b = 3, c = –1

15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

23 Addition and subtraction of fractions

Before starting this worksheet make sure you know what the following mean. Give an example of each.

Factors	HCF	Multiples	LCM	Improper fraction to mixed number	Mixed number to improper fraction	To simplify a fraction

1. Show why these fractions are equivalent.

Example: $\frac{3}{9} = \frac{1}{3}$

Factors of 3 = {1; 3}
Factors of 9 = {1; 3 ;9}
HCF = 3

$\therefore \frac{3}{9} \div \frac{3}{3} = \frac{1}{3}$

HCF staxnds for highest common factor.

a. $\frac{4}{28} = \frac{1}{7}$

b. $\frac{24}{60} = \frac{2}{5}$

c. $\frac{25}{125} = \frac{1}{5}$

2. Calculate and simplify fractions that are multiples of each other.

Example: $\frac{1}{2} + \frac{3}{4}$

$= \frac{1}{2} \times \frac{2}{2} + \frac{3}{4}$

$= \frac{2}{4} + \frac{3}{4}$ or $\frac{2+3}{4}$

$= \frac{5}{4}$

$= 1\frac{1}{4}$

Why did we multiply $\frac{1}{2} \times \frac{2}{2}$?

Can we add fractions with different denominators?

Yes, if we make the denominators the same.

a. $\frac{2}{4} + \frac{7}{8} - \frac{1}{6} =$

b. $\frac{9}{10} - \frac{3-2}{5} + \frac{7}{8} =$

c. $\frac{2}{6} + \frac{5+1}{12} =$

d. $\frac{8}{10} + \frac{2}{6} - \frac{9}{12} =$

e. $\frac{13}{15} - \frac{8}{10} + \frac{1}{5} =$

f. $\frac{3}{4} - \frac{5-3}{6} + \frac{7}{8} =$

3. Calculate and simplify fractions that are not multiples of each other.

Example: $2\frac{1}{5} + \frac{3}{6}$

$= \frac{11}{5} + \frac{3}{6}$

Multiples of 5 = {5; 10; 15; 20; 25; 30; 35}

Multiples of 6 = {6: 12; 18; 24; 30; 36}

LCM = 30

$= \frac{11}{5} \times \frac{6}{6} + \frac{3}{6} \times \frac{5}{5}$

$= \frac{66}{30} + \frac{18}{30}$

$= 2\frac{24}{30}$

$= 2\frac{4}{5}$

a. $3\frac{7}{10} - 1\frac{8}{9} =$

b. $- 2\frac{2}{10} + 1\frac{6}{7} =$

c. $8\frac{3}{4} - 6\frac{5}{6} + \frac{1}{2} =$

d. $5\frac{4}{10} - 8\frac{4}{5} =$

e. $3\frac{1}{2} + 2\frac{3}{9} + \frac{3}{8} =$

f. $9\frac{7}{8} - 7\frac{3}{7} =$

Problem solving

If the answer to a sum is ¾, what could the sum be? Create some of your own word sums like this.

24 Addition and subtraction of fractions that include squares, cubes, square roots and cube roots

Before starting this worksheet make sure you know what the following mean. Give an example of each.

Calculate a square number	Calculate a square root	Calculate a cube number	Calculate a cube root

1. Calculate the following fractions, using the example to guide you.

Example 1: $\frac{2^2}{2^3} + \frac{3^2}{4^2}$

$= \frac{4}{8} + \frac{9}{16}$

$= \frac{8}{16} + \frac{9}{16}$

$= \frac{17}{16}$

$= 1\frac{1}{16}$

Example 2: $-\frac{1^3}{3^2} - \frac{2^3}{4^2}$

$= -\frac{1}{9} - \frac{8}{16}$

$= -\frac{16}{144} - \frac{72}{144}$

$= -\frac{88}{144}$

$= -\frac{11}{18}$

9	3		16	2
3	3		2	2
1			2	2
			2	2

LCM: **3 x 3 x 2 x 2 x 2 x 2 = 144**

88	2		144	2
44	2		72	2
22	2		36	2
11	11		18	2
1			9	3
			3	3
			1	

HCF: **2 x 2 x 2 = 8**

HCF = 8

$= \frac{-88}{144}$

$= \frac{-11}{18}$

Look at example 2: Why is it important to understand LCM and HCF when we calculate fractions?

a. $\frac{8^2}{8^3} - \frac{10^2}{10^3} =$

b. $\frac{2^2}{2^3} + \frac{7^2}{7^3} =$

c. $\frac{4^2}{4^3} + \frac{4^2}{4^3} =$

d. $\frac{5^2}{5^3} - \frac{3^2}{3^3} =$

e. $\frac{1^2}{1^3} - \frac{9^2}{9^3} + \frac{11^2}{11^3} =$

f. $\frac{4^2}{4^3} + \frac{15^2}{15^3} =$

2. Complete the following:

Example: $\frac{\sqrt{9}}{\sqrt{16}} + \frac{\sqrt[3]{8}}{\sqrt[3]{27}}$

$= \frac{3}{4} + \frac{2}{3}$

$= \frac{9}{12} + \frac{8}{12}$ or $\frac{9+8}{12}$

$= \frac{17}{12}$

$= 1\frac{5}{12}$

a. $\frac{\sqrt{25}}{\sqrt{100}} + \frac{\sqrt[3]{1\,331}}{\sqrt{144}} =$

b. $\frac{\sqrt{36}}{\sqrt[3]{1000}} - \frac{\sqrt[3]{64}}{\sqrt{25}} =$

c. $\frac{\sqrt{1}}{\sqrt{9}} + \frac{\sqrt[3]{8}}{\sqrt{16}} =$

d. $\frac{\sqrt{1}}{\sqrt[3]{1000}} - \frac{\sqrt[3]{64}}{\sqrt{25}} =$

e. $\frac{\sqrt[3]{1\,331}}{\sqrt[3]{8}} + \frac{\sqrt{169}}{\sqrt{144}} =$

f. $\frac{\sqrt{81}}{\sqrt[3]{1000}} - \frac{\sqrt[3]{27}}{\sqrt[3]{64}} =$

Problem solving

Create your own word sums using cubes and cube roots.

25 Multiplication of fractions

What is the reciprocal of a number?

To get the reciprocal of a number divide 1 by the number.	If you multiply a number by its reciprocal you get 1.	Did you know that every number has a reciprocal except 0?	A reciprocal is also called the multiplicative inverse.
The reciprocal of 2 is $\frac{1}{2}$	... such as $3 \times \frac{1}{3} = 1$	... because $\frac{1}{0}$ is undefined.	

1. Calculate and simplify.

Example:

$6 \times \frac{1}{2}$

$= \frac{6}{1} \times \frac{1}{2}$

$= \frac{6}{2}$

$= 3$

a. $8 \times \frac{1}{2} =$

b. $9 \times \frac{1}{3} =$

c. $7 \times \frac{1}{14} =$

d. $5 \times \frac{2}{15} =$

e. $4 \times \frac{2}{12} =$

f. $9 \times \frac{1}{27} =$

2. Simplify.

You can simplify by finding the highest common factor (HCF) – if you cannot find the HCF straight away, keep on simplifying using smaller common factors.

Example:

$\frac{4}{8} \times \frac{7}{6}$

$\frac{4 \times 7}{8 \times 6} = \frac{28}{48}$

Simplify if needed:

$\frac{28}{48} \div \frac{4}{4}$

$= \frac{7}{12}$

How did I know to simplify by dividing by 4?

Factors of 28 = {1; 2; 4; 7; 14; 28}

Factors of 48 = {1; 2; 4; 6; 8; 12; 16; 24; 48} or

48	2	28	2
24	2	14	2
12	2	7	7
6	2	1	
3	3		
1			

48 = 2 x 2 x 2 x 2 x 3

28 = 2 x 2 x 7

HCF = 2 x 2 = **4**

a. $\frac{1}{6} \times \frac{2}{4} =$

b. $\frac{3}{4} \times \frac{2}{5} =$

c. $\frac{2}{7} \times \frac{1}{2} =$

3. Simplify.

Example:

$$-\frac{8}{9} \times \frac{7}{10}$$
$$= -\frac{8}{9} \times \frac{7}{10}$$
$$= -\frac{28}{45}$$

or

$$-\frac{8 \times 7}{9 \times 10}$$
$$= -\frac{56}{90}$$
$$= -\frac{56}{90} \div \frac{2}{2}$$
$$= -\frac{28}{45}$$

90	2	56	2
45	5	28	2
9	3	14	2
3	3	7	7
1		1	

90 = 2 x 5 x 3 x 3
56 = 2 x 2 x 2 x 7

HCF = 2

a. $\frac{2}{10} \times \frac{6}{8} =$

b. $\frac{2}{6} \times -\frac{3}{7} =$

c. $\frac{4}{8} \times \frac{2}{2} =$

continued ☛

Multiplication of fractions continued

Simplify by finding the highest common factor (HCF). If you cannot straight away find the HCF, keep on simplyfying

4. Simplify.

Example:

$$\frac{12}{14} \times \frac{7}{8} = \frac{12}{14} \times \frac{7}{8} = \frac{3 \times 1}{2 \times 2} = \frac{3}{4}$$

or

$$\frac{12 \times 7}{14 \times 8} \quad \frac{84 \div 24}{112 \div 24} = \frac{3}{4}$$

112	2
56	2
28	2
14	2
7	7
1	

84	2
42	2
21	7
3	3
1	

2 x 2 x 2 x 2 x 7
2 x 2 x 3 x 7

LCM = 28

HCF = 7

a. $\frac{3}{4} \times \frac{4}{7} =$

b. $\frac{2}{9} \times \frac{3}{10} =$

c. $\frac{4}{8} \times \frac{1}{6} =$

5. Simplify and write your answers as mixed numbers (use a calculator if needed):

Example:

$$4\frac{5}{6} \times 3\frac{2}{3} = \frac{29}{6} \times \frac{11}{3} = \frac{29 \times 11}{6 \times 3} = \frac{319}{18} = 17\frac{13}{18}$$

REVISION

To convert mixed numbers to improper fractions:

$4\frac{5}{6}$ (multiply 4 by 6 and add 5 = $\frac{29}{6}$ to get the numerator).

$3\frac{2}{3}$ (multiply 3 by 3 and add 2 to get the numerator = $\frac{11}{3}$).

To change an improper fraction to a mixed number:

$\frac{319}{18}$ (ask how many times 18 goes into 319 (319 ÷ 18 = 17 rem 13) = 17 $\frac{13}{18}$).

Use a calculator if necessary.

a. $2\frac{1}{3} \times \frac{1}{4} =$

b. $\frac{1}{2} \times 2 =$

c. $3\frac{4}{5} \times 4\frac{2}{20} =$

6. Simplify.

Example:

$-5\frac{1}{2} \times \frac{4}{10}$

$= -\frac{11}{2} \times \frac{4}{10}$

$= -\frac{11 \times 4}{2 \times 10}$

$= -\frac{44}{20}$

$= -2\frac{4}{20}$

Simplyfy = $-2\frac{1}{5}$

REVISION

Do you still remember?

(positive number) x (positive number) = positive number

(positive number) x (negative number) = negative number

(negative number) x (negative number) = positive number

a. $\frac{8}{9} \times -\frac{3}{4} =$

b. $-3\frac{3}{8} \times \frac{1}{2} =$

c. $-\frac{1}{4} \times -1\frac{1}{4} =$

Problem solving

A train has nine passenger wagons. Each passenger wagon has a seating capacity of 30. If these passenger wagons are replaced with wagons that have half the seating capacity, how many wagons will the train have to have to accommodate the same number of passengers?

26 Division of fractions

Revision: what does reciprocal mean?

Number	**Reciprocal**
8	$\mathbf{\frac{1}{8}}$

Compare what happens if you divide and multiply $\frac{3}{4}$ and $\frac{1}{4}$.

Multiply	Divide
$\frac{3}{4} \times \frac{1}{4}$ =	$\frac{3}{4} \div \frac{1}{4}$ =

What do you notice?

1. Simplify.

Example:

$$\frac{7}{9} \div \frac{4}{12}$$
$$= \frac{7}{9} \times \frac{12}{4}$$
$$= \frac{28}{12}$$
$$= \frac{14}{6}$$
$$= 2\frac{2}{6}$$
$$= 2\frac{1}{3}$$

- Turn the second fraction upside–down (this is its reciprocal) • Multiply the first fraction by that reciprocal.
- Simplify the fraction if necessary.

a. $\frac{8}{10} \div 3 =$

b. $\frac{2}{6} + \left(-\frac{8}{12}\right) =$

c. $\frac{1}{4} \div 1\frac{1}{12} =$

2. Simplify.

Example:

$-\frac{1}{9} \div 3\frac{1}{10}$

$= -\frac{1}{9} \div \frac{31}{10}$

$= -\frac{1}{9} \times \frac{10}{31}$

$= -\frac{10}{279}$

Is it possible to simplify this expression?

$$-9\frac{1}{3} \div 8\frac{3}{4} =$$

a. $-3\frac{1}{16} \div 1\frac{1}{8} =$

b. $-7\frac{2}{5} \div 5\frac{1}{10} =$

c. $-9\frac{1}{3} \div \left(-8\frac{3}{4}\right) =$

3. Simplify.

Example:

$4\frac{1}{16} \div \frac{2}{4}$

$= \frac{65}{16} \times \frac{4}{2}$

$= \frac{65}{8}$

$= 8\frac{1}{8}$

a. $2\frac{1}{4} \div 2 =$

b. $4\frac{3}{4} \div 2\frac{2}{3} =$

c. $\frac{7}{4} \div \frac{1}{4} =$

Problem solving

Ask one of your family members if they know how to divide fractions. If they don't know or can't remember, show them how to do it.

27 Percentages

What is 20% of R140?

20% x R140

$\frac{20}{100}$ x R140

$\frac{20}{100}$ x $\frac{R140}{1}$

= $\frac{R2\ 800}{100}$

= R28

What does 'of' mean in mathematics?

What does 20 % mean?

How can I write R140 as a fraction?

Why can I also say?
0,2 x R140 = R28

1. Calculate the following:

a. What is 15% of R600?

b. What is 20% of R250,00?

c. What is 10% of R1 000,00?

2. Complete the following:

Example: What percentage is R1,40 (of) R10,00?

$\frac{R1,40}{R10,00}$ of 100%

$= \frac{R1,40}{10} \times \frac{100}{1}$ %

$=$ 14 %

`of` tells me it is a multiplication sum.

a. What percentage is R10,00 of R200,00?

b. What percentage is 20c of R1,95?

continued

Percentages continued

3. Calculate the percentage increases. Round off your answers to the nearest hundredth.

Example: Calculate the percentage increase in the price of petrol if it increases from R9,15 per litre to R9,50 per litre.

R9,50 – R9,15 = R0,35

$\frac{0,35}{9,15} \times 100\ \%$

$= \frac{35}{915}\ \%$

$= 3,83\ \%$

Before you answer a and b, explain this example in your own words.

a. Calculate the percentage increase in the price of a computer game if it increases from R450,00 to R699,00.

b. Calculate the percentage increase in the price of milk if it increases from R8,50 per litre to R9,25 per litre.

4. **Calculate these percentage decreases. Round your answers off to the nearest hundredth.**

Example: Calculate the percentage decrease in the price of maize if it decreases from R1 280 per ton to R1 275 per ton.

R1 280 – R1 275 = R5

$\frac{5}{1\ 280} \times \frac{100}{1}$ %

$= \frac{500}{1\ 280}$ %

$=$ 0,39 %

a. Calculate the percentage decrease in the price of a laptop computer if it drops from R4 599 to R4 299.

b. Coffee goes on special at the supermarket. The price drops from R52,99 per tin to R38,99 per tin. What is the percentage decrease in price?

Problem solving

Find out what the last increase or decrease in petrol was. Calculate the percentage increase or decrease. Why do you think the price of petrol regularly increases or decreases?

28 Common fractions, decimal fractions and percentages

What do you need to multiply the following numbers by to make them 100? How fast can you do this?

2	4	5	8	10	20	25	70
x 50 = 100							

1. Write these fractions as percentages.

Example 1:

$\frac{2}{5}$

$= \frac{2}{5} \times \frac{20}{20}$

$= \frac{40}{100}$

= 0,4

= 40%

Example 2:

$\frac{6}{8}$

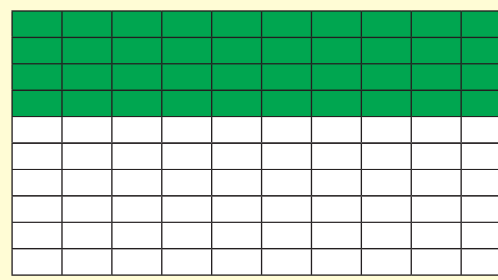
$= \frac{6}{8} \times \frac{125}{125}$

$= \frac{750}{1\,000}$

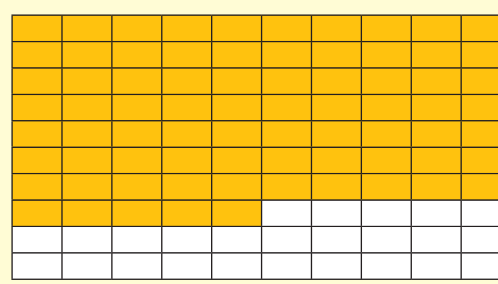
= 0,75

= 75 %

Note: $= \frac{40}{100} = 0{,}4 = 40\ \%$



Note: $= \frac{75}{100} = 0{,}75 = 75\ \%$



We can multiply 5 by 20 to get 100, so you multiply the top (numerator) and bottom (denominator) by 20.

We can multiply 8 by 125 to get 1 000, so you multiply the numerator (top) and denominator (bottom) by 125. Why did we make the denominator 1 000 and not 100?

a. $\frac{3}{4}$

b. $\frac{2}{3}$

c. $\frac{6}{7}$

d. $\frac{1}{2}$

e. $\frac{5}{7}$

f. $\frac{1}{8}$

Example 3:

There is another method for converting a fraction into a percentage. This is useful when the denominator cannot easily be multiplied by a number to get 100 or 1 000.

$\boxed{\frac{5}{23}}$

$\frac{5}{23} \times 100\,\%$

$= \frac{500}{23}\,\%$

$= 21{,}74\,\%$

5	0	0	÷	2	3

Use a calculator for this.

g. $\frac{4}{8}$

h. $\frac{5}{25}$

i. $\frac{15}{15}$

j. $\frac{18}{20}$

k. $\frac{3}{9}$

l. $\frac{4}{36}$

2. Write as a percentage and as a common fraction, revision.

a. 0,6

b. 0,25

c. 0,75

d. 0,1

e. 0,530

f. 0,36

3. Write as a percentage and as a common fraction, revision.

a. 0,325

b. 0,205

c. 0,723

d. 0,825

e. 0,125

f. 0,065

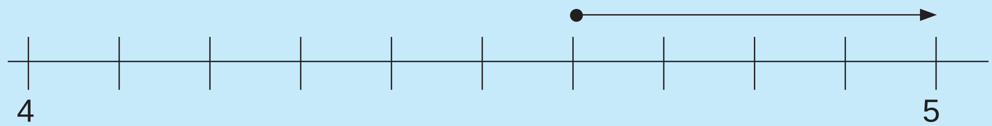
Problem solving

Write 35,4% as a common and as a decimal fraction.

29 Addition, subtraction and rounding of decimal fractions

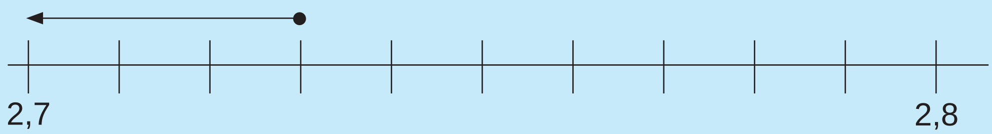
Revise:

Round off to the nearest unit. Round off 4,6 to 5.



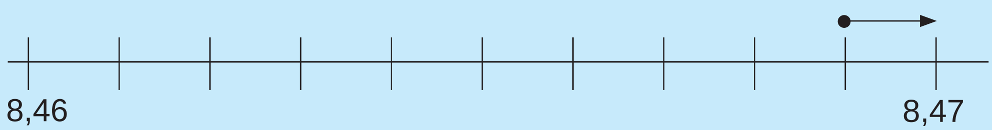
What is 4,4 rounded off to the nearest unit?

Round off to the nearest tenth. Round of 2,73 to 2,7



What is 2,76 rounded off to the nearest tenth?

Round off to the nearest hundredth. Round of 8,469 to 8,47



What is 8,469 rounded off to the nearest hundredth?

1. Round off to the nearest unit, tenth and hundredth.

Example:
Round off 5,9 to the nearest unit: 6
Round off 5,91 to the nearest tenth: 5,9
Round off 5,905 to the nearest hundredth: 5,91

a. 0,75
Unit:
Tenth:
Hundredth:

b. 0,123
Unit:
Tenth:
Hundredth:

c. 0,825
Unit:
Tenth:
Hundredth:

d. 0,795
Unit:
Tenth:
Hundredth:

e. 0,952
Unit:
Tenth:
Hundredth:

f. 0,468
Unit:
Tenth:
Hundredth:

2. **Calculate the following, using the expanded notation method and then the column method. Then test your answer. Round off your answer to the nearest unit, tenth and hundredth. (Use your own paper if necessary.)**

Example: expanded notation method:

3,765 + 2,143
= 3 + 2 + 0,7 + 0,1 + 0,06 + 0,04 + 0,005 + 0,003
= 5 + 0,8 + 0,1 + 0,008
= 5,908

Column method:	**Test your answer:**	
3,765	5,908	3,765 rounded off to the nearest
+ 2,143	– 2,143	Unit: 4
5,908	3,765	Tenth: 3,8
		Hundredth: 3,77

a. 2,354 + 7,265 =

Expanded notation	Column method	Testing	Rounded off to the nearest: Unit: Tenth: Hundredth:

b. 2,686 + 1,325 =

Expanded notation	Column method	Testing	Rounded off to the nearest: Unit: Tenth: Hundredth:

c. 1,765 + 3,925 =

Expanded notation	Column method	Testing	Rounded off to the nearest: Unit: Tenth: Hundredth:

d. 8,940 – 2,355 =

Expanded notation	Column method	Testing	Rounded off to the nearest: Unit: Tenth: Hundredth:

e. 7,495 + 2,105 =

Expanded notation	Column method	Testing	Rounded off to the nearest: Unit: Tenth: Hundredth:

f. 6,725 – 4,025 =

Expanded notation	Column method	Testing	Rounded off to the nearest: Unit: Tenth: Hundredth:

Problem solving

Why do we round off? Find ten examples in real life when we need to round off decimal fractions in daily life.

30 Multiple operations with decimals

How fast can you multiply or divide the following?

2 x 0,3 =	0,2 x 0,3 =	0,2 x 0,03 =	0,02 x 0,03 =	0,002 x 0,03 =	0,002 x 0,003 =
1 000 ÷ 5 =	100 ÷ 5 =	10 ÷ 5 =	0,1 ÷ 5 =	0,01 ÷ 5 =	0,001 ÷ 5 =

1. Calculate the following:

Example: (6 + 0,3) x (7 + 0,5)
= (6 + 0,3) x 7 + (6 + 0,3) x 0,5
= 6 x 7 + 0,3 x 7 + 6 x 0,5 + 0,3 x 0,5
= 42 + 3,1 + 3,0 + 0,15
= 47,25

a. (3,5 + 4,3) x (1,2 – 0,9) =

b. 1,2 x (1,3 + 8,6) =

c. (8,2 – 6,4) x (5,8 – 6,2) =

2. Calculate the following:

Example: 7,3 x 8,4

```
   8,4
 x 7,3
   252
+ 5 880
  6 1,32
```

a. 6,2 x 3,8 =

b. 2,6 x 4,9 =

c. 9,5 x 3,9 =

Term 1 – Week 6

3. Calculate the following:

Example:

```
    1,7
8 )13,6
    8
    56
    56
     0
```

a. $7\overline{)12,6}$ =

b. $9\overline{)29,7}$ =

c. $6\overline{)52,8}$ =

4. Calculate the following. Check your answer with a calculator.

Example: 2,576 ÷ 0,28

$= \frac{2\,576}{1\,000} \div \frac{28}{100}$

$= \frac{2\,576}{1\,000} \times \frac{100}{28}$

$= \frac{2\,576}{280} \div \frac{7}{7}$

$= \frac{368}{40} \div \frac{4}{4}$

$= \frac{92}{10}$

$= 9,2$

Example: 3,150 ÷ 0,24

$= \frac{3\,150}{1\,000} \div \frac{24}{100}$

$= \frac{3\,150}{1\,000} \times \frac{100}{24}$

$= \frac{3\,150}{240}$

$= \frac{1\,575}{120}$

$= \frac{315}{24}$

$= 13,125$

```
      13,125
= 24 )315
      24
       75
       72
        3 0
        2 4
          60
          48
          120
          120
            0
```

a. 1,715 ÷ 0,35 =

b. 2,756 ÷ 0,32 =

Problem solving

Choose one sum from questions 1, 2, 3, or 4. Write a word sum for each.

31 Calculate squares, square roots, cubes and cube roots

Can you use a scientific calculator to calculate exponents such as 3^5?

Press	3
Press	y^x
Press	5
Press	=

What does ^ mean when you write exponents?

Oh so 3^5 is the same as 3^5.

^ means "raised to the power of".

Note that different makes and models of calculator may require different steps.

1. Estimate these squares and then calculate with a calculator.

Example: If $5^2 = 25$ what is $5,5^2$?

Estimate

- $5^2 = 25$ then $5,5^2$ should be bigger than 25. Why?
- $6^2 = 36$ then $5,5^2$ should be smaller than 36. Why?

Calculator

Press	5,5
Press	y^x
Press	2
Press	=

= 30, 25

On some calculators you don't need to press the = button.

Calculate

5, 52

= 5, 5 x 5, 5

Use the distributive property of number.

(5 + 0,5) (5 + 0,5)

= 25 + 2,5 + 2,5 + 0,25

= 30,25

a. If $3^2 = 9$, what is $3,5^2$?

b. If $4^2 = 16$, what is $4,5^2$?

c. If $9^2 = 81$, what is $9,5^2$?

d. If $6^2 = 36$, what is $6,5^2$?

e. Do each one again showing all the steps of your calculation. (You can do this on a separate piece of paper.)

2. **Estimate these cubes and then calculate with a calculator.**

Example: If $4^3 = 64$ what is $4{,}5^3$?

Estimate	*Calculator*		*Calculate*
$4^3 = 64$	Press	4,5	(4,5)(4,5)(4,5)
$5^3 = 125$	Press	y^x	= [(4 + 0,5)(4 + 0,5)](4,5)
so the answer must be	Press	3	= (16 + 2 + 2 + 0, 25)(4, 5)
between 64 and 125	Press	=	= (20, 25)(4,5)
	= 90,125		= (20 + 0, 25)(4 + 0,5)
			= 80 10 + 1 + 0, 125
			= 90, 125

a. If $2^3 = 8$, what is $2{,}5^3$?

b. If $8^3 = 512$, what is $8{,}5^3$?

c. If $1^3 = 1$, what is $1{,}5^3$?

d. Do each one again showing all the steps of your calculation.

continued

Calculate squares, square roots, cubes and cube roots continued

3. Estimate these square roots and then calculate with a calculator.

Example: If $\sqrt{16}$ = 4 what is $\sqrt{18}$

Estimate

$\sqrt{16} = 4$
$\sqrt{25} = 5$
So $\sqrt{18}$ should be between 4 and 5.

Calculator

Press 18

Press √

Press =

= 4,24

(Rounded off)

Note that different makes and models of calculator may require different steps.

a. If $\sqrt{9}$ = 3 what is $\sqrt{12}$?

b. If $\sqrt{36}$ = 6 what is $\sqrt{42}$?

c. If $\sqrt{16}$ = 4 what is $\sqrt{20}$?

d. Do each one again showing all the steps of your calculation.

4. Estimate these cube roots and then calculate with a calculator.

Example: If $\sqrt[3]{27} = 3$ what is $\sqrt[3]{50}$

Estimate

$\sqrt[3]{27} = 3$
$\sqrt[3]{64} = 4$
So $\sqrt[3]{50}$ should be between 3 and 4.

Calculator

Press	50		Press	50
Press	$\sqrt[x]{\ }$	or	Press	$\sqrt[3]{x}$
Press	3		Press	=
Press	=		**= 3,68**	
= 3,68				

a. If $\sqrt[3]{64} = 4$ what is $\sqrt[3]{68}$?

b. If $\sqrt[3]{27} = 3$ what is $\sqrt[3]{20}$?

c. If $\sqrt[3]{216} = 6$ what is $\sqrt[3]{222}$?

d. Do each one again showing all the steps of your calculation.

Problem solving

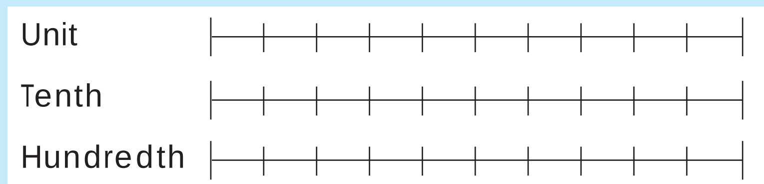
Give the steps you wrote down for auestion 1 a to d to a friend to go through and check.

32 Calculate more squares, square roots, cubes and cube roots

You need to know and revise the following:

How to calculate square roots using a calculator.

How to round off a decimal to the nearest unit, tenth or hundredth using the number lines below. Give and example of each.



1. Calculate and round off to the nearest unit, tenth and hundredth.

Example:		**unit**	**tenth**	**hundredth**
	$\sqrt{6} + \sqrt{12}$			
	$= 2{,}449 + 3{,}464$	6	5,9	5,91
	$= 5{,}913$			

a. $\sqrt{17} + \sqrt{24} =$

b. $\sqrt[3]{65} + \sqrt[3]{730} =$

c. $\sqrt{148} + \sqrt[3]{1430} =$

2. Calculate and round off to the nearest unit, tenth and hundredth.

Example:		**unit**	**tenth**	**hundredth**
	$2{,}5^2 + 2{,}5^3$	22	22,9	22,88
	$= 6{,}25 + 15{,}625$			
	$= 21{,}875$			

a. $2{,}9^2 + 1{,}4^3 =$

b. $1{,}3^3 + 11^2 =$

c. $1{,}2^2 + 8^2 =$

3. Calculate and round off to the nearest unit, tenth and hundredth.

Example:		unit	tenth	hundredth
	$(\sqrt{6})(\sqrt{12})$			
	$= (2{,}449)(3{,}464)$	8	8,5	8,48
	$= 8{,}483$			

Note that $(\sqrt{14} \cdot \sqrt{19})$ is the same as $(\sqrt{14})(\sqrt{19})$

a. $(\sqrt{13})(\sqrt{7})$

b. $(\sqrt{5})(\sqrt{8})$

c. $(\sqrt{14} \cdot \sqrt{19})$

4. Calculate and round off to the nearest unit, tenth and hundredth.

Example:				unit	tenth	hundredth
	$(2{,}5^2)(2{,}5^3)$		$(2{,}5^2)(2{,}5^3)$	98	97,7	97,66
	$= 6{,}25 \times 15{,}625$	or	$= 2{,}5^5$			
	$= 97{,}656$		$= 97{,}656$			

a. $(3{,}5)^2 (3{,}5)$

b. $(1{,}9)^2 (1.9)^2$

c. $(11{,}2)^3 (11{,}2)^2$

d. $(6{,}7)^2 (6{,}7)^3$

e. $(4{,}8)^2 (4{,}8)^3$

5. Calculate and round off to the nearest unit, tenth and hundredth.

Example:		unit	tenth	hundredth
	$(\sqrt{6} + (\sqrt{12} + \sqrt{20})$			
	$= 2{,}449 + 3{,}464 + 4{,}472$	10	0,4	10,39
	$= 10{,}385$			
		8	8,0	7,99
	$(\sqrt{6} + (\sqrt{12} + \sqrt[3]{9})$			
	$= 2{,}449 + 3{,}464 + 2{,}08$			
	$= 7{,}993$			

a. $\sqrt{79} - (\sqrt{13} + \sqrt{59})$

b. $\sqrt[3]{18} - (\sqrt[3]{500} - \sqrt[3]{210})$

c. $\sqrt[3]{34} - (\sqrt[3]{709} - \sqrt[3]{200})$

continued ☛

15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

32b Calculate more squares, square roots, cubes and cube roots continued

6. Calculate and round off to the nearest unit, tenth and hundredth.

Example: $2,5^2\ (1,5^2 + 1,2^2)$
$= (2,5^2 \times 1,5^2)\ (2,5^2 \times 1,2^2)$
$= (6,25 \times 2,25) + (6,25 \times 1,44)$
$= 14,062 + 9$
$= 23,162$

unit	tenth	hundredth
23	23,3	23,26

a. $3,2^2\ (11,6^2 + 7,8^3)$

b. $4,4^3(2,8^3 + 3,1^2)$

c. $8,1^3(3,9^3 + 7,4^3)$

d. $11,2^2\ (4,2^3 + 5,6^2)$

e. $9,6^2\ (8,2^3 + 10,3^2)$

7. Calculate and round off to the nearest unit, tenth and hundredth.

Example:		unit	tenth	hundredth
	$(\sqrt{6}\,(\sqrt{12}+\sqrt{20})$			
	$= (\sqrt{6} \times \sqrt{12}) + (\sqrt{6} \times \sqrt{20})$	**19**	**19,4**	**19,44**
	$\approx (2{,}449 \times 3{,}464) + (2{,}449 \times 4{,}472)$			
	$= 8{,}483 + 10{,}952$			
	$= 19{,}435$			

We write ≈ to show that we have already rounded off the numbers.

a. $\sqrt{26}\,\sqrt[3]{15} + \sqrt[3]{629}$

b. $\sqrt{21}\,\sqrt[3]{162} + \sqrt[3]{164}$

c. $\sqrt{325}\,\sqrt[3]{1000} + \sqrt[3]{137}$

Problem solving

Choose any sum you did in this lesson and make a word sum of it. This will need some careful thinking.

33 Exponential form

You need to revise the following:

Can you remember what scientific notation is?

How do we write $4{,}5 \times 10^0$?

7842,5	=	$7{,}8425 \times 10^3$

Ten to the power of three

$7842{,}5 = 7{,}8425 \times 1\,000 = 7{,}8425 \times 10^3$

$4{,}5 \times 1 = 4{,}5$

1. Revision: Compare the two numbers.

Example: $(-2)^2 = (-2)(-2) = 4$

$-(2)^2 = -(2)(2) = -4$

a. $-(4)^2$; $(-4)^2$

b. $-(6)^3$; $(-6)^3$

c. $(-3)^3$; $-(3)^3$

d. $(-8)^3$; $-(8)^3$

e. $(-6)^2$; $-(6)^2$

f. $(-4)^3$; $-(4)^3$

2. Revision : Fill in <, > or =. First write it as a number and then compare it.

Example:

$(-2)^2 > -(2)^2$

$(-3)^2 > -(3)^2$

$(-2) = -(2)^3$

a. $-(10)^2$ ____ $(-10)^2$

b. $-(6)$ ____ $(-6)^3$

c. $(-9)^3$ ____ $-(9)^3$

d. $(-8)^3$ ____ $(8)^3$

e. $(-6)^2$ ____ $-(6)^2$

f. $(-4)^3$ ____ $-(4)^3$

3. **Convert an ordinary number to scientific notation or scientific notation to an ordinary number.**

Example: 8 740 000 = $8,74 \times 10^{6}$ = 8 740 000

a. 256 000

b. 790 000 000

c. 5×10^{-6}

d. $8,1 \times 10^{6}$

e. 0,0000089

f. $3,12 \times 10^{-5}$

4. **Fill in <, > or =**

Example:

$4,32 \times 10^{4}$
$4,32 \times 10^{4} = 4,32 \times 10\ 000 = 43\ 200$
43 200

$4,32 \times 10^{-4}$
$4,32 \times 10^{-4} = 4,32 \times 0,0001 = 0,000432$
0,000433

43 200 > 0,000432

a. $2,24 \times 10^{4}$ ____ $0,25 \times 10^{-4}$

b. $2,5 \times 10^{3}$ ____ $2,5 \times 10^{-3}$

c. $1,75 \times 10^{-6}$ ____ $1,75 \times 10^{6}$

d. $1,95 \times 10^{-5}$ ____ $1,95 \times 10^{5}$

e. $0,75 \times 10^{-5}$ ____ $0,75 \times 10^{-5}$

f. $0,5 \times 10^{2}$ ____ $0,5 \times 10^{-2}$

Problem solving

Calculate: $2^{15} \times 2$ =

Show all your calculations.

34 Laws of exponents: $a^m \times a^n = a^{m+n}$

Revise the laws of exponents and give four examples of each using numbers.

$a^m \times a^n = a^{m+n}$				
$\frac{a^m}{a^n} = a^{m-n}$				

1. Use the laws of exponents to simplify the following:

Example: $s^2 \times s^4 = s^{2+4} = s^6$

a. $a^3 \times a^4 =$

b. $b^2 \times b^5 =$

c. $2^8 \times 2^9 =$

d. $f^8 \times f^3 =$

e. $d^2 \times d^6 =$

f. $y^5 \times y^4 =$

2. Calculate the following:

Example: $8^3 \times 8^2 = 8^{3+2} = 8^5 = 32\ 768$

a. $2^5 \times 2^2 =$

b. $5^2 \times 5^3 =$

c. $3^4 \times 3^2 =$

d. $7^3 \times 7^1 =$

e. $8^2 \times 8^3 =$

f. $3^2 \times 3 =$

3. Use the laws of exponents to simplify the following:

Example: $y^a \times y^b = y^{a+b}$

a. $a^m \times a^n =$

b. $d^e \times d^f =$

c. $v^g \times v^h =$

d. $e^j \times e^k =$

e. $x^r \times x^s =$

f. $b^p \times b^q =$

Problem solving

You need to explain to a friend who was absent from class how to do the following: multiply 5^9 by 5^7 using a calculator. What will you say?

35 Laws of exponents: $a^m \div a^n = a^{m-n}$

Revise the laws of exponents and give four examples of each using numbers.

$a^m \times a^n = a^{m+n}$				
$\frac{a^m}{a^n} = a^{m-n}$				

1. Use the laws of exponents to simplify the following:

Example: $m^5 \div m^3 = m^{5-3}$ or $\frac{m^5}{m^3} = m^{5-3}$

a. $a^4 \div a^3 =$

b. $\frac{f^9}{f^8} =$

c. $x^5 \div x^2 =$

d. $\frac{b^8}{b^2} =$

e. $\frac{e^6}{e^4} =$

f. $h^7 \div h^3 =$

2. Calculate the following:

Example: $2^4 \div 2^3 = 2^{4-3} = 4^1 = 4$

a. $\frac{2^3}{2^2}$

b. $4^4 \div 4^2 =$

c. $\frac{4^5}{4^4}$

d. $6^6 \div 6^3 =$

e. $3^7 \div 3^3 =$

f. $\frac{5^8}{5^5}$

3. Write as a fraction and then use the laws of exponents to simplify the following:

Example: $g^5 \div g^3 = \frac{g^5}{g^3} = g^{5-3} = g^2$

a. $a^4 \div a^3 =$

b. $d^6 \div d^5 =$

c. $g^6 \div g^4 =$

d. $f^9 \div f^6 =$

e. $c^8 \div c^2 =$

f. $j^{12} \div j^{10} =$

4. Use the laws of exponents to simplify the following:

a. $6^2 \times 6^3 =$

b. $4^2 \times 4^3 =$

c. $2^4 \times 2^5 =$

d. $10^3 \div 10^2 =$

e. $4^3 \div 4^2 =$

f. $2^5 \div 2^4 =$

Problem solving

You need to explain to your friend who was absent how to do this: $5^9 \div 5^7$ without using a calculator. How will you explain it?

36 Laws of exponents: $a^m \div a^n = a^{m-n}$ if $m<n$

Revise: give an example using numbers and and example using variables.

$a^m \times a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{a^m}{a^n} = a^{m-n}$ if $m < n$	$(a^m) = a^{mn}$

1. Use the laws of exponents to calculate the following:

Example: $w^4 \div w^6 = \frac{w^4}{w^6} = \frac{w\,w\,w\,w}{w\,w\,w\,w\,w\,w} = \frac{1}{w^2} = w^{-2}$ or using the laws of exponents: $w^4 \div w^6 = w^{4-6} = w^{-2}$

a. $x^2 \div x^3 =$

b. $\frac{e^7}{e^9} =$

c. $\frac{f^3}{f^7} =$

d. $a^8 \div a^{10} =$

e. $\frac{k^4}{k^6} =$

f. $a^6 \div d^{11} =$

2. Calculate the following:

Example: $\frac{2^4}{2^5} = 2^4 \div 2^5 = 2^{4-5} = 2^{-1} = \frac{1}{2}$

a. $\frac{2^3}{2^4} =$

b. $5^{10} \div 5^{12} =$

c. $7^6 \div 7^8 =$

d. $\frac{10^8}{10^{10}} =$

e. $11^9 \div 11^{11} =$

f. $\frac{8^6}{8^9} =$

3. Use the laws of exponents to simplify the following:

Example: $x^m \div x^n = \frac{x^m}{x^n} = x^{m-n}$

a. $a^m \div a^n =$

b. $d^f \div d^9 =$

c. $\frac{m^a}{m^b} =$

d. $f^x \div f^y =$

e. $\frac{e^k}{e^l} =$

f. $\frac{d^t}{d^u} =$

4. Use the following laws of exponents to simplify the following:

Example: $(q^2)^3 = (q^{2\times3}) = q^6$

a. $(m^3)^3 =$

b. $(k^5)^7 =$

c. $(n^4)^2 =$

d. $(r^3)^9 =$

e. $(z^5)^8 =$

f. $(s^7)^8 =$

5. Calculate the following:

Example: $(3^2)^3 = (3^6) = 729$

a. $(2^3)^3 =$

b. $(8^3)^3 =$

c. $(4^3)^2 =$

d. $(2^4)^7 =$

e. $(3^4)^8 =$

f. $(3^5)^5 =$

6. Use the laws of exponents to simplify the following:

Example: $(a^m)^n = a^{mn}$

a. $(a^m)^n =$

b. $(d^d)^y =$

c. $(h^p)^q =$

d. $(b^a)^b =$

e. $(c^r)^s =$

f. $(e^d)^e =$

Problem solving

a. What is the difference between $x^2 \div x^3$ and $x^3 \div x^2$?
b. Solve: $(12^4)^2$

37 Laws of exponents: $a^0 = 1$ and $(a \times t)^n = a^n t^n$

Substitute numbers for the variables and exponents in each of these examples.

$a^m \times a^n = a^{m+n}$		$xy^n = x^n y^n$	
$\frac{a^m}{a^n} = a^{m-n}$		$x^1 = x$	
$\frac{a^m}{a^n} = a^{m-n}$ if m < n		$x^0 = 1$	
$(a^m)^n = a^{mn}$		$a^{-n} = \frac{1}{g^n}$	

1. Simplify:

Example: $(a \times t)^3 = a^3t^3$

a. $(b \times c)^5 =$ b. $(r \times s)^s =$ c. $(c \times d)^3 =$

d. $(t \times s)^9 =$ e. $(f \times a)^4 =$ f. $(k \times n)^6 =$

2. Calculate the following:

Example: $(2 \times 5)^2 = 2^2 \times 5^2 = 2^25^2 = 4 \times 25 = 100$

a. $(2 \times 3)^2 =$ b. $(6 \times 7)^2 =$ c. $(2 \times 10)^2 =$

d. $(4 \times 3)^3 =$ e. $(2 \times 8)^4 =$ f. $(11 \times 3)^3 =$

3. Use the laws of exponents to simplify the following:

a. $(a \times c)^b =$ b. $(y \times b)^c =$ c. $(m \times p)^n =$

d. $(z \times t)^q =$ e. $(d \times f)^e =$ f. $(q \times t)^x =$

4. Use the law of exponents to simplify the following:

Example: $a^0 = 1$ and $a^1 = a$

a. $a^0 =$ b. $c^0 =$ c. $d^0 =$

d. $j^1 =$ e. $h^1 =$ f. $g^1 =$

5. Calculate the following:

Example: $12^0 = 1$ and $12^1 = 12$

a. $4^0 =$ b. $3^1 =$ c. $10^0 =$

d. $5^1 =$ e. $8^0 =$ f. $11^1 =$

6. Use the law of exponents to simplify the following:

Example: $5^{-3} = \frac{1}{5^3}$

a. $a^{-2} =$ b. $e^{-7} =$ c. $d^{-10} =$

d. $x^{-3} =$ e. $b^{-8} =$ f. $g^{-7} =$

7. Calculate the following:

Example: $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

a. $3^{-2} =$ b. $2^{-1} =$ c. $7^{-2} =$

d. $2^{-4} =$ e. $4^{-2} =$ f. $3^{-1} =$

8. Use the law of exponents to simplify the following:

Example: $a^{-n} = \frac{1}{a^n}$

a. $a^{-b} =$ b. $d^{-1} =$ c. $k^{-c} =$

d. $n^{-x} =$ e. $b^{-n} =$ f. $r^{-b} =$

Problem solving

Form a group of 4 to 6 friends and explain the laws of exponents to each other. Help each other.

38 Application of the law of exponents

Revise these laws.

$a^1 = a$	$a^0 = 1$	$a^{-1} = \frac{1}{a^1}$	$a^{-n} = \frac{1}{a^n}$
$a^m \times a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}$	$(a^m)^n = a^{mn}$	$(ab)^n = a^n b^n$
	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\frac{a^m}{a^n} = a^{m-n}$ if m < n	

Remember the sequence of operations

1. Use the laws of exponents to simplify the following:

a. $(a^3 \times a^4) + (a^4 \div a^3) =$

b. $x^3 \times x^4 \div x^4 =$

c. $y^7 \div y^5 + y^2 =$

d. $c^1 \times c^3 \div c^2 =$

e. $(e^3 \times e^5) =$

f. $(5^2 \times 5^3) \div 5^5 =$

2. Use the laws of exponents to calculate the following:

a. $3^2 \times 3^3 =$

b. $4^8 \div 2^3 =$

c. $5^3 \times 5^3 \div 5^2 =$

d. $4^2 \div 2^2 =$

e. $(12^5 \times 5^{-3}) =$

f. $7^5 \div 7^9 =$

3. Use the laws of exponents to calculate the following:

a. $3a \times 9a^4 =$

b. $14c \times 7c^5 =$

c. $2e^5 \div 4e^3 =$

d. $8z^4 \div 2z^3 =$

e. $125x^3 \div 25x^5 =$

f. $32d^3 \div 422d =$

4. Revision: simplify.

Example: $2x^{-2} = 2 \times x^{-2} = 2 \times \frac{1}{x^2} = \frac{2}{x^2}$

a. $3x^{-2} =$

b. $9x^{-3} =$

c. $7x^{-3} =$

d. $4x^{-3} =$

e. $5x^{-2} =$

f. $8x^{-5} =$

5. Revision: simplify:

Example: $4^n = (4)^n = (2 \times 2)^n$
$= (2^2)^n$
$= 2^{2n}$

a. $64^n =$

b. $16^x =$

c. $100^x =$

d. $121^n =$

e. $4^x =$

f. $144^n =$

continued

38b Application of the law of exponents continued

6. Revision: simplify.

Example: $9^n\ 2^{n-1}$
$= (3^2)^n\ 2^{n-1}$
$= 3^{2n}\ 2^{n-1}$

a. $16^x \cdot 3^{x+1} =$

b. $36^x \cdot 3^{x+2} =$

c. $121^x \cdot 2^{x+1} =$

d. $9^x \cdot 4^{x+2} =$

e. $25^x \cdot 5^{x+1} =$

f. $100^x \cdot 3^{x+4} =$

7. Simplify.

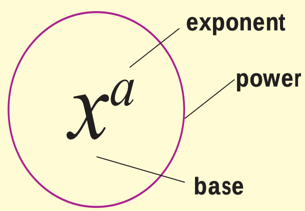
Example: $\frac{9^n \cdot 2^{n+1}}{4^n}$

$= \frac{3^{2n} \cdot 2^{n+1}}{2^{2n}}$

$= \frac{3^{2n} \cdot 2^{n+1-n}}{2^{2n-n}}$

$= \frac{3^{2n} \cdot 2}{2^n}$

$= \frac{2 \cdot 3^{2n}}{2^n}$



a. $\frac{8^n \cdot 2^{n+1}}{4^n}$

b. $\frac{16^n \cdot 3^{n+1}}{25^n}$

c. $\frac{36^n \cdot 4^{n+21}}{64^n}$

8. Factorise.

Example: $12^n = (12)^n = (2 \times 2 \times 3)^n = (2^2 \times 3^1)^n$
$= 2^{2n} \times 3^n$
$= 2^{2n}.3^n$

a. $20^n =$

b. $24^n =$

c. $54^n =$

d. $45^n =$

e. $18^n =$

9. Simplify.

Example: $\frac{9^{n-1}.12^n}{4^{n+1}.27^n}$

Try to get exponents with the same base on top and bottom of the line

$\frac{(3^2)^{n-1}.(2^2.3)^n}{(2^2)^{n+1}.(3^3)^n}$

$= \frac{3^{2n-2}.2^{2n}.3^n}{2^{2n+2}.3^{3n}}$

Now we can simplify by multiplying the exponents with the same base. Use the laws of exponents to do this.

$= \frac{(3^{2n-2}.3^n).2^{2n}}{2^{2n+2}.3^{3n}}$

$= \frac{3^{2n-2+n}.2^{2n}}{3^{3n}.2^{2n+2}}$

$= \frac{3^{3n-2}.2^{2n}}{3^{3n}.2^{2n+2}}$

Now let us divide the exponents with the same base.

$= \frac{3^{3n-2-3n}.2^{2n-2n}}{3^{3n-3n}.2^{2n+2-2n}}$

$= \frac{3^{-2}.2^0}{3^0.2^2}$

$= \frac{3^{-2}.1}{1.2^2}$

$= \frac{1}{3^2}.\frac{1}{2^2}$

$= \frac{1}{9}.\frac{1}{4}$

$= \frac{1}{36}$

$\frac{16^{n-1}.18^n}{4^{n+1}.36^n}$

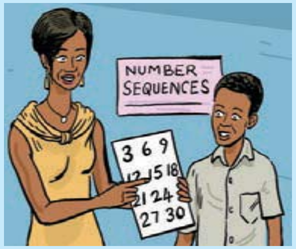
Problem solving

Write down all the **laws of exponents** that you used today.

Create your own sum using all these laws and solve it.

39 Sequences

Revision: What does each statement tell you? Give two more examples of each.



Constant difference e.g. –3; –7; –11; –15; ... Counting in "–4s" or "add –4 to the previous term".

Constant ratio e.g. –2; –4; –8; –16; –32; ... "Multiply the previous term with 2."

Variable difference or ratio e.g. 1; 2; 4; 7; 11; 16; ... "Increase the difference between consecutive terms by 1 each time."

1. Describe the pattern by giving the rule and then extend it with three terms.

a. 2; 4; 6; 8; 10

Add 2 to the previous term.
12; 14; 16

b. 1; 5; 9; 13; 17

c. –6; –8; –10; –12

d. –30; –20; –10; 0; 10

e. –1; 5; 11; 17

f. 15; 12; 9; 6; 3

2. Describe the pattern by giving the rule and then extend it by three terms.

a. 2; 4; 8; 16; 32; 64

b. 5; –20; 80; –320; 1 280

c. 729; 81; 9; 1; $\frac{1}{9}$; $\frac{1}{81}$

d. 25; 5; 1; 0,2; 0,04

3. Describe the pattern by giving the rule and then extend it by three terms.

a. 2; 4; 12; 48; 240

b. 1; 5; 13; 29; 61; 125

c. 16; 19; 23; 28; 34

d. 1; 5; 2; 6; 3; 7

4. Complete the table:

a.

Position in sequence	2	4	6	8	10	n
Term	–10	–8	–6	–4		

b.

Position in sequence	1	3	5	7	10	n
Term	–14	–12	–10	–8		

c.

Position in sequence	3	6	9	10	12	n
Term	–15	–12	–9		–6	

5. Determine the 10th and nth position of the term using a table and number sentences.

a.

Position in sequence	1	3	5	7	10	n
Term	1	9	25	49		

b.

Position in sequence	1	2	4	8	10	n
Term	1	4	16	64		

c.

Position in sequence	2	4	6	8	10	n
Term	6	18	38	66		

d.

Position in sequence	3	4	5	6	10	n
Term	27	64	125	216		

e.

Position in sequence	– 5	0	5	10	15	n
Term	– 126	– 1	124		3 374	

f.

Position in sequence	3	5	7	9	10	n
Term	26	124	342	728		

Problem solving

Create your own sequences as follows:
- Constant difference between the consecutive terms
- Constant ratio between the consecutive terms
- Neither a constant difference nor a constant ratio

40 Geometric and numeric patterns

Revision: Talk about this.

Position	1	2	3	4	5	n
Value of the term		16	24	32	40	

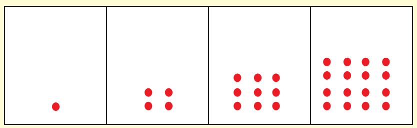
(1st term, 2nd term, 3rd term, 4th term, 5th term; 1 x 8, 2 x 8, 3 x 8, 4 x 8, 5 x 8)

Read the top row.
The position: 1st term, 2nd term, 3rd term, 4th term, 5th term, nth term
If the 2nd term position is 2 and its value is 16 the rule is 2 x 8 = 16.
What is the 1st term?

You need to make sure that you know what these geometric figures are.

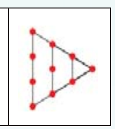
1. **Create and complete the following geometric patterns.**
   - Draw the first four terms in each of the following geometric patterns.
   - Write them in a table determining the 1st, 2nd, 3rd, 4th, 10th and nth terms, where applicable.

Example: Square



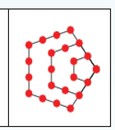
Position	1st	2nd	3rd	4th	10th	nth
Value	1	4	9	16	100	n^2

a. Triangle



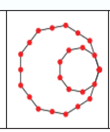
Position	1st	2nd	3rd	4th	nth
Value				10	

b. Pentagon



Position	1st	2nd	3rd	4th	nth
Value				22	

c. Nonagon



Position	1st	2nd	3rd	4th	nth
Value			24		

2. **Use the rule to complete each table:**

Example: Rule is 2x + 1

x	– 2	– 1	0	1	2	5	10
y	– 3	– 1	1	3	5	11	21

a. Rule: $y = 3x - 1$

x	– 2	– 1	0	1	2		10	50
y								

b. Rule: $y = \frac{1}{2}x + 2$

x	0	2	3	50	75	100
y						

c. Rule: $y = x - 5$

x	–3	–2	–1	0	1	13	25
y							

d. Rule: $y = 5x - 4$

x	1	3	5	7	27	47
y						

3. **Use the rule to complete each table.**

a. Rule: $y = x \times -2$

x	– 2	– 1	0	1	2	5
y						

b. Rule: $y = 10(x + 2)$

x	– 3	5	13	21	29	37
y						

Problem solving

Make your own rule and give a table to a friend to solve.

41 Addition & subtraction of like terms

Look at and discuss.

exponents constant variable

$2x^4 + x^2 + 6x - 1$

terms

Examples

monomial (1 term)	$8x^4$	(a + b)
binomial (2 terms)	$3x^2 + 4$	a + b
trinomial (3 terms)	$4x^2 + x^2 + 3$	

Polynomial: an algebric expression containing 1 or more terms with non negative integer exponents. $4x^2 + 2y^2$

Terms are separated by + and – and not by ÷ and x.

1. Revision: Simplify

Example: $3a^2 + 4a^2$
$= 7a^2$

a. $8w^3 + 7w^3 =$

Example: $3a^2 - 2a + 4a^2 + 6a$
$= 3a^2 + 4a^2 - 2a + 6a$
$= 7a^2 + 4a$

b. $5b^2 - 6b + 7b + 2b^2 =$

Example: $3a^2 + 2a - 5 + 4a^2 - 6a + 6$
$= 3a^2 + 4a^2 + 2a - 6a - 5 + 6$
$= 7a^2 - 4a + 1$

c. $4x^2 + 5x + 8 + 3x^2 + 6x + 4 =$

Example: $4ab^2 + 3ab + 3ab^2 + 2ab$
$= 4ab^2 + 5ab$

d. $4uv + 3uv^2 - 5uv + 4uv^2 =$

2. Match column A with column B.

A	B
Monomial	$3xy^2 + 2x + 4x - 5$
Binomial	$3xy^2 + 2x$
Trinomial	$3xy^2$
Polynomial	$3xy^2 + 2x + 5$

Like terms
Two terms with the **same variable** raised to the **same exponent** are called "like terms", e.g. $6z^2$ and $8Z^2$ are like terms as there are two terms with the same variable (z) with the same exponent (2).

3. Circle the following in each algebraic expression.

Example: A monomial: (3ab²) + 4ab + 6b – 8

a. A binomial: $8xy^2 + 5xy + 2x + 7xy^2$

b. A polynomial: $5ab^2 + 6ab + 7a + 6ab^2$

c. A trinomial: $7cd^2 + 8cd + 8cd^2 + 8cd$

d. A monomial: $9ef^3 + 4ef^2 + 5ef^2 + 5ef^3$

4. Revision: simplify:

Example: $3x^2 + 5x + 4 + 5x^2 - 2x - 1$
$= 8x^2 + 3x + 3$

a. $5x^2 + 3x + 4x^2 + 8x + 4 + 5 =$

b. $6a^2 + 8a + 5a^2 + 2 - 3 + 7a =$

c. $4b + 9b^2 - 7 - 5b + 6 - b^2 =$

d. $5x - 4 - 7x - 8x^2 - 2 - 3x^2 =$

e. $3 + 6a + 9a^2 + 2 + 3a^2 + 4a =$

5. Simplify:

Example: $2x^3 + 4x + 5x^2 + 8 + 6 + 5x^3$
$= 7x^3 + 5x^2 + 4x + 14$

a. $4x^3 + 2x^2 + 8 - 5x^3 - 4x^2 =$

b. $4x - 2x^2 + 2x^3 - 7 - 4x^2 =$

6. Simplify:

Example: $4x^2 + 4x + 2x + 3y^2 + 5x^2$
$= 9x^2 + 3y^2 + 6x$

a. $4x^2 + 2y^3 + 2y^2 + 3x^2 + 3y^3 =$

b. $8a^3 + 8a^2 - a^3 - 8b^3 + b^3 =$

7. Simplify:

Example: $3ab + 4ab^2 + 2ab + ab^2 + ab$
$= 5ab^2 + 6ab$

a. $3xy^2 + 5xy + 4xy^2 + 8xy + 6xy =$

b. $5ab^4 + 7ab^3 - 9ab^2 + 6ab^4 - 3ab^2 =$

8. Simplify:

Example: $5a^2b^3 + 6ab + 2a^2b^3 + 2ab + ab$
$= 7a^2b^3 + 9ab$

a. $3x^2y^3 + 7xy + 4xy^2 + 5x^2y^3 + 5xy^2 =$

b. $4a^2b^4 + 5a^3b^2 + 7ab - 3a^2b^4 + 2a^3b^2 =$

Problem solving

a. Create an algebraic expression with three different **like terms** and simplify it.
b. Write a polynomial with five terms, where two pairs are like terms. Simplify your answer.
c. If the answer is $5x^2 + 7y^2 + 9x$ and the original sum had seven terms, what could the original sum be?
d. Write a polynomial with fifteen terms and then simplify it. Note that you should have like terms in your polynomial.

42 The product of a monomial and binomial or trinomial

Revise.

$-2x(x + 2)$

$-2x$	x	2
	$-2x^2$	$-4x$

Remember to multiply the monomial with every term of the binomial.

$5x(2x^2 + 3x - 4)$

$5x$	$2x^2$	$3x$	-4
	$10x^3$	$15x^2$	$-20x$

Remember to multiply the monomial with every term of the trinomial.

1. Revision: simplify.

Example 2(3 + 4)
= (2 x 3) + (2 x 4)
= 6 + 8
= 14

a. 2(1 + 7) =

b. 2(6 + 2) =

2. Revision: simplify.

Example a(b + c)
= a x b + a x c
= ab + ac } or

a. x(y + z) =

b. x(y – z) =

3. Revision: simplify.

Example 3(a + b)
= (3 x a) + (3 x b)
= 3a + 3b

a. 2(a + b) =

b. 4(–k + l) =

4. Revision: simplify.

Example $x(2 + 4)$
$= (x \times 2) + (x + 4)$
$= 2x + 4x$
$= 6x$

a. a(3 + 4) =

b. $x(-2x + 3)$ =

5. Simplify.

Example: Method 1

$2x(3x^2 - 4x + 5)$
$= 6x^{1+2} - 8x^{1+1} + 10x$
$= 6x^3 - 8x^2 + 10x$

Method 2

$2x(3x^2 - 4x + 5)$

$2x$	$3x^2$	$-4x$	5
	$6x^3$	$-8x^2$	$+10x$

a. $4x(x^2 - 2x + 2)$ =

b. $7x(2x^2 - 4x + 10)$ =

c. $x(3x^2 + 4x + 5)$ =

d. $3x(5x^2 - 2x + 6)$ =

e. $5x(x^2 - 3x - 2)$ =

f. $6x(2x^2 + 4x + 7)$ =

6. Simplify using both methods.

Example: Method 1

$2x(3x^2 - 4x + 5)$
$= 6x^3 - 8x^2 + 10x$

Method 2

$2x(3x^2 - 4x + 5)$
$= (2x.3x^2) + (2x.-4x) + (2x.5)$
$= 6x^{1+2} - 8x^{1+1} + 10x$
$= 6x^3 - 8x^2 + 10x$

a. $-x(2x^2 + 3x + 2)$ =

b. $-4x(-3x^2 - 5x - 4)$ =

continued ☛

42b The product of a monomial & binomial or trinomial continued

c. $-3x(-x^2 + 2x - 6) =$

d. $-2x(3x^2 + 7x + 1) =$

e. $-5x(2x^2 - 4x - 8) =$

f. $-6x(-3x^2 - 6x + 3) =$

7. If $x = 3$, evaluate:

a. $4x^2 + 3x + 2 =$

b. $5x^2 - 6x + 8 =$

c. $-3x^2 - 2x + 3 =$

d. Now try it with a, b and c with $x = -3$

8. Simplify and then evaluate if $x = -2$.

a. $2x(6x^2 + 3x + 5) =$

b. $-3x(2x^2 + 6x + 9) =$

c. $4x(3x^2 - 2x - 2) =$

Make sure you multiply the monomial with all the terms of the trinomial.

Problem solving

The *a* x (times) can be "distributed" across the 2 + 4 into an a × 2 plus an a × 4. What did the original sum look like?

Create your own monomial multiplied by a trinomial and simplify it.

Create your own monomial multiplied by a trinomial and simplify it through substitution.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

43 The product of two binomials

Compare the following:

Did you know that your knowledge of map work can help you to calculate the product of two binomials? Use of the columns and rows to multiply two binomials.

	x	$+ 2$
x	x^2	$2x$
$+ 2$	$2x$	4

$x^2 + 4x + 4$

	x	$- 2$
x	x^2	$-2x$
$- 2$	$-2x$	4

$x^2 - 4x + 4$

	x	$- 2$
x	x^2	$-2x$
$+ 2$	$3x$	-6

$x^2 + x - 6$

1. Simplify the following:

Example $(x + 2)(x + 3)$

$= (x + 2)(x + 3)$
$= (x \times x) + (x \times 3) + (2 \times x) + (2 \times 3)$
$= x^{1+1} + 3x + 2x + 6$
$= x^2 + 3x + 2x + 6$
$= x^2 + 5x + 6$

	x	$+ 3$
x	x^2	$3x$
$+ 2$	$2x$	6

$x^2 + 5x + 6$

a. $(x + 1)(x + 2) =$

b. $(a + 2)(a + 7) =$

c. $(x + 5)(x + 4) =$

2. Simplify.

Example $(x - 2)(x - 3)$

$= (x - 2)(x - 3)$
$= (x \times x) + (x \times -3) + (-2 \times x) + (-2 \times -3)$
$= x^{1+1} - 3x - 2x + 6$
$= x^2 - 5x + 6$

	x	$- 3$
x	x^2	$-3x$
$- 2$	$-2x$	6

$x^2 - 5x + 6$

a. $(x - 5)(x - 2) =$

b. $(a - 10)(a - 3) =$

c. $(x - 7)(x - 6) =$

3. Multiply:

Example $(x + 2)(x - 3)$

$= (x + 2)(x - 3)$
$= (x \times x) + (x \times -3) + (-2 \times x) + (2 \times -3)$
$= x^{1+1} - 3x + 2x - 6$
$= x^2 - x - 6$

	x	$- 3$
x	x^2	$-3x$
$+ 2$	$2x$	-6

$x^2 - x + 6$

a. $(x + 1)(x - 4) =$

b. $(4a + 3)(a - 8) =$

c. $(2x + 3)(x - 2) =$

continued

43b The product of two binomials continued

4. Multiply.

Example $(x - 2)(x + 3)$

$= (x - 2)(x + 3)$

$= (x \times x) + (x \times 3) + (-2 \times x) + (-2 \times -3)$

$= x^{1+1} + 3x - 2x + 6$

$= x^2 - x - 6$

	x	$+ 3$
x	x^2	$3x$
-2	$-2x$	-6

$x^2 + x - 6$

a. $(x - 3)(x + 4) =$

b. $(2a - 3)(a + 1) =$

c. $(x - 5)(x + 1) =$

5. Multiply.

Example

$(x \pm 2)^2$

$= (x + 2)(x + 2)$ and $(x - 2)(x - 2)$

$= x^2 + 2x + 2x + 4$ and $x^2 - 2x - 2x + 4$

$= x^2 + 4x + 4$ and $x^2 - 4x + 4$

$= x^2 \pm 4x + 4$

	x	$+ 2$
x	x^2	$-2x$
$+ 2$	$2x$	4

	x	$- 2$
x	x^2	$-2x$
$- 2$	$-2x$	4

a. $(x \pm 1)^2 =$

$(x + 1)^2$	$(x - 1)^2$

b. $(a \pm 6)^2 =$

$(a + 6)^2$	$(a - 6)^2$

6. Simplify.

Example $2(x - 3)^2$

$= 2[(x - 3)(x - 3)]$

$= 2[x^2 - 3x - 3x + 9]$

$= 2[x^2 - 6x + 9]$

$= 2x^2 - 12x + 18$

	x	$- 3$
x	x^2	$-3x$
$- 3$	$-3x$	9

a. $2(x + 2)^2 =$

b. $2(x + 7)^2 =$

7. Simplify.

a. $2(x - 3)^2 - 3(x + 1)(2x - 5) =$

b. $3(x + 4)^2 - 2(x + 3)(3x - 6) =$

Problem solving: be creative

- Create and solve two binomials multiplied.
- Create and solve two binomials multiplied. Use integers.
- Create and solve two binomials multiplied. Use the +/– operations.
- Create and solve two binomials multiplied. Use the +/– operation and coefficients.

44 The product of two binomials

Can you remember what a factor is?

Factors are numbers you multiply together to get another number.

Oh, yes the factor s of 12 will be: 1, 2, 3, 4, 6 and 12 , since 1 x 12 = 2 x 6 = 3 x 4 = 12

What are the factors of $x^2 + 7x + 12$?

You should ask which two binomials, when multiplied together, will give you this trinomial.

- Write two brackets ( ) ( ).
- Factorise $x^2 = (x)(x)$.
- Factorise 12 = (3)(4) and make sure that the sum of these two factors gives you 7.
- Fill in your operators $(x + 3)(x + 4)$.

1. Factorise.

Example $x^2 + 5x + 6$
$= x^2 + 5x + 6$
$= x^2 + 5x + 6$
$= (x + 3)(x + 2)$

→

x^2	
	6

→

x^2	$2x$
$3x$	6

Test: $x^2 + 2x + 3x + 6$
$x^2 + 5x + 6$

The product of the two factors gives me 6 but when added they give me 5.

a. $x^2 + 5x + 6 =$

b. $x^2 + 6x + 8 =$

c. $x^2 + 9x + 14 =$

d. $x^2 + 11x + 10 =$

e. $x^2 + 15x + 54 =$

f. $x^2 + 12x + 27 =$

2. Factorise.

Example $x^2 – 5x + 6$
$= x^2 – 5x + 6$
$= x^2 – 5x + 6$
$= (x – 3)(x – 2)$

→

x^2	
	6

→

x^2	$–3x$
$–2x$	6

$–3x –2 = 6$
$–3$ and $– 2 = – 5$

The product of the two factors gives me 6 but when added they give me 5.

a. $x^2 + x – 6 =$

b. $x^2 + 3x – 54 =$

c. $x^2 + 4x – 60 =$

d. $x^2 + 5x – 14 =$

e. $x^2 – x – 56=$

f. $x^2 + 7x – 8 =$

3. Factorise.

Example $x^2 – 7x + 12$
$= (x –4)(x – 3)$

→

x^2	
	12

→

x^2	$–4x$
$–3x$	12

a. $x^2 – 7x +12 =$

b. $x^2 – 13x + 42 =$

c. $x^2 – 11x + 30 =$

d. $x^2 – 9x + 20 =$

e. $x^2 – 15x + 56 =$

f. $x^2 – 8x + 15 =$

Problem solving

Find the factors of $x^2 + 11x + 24$

45 Divide monomials and binomials

Revise.

Law of exponents

- with variables

$\frac{x^m}{x^n} = x^{m-n}$

- with constants

$\frac{2^3}{2^2} = 2^{3-2}$

How fast can you simplify this?

$\frac{16}{8}$ = **2**	$\frac{20}{4}$ =	$\frac{12}{3}$ =	$\frac{21}{7}$ =
$\frac{25}{5}$ =	$\frac{30}{3}$ =	$\frac{9}{3}$ =	$\frac{15}{5}$ =

1. Simplify.

Example: using laws of exponents

$\frac{6x^3}{2x}$

$= \frac{6x^3}{2x}$

$= \frac{3x^3}{x}$

$= 3x^{3-1}$

$= 3x^2$

$\frac{6x^2}{2{\cdot}x}$

$\frac{6{\cdot}x{\cdot}x{\cdot}x}{2{\cdot}x}$

$= \frac{2{\cdot}3{\cdot}x{\cdot}x{\cdot}x}{2{\cdot}x}$

$= 3x^2$

Use a different method to check your answer.

a. $\frac{8x^3}{2x} =$

b. $\frac{16x^2}{8x} =$

c. $\frac{12x^4}{3x} =$

d. $\frac{20x^5}{4x} =$

e. $\frac{18x^5}{9x^4} =$

f. $\frac{21x^5}{7x^2} =$

2. Simplify.

Example: using laws of exponents

$\frac{6x^3 + 8x^2}{2x}$

$= \frac{6x^3}{2x} + \frac{8x^2}{2x}$

$= 3x^{3-1} + 4x^{2-1}$

$= 3x^2 + 4x$

$\frac{6x^3}{2x} + \frac{8x^2}{2x}$

$= \frac{2{\cdot}3{\cdot}x{\cdot}x{\cdot}x}{2{\cdot}x} + \frac{2{\cdot}4{\cdot}x{\cdot}x}{2{\cdot}x}$

$= 3x^2 + 4x$

Use a different method to check your answer.

$\frac{6x^3 + 8x^2}{2x}$

$= \frac{6x^3}{2x} + \frac{8x^2}{2x}$

$= 3x^{3-1} + 4x^{2-1}$

$= 3x^2 + 4x$

a. $\frac{6x^3 + 9x^2}{3x} =$

b. $\frac{16x^3 + 8x^2}{4x} =$

c. $\frac{25x^3 - 15x^2}{5x} =$

d. $\frac{24x^4 - 12x^3}{6x} =$

e. $\frac{8x^5 + 10x^3}{2x} =$

f. $\frac{30x^5 - 9x^4}{3x^2} =$

Problem solving

Create five of your own examples of binomials divided by a monomial.

46 Substitution

Do you remember what substitution is? How can you use substitution to evaluate the following if $x = -5$?

$x + 0{,}2$	x^2	$5x$	$2x^2 + x$

1. Revision: If x = 2, evaluate:

Example: $x + 5$
$= 2 + 5$
$= 7$

a. $x + 9 =$

b. $-x \times 2 =$

Why do these answers differ?

2. If $x = 2$, evaluate:

Example: $x^2 + 3x + 4$
$= (2)^2 + 3(2) + 4$
$= 4 + 6 + 4$
$= 14$

If $x = -2$, then:
$x^2 + 3x + 4$
$= (-2)^2 + 3(-2) + 4$
$= 4 - 6 + 4$
$= 2$

a. $x^2 + 6x + 5 =$

b. $2x^2 + 9x + 1 =$

c. $x^2 + 9x + 6 =$

d. $5x^2 + 3x + 2 =$

e. $8x + x^2 - 5 =$

f. $8 - x^2 - 5x =$

3. Evaluate the expression if $x = -3$, and if $x = \frac{1}{3}$:

Example: $-x^2 + 3x + 4$

If $x = -3$, then
$= -(-3)^2 + 3(-3) + 4$
$= -9 - 9 + 4$
$= -18 + 4$
$= -14$

If $x = \frac{1}{3}$, then:
$-(\frac{1}{3})^2 + 3(\frac{1}{3}) + 4$
$= -\frac{1}{9} + 1 + 4$
$= 5 - \frac{1}{9}$
$= 4\frac{8}{9}$

a. $-x^2 + 4x + 2 =$

b. $-x^2 + 5x + 3 =$

c. $6 + 5x - 4x^2 =$

d. $7 + 2x^2 - 5x =$

e. $-2x^2 - x + 5 =$

f. $-3 - 5x^2 + 5 =$

Problem solving

If your answer is –15, write down a possible trinomial.
If your answer is 15, write down a possible trinomial.

47 Factorise algebraic expressions

How fast can you factorise the following?

Example: 3 – 27
= 3(1 – 9)

4 + 16 =　　5 – 25 =　　6 + 42 =

7 + 56 =　　9 + 99 =　　48 – 6 =

1. Factorise.

a.

Example $2x(a + b) + 3(a + b)$
$= (a + b)(2x + 3)$

(a + b) is the common factor.

What is the common factor?

$4x$ (c + d) + 2(c + d) =

b.

Example $2x(a - b) + 3(a - b)$
$= (a - b)(2x + 3)$

(a – b) is the common factor.

What is the common factor?

$4x$ (a – b) – 5(a – b) =

c.

Example $(3a + b)(x) + (3a + b)(y)$
$= (3a + b)(x + y)$

(3a + b) is the common factor.

What is the common factor?

(3a + b)(m) + (3a + b)(n) =

2. Factorise.

Example (3a + b)(p – 2t) – (3a + b)(2p+ 2t)
= (3a + b)[(p – 2t) – (2p + 2t)]
= (3a + b)(p–2t–2p–2t)
= (3a + b)(–p–4t)
= –(3a + b)(p + 4t)

(3a + b) is the common factor.

a. (2a + b)(p – 3t) + (2a + b)(p + 3t) =

b. $(3x + y)(a + b) - (3x + y)(a - b)$ =

Example $ax - bx$
$= x(a - b)$

c. $cx + dx$ =

d. $mx + nx$ =

Example $ax - bx + 2a - 2b$
$= x(a - b) + 2(a - b)$
$= (a - b)(x + 2)$

e. $ax + bx + 4a + 4b$ =

f. 2m – 2n + 3m + 3n =

Example 1: a – 4b = 1(a – 4b)

Example 2: 4b – a = –1(a – 4b)

g. $x - 2y$ =

h. $2y - x$ =

Example $3a^2 - 27$
$= 3(a^2 - 9)$

i. $2a^2 - 18$ =

j. $5a^2 + 30$ =

3. Factorise.

Example: $a^4 - a^2$
$= a^2 [\frac{a^4}{a^2} - \frac{a^2}{a^2}]$
$= a^2 [a^{4-2} - 1]$
$= a^2 [a^2 - 1]$
or
$a^4 - a^2$
$= (a.a.a.a) - (a.a)$
$= a.a(a.\ a - 1)$
$= a^2(a^2 - 1)$

a. $x^5 - x^3$ =

b. $d^8 + d^4$ =

Example: $6a^4 - 4a^2$
$= 2a^2(3a^2 - 2)$
or
$6a^4 - 4a^2$
$= 2a^2 (\frac{6a^4}{2a^2} - \frac{4a^2}{2a^2})$
$= 2a^2 - (3a^{4-2} - 4a^{2-2})$
$= 2a^2 (3a^2 - 2a^0)$
$= 2a(3a^2 - 2)$

c. $6b^4 - 3b^2$ =

d. $8a^6 - 6a^4$ =

continued

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

Factorise algebraic expressions
continued

4. Factorise.

Example $(a + b)^2$
$= (a + b)(a + b)$

a. $(a + b)^3 =$

b. $(x + y)^2 =$

Example $(a + b)^2 - 5(a + b)$
$= (a + b)(a + b) - 5(a + b)$
$= (a + b)[(a + b) - 5]$
$= (a + b)(a + b - 5)$

c. $(x + y)^2 - 6(x + y) =$

d. $(d + e)^2 - 2(d + e) =$

5. Factorise.

Example: $25a^2$
$= (5a)^2$

Note that: $1 = 1^2 = 1 \times 1$

Example: $25a^2 - 1$
$= (5a)^2 - 1$

a. $16a^2 =$

b. $64a^2 =$

c. $9a^2 - 1 =$

d. $49a^2 - 1 =$

6. Revision: use the example to guide your factorisation

Example: $a^2 + b^2$
$= (a)^2 + (b)^2$

a. $x^2 + y^2 =$

b. $c^2 + d^2 =$

7. Factorise.

Example: $a^4 - b^4$
$= (a^2 - b^2)(a^2 + b^2)$
$= (a - b)(a + b)(a^2 + b^2)$

a. $x^4 - y^4 =$

b. $c^4 - d^4 =$

Example: $9(a + b)^2 - 1$
$= [3(a + b)]^2 - 1^2$
$= [3(a + b) + 1][3(a + b) - 1]$
$= (3a + 3b + 1)(3a + 3b - 1)$

c. $64(x + y)^2 + 1 =$

d. $25(a + b)^2 - 1 =$

8. Simplify using factorisation.

Example: $3x - 3y$
$= 3(x - y)$

a. $5x + 5y =$

b. $7a + 7b =$

Example: $\frac{3x-3y}{6x-6y}$
$= \frac{3(x-y)}{6(x-y)}$
$= \frac{3}{6}$
$= \frac{1}{2}$

c. $\frac{4x + 4y}{16x + 16y} =$

d. $\frac{5x - 5y}{10x + 10y} =$

Problem solving

Create an algebraic expression where the common expression is:

a. $4a + b$ b. $(x^2 + y^2)$ c. $(x + y)^2$

48 Divide a trinomial and polynomial by a monomial

Give an example of each.

$\frac{\text{trinomial}}{\text{monomial}}$

$\frac{\text{polynomial}}{\text{monomial}}$

Write down a few keywords to help you to remember how to:

Simplify:

Factorise:

1. Simplify the fractions using factorisation.

Example: $3x - 3y$
$= 3(x - y)$

a. $5x + 5y =$

b. $7a + 7b =$

Example: $\frac{3x-3y}{6x-6y}$
$= \frac{3(x-y)}{6(x-y)}$
$= \frac{3}{6}$
$= \frac{1}{2}$

c. $\frac{4x + 4y}{16x + 16y} =$

d. $\frac{5x - 5y}{10x + 10y} =$

2. Simplify and factorise.

Example: **Simplify:** $\frac{4x^4 - 2x^3}{2x^2}$
$= \frac{4x^4}{2x^2} - \frac{2x^3}{2x^2}$
$= 2x^{4-2} - x^{3-2}$
$= 2x^2 - x$

Factorise: $\frac{4x^4 - 2x^3}{2x^2}$
$= \frac{2x^2(2x^2 - x)}{2x^2}$
$= 2x^2 - x$

a. $\frac{6x^5 - 63}{3x^2} =$

b. $\frac{8x^{12} + 16x^6}{4x^3} =$

Example: **Simplify:** $\frac{6x^3 - 8x^2 + 2x}{2x}$
$= \frac{6x^3}{2x} - \frac{8x^2}{2x} + \frac{2x}{2x}$
$= 3x^{3-1} - 4x^{2-1} + 1$
$= 3x^2 - 4x + 1$

Factorise: $\frac{6x^3 - 8x^2 + 2x}{2x}$
$= \frac{2x(3x^2 - 4x + 1)}{2x}$
$= 3x^2 - 4x + 1$

c. $\frac{9x^4 + 6x^2 + 3x}{3x} =$

d. $\frac{8x^5 - 4x^3 - 4x}{2x} =$

Example: $\frac{6x^3 - 8x^2 + 2x + 10}{2x}$
$= \frac{6x^3}{2x} - \frac{8x^2}{2x} + \frac{2x}{2x} + \frac{10}{2x}$
$= 3x^{3-1} - 4x^{2-1} + 1 + \frac{5}{x}$
$= 3x^2 - 4x + 1 + \frac{5}{x}$

Factorise: $\frac{2x(3x^2 - 4x + 10}{2x}$
$= 3x^2 - 4x + 1 + \frac{10}{2x}$
$= 3x^2 - 4x + 1 + \frac{5}{x}$

e. $\frac{6x^3 + 4x^2 + 2x + 6}{2x} =$

f. $\frac{9x^4 + 6x^3 - 3x - 9}{3x} =$

Problem solving

Create a polynomial divided by a monomial. Simplify and factorise the expression.

49 Linear equations that contain fractions

Look at the three examples. Discuss.

$4a + 5 = 17$	$3x = 6$	$3(x - 2) = x + 1$
$4a = 17 - 5$	$\frac{3x}{3} = \frac{6}{3}$	$3x - 6 = x + 1$
$4a = 12$	$x = 2$	$3x - x - 6 = 1$
$\frac{4a}{4} = \frac{12}{4}$		$2x = 7$
$a = 3$		$\frac{2x}{2} = \frac{7}{2}$
		$x = 3\frac{1}{2}$

1. Solve the linear equation.

Example: $4x = 2$

$\frac{4x}{4} = \frac{2}{4}$

$x = \frac{1}{2}$

a. 6a = 3

b. 9b = 10

2. Solve for x.

Example: $7 = \frac{3}{x}$

$\frac{7}{1} \times \frac{x}{1} = \frac{3}{x} \times \frac{x}{1}$

$7x = 3$

$\frac{7x}{7} = \frac{3}{7}$

$x = \frac{3}{7}$

a. $8 = \frac{4}{x}$

b. $9 = \frac{3}{x}$

A linear equation is an equation that makes a straight line when it is graphed. It has only one unknown and that is only to the power of 1.

3. Solve for x.

Example: $\frac{7}{x-2} = \frac{3}{x}$

$\frac{7}{\cancel{x-2}} \times \frac{\cancel{x-2}}{1} = \frac{3}{x} \times \frac{x-2}{1}$

$7 = \frac{3}{x} \times \frac{x-2}{1}$ *or* $7 = \frac{3(x-2)}{x}$

$7 \times \frac{x}{1} = \frac{3}{x} \times \frac{x}{1} \times \frac{x-2}{1}$ *or* $7 \times \frac{x}{1} = \frac{3(x-2)}{1} \times \frac{x}{1}$

$7x = 3(x - 2)$

$7x = 3x - 6$

$7x - 3x = 3x - 3x - 6$

$4x = -6$

$\frac{4x}{4} = \frac{-6}{4}$

$x = -\frac{3}{2}$

a. $\frac{8}{x+2} = \frac{4}{x}$

b. $\frac{5}{x-3} = \frac{2}{x}$

4. Solve linear equations containing fractions.

Example: $\frac{x}{3} = 1$

$\frac{x}{3} \times \frac{3}{1} = 1 \times \frac{3}{1}$

$x = 3$

a. $\frac{x}{2} = 1$

b. $\frac{x}{5} = 1$

continued

49b Linear equations that contain fractions continued

Example:
$\frac{2x-1}{4} = 1$
$\frac{2x-1}{4} \cdot \frac{4}{1} = 1 \cdot \frac{4}{1}$
$2x - 1 = 4$
$2x = 4 + 1$
$2x = 5$
$\frac{2x}{2} = \frac{5}{2}$
$x = \frac{5}{2}$
$x = 2\frac{1}{2}$

c. $\frac{3x+1}{5} = 1$

d. $\frac{4x-2}{6} = 1$

Example:
$\frac{x}{3} + \frac{x}{4} = 1$
$\frac{x}{3} \times \frac{4}{4} + \frac{x}{4} \times \frac{3}{3}$
$\frac{4x}{12} + \frac{3x}{12} = 1$
$\frac{7x}{12} = 1$
$\frac{7x}{12} \times \frac{12}{1} = 1 \times 12$
$7x = 12$
$x = \frac{12}{7}$

e. $\frac{x}{2} + \frac{x}{3} = 1$

f. $\frac{x}{5} + \frac{x}{3} = 1$

Example:
$\frac{x}{3} + \frac{2x-1}{4} = 1$
$\frac{x}{3} \times \frac{4}{4} + \frac{2x-1}{4} \times \frac{3}{3} = 1$
$\frac{4x}{12} + \frac{6x-3}{12} = 1$
$\frac{4x+6x-3}{12} \times \frac{12}{1} = 1 \times \frac{12}{1}$
$4x + 6x - 3 = 12$
$10x - 3 = 12$
$10x = 15$
$\frac{10x}{10} \times \frac{15}{10}$
$x = \frac{3}{2}$
$x = 1\frac{1}{2}$

g. $\frac{x}{4} + \frac{2x+1}{2} = 1$

h. $\frac{x}{5} + \frac{3x-2}{2} = 1$

Problem solving

Create algebraic equations that give you an answer of:

a. $x = \frac{3}{4}$

b. $x = \frac{1}{2}$

c. $x = \frac{5}{2}$

50 Solve equations of the form: a product of factors equals zero

Revise the following:

$x^2 + 6x + 8$

2 x 4 = 8
2 + 4 = 6

	x	$+ 2$
x	x^2	$2x$
$+ 4$	$4x$	8

1. Factorise.

Example: $x^2 + 5x + 6$
$= (x + 2)(x + 3)$

a. $x^2 - x - 6 =$

b. $x^2 + 9x + 14 =$

2. Solve x.

$x + 1 = 0$
$x + 1 - 1 = 0 - 1$
$x = -1$

$x + 3 = 0$
$x + 3 - 3 = 0 - 3$
$x = -3$

Example $(x + 1)(x + 3) = 0$
$x + 1 = 0$ or $x + 3 = 0$
$x = -1$ or $x = -3)$

a. $(x + 2)(x + 3) = 0$

b. $(x + 4)(x - 1) = 0$

3. Factorise.

$x.x - 3x$
$x(x - 3x)$

Example: $x^2 - 3x$
$= x(x - 3)$

a. $x^2 + 2x =$

b. $x^2 + 5x =$

Why is it so important to know how to factorise a polynomial?

4. Solve for x.

Example: $x^2 - 3x = 0$
$= x(x - 3) = 0$
$x = 0$ or $x - 3 = 0$
$x = 0$ or $x = 3$

a. $x^2 + 2x = 0$

b. $x^2 - 6x = 0$

5. Factorise:

$= (x + 5)(x - 5)$
$= x^2 + 5x - 5x - 25$
$= x^2 - 25$

Example: $x^2 - 25$
$= x^2 - 5^2$

a. $x^2 - 36 =$

b. $x^2 - 16 =$

6. Calculate the square root and use the example to show positive and negative numbers:

Example: $\sqrt{25}$
$\sqrt{5.5}$ or $\sqrt{-5.-5}$
$= 5$ or -5

Why is $\sqrt{25} = \sqrt{5.5}$ or $\sqrt{-5,-5}$?

5 x 5 = 25
and
–5 x –5 = 25

a. $\sqrt{36}$

b. $\sqrt{16}$

7. Solve for x:

Example: $x^2 - 25 = 0$
$x^2 = 25$
$x^2 = 5^2$
$\sqrt{x^2} = \sqrt{5^2}$
$x = \pm 5$
$x = +5$ *or* $x = -5$

Test:
$x^2 - 25 = 0$
$(5)^2 - 25 = 0$
$25 - 25 = 0$
$0 = 0$

$(5)^2 - 25 = 0$
$25 - 25 = 0$
$0 = 0$

a. $x^2 - 25 =$

b. $x^2 - 36 =$

Problem solving

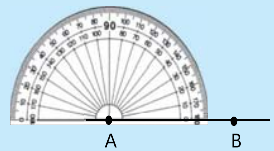
Create a sum where the product of factors equals zero and solve it.

51 Construct angles and polygons using a protractor

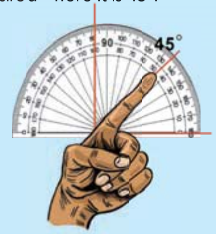
Revise the following:

Step 1: Draw a line segment. Label it AB.

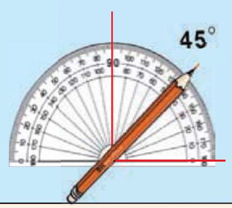
Step 2: Place the protractor so that the origin (small hole) is over point A. Rotate the protractor so that the base line is exactly along the line AB.



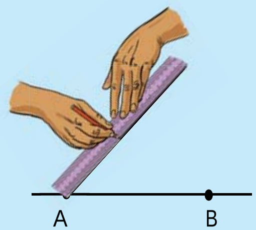
Step 3: Using (in this case) the inner scale, find the angle desired - here it is 45°.



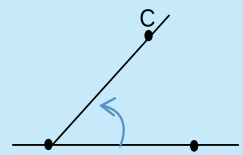
Step 4: Make a mark at this angle, and remove the protractor.



Step 5: With a ruler, draw a line from A to the mark you just made. Label this point C.



Step 6: The line drawn makes an angle BAC with a measure of 45°.



1. Construct the following with a protractor as a revision activity. Label the angles. Do this on separate piece of paper or exercise book.

a. Obtuse angle
b. Acute angle
c. Reflex angle
d. Straight line
e. Right angle
f. Revolution

2. Name all the quadrilaterals and triangles. Label their angles.

a. Quadrilaterals

b. Triangles

3. Draw the following angles and polygons. Label them.

a. A 60º angle ABC.

b. A 270º angle.

c. A triangle with one 45º angle and one 65º angle.

d. A triangle with an 80º and 35º angle.

e. A quadrilateral with one 70º angle and one 121º angle.

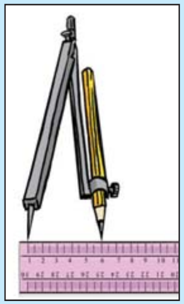
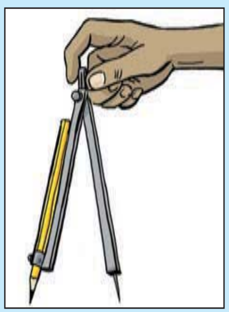
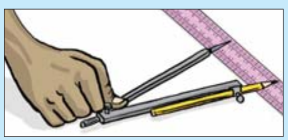
f. A quadrilateral with two 85º angles.

Problem solving

Construct the top view of a very modern house using a protractor.

52 Using a pair of compasses

Revision

To draw a circle accurately, use a pair of compasses. 	Align the pencil lead with the compass point. 	Tighten the hold for the pencil so that it does not slip. 
Make sure that the hinge at the top of the compass is tightened so that it does not slip. 	Set the compass to the radius of the circle. The radius is the distance between the centre and the circumference; it is half the diameter. 	Press down the compass point and turn the knob at the top of the compass to draw a circle. 

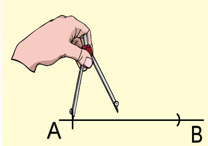
1. Draw a circle. Give an everyday example of a circle this size.

a. with a radius of 1,8 cm.	b. with a diameter of 3,2 cm.	c. with a radius of 16 mm.
Everyday example:	Everyday example:	Everyday example:

2. Revision: construct perpendicular lines from both sides.

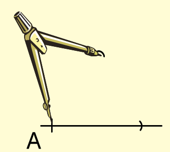
Step 1

Draw a line and mark A and B on it. Put the compass point on A and open it so that the pencil touches B. (So you have "measured" the length of AB with the pair of compasses.)



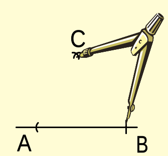
Step 2

Leaving the compass point on A, draw an arc with the compass roughly where you think the perpendicular line is going to be.



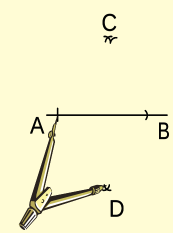
Step 3

Now move the compass point to B and draw another arc which crosses the first. Label the crossing point C.



Step 4

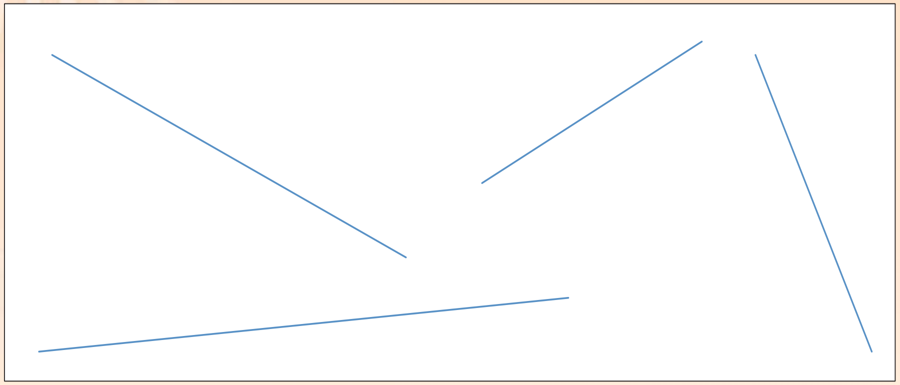
Look at the drawing and do the same to get point D below the horizontal line



continued ☛

52b Using a pair of compasses continued

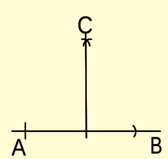
Draw lines perpendicular to these using a protractor.



3. Revision: construct a 45° angle on a separate piece of paper.

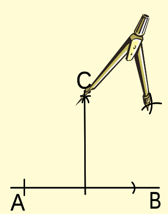
Step 1

Follow the steps for drawing a perpendicular line.



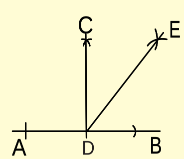
Step 2

Leaving the compass point on C, draw an arc with the compass roughly halfway between C and B. Place it on B and draw an arc crossing the first one.



Step 3

Mark it as E and draw the line from D to E which creates two 45° angles.



4. Use your knowledge of how to construct a 45° angle to help you construct these angles.

a. 22,5° angle	b. 11,25° angle
c. 135° angle	d. 112,5° angle

Problem solving

Show in 4 steps how to draw a 225° angle.

53 Constructing triangles

Who constructs triangles in everyday life? Use some of the guidance below.

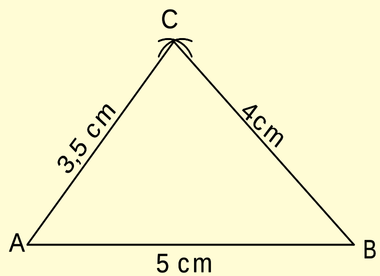
A triangle is a very strong structure. The triangle is used in structural designs to reinforce and support weight.

The right triangle is one of the most important geometrical figures and has been used in many applications for thousands of years.

1. Construct ΔABC in which AB = 5cm, AC = 3,5 cm and BC = 4 cm. Follow the steps.

How to construct a triangle when three sides are given (SSS).

Step 1: Draw AB = 5 cm.	**Step 2:** With A as centre and radius 3,5 cm, draw an arc.
Step 3: With B as centre and radius 4 cm draw another arc intersecting the arc of C.	**Step 4:** Join AC and BC



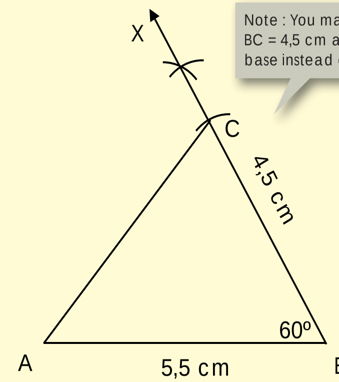
Practise.

Construct ΔCDE in which CD = 4,2 cm, CE = 3,6 cm and DE = 3,6 cm.

2. Construct a triangle ABC in which AB = 5,5cm, BC = 4,5 cm and ∠ABC = 60°.

How to construct a triangle when two sides and the included angle is given (SAS).

Step 1: Draw AB = 5,5 cm.	**Step 2:** At B, construct an angle ABX = 60°.
Step 3: With B as centre and radius 4,5 cm draw an arc cutting BX at C.	**Step 4:** Join AC. Then ΔABC is the required triangle.



Note : You may take BC = 4,5 cm as the base instead of AB.

Practise.

continued ☛

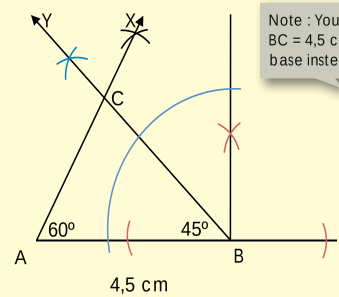
Constructing triangles continued

Construct a triangle DEF in which DE = 3,7 cm, EF = 41 mm and ∠DEF = 55°.

3. Construct a ΔABC in which ∠A = 60°, ∠B = 45° and BC = 4,5 cm.

How to construct a triangle when two angles and the included side are given (ASA).

Step 1: Draw AB = 4,5 cm.	**Step 2:** At A, construct ∠BAX = 60°.
Step 3: At B, construct ∠ABY = 45° with BY crossing AX at C.	Then ΔABC is the required triangle



Note : You may take BC = 4,5 cm as the base instead of AB.

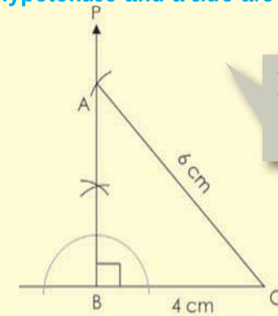
Practise.

Construct a ΔKLM in which ∠K = 48°, ∠L = 48° and side KL = 3,9 cm.

4. Construct a right triangle ABC, right-angled at B, side BC = 4cm and hypotenuse AC = 6 cm.

How to construct a right triangle when its hypotenuse and a side are given.

Step 1: Draw BC = 4 cm.	**Step 2:** At B, construct ∠CBP = 90°.
Step 3: With C as centre and radius 6 cm draw an arc cutting BP at A.	**Step 4:** Join AC.



The hypotenuse is the side opposite the right angle in a right-angled triangle

Practise.

Construct a right triangle XYZ, right-angled at Y, side YZ = 5 cm and hypotenuse XZ = 7 cm

Problem solving

Construct a kite.

54 Constructing quadrilaterals

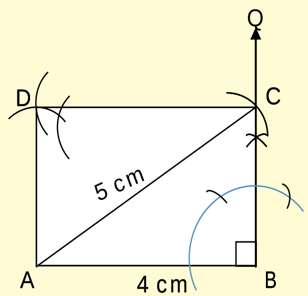
How did the designers of these rooms use quadrilaterals?

1. Construct a rectangle.

How to construct a rectangle when one of its diagonals and a side are given.

Step 1: Draw AB = 4 cm.	**Step 2:** At B, draw ∠ABQ = 90°.
Step 3: With A as centre and radius 5 cm, draw an arc cutting BQ at C.	**Step 4:** With C as centre and radius 4 cm, draw an arc.
Step 5: With A as centre and radius = BC, draw an arc cutting the arc drawn in Step 4 at D.	**Step 6:** Join DC and AD.

Remember that in a rectangle each angle is 90°.



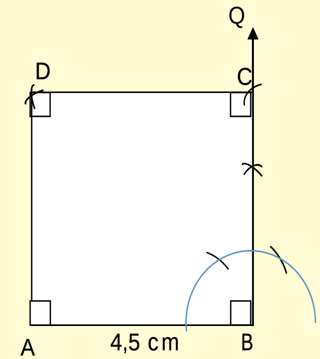
Practise.

Construct a rectangle KLMN in which KL = 3,6 cm and KM = 4,5 cm.

2. Construct a square.

How to construct a square when its side is given.

Step 1: Draw AB = 4,5 cm.	**Step 2:** Construct ∠ABQ = 90° at B.
Step 3: From BQ cut off BC = 4,5 cm.	**Step 4:** From A and C, draw two arcs of radii 4,5 cm each to cut each other at D.
Step 5: Join AD and CD.	



Practise.

continued ☛

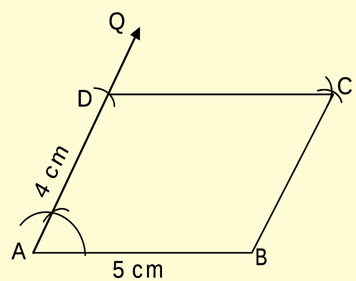
54b Constructing quadrilaterals continued

Construct a square GHIJ in which GH = 32 mm.

3. Construct a parallelogram.

How to construct a parallelogram when two adjacent sides and the included angle are given.

Step 1: Draw AB = 5 cm.	**Step 2:** At A, construct $\angle BAQ = 60°$.
Step 3: From AQ cut off AD = 4 cm.	**Step 4:** With B and D as centres and radii equal to 4 cm and 5 cm respectively, draw two arcs cutting each other at C.
Step 5: Join CD and BC.	



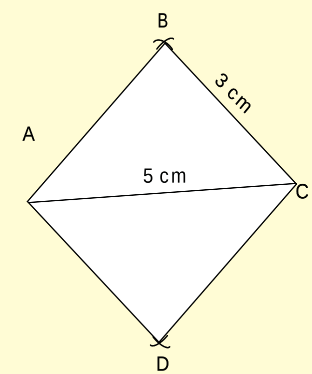
Practise.

Construct a parallelogram in which the adjacent sides are 6 cm and 3 cm and the included angle is 60°.

4. Construct a rhombus.

How to construct a rhombus when one diagonal and side are given.

Step 1: Draw AC = 5 cm.	**Step 2:** With A as centre and radius 3 cm, draw two arcs - one above AC and the other below AC.
Step 3: With C as centre and radius 3 cm draw two arcs - one above AC and the other below AC intersecting the arcs of Step 2 in B and D respectively.	**Step 4:** Join AB, BC, CD and AD.



Practise.

Construct a rhombus, when one of its diagonals is 4 cm and the side is 3 cm.

Problem solving

Construct a kite.

55 Regular and irregular polygons

Draw three examples each of regular and irregular polygons. Remember to label your polygons.

If all the angles are equal and all sides are equal than it is a **regular** polygon.

If the angles and sides are not equal then it is an **irregular** polygon.

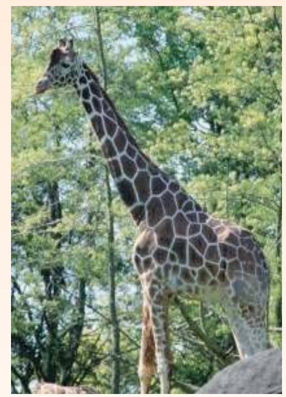
Are the total sum of the interior angles of each kind of polygon triangles/ quadrilaterals, etc.) always the same?

1. **Complete the table.**

Polygon	Total number of sides	Angle sizes	Total:
Regular triangle	3	60º + 60º + 60º	180º
Irregular triangle			
Regular quadrilateral			
Irregular quadrilateral			
Regular pentagon			
Irregular pentagon			
Regular hexagon			
Irregular hexagon			
Regular heptagon			
Irregular octagon			
Regular nonagon			
Irregular nonagon			
Regular decagon			
Irregular decagon			

2. **What is this? What polygon (s) can you identify? Describe the polygons.**

3. **Look at the giraffe. Identify all the regular and irregular polygons. Describe them.**



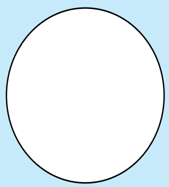
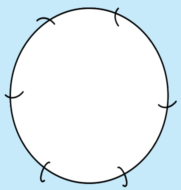
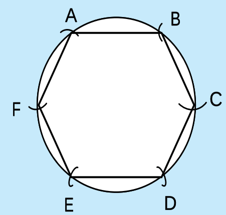
4. **What type of art is this? Identify all the geometric figures. Describe each.**

Problem solving

Construct an irregular hexadecagon. Measure all the angles.

56 Construct a hexagon

Revise the following:

Step 1: Draw a circle. Measure the radius with a pair of compasses.	Step 2: Make markings the same distance apart on the circumference, using the compasses.	Step 3: Label and join the points.
		

Identify the hexagons and explain how each one is used.




1. Construct a hexagon and label the vertices A to F.

a. Is this a regular or irregular hexagon? Why?

b. What is the length of the sides? How will you measure it using a compass?

c. What is the size of the angles? How will you determine it (i) without a protractor and (ii) with a protractor?

d. What is the distance from AD, FC, or BE? What is this of the circle?

e. What is the relationship between AD and AB?

2. Construct a regular hexagon with the sides equal to 3,2 cm.

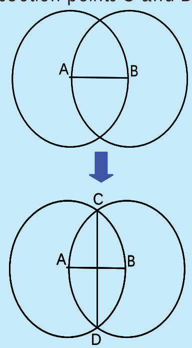
Problem solving

Construct a dodecagon using a similar method to that used in this worksheet.

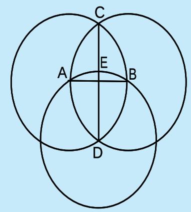
57 Constructing a pentagon

Step 1: Draw a circle around A with radius AB.

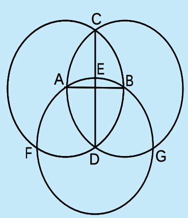
Step 2: Draw a circle around B with radius AB. Call their intersection points C and D.



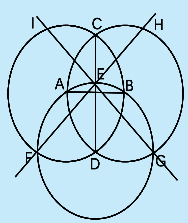
Step 3: Draw a circle around D with radius DA. Circle D intersects line CD at E.



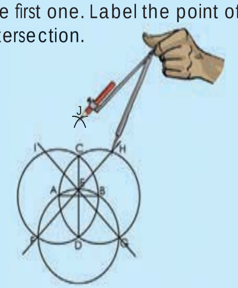
Step 4: Circle D intersects circle A at F and intersects circle B at G.



Step 5: Draw a line through FE and a line through GE. Line FE intersects circle B at H. Line GE intersects circle A at I.



Step 6: Set your pair of compasses to the length of AI. Place the compass point on I, and make a small arc above C. Then place your compass point on H and make an arc crossing the first one. Label the point of intersection.



Step 7: All the points A, B, I, H and J are points of the pentagon.

Where will you find this pentagonal-shaped castle?

1. Construct a pentagon and label its vertices A, B, H, J, and I

2. Answer the following:

a. Complete the following: JH = _____ = _____ = _____ = _____ = _____

b. Is the pentagon regular or irregular? Why?

c. Describe AB, DA and DB.

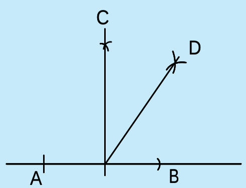
3. Draw a regular pentagon with sides equal to 2,3 cm.

Problem solving

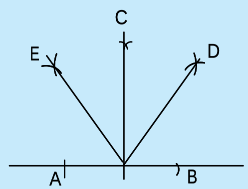
Write down step by step how you would construct a pentagon using a protractor.

58 Constructing an octagon

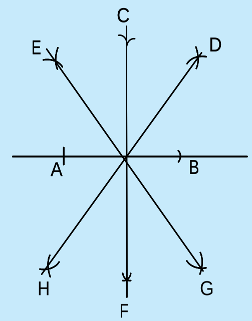
Step 1: Draw a 45º angle.



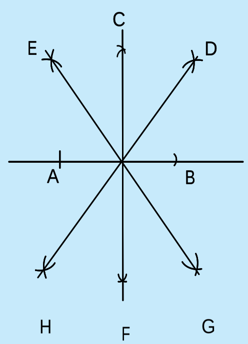
Step 2: Draw another 45º angle between A and C. You now have four 45º angles.



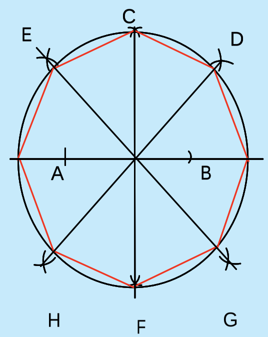
Step 3: Draw a another four 45º angles below line AB.



Step 4: Draw a circle using the middle point where all lines intersect.



Step 5: You might need to extend line AB. Where the lines intersect with the circle, draw lines from the one intersection to the next.



How are octagons used?

1. Now construct an octagon by yourself.

2. Complete the following:

a. OF = _____ = _____ = _____ = _____ = _____ = _____ = _____ = _____

b. What is this of the circle?

c. HF = _____ = _____ = _____ = _____ = _____ = _____ = _____ = _____

d. What is this of the circle?

3. Draw a regular octagon with equal radius of 2,8cm.

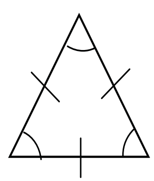
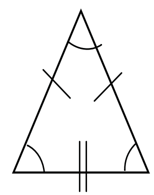
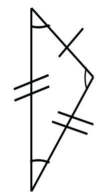
Problem solving

Write down step by step how you will construct a pentagon using a protractor

59 Interior angles of a triangle

Revise: There are special names for triangles according to:

Sides

Equilateral	Isosceles	Scalene
		

Angles

Acute:	all angles are less than 90º.
Right:	has a right angle (90º).
Obtuse:	has an angle more than 90º.

1. Read the following and label the triangle.

To prove: ∠A + ∠B + ∠C = 180°

Construction: through A, draw a line DAE parallel to BC.

Proof: Since DE is parallel to BC, and AB is a transversal

∠∠B = ∠DAB (pair of alternate angles)

Similarly ∠C = ∠EAC (pair of alternate angles)

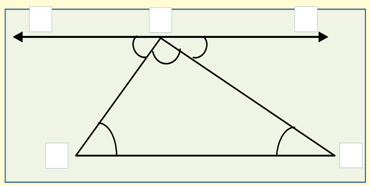
∠∠B + ∠C = ∠DAB + ∠EAC (two pairs of alternate angles)

Now ∠ A + ∠DAB + ∠EAC = 180º (straight line)

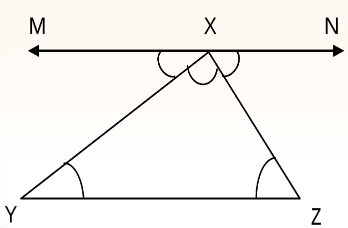
and because ∠DAB + ∠EAC = ∠B + ∠C
∠∠A + ∠B + ∠C = 180°

The sum of the three angles of a triangle is 180°.

Ask your teacher to calculate alternate transversals with you.



2. Prove that the sum of the three angles of a triangle is 180º for this triangle.



3. Calculate and construct these triangles. Classify the triangle.

Note that your answers and constructions could be different from those of your fellow classmates.

a. ∠A + ∠B + 90º = 180°

b. ∠A + 45º + ∠C = 180°

c. 100º + ∠B + ∠C = 180°

d. ∠A + 65º + ∠C = 180°

e. ∠A + 60º + ∠C = 180°

f. 120º + ∠B + ∠C = 180°

Problem solving

If one angle of a triangle equals 45º, what could the sizes of the other angles be? Give five different possibilities.

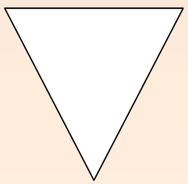
60 Triangles

Symbols in geometry. Identify the symbols we will use when we work with triangles. Give a reason for each.

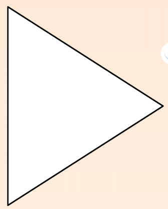
Triangle	Angle	Perpendicular	Parallel	Degrees	Right angles
△	∠	⊥	∥	°	∟
Line segments	**Line**	**Ray**	**Congruent**	**Similar**	**Therefore**
$\overline{AB}$	$\overleftrightarrow{AB}$	$\overrightarrow{AB}$	≅	~	∴

1. Measure the sides of the triangles. Label the triangles and describe them.

a.

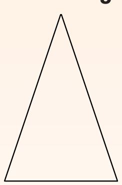


b.



Think of words such as: types of triangles, congruent, transformations, etc.

2. Label the triangle ABC.



a. What do these symbols mean?

ΔABC

$\overline{AB} = \overline{AC}$

$\angle ABC = \angle ACB$

3. Using geometric symbols, how would one change an isosceles triangle to an equilateral triangle?

4. Describe a scalene triangle according to sides and angles, and label it.

a. What do these symbols mean?

ΔABC

$\overline{AB} \neq \overline{AC} \neq \overline{BC}$

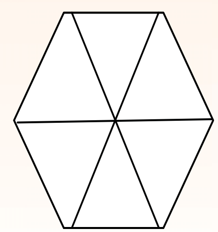
$\angle ABC \neq \angle ACB \neq \angle BAC$

5. Using geometric symbols, how would one change the scalene triangle above to a regular triangle?

6. a. What is a right-angled triangle? Construct and label the triangle.

b. Change this irregular triangle to a regular triangle.

7. Look at the photograph and complete the questions.



a. What is this? ____________________

continued ☛

60b Triangles continued

b. Where will you find these triangles? ____________________

c. What type of triangles are they? ____________________

d. Do you only see triangles? ____________________

e. Can I divide these other shapes into triangles? ____________________

f. What type of triangles will they form? ____________________

g. Are the triangles regular or irregular? ____________________

8. Look at the patterns in this stained glass window.

Do you see any triangles? Do you see any irregular geometric shapes?

9. Choose the correct answer and put a tick (√) next to it:

a. Which of the following could be the angles of a triangle?
i. 65°, 45° and 80°
ii. 90°, 30° and 61°
iii. 60°, 60° and 59°
iv. 60°, 60° and 60°

b. The hypotenuse of a triangle is:
i. The side opposite the right angle in a right-angled triangle.
ii. The side next to the right angle in a right-angled triangle.
iii. The angle of a right-angled triangle.
iv. All three sides of a right-angled triangle.

c. An equilateral triangle has:
i. Two sides that are equal.
ii. All the sides are equal but not the angles.
iii. All the sides and the interior angles are equal.
iv. All the angles are equal but not the sides.

d. An isosceles triangle has:
i. All the sides equal.
ii. At least two sides that are equal and its base angles are equal.
iii. At least two sides that are equal but no angles are equal.
iv. Two angles that are equal but no sides are equal.

e. A right-angled triangle has:
i. No angles that are right angles.
ii. All angles that are 60º.
iii. Two angles that are 90º.
iv. One angle that is a right angle.

Problem solving

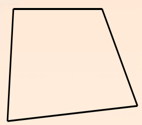
Create your own stained glass pattern. You should use as many irregular triangles as you can.

61 Polygons

Talk about this flow diagram.

- Triangles
  - Regular
    - Equilateral triangle
  - Irregular
    - Scalene triangle
    - Isosceles triangle
- Quadrilaterals
  - Regular
    - Square
    - Rhombus
  - Irregular
    - Rectangle
    - Parallelogram
    - Trapezium

1. Use the symbols and colours to answer the questions. Paste or draw everyday example pictures next to each.

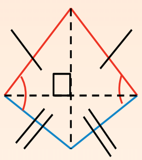

Quadrilateral

a. What is a quadrilateral?

____
____
____

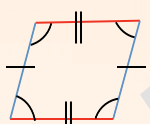
b. Describe a trapezium?

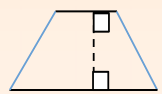
____
____
____


Kite

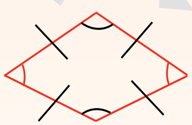
f. Describe a kite?

____
____
____


Parallelogram

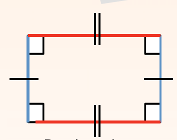

Trapezium

In the United States of America they call it a trapezoid.


Rhombus

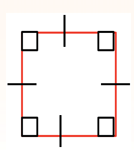
g. Describe a parallelogram

____
____
____


Rectangle

c. Describe a rectangle?

____
____
____


Square

e. Describe a rhombus?

____
____
____

d. Describe a square?

____
____
____

2. Construct a kite, label it and divide it into two triangles. Are these triangles regular or irregular?

3. Divide a trapezium into irregular triangles. Label it.

4. Identify and then name the regular and irregular polygons in this mosaic.

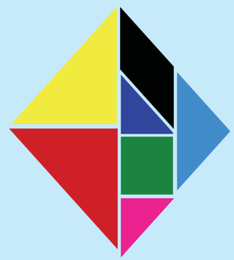
Problem solving

Make a mosaic (you can use old paper pieces) using different types of polygons.

62 Polygons

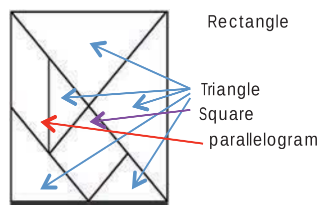
What is a tangram?

The **tangram** is a dissection puzzle consisting of seven flat shapes, called tans, which are put together to form shapes. The objective of the puzzle is to form a specific shape using all seven pieces, which may not overlap. It was originally invented in China.



1. **Make a geometric shape with all the pieces from the tangram from Cut-out 1. Draw a sketch of it in the answer block and say whether it is a regular or irregular shape. Label the shapes of its component parts.**

a. Make a large square.



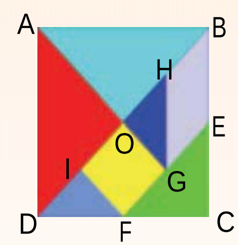
b. Make a rectangle.

c. Make a parallelogram .

d. Make a trapezium.

e. Make any other quadrilateral .

2. **Complete the table:**

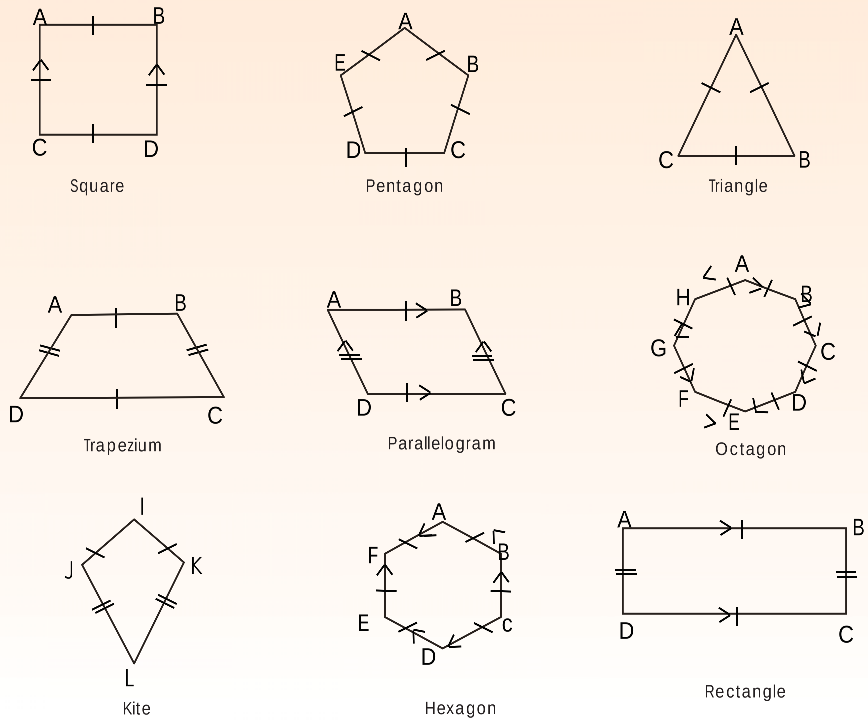


Geometric figure	What fraction of the square is it?	Name the shape	Is the shape regular or irregular
a. AOD			
b. ADB			
c. OGFI			

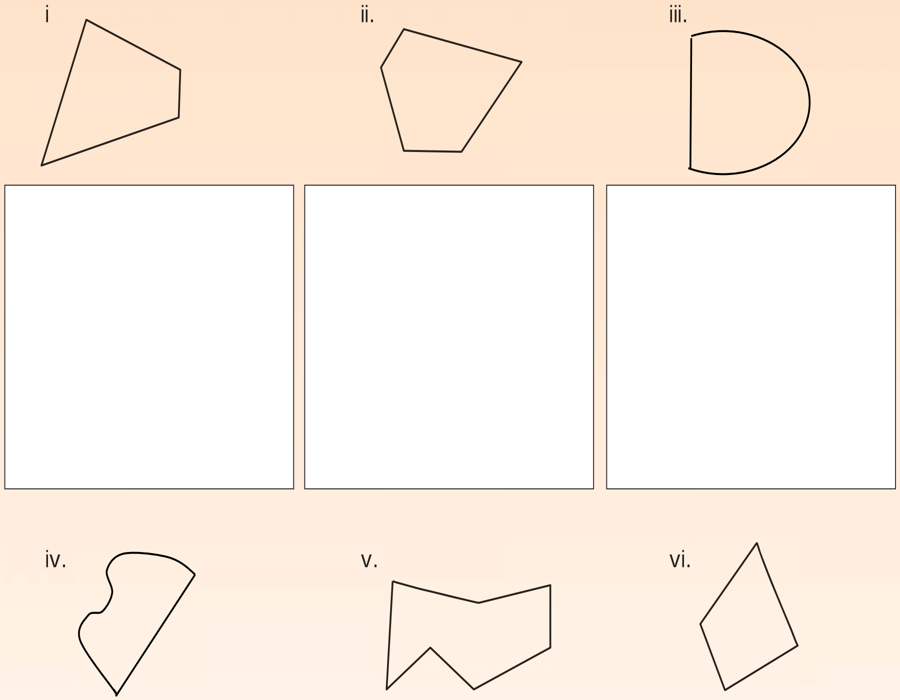
continued ☛

62b Polygons continued

3. Look at the shapes below. What are the differences and similarities between the polygons?



4. Identify whether the following shapes are polygons and whether they are regular or irregular. Give reasons for your answer.



Problem solving

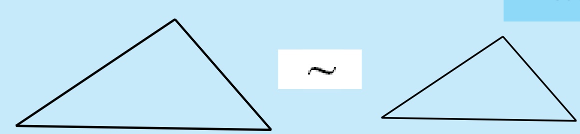
Create any other polygon using all seven tangram pieces. Draw and describe it.

63 Similar triangles

What is similarity?
Similar triangles have the following properties:

- Each corresponding pair of angles is equal.
- The ratio of any pair of corresponding sides is the same.
- They have the same shape but not the same size.

These triangles are similar:



We can tell whether two triangles are similar without testing all the sides and all the angles of the two triangles.

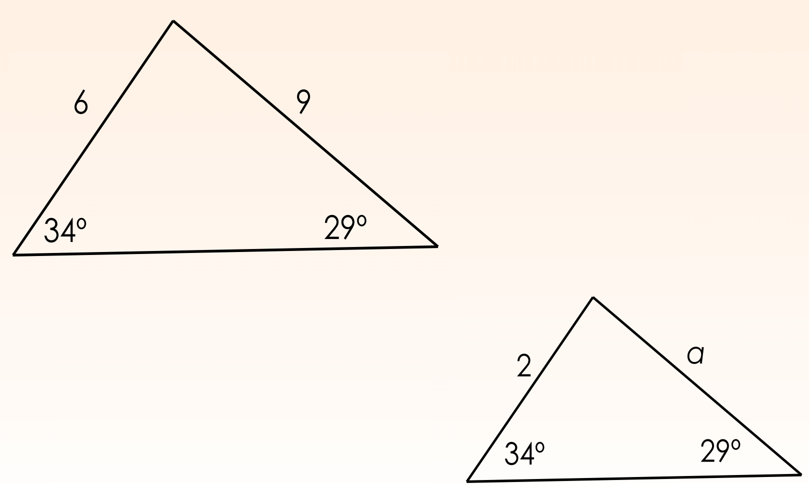
There are two rules to check for similar triangles. They are called the AA rule and RAR rule.

As long as one of the rules is true, it is sufficient to prove that the two triangles are similar

1. Discuss these rules.

AA rule
If two angles of one triangle are equal to two angles of another triangle, then the triangles are similar.

a. Given the following triangles, find the length of ***a***



Solution:

Step 1: The triangles are similar because of the _____ rule.

A ratio shows the relative sizes of two or more values.Example: 2:3

Step 2: The ratios of the lengths are equal. $\frac{6}{2} = \frac{9}{a}$

Step 3: Make use of cross multiplication to simplify.

Here is an example of cross multiplication:

8/4 = 12/x

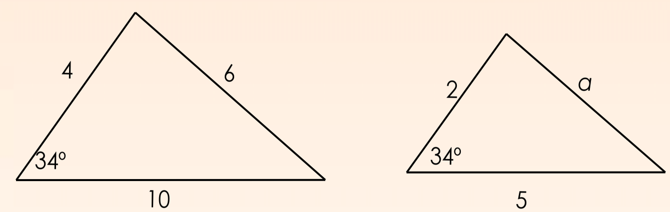
2/1=12/x

2x=12

x=6

RAR rule
If the angle of one triangle is the same as the angle of another triangle and the sides containing these angles are in the same ratio, then the triangles are similar.

b. Given the following triangles, find the length of a.



Solution:

Step 1: The triangles are similar because of the ________ rule.

Step 2: The ratios of the lengths are equal.

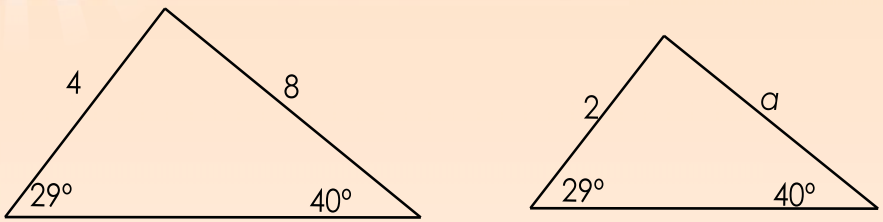
Step 3: The length of a is

continued ☛

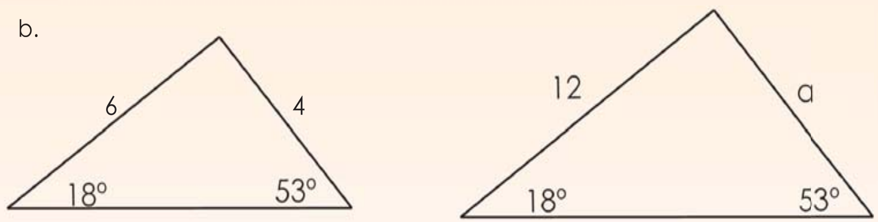
63b Similar triangles continued

2. Find the length of a. State the rule you are using.

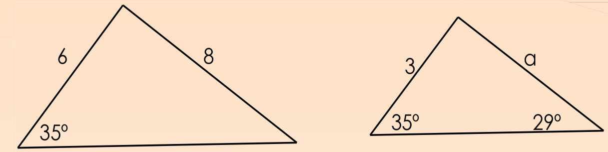
a.



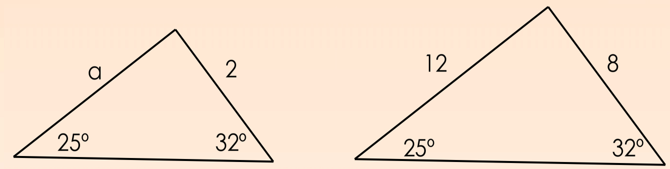
b.



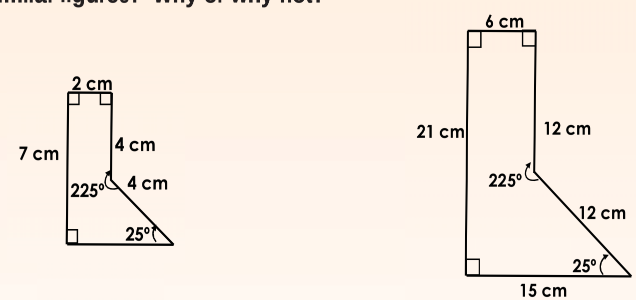
c.



d.



3. Are these similar figures? Why or why not?

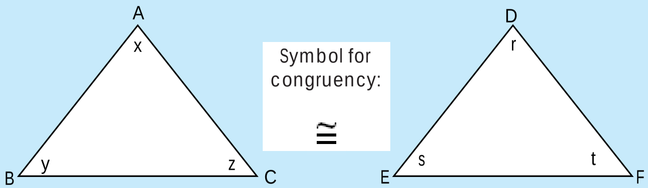


Problem solving

Find two figures in everyday life that are similar. Construct it.

64 Congruent triangles

Congruent triangles are triangles that have the same size and shape. This means that the corresponding sides are equal and the corresponding angles are equal.



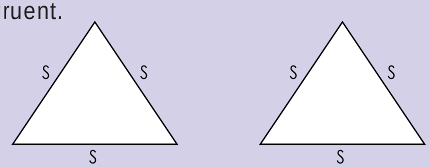
Symbol for congruency:

≅

- The corresponding sides are: AB and DE, AC and DF and BC and EF
- The corresponding angles are: x and r, y and s, z and t.
- There are five rules to check for congruent triangles.
- These are the rules: **SSS, SAS, ASA, AAS** and **RHS**.

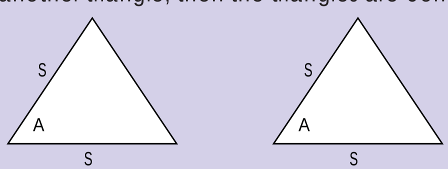
1. Discuss the following and draw examples:

SSS rule (Side – Side – Side)
If three sides of one triangle are equal to three sides of another triangle then the triangles are congruent.



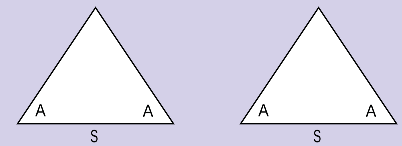
a. Draw congruent triangles using the SSS rule. Indicate the length of the sides of the triangles.

SAS rule (Side – Angle – Side)
If two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle, then the triangles are congruent.



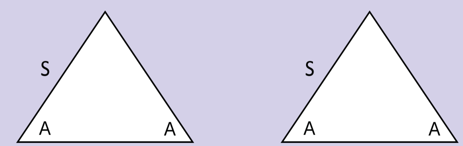
b. Draw congruent triangles using the SAS rule. Indicate the length of the sides of the triangles.

ASA rule (Angle – Side – Angle)
If two angles and the included side of one triangle are equal to two angles and the included side of another triangle, then the triangles are congruent.



c. Draw congruent triangles using the ASA rule. Indicate the length of the sides of the triangles.

AAS rule (Angle – Angle – Side)
If two angles and a non-included side of one triangle are equal to two angles and a non-included side of another triangle, then the triangles are congruent.

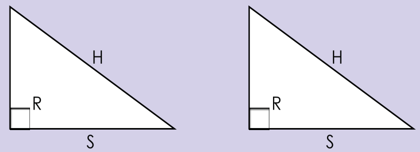


d. Draw congruent triangles using the AAS rule. Indicate the length of the sides of the triangles.

continued

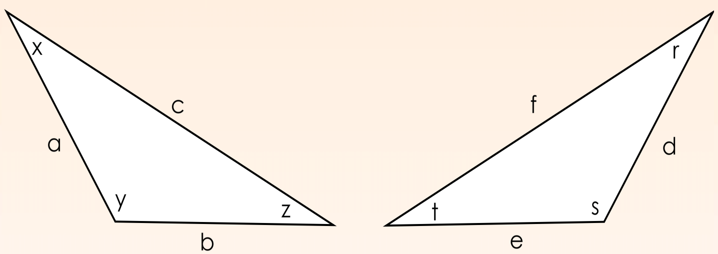
64b Congruent triangles continued

RHS rule (Right angle – Hypotenuse – Side)
If two angles and the included side of one triangle are equal to two angles and the included side of another triangle, then the triangles are congruent.



e. Draw congruent triangles using the RHS rule. Indicate the length of the sides of the triangles if the hypotenuse if the two other sides are 3 cm and 4 cm long.

2. Which of the following conditions would be sufficient for the above triangles to be congruent? Give an explanation for each.



a. a=d, x=r, b=e

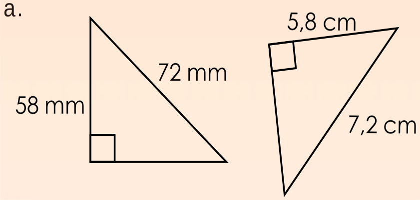
b. a=d, y=s, z=t

c. c=f, y=t, b=e

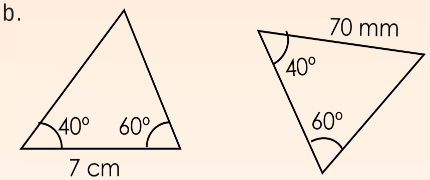
d. a=e, y=t, z=s

3. State whether the following pairs of triangles are congruent.
If they are, give a reason for your answer using the SSS, ASA, SAS, SAA or RHS rules.

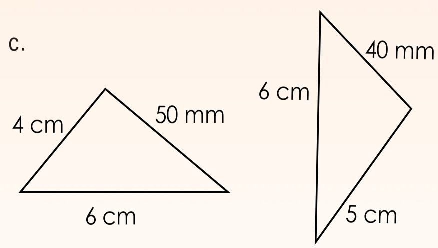
a.



b.



c.



Problem solving

Find any congruent shapes in a nature and make a drawing of them.

65 Lines and angles

Look at these pictures and identify all the lines and angles.

What impact does perspective have on the 2nd and the 3rd picture?

Words that may help you:
- Line
- Line segment
- Ray
- Perpendicular lines
- Parallel lines
- Angle
- Acute angle
- Right angle
- Obtuse angle
- Straight line
- Reflex angle

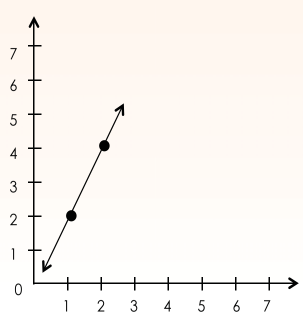
1. Label these symbols that you use when you work with angles and lines.

______ Δ	______ $\angle$	______ $\perp$	______ $\parallel$	______ $^{\circ}$	______ ∟
______ $\overline{AB}$	______ $\overleftrightarrow{AB}$	______ $\overrightarrow{AB}$	______ $\cong$	______ $\sim$	______ $\therefore$

a. Say why you will use these:

b. Say why you will not use these:

2. What helped us to draw this line?



Draw the following lines.

Give any other point on this line.

a. (1,1) and (3,3)

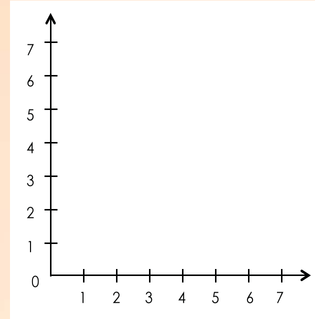
b. (2,7) and (5,5)

c. (6,5) and (7,6)

d. (4,1) and (7,3)

e. (1,4) and (3,4)

3. Use the graph to answer the questions.



a. Why is line x = 4 is a vertical line?

b. Show it on the graph.

c. Why is line y = 3 is a horizontal line?

d. Show it on the graph.

e. Where will these two lines be perpendicular to each other? _____

f. Draw a line parallel to the line in a. (x = 4) and then one parallel to the line in c. (y = 3). Describe it.

4. What is another name for a 180° angle?

5. Give a description of each of the following words: acute, obtuse, right and reflex. Where in everyday life do we find these angles. Which one is most commonly used?

Problem solving

Be creative and write a paragraph on what the world would be without lines and angles.

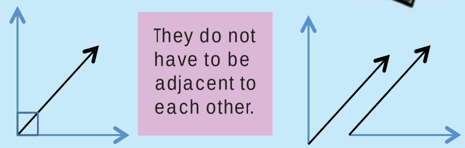
66 Complementary and supplementary angels

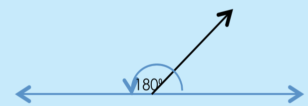
Give an example of each using variables and then substitute it with a number.

Two angles are complementary if they add up to 90 degrees.

They do not have to be adjacent to each other.

Two angles are supplementary if they add up to 180 degrees.





1. Use the following 1. Draw the following angles and say if they are complementary or supplementary angles. Determine the size of the second angle.

a. $\angle 1 + 30^\circ = 90^\circ$ b. $48^\circ + \angle 2 = 180^\circ$ c. $\angle 1 + \angle 2 = 90^\circ$

d. $\angle 1 + 100^\circ = 190^\circ$ e. $36^\circ + \angle 2 = 90^\circ$ f. $\angle 1 + \angle 2 = 180^\circ$

2. Look at this picture of girders and identify and label the complementary and supplementary angles.

2. Draw five different complementary angles and label them.

3. Draw five different supplementary angles and label them.

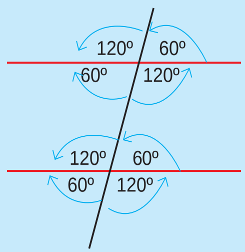
4. Find any complementary and supplementary angles in your everyday environment. Draw and label them.

Problem solving

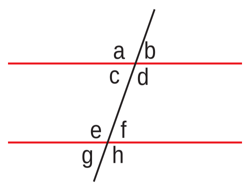
Can two obtuse angles be complementary? Can they be supplementary?

67 Transversals

Transversals are straight lines that cut across other (usually parallel) straight lines. Why are many angles the same in this drawing of a transversal crossing two parallel lines?

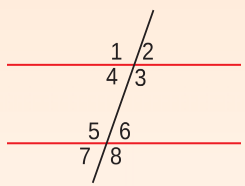


These angles can be made into pairs of angles which have special names.



Parallel lines
Transversal
Vertical angles:
a = d; b = c; e = h; f = g
Corresponding angles:
a = e; b = f; c = g; d = h
Alternate interior angles
c = f; d = e
Alternate exterior angles
a = h; b = g
Consecutive interior angles
c + e = 180ºd + f = 180º

1. Measure each angle.



∠1 = ___ ∠2 = ___ ∠3 = ___
∠4 = ___ ∠5 = ___ ∠6 = ___
∠7 = ___ ∠8 = ___
∠1+ ∠2 = _____ and are called _________ angles.
That will be the same for ____________________
____.

a. Find all the vertically opposite angles. Write them down.

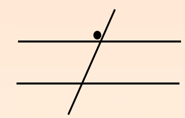
b. Find all the corresponding angles. Write them down.

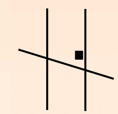
c. Find all the alternate angles. Write them down.

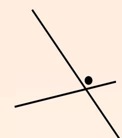
d. Find the co-interior angles. Write them down.

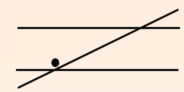
e. Why are angles 2 and 7 equal?

2. Identify and mark the vertically opposite angle.

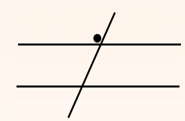
a.


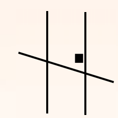
b.


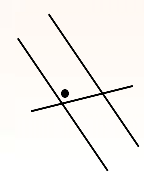
c.


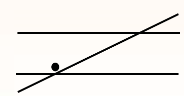
d.


3. Identify the corresponding angle.

a.


b.


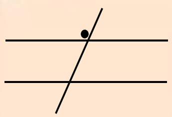
c.


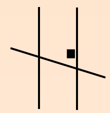
d.


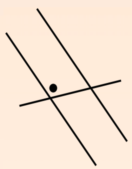
continued ☛

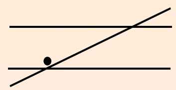
67b Transversals continued

4. Identify and mark the alternate angle.

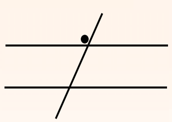
a. 

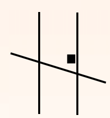
b. 

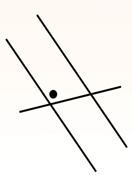
c. 

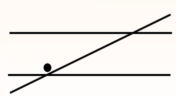
d. 

5. Identify all the angles that will be equal to the one marked.

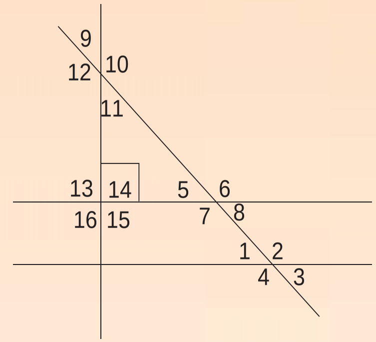
a. 

b. 

c. 

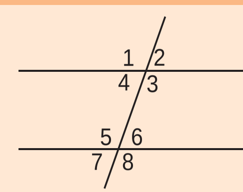
d. 

6. How would you work out each angle, if angle 1 was given?



Problem solving

If $\angle 1 = 105^\circ$ and $\angle 2 = 75^\circ$, what could the sizes of $\angle$s 3 to 8 be?



68 Pairs of angles

Look at this photograph and discuss it.

1. Use the knowledge learnt in previous worksheets to work out angles BCD, CDB, DBC, ABD, BDE, ABD and BAE. You can work out the angles in any order you like. Triangle BCD is an equiangular triangle. Angle AEB is a right angle.

2. Make a similar 'roller coaster problem'. Try to use all the concepts that you have learnt so far. Construct and draw or paste your picture here.

Concepts to be used when creating your problems:

- Parallel lines
- Transversal
- Vertical angles
- Corresponding angles
- Alternate exterior angles
- Consecutive interior angles

Problem solving

Solve your own created problem (Question 2) with a family member.

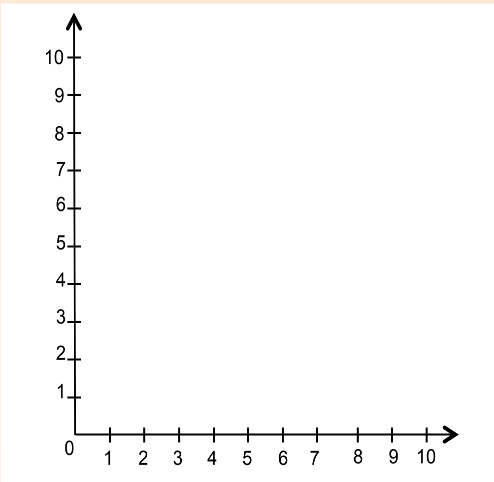
69 Application of geometric figures and lines

To be able to answer this worksheet you need to know the following concepts. Revise your knowledge of them by writing a definition for each.

Congruency	Translation
A line	Rotation
To plot	Reflection

Here you need to think back what you did in grade 8.

1. Complete the following



a. Plot (2,5), (6,5), (2,9) and (6,9) on the grid.

b. What geometric figure does it form?

c. Label its vertices.

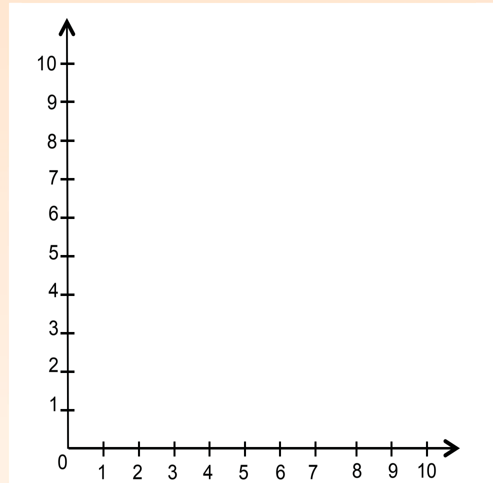
d. Draw a line from (1,10) to (7,4).

e. What is your geometric figure now divided into?

f. What are the sizes of the angles?

g. Are the two figures congruent to each other and why?

2. Complete the following



a. Plot (1,9), (9,9) and (5,5). Join them up. What geometric figure does it form?

b. Plot (1,1). Can you form another geometric figure that is congruent to the shape in Question a, using the existing points?

c. Plot (5,1) and (3,3). Use those points to draw a figure similar to the ones in Questions 2a and 2b.

69b Application of geometric figures and lines continued

e. Plot (7,7). Draw a line from (7,7) to (7,3). What geometric figure does this form? Is this geometric figure congruent to any other shape?

f. Plot (9,5). Draw a line from (7,3) to (9,5). How would you create a parallelogram?

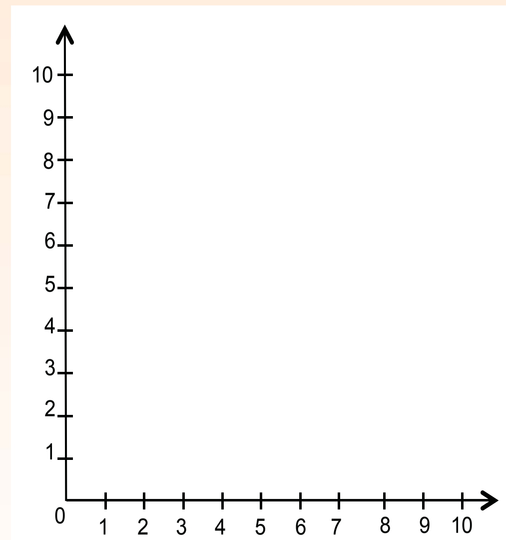
3. In nature and art we often find congruent geometric figures. Identify such shapes in the photographs.

a. Write down in your own words what translation mean.

b. Look at the photo of the butterfly. Identify the congruent shapes. What do you notice about the shapes if you compare them to the shapes on the snake?

c. Look at what the jewellery designer made. Identify all the congruent shapes. What type of transformations were made?

4. Draw congruent figures on this graph. Use the colours indicated for each figure: Translation (black), reflection (blue) and rotation (red).

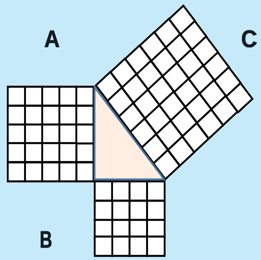


Problem solving

Discuss Question 3 with a family member.

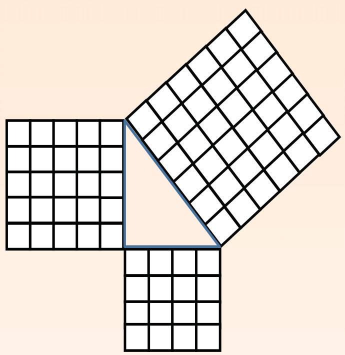
70 Pythagorean theorem

Revise the Pythagoras's theorem. About 2 500 years ago, a man named Pythagoras discovered an amazing fact about triangles. Can you still remember it?



What is the size of block A? (5^2)
What is the size of block B? (4^2)
What is the size of block C? (6^2)
What do you notice?

1. Write an equation for the following and solve it.

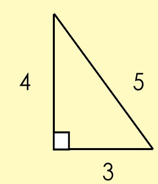


2. Here are the length of the sides of some right angled triangles.
Make drawings to show that the area of the square drawn on the longest side of each right-angled triangle is equal to the total area of the squares drawn on the other two sides. This will require some clever thinking. You may need extra paper.

	Side	Side	Side	
a.	6	8	10	
b.	15	25	20	
c.	45	36	27	
d.	20	12	16	
e.	9	15	12	

3. Write an equation for the following and calculate each side:

Example

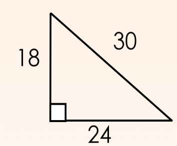


$4^2 + 3^2 = 5^2$

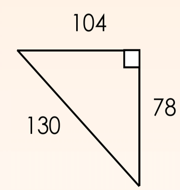
$16 + 9 = 25$

$25 = 25$

a.



b.

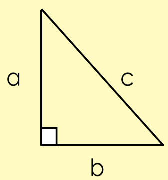


continued

70b Pythagorean theorem continued

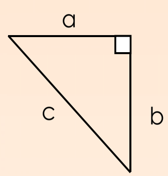
4. Write an equation for each of the following:

Example

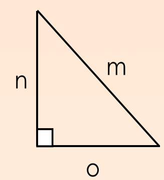


$a^2 + b^2 = c^2$

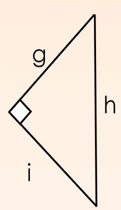
a.



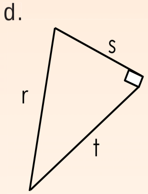
b.



c.

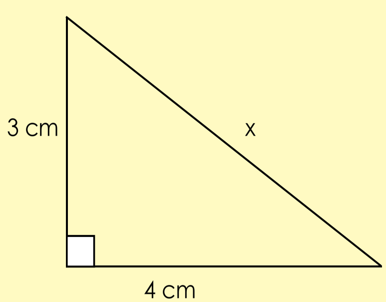


d.



5. Find the lengths of the unknown sides in the following right-angled triangles. You may use a calculator.

Example



$x^2 = (3\ cm)^2 + (4\ cm)^2$

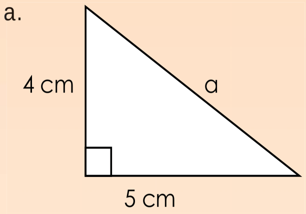
$x^2 = 9\ cm^2 + 16\ cm^2$

$x^2 = 25cm^2$

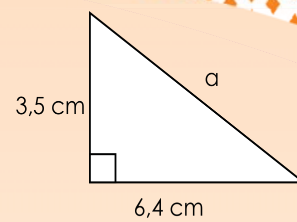
$x^2 = \sqrt{25cm^2}$

$x = 5\ cm$

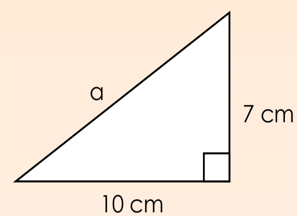
a.



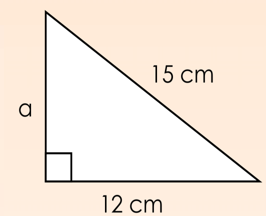
b.



c.



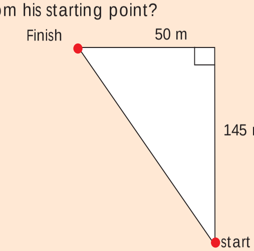
d.



Problem solving

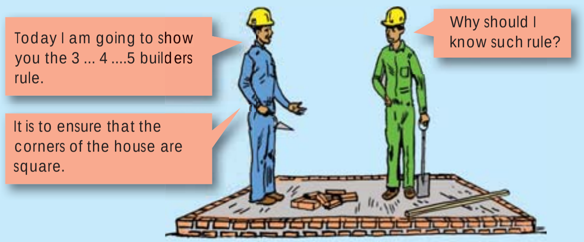
a. Give two example of where we can use Pythagoras in everyday life.

b. Themba walks as shown in the diagram. He finishes 145 m north and 50 m west from his starting point. How far is Themba from his starting point?



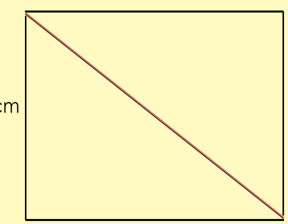
71 More on the theorem of Pythagoras

Read the conversation between these two builders.



1. Find the lengths of the of the diagonal of the rectangle.

Example



$x^2 = (3\ cm)^2 + (4\ cm)^2$

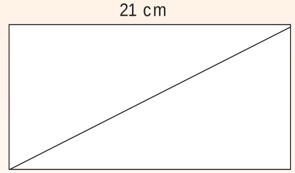
$x^2 = 9\ cm^2 + 16\ cm^2$

$x^2 = 25 cm^2$

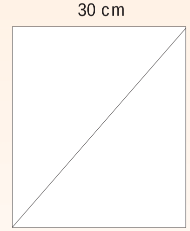
$x^2 = \sqrt{25 cm^2}$

$x = 5\ cm$

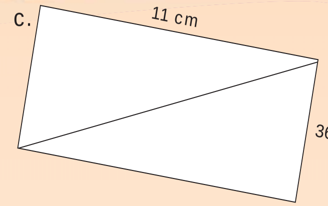
a.



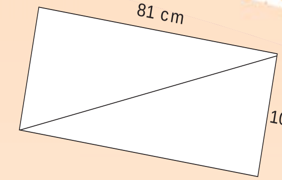
b.



c.

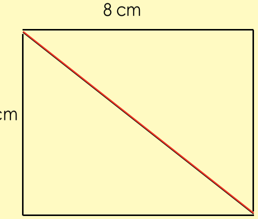


d.



2. Find the length of the diagonal of the rectangle.

Example



$x^2 = (5\ cm)^2 + (8\ cm)^2$

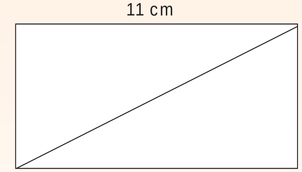
$x^2 = 25\ cm^2 + 64\ cm^2$

$x^2 = 89\ cm^2$

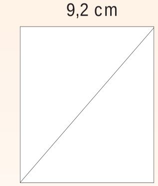
$x^2 = \sqrt{89 cm^2}$

$x = 9{,}40\ cm$

a.



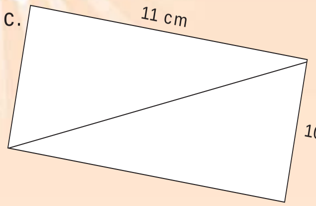
b.



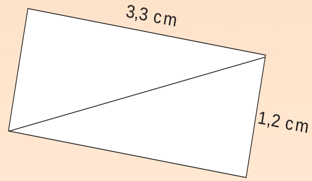
continued ☛

71b More on the theorem of Pythagoras continued

c.

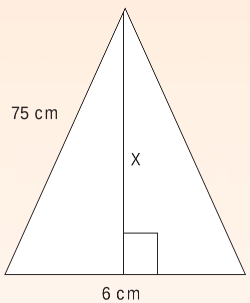


d.

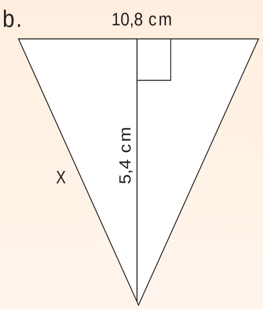


3. Find the unknown side.

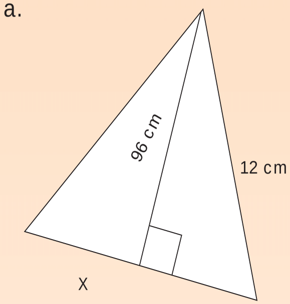
a.



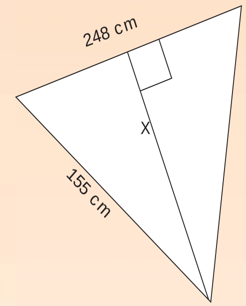
b.



a.

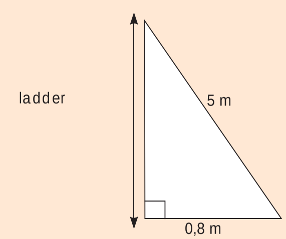


b.



Problem solving

a. Lindiwe has put her ladder against the wall. How far up the wall does the ladder reach?



b. A triangular area is being tiled. The sides of the area are 8 cm, 12 cm and 18 cm. Is this a right-angled triangle? Explain your answer.

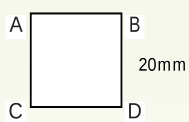
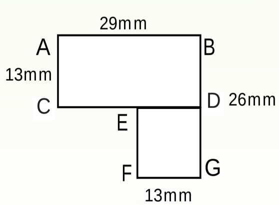
72 Perimeter of a square and rectangle, area of a square and rectangle

What do these formulae mean? Link it with the words on the right.

P = 4S	perimeter of square
P = 2(l + w) or P = 2l + 2w	perimeter of rectangle
A = l^2	area of square
A = l x w	area of rectangle

W = Width = Breadth = B

1. Complete the table. Give your answers in mm and cm.

Figure	What formula will you use to calculate the:	
	Perimeter	**Area**
	Formula: P = mm = cm	Formula: A = mm^2 = cm^2
	Formula: P = mm = cm	Formula: A = mm^2 = cm^2
	Formula: P = mm = cm	Formula: A = mm^2 = cm^2

2. Construct and calculate the area and the perimeter of the following:

a. Rectangle ABCD where AB = 2,4 cm and BD = 1,6 cm.

b. Square ABCD where AB = 3,9 cm.

c. Rectangle ABCD and square BEFD, where the rectangle and square share the same side BD. EF = 2,7 and AB = 4,1.

Problem solving

If the perimeter of a square is 24 cm, what is the length of each side?

The perimeter of a rectangular plot of land is 29,5 m. If the length is increased by 2 m and the breadth is reduced by 1 m, the area of the plot remains unchanged. Show this.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

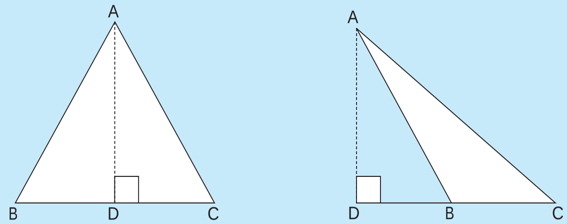
73 Area of a triangle

Revise the formulas :

A = ½ (b x h)

Area of a triangle = ½ (base x perpendicular height

Every triangle has three bases (or sides), each with a related height or altitude. This height of a triangle is a line segment drawn from any vertex perpendicular to the opposite side.



Note: AD is the perpendicular height on BC of ΔABC

1. What is the formula for calculating the area of a triangle?

2. Construct and then draw the following triangles and calculate the area by measuring the base and the perpendicular height of each triangle.

a. An isosceles triangle.

b. A right-angled triangle.

c. A scalene triangle.

3. What is the area of a triangle that has a:

a. Base of 4 cm and a height of 2,3 cm?

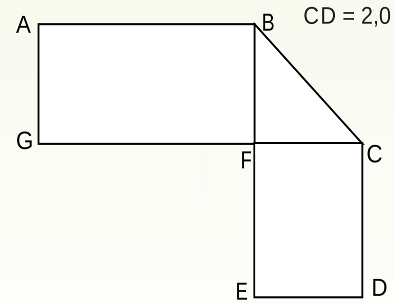
b. Base of 2,8 cm and a height of 3,6 cm?

c. Base of 34 mm and a height of 4,2 cm?

4. What is the length of the base of a triangle that has an area of 40 cm^2 and a height of 4 cm?

5. Calculate the area:

AB = 3,0 cm
AG = 1,5 cm
AG = ED
CD = 2,0 cm



Problem solving

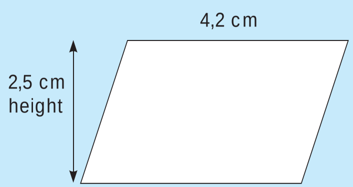
If the area of a triangle is 5,635 cm2 what could the height be?

74 Area of parallelograms and trapeziums

Revise:
A = l x w
A = ½ (b x h)

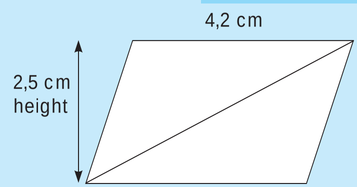
Area of a rectangle
Area of a triangle

To find the area of a parallelogram, we can use a similar formula to that used for the area of a rectangle, multiplying the length of the base (length) by the perpendicular height.



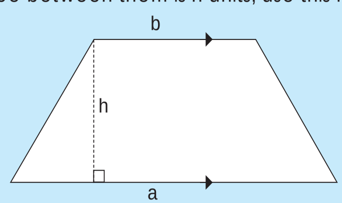
A = b x h
= 4,12 cm x 2,5 cm
= 10,3 cm²

The parallelogram can be divided into triangles.



To find the area of a trapezium of which the length of the parallel sides are a units and b units, and the perpendicular distance between them is h units, use this formula:

$A = \frac{1}{2}(a+b)h$



1. What is the formula for calculating the:

a. Area of a parallelogram.

b. Area of a trapezium.

2. Find the area of a trapezium of which the parallel sides are 10,5 cm and 8,2 cm, and the perpendicular distance between the sides is 4 cm.

3. Find the area of a parallelogram with base 6,4 cm and height 3,8 cm.

Problem solving

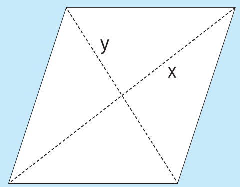
If the area of the trapezium is 39 cm², what could the height be?

75 Area of a rhombus and a kite

Area of a rhombus

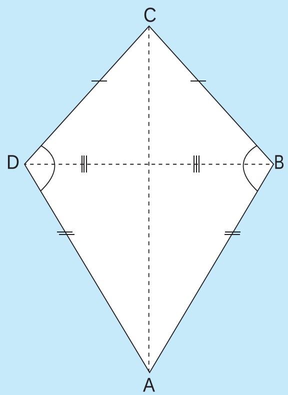
A rhombus is a special kind of parallelogram and its area can be found with the same formula (A = b x h) or with this formula where the area is half the two diagonals multiplied together.

$A = \frac{1}{2}xy$



Area of a kite

A kite has two pairs of adjacent sides that are equal and one pair of opposite angles that are equal. Diagonals intersect at right angles. One diagonal is bisected by the other. Consider the area of the following kite.



Diagonal AC = x
Diagonal BD = y

$A = \frac{1}{2}xy$

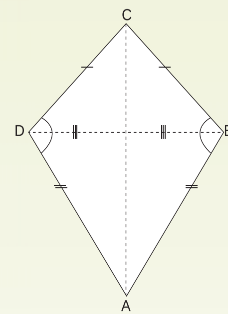
1. What is the formula for calculating the:

a. Area of a rhombus

b. Area of a kite

2. Find the area of a rhombus with diagonals measuring 12,5 cm and 18,5 cm.

3. Find the area of this kite:



Diagonal AC = 22,5 cm
Diagonal BD = 16,5 cm

a. Using the formula.

b. Using the formula of a triangle.

Problem solving

If the area of the kite is 112 cm^2, what could the diagonals be?

76 Area of a circle

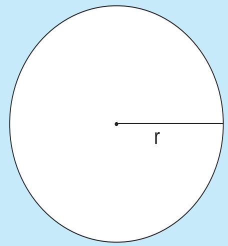
Revise the formulae for all polygons learnt so far. Is a circle a polygon or not? Why?

Area of a circle

The area of a circle is given by a formula:

Area = πr^2 where $\pi = \frac{22}{7}$ and r is the radius.

Note: the value of π is a decimal that goes on for ever but we usually take it to 3 decimal places: 3,142



1. What is the formula for calculating the area of a circle? Test the formula.

2. Construct and label the following circles with these diameters.

a. 14 cm

b. 10,4 cm

b. 78 cm

Problem solving

If the area of the circle is 154 cm^2, what will the radius be?

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

77 Finances – Budget, Loans and Interest

Can you still remember what a **budget** is?

Budget is the estimate of cost and revenues over a specified period.

What are **loans** and **interest**?

A **loan** is sum of money that an individual or a company lends to an individual or company with the objective of gaining profits when the money is paid back.

Interest is the fee charged by a lender to a borrower for the use of borrowed money. The rate of interest is usually expressed as an annual percentage of the amount borrowed (the principal amount).

Do you know what the difference is between **simple** and **compound** interest?

Interest can be calculated in two ways:

- **Simple Interest**
  The formula for simple interest is:

$$\frac{\text{Principal amount} \times \text{rate of interest (\%)} \times \text{number of periods}}{100}$$

- **Compound Interest**
  Compound interest means that the interest will include interest calculated on interest.

- The formula for calculating the Total future amount owed is:

$$\text{Principal amount} \times \left(1 + \frac{\text{Rate of interest (\%)}}{100}\right)^{\text{Number of periods}}$$

Example of compound interest

- An amount of R100 is invested for two years with 10 % compounded (added) yearly.
- The interest at the end of the first year would be: R100 x 10 % = R10
- In the second year the interest rate of 10 % would apply not only to the R100, but also to the R10 interest of the first year.
- In the second year the interest would be: R110 x 10 % = R11
- Total interest earned over the two years will be: R10 (year 1) + R11 (year 2) = R21
- Total investment after two years: R100 (principal amount) + R21 (interest) = R121
- Using the formula: Total future amount $= R100\,(1 + 0{,}10)^2$
  $= R100\,(1{,}1)^2$
  $= R100\,(1{,}21)$
  $= R121$

1. Palesa needs to earn R500 in interest so she will have enough to buy a used bicycle. She puts R2 000 into an account that earns 5 % per year simple interest. How long will she need to leave her money in the account to have enough money for the bicycle?

2. Thabo has R500 that he invests in an account that pays 8 % interest compounded yearly. How much money does Thabo have at the end of 3 years?

3. Susan has R1 000 that she invests in an account that pays 7.5 % interest compounded yearly. How much money does Susan have at the end of 5 years?

4. You saved R4 750 during the last year. You decide that it will be the best to invest the money. At your local bank they have two investment options:Option 1: A 5 Year fixed deposit with 3,25 % simple interest per year.Option 2: A 5 Year fixed deposit with 3,10 % compound interest per year.Which 5 year investment will be the best?

Problem solving

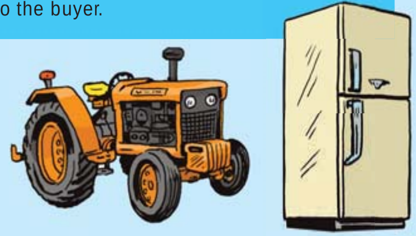
Suppose that you want to have R100 000 in thirty-six months' time when plan to enrol at a university. You want to invest in a plan yielding 3.5% interest per year, compounded monthly. How much should you invest?

78 Finances – Hire Purchase

Can you still remember the meaning of **hire purchase**?

Hire purchase is a system by which a buyer pays for an asset in regular installments, while enjoying the use of it.

During the repayment period, ownership of the item does not pass to the buyer. Upon the full payment of the loan, the title passes to the buyer.



Many organisations enter into hire purchase or leasing agreements to pay for and use equipment over a period of time rather than paythe full cost up front.

The repayment period is normally the same as the production life of the machine. For example: a farmer buys a tractor and pays it off over 5 years. After 5 years he typically has to replace the tractor.

1. The hire purchase price of a refrigerator is R6 500. The deposit of R500 is made and the remainder is paid in equal monthly payments of R250.

   a. Calculate the number of monthly payments that must be made.

   b. If the cash price is R4 000, express as a percentage of the cash price, the extra cost of buying on hire purchase.

   c. What is the interest rate (simple interest) charged on this transaction?

2. A new TV costs R6 900 cash. It is available on hire purchase with a deposit of 15% followed by 12 instalments of R558,50.Find the total hire purchase price and the extra amount that you would pay (on top of the cash price) using hire purchase.

3. The cash price of a bike is R220. The hire purchase price is R300. If the deposit is 10% followed by 10 equal monthly instalments, find the amount you will pay each month.

Problem solving

A DVD player costs R240 cash. It is available on hire purchase by paying a deposit of 20% followed by 12 instalments of R18,50. Find the extra amount paid by hire purchase.

If you save R18,50 per month at 12% interest per year compounded monthly. How long must you save to buy the DVD in cash? How much will you save?

Remember interest is compounded monthly. Draw a table to help you.

79 Finances – Exchange rates

Do you know what exchange rate means?

An **exchange rate** is the current market price for which one currency can be exchanged for another.

The **Rand** (sign: R; code: ZAR) is the currency of South Africa.

In modern China, people use **Renminbi** as their money. In Chinese, "Renminbi" means "people's money". A unit of this currency is called the **Yuan**.

The symbol for the Yuan looks like this: ¥ (code: CNY)

The **Canadian Dollar** (sign: $; code: CAD) is the currency of Canada.

Use the exchange rates in the table to help you solve the word problems. Show your work in the space provided.

	ZAR (R)	USD ($)	GBP (£)	CAD ($)	EUR (€)	AUD ($)
ZAR	1,00	6,76	11,06	6,89	9,88	7,17
USD	0,15	1,00	1,60	0,92	1,46	0,87
GBP	0,09	1,09	1,00	0,58	0,91	0,55
CAD	0,15	1,09	1,74	1,00	1,59	0,95
EUR	0,10	0,69	1,10	0,63	1,00	0,60
AUD	0,14	1,15	1,83	1,05	1,67	1,00

1. Suzanne wants to order a new CD from Germany. She has R250 in her savings account. The CD costs €5. Once she has bought the CD, how much money will she have left in ZAR?

   If she can order the same CD from Canada for $7, where must she order it from for the best price provided the shipment cost is the same.

2. Reinette lives in Worcester, South Africa. Her uncle lives in Sydney, Australia. For her birthday, Reinette received $50 from her uncle. How many South African Rands (ZAR) can she buy with her birthday money?

3. Reinette takes the money she received from her uncle and orders a new computer programme from America. After she has bought the programme, she will still have R150 left. How much does the programme cost in US$?

4. Reinette wants to order another programme from England. The programme costs £15. Will Reinette have enough money to buy this programme?

Problem solving

Which currency in the table has the highest valued currency unit?

80 Finances – Commissions and Rentals

**Do you know what commission means?
What are rentals?**

Commission is the fee charged by a broker or an agent for his/her service to facilitate a transaction, such as the buying or selling of goods.

Rental is when an item is leased out for a specific period of time.

Many employees are paid salaries based on the number of hours they have worked over a given period of time plus a commission.

1. Andrew lives in Johannesburg. His parents are planning a vacation to Cape Town. They decide to fly to Cape Town and then rent a car. The car rental company charge R200 per day (including 200 km free) and R1,80 per km. The insurance will be 7,5% of the daily rental amount and the GPS an additional R45 per day.

   What will the total cost be for the vacation if they spent 6 days in Cape Town and travelled 1650 km in total?

   A truck rental agency charges a daily fee plus a kilometre fee. Julie was charged R460 for two days and 100 kilometres and Christina was charged R 1 050 for three days and 400 kilometres. What is the agency's daily fee and what is the kilometre fee?

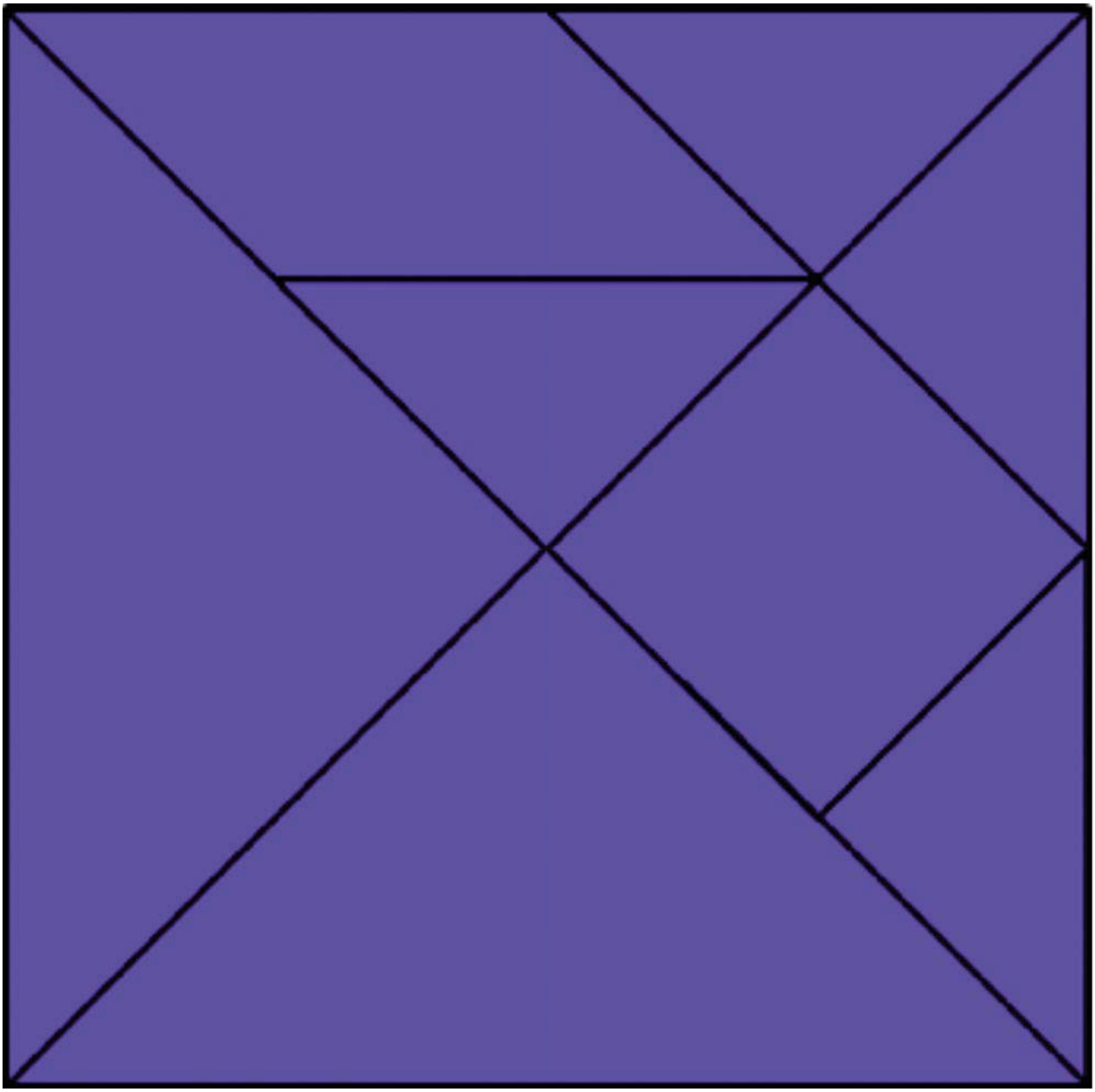
2. Hertz has a processing fee of R115,00 and charges R210 per day for car rental. Avis Car Rental has a processing fee of R255,00 and charges R190 per day for a car. When will the cost of the rentals be equal?

3. Tara is a sales representative for a cosmetic company. She is paid R5,15 per hour each week plus a commission of 10% on the amount of sales over R5 000. She works 40 hours one week, and she sells R7 260 worth of cosmetics during that week. She has been offered a job at another cosmetic company that pays R5,00 per hour for a 40 hour work week plus a commission of 4% on total sales. Which job would pay more? Should she change jobs?

4. Two furniture salesmen are comparing their salaries. Gert is paid R25,00 per hour plus a 15% commission on his total sales. Ben is paid R29,00 per hour plus a 10% commission on his total sales. Suppose each has sold R5 000 worth of furniture, compare their income over various periods of time to find out when they will earn the same. What will happen after that point? Who would have earned more before that point?

Problem solving

A real estate agent received a 6% commission on the selling price of a house. If his commission was R8 650, what was the selling price of the house?



Notes

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

Notes

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