NATIONAL SENIOR CERTIFICATE

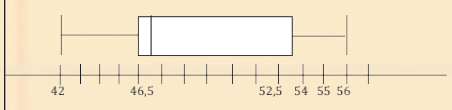
GRADE 12: MATHEMATICS P2

FEBRUARY/MARCH 2011

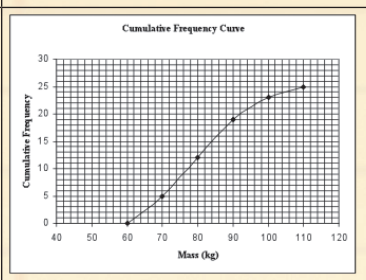
MARKS: 150

This memorandum consists of 5 pages.

QUESTION 1

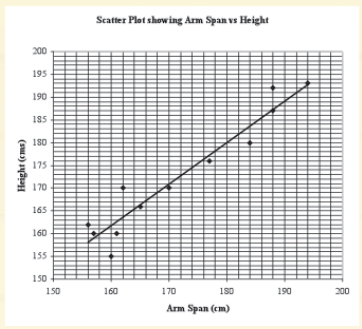
1.1	$\frac{55+55+50+47+42+3x}{8} = 48{,}375$ $\frac{249+3x}{8} = 48{,}375$ $3x = 138$ $3x = 138$ $x = 46$	✓ $\frac{249+3x}{8} = 48{,}375$ $3x = 138$ (2)
1.2		✓ max and min ✓ median ✓ Q_1 and Q_3 ✓ whiskers (4) **[6]**

QUESTION 2

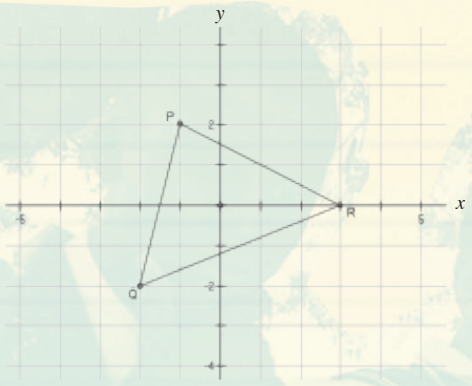
2.1 2.1		✓✓ Frequencies ✓✓ Cumulative Frequencies (4)
2.2		✓✓ plotting points 1 mark: 3 – 5 points correctly 0 marks : 2 or less points correctly plotted ✓ graph (3)
2.3	Mean = 79,28	✓✓ answer (2)
2.4	Standard Deviation = 11,02 79,28 – 11,02 = 68,26 79,28 + 11,02 = 90,3 17 players lie in this interval. $\frac{17}{25} = 68\%$	✓✓✓ sd = 11,02 ✓ 17 players ✓ 68% (5) **[14]**

Mass (kg)	Frequency	Cumulative Frequency
$60 \le x < 70$	5	5
$70 \le x < 80$	7	12
$80 \le x < 90$	7	19
$90 \le x < 100$	4	23
$100 \le x < 110$	2	23

QUESTION 3

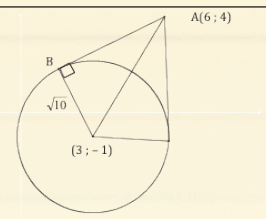
3.1 & 3.2		Question 3.1 4 marks: All points plotted correctly. 3 marks: 9 – 11 points correct 2 marks: 6 – 8 points correct 1 marks: 3 – 5 points correct 0 marks if less than 3 points plotted correctly. (4) Question 3.2 ✓✓ linear best fit Line (2)
3.3	Yes. The relationship between arm span and height is a positive, linear one so we can expect a person with below average arm span to have below average in height.	✓ Yes ✓ Reason (2) **[8]**

QUESTION 4

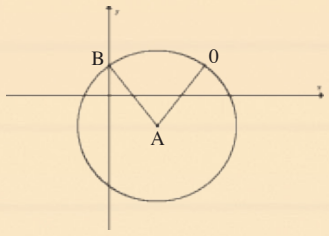


4.1	Let β be the angle of inclination of PQ. $\tan\beta = m_{PQ}$ $\tan\beta = \frac{2-(-2)}{-1-(-2}$ $\tan\beta = 4$ $\beta = 75{,}96°$	✓ $\tan\beta = m_{PQ}$ ✓ $\tan\beta = 4$ ✓ answer (3)
4.2	M $\left(\frac{-1+3}{2}; \right.$ M (1 ; 1)	✓ x-value ✓ y-value (2)
4.3	PQ $\sqrt{(-1+2)^2+(2+2)^2}$ $= \sqrt{17}$ PR = $\sqrt{(-1-3)^2+(2-0)^2}$ $= \sqrt{20}$ QR = $\sqrt{(0-(-2))^2+(3-(-2))^2}$ $= \sqrt{29}$ Perimeter $= \sqrt{29}+\sqrt{20}+\sqrt{17}$ $= 13{,}98$ units = 14 to the nearest whole number	✓ substitution into correct formula ✓ answer ✓ answer ✓ sum ✓ answer (5)
4.4	$y-1 = 4(x-1)$ $y = 4x-3$	✓ m = 4 ✓ substitution of (1 ; 1) ✓ answer (3) **[13]**

QUESTION 5

5.1.1	$x^2+y^2-8x+6y$ $= (2)^2+(-9)^2-8(2)+6(-9)$ $= 4+81-16-54$ $= 15$ Hence, the point lies on the circumference of the circle. **OR** $x^2+y^2-8x+6y = 15$ $(x-4)^2+(y+3)^2 = 15+16+9$ $(x-4)^2+(y+3)^2 = 40$ $(x-4)^2+(y+3)^2$ $= (2-4)^2+(-9+3)^2$ $= 2^2+6^2$ $= 40$ ∴ The point lies on the circumference of the circle.	✓ substitution ✓ answer (2) ✓ substitution ✓ answer (2)
5.1.2	$x^2+y^2-8x+6y = 15$ $(x-4)^2+(y+3)^2 = 15+16+9$ $(x-4)^2+(y+3)^2 = 40$ Circle centre (4; –3) $m_{rad} = \frac{-3-(-9)}{4-2}$ $m_{rad} = 3$ $m_{tan} = -\frac{1}{3}$ $y+9 = -\frac{1}{3}(x-2)$ $y = -\frac{1}{3}x-\frac{25}{3}$	✓✓ $(x-4)^2+(y+3)^2 = 40$ ✓ centre ✓ gradient of radius ✓ gradient of tangent ✓ substitution ✓ answer (7)
5.2	 Radius AB = $\sqrt{10}$ Distance from A to centre of circle is $= \sqrt{(6-3)^2+(4+1)^2}$ $= \sqrt{9+25}$ $= \sqrt{34}$ $AB^2 = 34-10$ $AB^2 = 24$ $AB = \sqrt{24}$ $AB = 2\sqrt{6}$ $AB = 4{,}90$ **OR** $r^2 = 10$ $r = 10$ Radius ⊥ tangent By Pythagoras $AB^2 = (6-3)^2+(4+1)^2-10$ $= 24$ $AB = 4{,}90$	✓ radius = $\sqrt{10}$ ✓ subs into distance formula ✓ $\sqrt{34}$ ✓ $AB^2 = 34-10$ ✓ answer ✓ radius = $\sqrt{10}$ ✓✓ $AB^2 = (6-3)^2+(4+1)^2-10$ ✓ AB = 4,90 (5) **[14]**

QUESTION 6

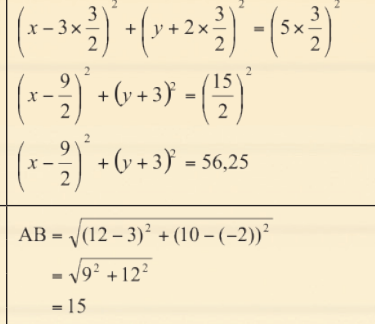


6.1	$9+(y+2)^2=25$ $(y+2)^2=16$ $y+2=\pm4$ y = 2 or y = – 6 B(0 ; 2) **OR** x = 0 $(0)^2-6(0)+y^2+4y=12$ $y^2+4y-12=0$ $(y+6)(y-2)=0$ y = – 6 or y = 2 B(0 ; 2)	✓ x = 0 ✓ factors ✓ answers ✓ answer for B (4) ✓ x = 0 ✓ factors ✓ answers ✓ answer for B (4)
6.2	C(6 ; 2)	✓✓ answer (2)
6.3	$\left(x-3\times\frac{3}{2}\right)^2+\left(y+2\times\frac{3}{2}\right)^2=\left(5\times\frac{3}{2}\right)^2$ $\left(x-\frac{9}{2}\right)^2+(y+3)^2=\left(\frac{15}{2}\right)^2$ $\left(x-\frac{9}{2}\right)^2+(y+3)^2=56{,}25$	✓ each part $\times\frac{3}{2}$ ✓ answer (2)
6.4.1	$AB=\sqrt{(12-3)^2+(10-(-2))^2}$ $=\sqrt{9^2+12^2}$ $=15$	✓ substitution ✓ answer (2)
6.4.2	The radii are 5 and 10. $r_A+r_B=5+10$ $=15$ $=AB$ The circles will only intersect at one point.	✓ addition of radii ✓ answer (2) **[12]**

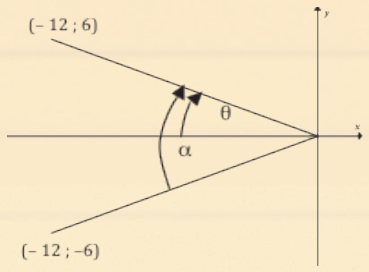
QUESTION 7

	$-3=x\cos150°-2\sin150°$ $-3=-x.\frac{\sqrt{3}}{2}-2.\frac{1}{2}$ $\frac{\sqrt{3}}{2}x=2$ $x=\frac{4}{\sqrt{3}}$ $y=x.\sin150°+2.\cos150°$ $y=\frac{4}{\sqrt{3}}.\frac{1}{2}+2.\left(-\frac{\sqrt{3}}{2}\right)$ $=\frac{2}{3}.\sqrt{3}-\sqrt{3}$ $=-\frac{\sqrt{3}}{3}$	✓ expansion ✓ substitution ✓ answer ✓ expansion ✓ answer **[5]**

QUESTION 8

8.1		✓✓✓ coordinates of new points (3)
8.2.1	$\frac{MN}{M'N'}=\frac{2}{3}$	✓✓ (2)
8.2.2	$\frac{\text{area }\Delta MNP}{\text{area }\Delta M'N'P'}=\frac{4}{9}$	✓✓ (2)
8.2.3	$\frac{\text{area }\Delta MNP}{\text{area }\Delta M''N''P''}=\left(\frac{4}{9}\right)^{n+1}$	✓✓ (2) **[9]**

QUESTION 9

9.1	A′ (– 12; – 6)	✓ answer
9.2	$x'=x\cos\alpha-y\sin\alpha$ $-12\cos\alpha-6\sin\alpha=-12$ $-2\cos\alpha-\sin\alpha=-2$.....(1) $y'=y\cos\alpha+x\sin\alpha$ $6\cos\alpha-12\sin\alpha=-6$ $\cos\alpha=2\sin\alpha-1$ ...(2) Substitute (2) into (1) $-2(2\sin\alpha-1)-\sin\alpha=-2$ $-4\sin\alpha+2-\sin\alpha=-2$ $-5\sin\alpha=-4$ $\sin\alpha=\frac{4}{5}$ $\alpha=53{,}13°$ **OR**	✓ substitution ✓ substitution ✓ simplification ✓ substitution ✓ simplification ✓ answer (6)
	 $\tan\theta=\frac{1}{2}$ $\theta=26{,}565°$ $\alpha=2(26{,}565°)$ $\alpha=53{,}13°$	✓✓ $\tan\theta=\frac{1}{2}$ ✓ $\theta=26{,}565°$ ✓✓ $\alpha=2(26{,}565°)$ ✓ answer (6) **[7]**

QUESTION 10

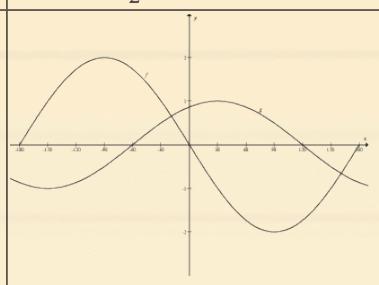
10.1.1	$\cos2 = \sqrt{1-\sin^2 28°}$ $\sqrt{1-\sin^2 28°}$ $= \sqrt{1-a^2}$	✓ ✓ answer (2)
10.1.2	$\cos64°$ $=\cos2(32°)$ $=2\cos^2 32°-1$ $=2b^2-1$	✓ $\cos2(32°)$ ✓ $2\cos^2 32°-1$ ✓ answer (4)
10.1.3	$\sin4°$ $=\sin(32°-28°)$ $=\sin32°\cos28°-\cos32°\sin28°$ $=\sqrt{1-b^2}.\sqrt{1-a^2}-ab$ **OR** $\sin4°$ $=\sin(60°-2\times28°)$ $=\sin60°\cos(2\times28°)-\cos60°\sin(2\times28°)$ $=\frac{\sqrt{3}}{2}\left(1-2a^2\right)-\frac{1}{2}(2a)\sqrt{1-a^2}$ $=\frac{\sqrt{3}}{2}-\sqrt{3}a^2-a\sqrt{1-a^2}$ **OR** $\sin4°$ $=\sin(2\times32°-60°)$ $=\sin(2\times32°)\cos60°-\cos(2\times32°).\sin60°$ $=2.b\sqrt{1-b^2}.\frac{1}{2}-\frac{\sqrt{3}}{2}\left(2b^2-1\right)$ $=b\sqrt{1-b^2}-\sqrt{3}b^2+\frac{\sqrt{3}}{2}$ **OR** Using sin(A + B) + sin(A – B) = 2.sinA.cosB With A = 28° and B = 32° $\sin60°+\sin(-4°)=2ab$ $\sin4°=\frac{\sqrt{3}}{2}-2ab$ **OR** Using sin(A + B) + sin(A – B) = 2.sinA.cosB With A = 32° and B = 28° $\sin60°+\sin(4°)=2\sqrt{1-b^2}.\sqrt{1-a^2}$ $\sin4°=2\sqrt{1-b^2}.\sqrt{1-a^2}-\frac{\sqrt{3}}{2}$ **OR** Using $\sin4°=2\sin2°.\cos2°$ and $\sin2°=\sin(30°-28°)=\frac{1}{2}\left(\sqrt{1-a^2}-\sqrt{3}a\right)$ and $\sin2°=\sin(32°-30°)=\frac{1}{2}\left(\sqrt{3}\sqrt{1-b^2}-b\right)$ and $\cos2°=\cos(30°-28°)=\frac{1}{2}\left(\sqrt{3}\sqrt{1-a^2}+a\right)$ and $\cos2°=\cos(32°-30°)=\frac{1}{2}\left(\sqrt{3}b+\sqrt{1-b^2}\right)$ then $\sin4°=\frac{1}{2}\left\{\sqrt{3}b\sqrt{1-a^2}-3ab+\sqrt{1-a^2}.\sqrt{1-b^2}-\sqrt{3}a\sqrt{1-b^2}\right\}$	✓ $\sin(32°-28°)$ ✓ expansion ✓ answer (4)

	OR $\sin 4° = \frac{1}{2}\left\{3\sqrt{1-b^2}\sqrt{1-a^2} + \sqrt{3}a\sqrt{1-b^2} - \sqrt{3}b\sqrt{1-a^2} - ab\right\}$	
10.2	$b\sqrt{1-a^2} - a\sqrt{1-b^2}$ $= \cos 32°.\sqrt{1-\sin^2 28°} - \sin 28°\sqrt{1-\cos^2 32°}$ $= \cos 32°.\cos 28° - \sin 28°.\sin 32°$ $= \cos(32° + 28°)$ $= \cos 60°$ $= \frac{1}{2}$	✓ substitution ✓ cos 28° ✓ cos 32° ✓ compound angle formula (4)
10.3.1	$\frac{\sin 130°.\tan 60°}{\cos 540°.\tan 230°.\sin 400°}$ $= \frac{\sin 50° \times \tan 60°}{\cos 180° \times \tan 50° \times \sin 40°}$ $= \frac{\sin 50° \times \sqrt{3}}{-1 \times \frac{\sin 50°}{\cos 50°} \times \cos 50°}$ $= -\frac{\sqrt{3}\cos 50°}{\cos 50°}$ $= -\sqrt{3}$	✓ sin50° ✓ tan50° ✓ sin40° ✓ cos50° ✓ $\frac{\sin 50°}{\cos 50°}$ ✓ – 1 ✓ answer (7)
10.3.2	$(1-\sqrt{2}\sin 75°)(1+\sqrt{2}\sin 75°)$ $= 1 - 2\sin^2 75°$ $= \cos 150°$ $= \frac{-\sqrt{3}}{2}$ **OR** $\sin 75°$ $= \sin(45° + 30°)$ $= \sin 45°.\cos 30° + \cos 45°.\sin 30°$ $= \frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}.\frac{1}{2}$ $\sqrt{2}\sin 75° = \frac{\sqrt{3}}{2} + \frac{1}{2} = a$ $(1-\sqrt{2}\sin 75°)(1+\sqrt{2}\sin 75°)$ $= (1-a)(1+a)$ $= 1 - a^2$ $= 1 - \left(\frac{3}{4} + \frac{1}{4} + 2.\frac{\sqrt{3}}{2}.\frac{1}{2}\right)$ $= -\frac{\sqrt{3}}{2}$	✓ simplification ✓ $1 - 2\sin^2 75°$ ✓ cos 150° ✓ answer (4) ✓ simplification ✓ $1 - 2\sin^2 75°$ ✓ cos 150° ✓ answer (4)
10.4	$\sin^2 x + \cos 2x - \cos x = 0$ $\sin^2 x + (\cos^2 x - \sin^2 x) - \cos x = 0$ $\cos^2 x - \cos x = 0$ $\cos x(\cos x - 1) = 0$ $\cos x = 0 \ \ or \ \cos x = 1$ $x = \pm 90° + k.360° \quad or \ x = 0° + k.360° \quad k \in Z$ $= k.360°$ (i.e. $x = 90° + k.180°$ or $x = k.360° \pm 90°, k \in Z$)	✓ $(\cos^2 x - \sin^2 x)$ ✓ $\cos^2 x - \cos x = 0$ ✓ factors ✓ $\cos x = 0 \ \ or \ \cos x = 1$ ✓ $90° + k.360°$ ✓ $k.360°$ ✓ $x = -90° + k.360°$ (7)
10.5.1	x = 0°; 90°; 180°	✓✓✓ each value (3)
10.5.2	$\frac{\cos 2x.\tan x}{\sin^2 x} = \frac{(\cos^2 x - \sin^2 x).\frac{\sin x}{\cos x}}{\sin^2 x}$ $= \frac{\cos^2 x - \sin^2 x}{\cos x.\sin x}$ $= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$ $= \frac{\cos x}{\sin x} - \tan x$	✓ $(\cos^2 x - \sin^2 x)$ ✓ $\frac{\sin x}{\cos x}$ ✓ answer ✓ $\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$ ✓ answer (5) **[39]**

QUESTION 11

11.1	$EC^2 = DE^2 + DC^2 - 2DE.DC\cos\hat{C}$ $= (7,5)^2 + (9,4)^2 - 2.(7,5)(9,4)\cos 32°$ $= 25,03521844...$ EC = 5,0 metres	✓ substitution into cosine rule ✓ 25,03521844... ✓ answer (3)
11.2	$\frac{\sin D\hat{C}E}{7,5} = \frac{\sin 32°}{5,0}$ $\sin D\hat{C}E = \frac{7,5.\sin 32°}{5,0}$ $= 0,7948788963$ $D\hat{C}E = 52,6°$	✓ sin rule ✓ 0,7948788963 ✓ answer (3)
11.3	Area of ΔDEC $= \frac{1}{2}DE.DC\sin\hat{D}$ $= \frac{1}{2}(7,5)(9,4)\sin 32°$ $= 18,7m^2$ **OR** $= \frac{1}{2}CE.DC\sin 52,6°$ $= \frac{1}{2}(5,0)(9,4)\sin 52,6°$ $= 18,7m^2$	✓ substitution ✓ answer (2)
11.4	$\sin 32° = \frac{EG}{7,5}$ $EG = 7,5.\sin 32°$ $= 4,0$ $EF = (4 + 3,5)$ $= 7,5$ metres **OR** $EG = EC.\sin 52,6°$ $= (5,0).\sin 52,6°$ $= 4,0$ $EF = 4,0 + 3,5$ $= 7,5$ **OR** $\frac{1}{2}.DC.EG = \text{area } \Delta DEC$ $\frac{1}{2}(9,4)EG = 18,7$ $\therefore EG = \frac{18,7 \times 2}{9,4}$ $= 4,0$ $EF = 4,0 + 3,5$ $= 7,5$	✓ ratio ✓ substitution ✓ answer (3) **[11]**

QUESTION 12

12.1	Period = 360°	✓ answer (1)
12.2	Amplitude = $\frac{1}{2}$	✓✓ answer
12.3		✓ shape ✓ *x* intercepts ✓ amplitude (3)
12.4	2 solutions	✓ answer (1)
12.5	$-60° \le x \le 120°$ or $x \in [-60°; 120°]$	✓ – 60°; 120° ✓ notation
12.6	$-90° < x < 30°$ or $x \in (-90°; 30°)$	✓✓ – 90°; 30° ✓ notation (3) **[12]**

TOTAL: 150