

NATIONAL SENIOR CERTIFICATE

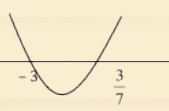
GRADE 12: MATHEMATICS P1

FEBRUARY/MARCH 2011

MARKS: 150

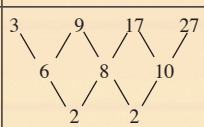
This memorandum consists of 6 pages.

QUESTION 1

1.1.1	$x^2 - x = 12$ $x^2 - x - 12 = 0$ $(x - 4)(x + 3) = 0$ $x = 4$ or $x = -3$ OR $x(x - 1) = 12$ $4(3) = 12$ $x^2 - x = 12$ By inspection $x = 4$ or $x = -3$	✓ standard form ✓ factors ✓ answers ✓ factors ✓ ✓ answers	(3)
1.1.2	$2x^2 + 3x - 7 = 0$ $x = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-7)}}{2(2)}$ $= \frac{-3 \pm \sqrt{65}}{4}$ $x = 1,27$ or $x = -2,77$	✓ substitution into correct formula ✓ 65 ✓ ✓ answers	(4)
1.1.3	$7x^2 + 18x - 9 > 0$ $(7x - 3)(x + 3) > 0$  $x < -3$ or $x > \frac{3}{7}$ OR $x \in (-\infty; -3) \cup \left(\frac{3}{7}; \infty\right)$	✓ factors ✓ $\frac{3}{7}$ and -3 ✓ correct intervals	(4)
1.2	$2x - y = 7$ $y = 2x - 7$ Substitute $y = 2x - 7$ into $x^2 + xy = 21 - y^2$ $x^2 + x(2x - 7) = 21 - (2x - 7)^2$ $x^2 + 2x^2 - 7x = 21 - 4x^2 + 28x - 49$ $7x^2 - 35x + 28 = 0$ $x^2 - 5x + 4 = 0$ $(x - 4)(x - 1) = 0$ $x = 4$ or $x = 1$ $y = 1$ or $y = -5$	✓ $y = 2x - 7$ ✓ substitution ✓ multiplication ✓ standard form ✓ factors ✓ x-answers ✓ y-answers	(7)
	OR $2x - y = 7$ $x = \frac{7 + y}{2}$ Substitute $x = \frac{7 + y}{2}$ into $x^2 + xy = 21 - y^2$ $\left(\frac{7 + y}{2}\right)^2 + \left(\frac{7 + y}{2}\right)y = 21 - y^2$ $\frac{49 + 14y + y^2}{4} + \frac{7y + y^2}{2} = 21 - y^2$ $49 + 14y + y^2 + 2(7y + y^2) = 84 - 4y^2$ $49 + 14y + y^2 + 14y + 2y^2 = 84 - 4y^2$ $7y^2 + 28y - 35 = 0$ $y^2 + 4y - 5 = 0$ $(y + 5)(y - 1) = 0$ $y = -5$ or $y = 1$ $x = 1$ or $x = 4$	✓ $x = \frac{7 + y}{2}$ ✓ substitution ✓ multiplication ✓ standard form ✓ factors ✓ x-answers ✓ y-answers	(7)
1.3	$\left(\sqrt[5]{\sqrt{35} + \sqrt{3}}\right)\left(\sqrt[5]{\sqrt{35} - \sqrt{3}}\right)$ $\sqrt[5]{(\sqrt{35} + \sqrt{3})(\sqrt{35} - \sqrt{3})}$ $= \sqrt[5]{35 - 3}$ $= \sqrt[5]{32}$ $= 2$	✓ $\sqrt[5]{(\sqrt{35} + \sqrt{3})(\sqrt{35} - \sqrt{3})}$ ✓ $\sqrt[5]{35 - 3}$ ✓ answer	(3)

[21]

QUESTION 2

2.1	39	✓ answer	(1)
2.2	 Let $T_n = an^2 + bn + c$ $2a = 2$ $a = 1$ $3a + b = 6$ $3(1) + b = 6$ $b = 3$ $a + b + c = 3$ $1 + 3 + c = 3$ $c = -1$ $T_n = n^2 + 3n - 1$ OR $2a = 2$ $a = 1$ $c = 3 - 4 = -1$ $T_n = n^2 + bn - 1$ $3 = (1)^2 + b(1) - 1$ (using $T_1 = 3$) $b = 3$ $T_n = n^2 + 3n - 1$	✓ formula ✓ $a = 1$ ✓ $b = 3$ ✓ $c = -1$ ✓ $a = 1$ ✓ $c = -1$ ✓ formula ✓ $b = 3$	(4)
2.3	$n^2 + 3n - 1 > 269$ $n^2 + 3n - 270 > 0$ $(n + 18)(n - 15) > 0$ The first value of n is 16 The term is $16^2 + 3(16) - 1 = 303$	✓ $n^2 + 3n - 1 > 269$ ✓ factors ✓ $n = 16$ ✓ answer	(4)

QUESTION 3

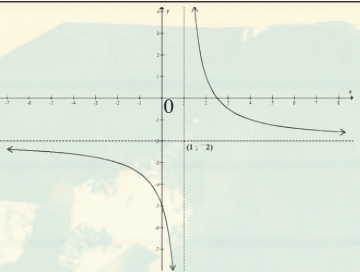
3.1	$S_{\infty} = 8 + \frac{8}{\sqrt{2}} + \dots$ $r = \frac{1}{\sqrt{2}}$ and $S_{\infty} = \frac{a}{1 - r}$ $= \frac{8}{1 - \frac{1}{\sqrt{2}}}$ $= \frac{8\sqrt{2}}{\sqrt{2} - 1}$ $= \frac{8\sqrt{2}(\sqrt{2} + 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)}$ $= 8\sqrt{2}\sqrt{2} + 8\sqrt{2}$ $= 16 + 8\sqrt{2}$ OR $S_{\infty} = 8 + \frac{8}{\sqrt{2}} + \dots$ $r = \frac{1}{\sqrt{2}}$ and $S_{\infty} = \frac{a}{1 - r}$ $= \frac{8}{1 - \frac{1}{\sqrt{2}}}$ $= \frac{8\left(1 + \frac{1}{\sqrt{2}}\right)}{\left(1 - \frac{1}{\sqrt{2}}\right)\left(1 + \frac{1}{\sqrt{2}}\right)}$ $= \frac{8\left(1 + \frac{1}{\sqrt{2}}\right)}{\frac{1}{2}}$ $= 16\left(1 + \frac{1}{\sqrt{2}}\right)$ $= 16 + \frac{16\sqrt{2}}{2}$	✓ $r = \frac{1}{\sqrt{2}}$ ✓ substitution ✓ substitution ✓ rationalisation ✓ simplification ✓ $r = \frac{1}{\sqrt{2}}$ ✓ ✓ substitution ✓ rationalisation ✓ simplification	(4)
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3.2.1	$5 + 15 + 45 + \dots + T_{20}$ $= \sum_{n=1}^{20} 5(3)^{n-1}$ OR $5 + 15 + 45 + \dots + T_{20}$ $= 5 \sum_{n=0}^{19} (3)^n$ OR $5 + 15 + 45 + \dots + T_{20}$ $= 5 \sum_{l=1}^{l+19} (3)^{l-1}$ for any $l \in \mathbb{Z}$	✓ ✓ answer ✓ ✓ answer ✓ ✓ answer	(2) (2) (2)
3.2.2	$5 + 15 + 45 + \dots + T_{20}$ $= \frac{5(3^{20} - 1)}{3 - 1}$ $= 8\,716\,961\,000$	✓ formula ✓ substitution ✓ answer	(3) [9]

QUESTION 4

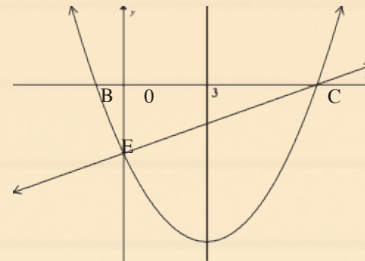
4.1.1	$S_{23} = \frac{23}{2}(5(23) + 9)$ $= 1426$	✓ substitution ✓ answer	(2)
4.1.2	$T_{23} = S_{23} - S_{22}$ $= 1426 - \frac{22}{2}(5(22) + 9)$ $= 1426 - 1309$ $= 117$	✓ statement ✓ $S_{22} = 1309$ ✓ answer	(3) (3)
4.2	Arithmetic Sequence: $12; 12 + d; 12 + 2d$ Geometric Sequence: $12; 12r; 12r^2$ $12 + d = 12r$ $d = 12r - 12$ $12 + 12r + 12r^2 = 12 + 12 + d + 12 + 2d + 3$ $12r^2 = 12 + 2(12r - 12) + 3$ $12r^2 = 12 + 24r - 24 + 3$ $12r^2 - 24r + 9 = 0$ $4r^2 - 8r + 3 = 0$ $(2r - 3)(2r - 1) = 0$ $r = \frac{3}{2}$ or $r = \frac{1}{2}$ OR The 3 rd term of GP = 3 + 3 rd term of AP $12r^2 = 3 + 12 + 2d$ $12r^2 = 15 + 24r - 24$ $12r^2 - 24r + 9 = 0$ $4r^2 - 8r + 3 = 0$ $(2r - 3)(2r - 1) = 0$ $r = \frac{3}{2}$ or $r = \frac{1}{2}$	✓ equation ✓ equation ✓ standard form ✓ factors ✓ answer ✓ equation ✓ equation ✓ standard form ✓ factors ✓ answer	(6) (6) [11]

QUESTION 5

5.1	$x = 1$ $y = -2$	✓ ✓ answers	(2)
5.2	y-intercept: $y = \frac{3}{0-1} - 2 = -5$ x-intercept: $\left(\frac{5}{2}; 0\right)$ $0 = \frac{3}{x-1} - 2$ $2 = \frac{3}{x-1}$ $2x - 2 = 3$ $2x = 5$ $x = \frac{5}{2}$	✓ $y = -5$ ✓ substitute $y = 0$ ✓ answer	(3) (3)
5.3		✓ asymptotes ✓ y-intercept ✓ shape	(3)
5.4	$-f(x) = \frac{-3}{x-1} + 2$ $y \in R - \{2\}$ OR $(-\infty; 2) \cup (2; \infty)$ OR $y \in R; y \neq 2$	✓ answer	(1)

5.5	$g(x) = \frac{-3}{x+1} - 2$ $= \frac{3}{-x-1} - 2$ Reflection of f about the y-axis. OR (i) horizontal shift 2 units to the left followed by (ii) reflection in x-axis, followed by (iii) vertical downward shift of 4 units	✓ manipulation ✓ answer	(2) [11]
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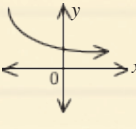
QUESTION 6



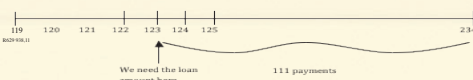
6.1	$\frac{x}{2} - \frac{7}{2} = 0$ $x = 7$ $C(7 ; 0)$ OR $y = \frac{7}{2} - \frac{7}{2}$ $y = 0$ $C(7 ; 0)$	$\frac{x}{2} - \frac{7}{2} = 0$
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	OR Maximum value = $1 - (-8)$ = 9 OR $t(x) = -\frac{1}{2}x^2 + 3x + \frac{9}{2}$ $t'(x) = -x + 3 = 0$ Max V at $x=3 = -\frac{1}{2}(3)^2 + 3(3) + \frac{9}{2} = 9$	
6.6	$f(x^2 - 2) = 0$ $f(x) = 0$ if $x = -1$ or $x = 7$ $\therefore f(x^2 - 2) = 0$ if $x^2 - 2 = -1$ or $x^2 - 2 = 7$ $\therefore x^2 = 1$ or $x^2 = 9$ $\therefore x = 1$ or $x = -1$ or $x = 3$ or $x = -3$ $\frac{1}{2}(x^2 - 2 - 3) - 8 = 0$ $\frac{1}{2}(x^2 - 5) = 8$ $(x^2 - 5) = 16$ $x^2 - 5 = 4$ or $x^2 - 5 = -4$ $x^2 = 9$ or $x^2 = 1$ $x = 3$ or $x = -3$ or $x = 1$ or $x = -1$ OR $f(x^2 - 2) = 0$ $\frac{1}{2}(x^2 - 2 - 3) - 8 = 0$ $\frac{1}{2}(x^2 - 5) = 8$ $(x^2 - 5) - 16 = 0$ $(x^2 - 5 - 4)(x^2 - 5 + 4) = 0$ $(x^2 - 9)(x^2 - 1) = 0$ $(x - 3)(x + 3)(x - 1)(x + 1) = 0$ $x = 3$ or $x = -3$ or $x = 1$ or $x = -1$	✓ substitution ✓ simplification ✓ answer ✓ answer ✓ substitution ✓ simplification ✓ factors ✓ answer ✓ substitution ✓ simplification ✓ factors ✓ answer

QUESTION 7

7.1	Decreasing function Since $0 < a < 1$ OR As x increases, $f(x)$ decreases	✓ decreasing ✓ $a < 1$
7.2	$f^{-1}: x = \left(\frac{1}{3}\right)^y$ OR  OR $f^{-1}: x = \left(\frac{1}{3}\right)^y$ $y = -\log_3 x$	✓ $x = \left(\frac{1}{3}\right)^y$ ✓ $y = \log_{\frac{1}{3}} x$ $y = -\log_3 x$
7.3	$y = -5$	✓ answer
7.4	Reflection about $y = x$. Reflection about the x-axis. OR Reflection about the y-axis. Then reflection about the line $y = x$. OR Reflection about the line $y = -x$ followed by reflection about the y-axis. OR Rotation through 90° in a clockwise direction. OR Rotation through 90° in an anti-clockwise direction. Reflection through the origin.	✓ reflection about $y = x$ ✓ reflection about y-axis ✓ reflection about y-axis ✓ reflection about $y = x$ ✓ rotation through 90° ✓ clockwise direction ✓ answer ✓ answer

QUESTION 8

8.1	$A = P(1+i)^n$ $1711,41 = 1430,77\left(1 + \frac{i}{12}\right)^{18}$ $\left(1 + \frac{i}{12}\right)^{18} = 1,196146131...$ $1 + \frac{i}{12} = 1,009999937...$ $i = 0,1199992431...$ OR $\left[\frac{1711,41}{1430,77}\right]^{\frac{1}{18}}$ = 1,00999... $\therefore i = 12(1,01 - 1)$ = 0,12 = 12% Rate = 12, 00% p.a. compounded monthly.	✓ substitution ✓ $\left(1 + \frac{i}{12}\right)^{18} = 1,196146131...$ ✓ $1 + \frac{i}{12} = 1,009999937...$ ✓ answer
8.2.1	$P_v = \frac{x[1 - (1+i)^{-n}]}{i}$ $800000 = \frac{10000\left[1 - \left(1 + \frac{0,14}{12}\right)^{-n}\right]}{\frac{0,14}{12}}$ $1 - \left(1 + \frac{0,14}{12}\right)^{-n} = \frac{14}{15}$ (= 0,9333333) $\left(1 + \frac{0,14}{12}\right)^{-n} = \frac{1}{15}$ (= 0,066666666) $\log\left(1 + \frac{0,14}{12}\right)^{-n} = \log \frac{1}{15}$ $-n \log\left(1 + \frac{0,14}{12}\right) = \log \frac{1}{15}$ $\left(\begin{array}{l} -n = \frac{\log \frac{1}{15}}{\log\left(1 + \frac{0,14}{12}\right)} \\ = -233,47 \end{array}\right)$ $n = 233,47$ $\therefore$ the loan will be paid off at the end of the 234th month OR Balance outstanding after 233rd month $= 800000\left(1 + \frac{0,14}{12}\right)^{233} - \frac{10000\left[\left(1 + \frac{0,14}{12}\right)^{233} - 1\right]}{\frac{0,14}{12}}$ = R4 660,04 which is less than R10 000 Therefore the loan will be paid off after 234 months.	✓ substitute into Pv ✓ $i = \frac{0,14}{12}$ ✓ using logs ✓ answer
OR	Total value of the loan after 234 payments $\frac{10000\left(1 - \left(1 + \frac{0,14}{12}\right)^{-234}\right)}{\frac{0,14}{12}}$ = R800 350,21 > R800 000 and the differences is less than R10 000 Therefore the loan will be paid off after 234 months.	✓ substitution into P formula ✓ 234 ✓ answer ✓ argument
8.2.2	Balance Outstanding after 119 months $= 800000\left(1 + \frac{0,14}{12}\right)^{119} - \frac{10000\left[\left(1 + \frac{0,14}{12}\right)^{119} - 1\right]}{\frac{0,14}{12}}$ = R629 938,11  Total Payable at the end of the 123rd month $629938,11\left(1 + \frac{0,14}{12}\right)^4$ = R659 853,68 New instalment: $659\,853,68 = \frac{x\left[1 - \left(1 + \frac{0,14}{12}\right)^{-111}\right]}{\frac{0,14}{12}}$ $x = R10\,632,39$	✓ $= 800000\left(1 + \frac{0,14}{12}\right)^{119} - \frac{10000\left[\left(1 + \frac{0,14}{12}\right)^{119} - 1\right]}{\frac{0,14}{12}}$ ✓ R629 938,11 ✓ $629938,11\left(1 + \frac{0,14}{12}\right)^4$ ✓ R659 853,68 ✓ substitution into P_v ✓ answer

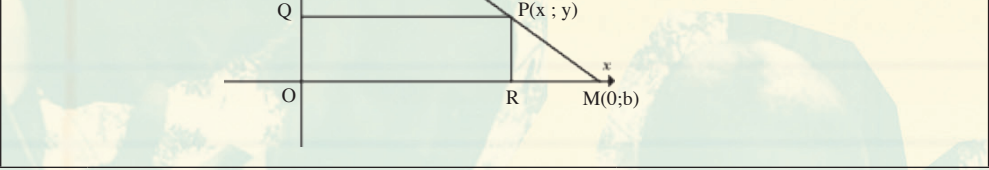
QUESTION 9

9.1	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{1 - 3(x+h)^2 - (1 - 3x^2)}{h}$ $= \lim_{h \rightarrow 0} \frac{1 - 3x^2 - 6xh - 3h^2 - 1 + 3x^2}{h}$ $= \lim_{h \rightarrow 0} \frac{-6xh - 3h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-6x - 3h)}{h}$ $= \lim_{h \rightarrow 0} (-6x - 3h)$ $= -6x$	✓ substitution into formula ✓ $1 - 3x^2 - 6xh - 3h^2$ ✓ $h(-6x - 3h)$ ✓ answer
9.2	$D_x \left[4 - \frac{4}{x^3} - \frac{1}{x^4} \right]$ $= D_x [4 - 4x^{-3} - x^{-4}]$ $= 12x^{-4} + 4x^{-5}$	✓ simplification ✓✓ answer
9.3	$y = (1 + \sqrt{x})^2$ $y = 1 + 2\sqrt{x} + x$ $y = 1 + 2x^{\frac{1}{2}} + x$ $\frac{dy}{dx} = x^{\frac{1}{2}} + 1$	✓ expansion ✓ ✓ 1

QUESTION 10

10.1	$(-6)(-3)(+2) = 36$ y-intercept is 36 OR $g(x) = (x-6)(x^2 - x - 6)$ $g(x) = x^3 - 7x^2 + 36$ y-intercept : (0;36)	$x^{-\frac{1}{2}}$ ✓ $(-6)(-3)(+2)$ ✓ y-intercept is 36 ✓ trinomial ✓ 36
10.2	$g(x) = 0$ $x = 6 \text{ or } x = 3 \text{ or } x = -2$ intercepts are (6 ; 0) and (3 ; 0) and (-2 ; 0)	✓ $g(x) = 0$ ✓ all x-intercepts
10.3	$g(x) = (x-6)(x^2 - x - 6)$ $= x^3 - 7x^2 + 36$ $g'(x) = 3x^2 - 14x$ $0 = x(3x - 14)$ $x = 0 \text{ or } x = \frac{14}{3}$ Turning points are (0 ; 36) and $(\frac{14}{3}; -\frac{400}{27})$	✓ $x^3 - 7x^2 + 36$ ✓ $g'(x) = 3x^2 - 14x$ ✓ $g'(x) = 0$ ✓ answers ✓✓ points
10.4		✓ x-intercepts ✓✓ turning points ✓ shape
10.5	$g(x).g'(x) < 0$ $x < -2 \text{ or } 0 < x < 3 \text{ or } \frac{14}{3} < x < 6$	1 mark for each inequality

QUESTION 11



11.1	$m = -\frac{b}{a}$ $y - b = \frac{-b}{a}(x - 0) \text{ OR } y = \frac{-b}{a}x + b$ $\text{OR } y = -\frac{b}{a}x + b$ $\frac{x}{a} + \frac{y}{b} = 1$	✓ $m = -\frac{b}{a}$ ✓ answer ✓ answer
11.2	$A = xy$ $A = x\left(\frac{-bx}{a} + b\right)$ $= -\frac{b}{a}x^2 + bx$ $\frac{dA}{dx} = -\frac{2b}{a}x + b$ $0 = -\frac{2b}{a}x + b$ $-ba = -2bx$ $x = \frac{a}{2}$ $y = -\frac{b}{a}\left(\frac{a}{2}\right) + b$ $= \frac{b}{2}$ P $\left(\frac{a}{2}; \frac{b}{2}\right)$ which is the midpoint of MN OR $\frac{x}{a} + \frac{y}{b} = 1$ $\frac{y}{b} = 1 - \frac{x}{a}$ To maximise xy, we maximise $\frac{xy}{ab} = \frac{x}{a}\left(\frac{y}{b}\right) = \frac{x}{a}\left(1 - \frac{x}{a}\right)$ This is a maximum when $\frac{x}{a} = \frac{1}{2}$ i.e. $x = \frac{a}{2}$ By the midpoint theorem, P is then the midpoint of MN.	✓ area formula ✓ substitution ✓ $\frac{dA}{dx} = -\frac{2b}{a}x + b$ ✓ $\frac{dA}{dx} = 0$ ✓ x-value ✓ y-value

QUESTION 12

12.1	$x \geq 1$ $y \leq 12$ $x + y \geq 10$ $x + y \leq 15$ $y \geq 2x$ $x, y \in N_0$	✓ $x \geq 1$ ✓ $y \leq 12$ ✓ $x + y \geq 10$ ✓ $x + y \leq 15$ ✓✓ $y \geq 2x$
12.2		✓ $x \geq 1$ ✓ $y \leq 12$ ✓ $x + y \leq 15$ ✓ $x + y \geq 10$ ✓ $y \geq 2x$ ✓✓ feasible region
12.3	No. The point (5 ; 8) lies outside the feasible region OR 8 is not greater than $2(5) = 10$	✓ No ✓ Reason
12.4	$I = 600x + 900y$ $y = -\frac{2}{3}x + \frac{I}{900}$ Maximum Income at (3 ; 12) 3 single bedrooms and 12 double bedrooms OR To optimise profit, the group must build as many rooms as possible and then, as many double rooms as possible. So 15 rooms, 12 double rooms, hence 3 single rooms.	✓ objective function