NATIONAL SENIOR CERTIFICATE

GRADE 12: MATHEMATICS P1

FEBRUARY/MARCH 2011

MARKS: 150

This memorandum consists of 6 pages.

QUESTION 1

1.1.1	$x^2 - x = 12$ $x^2 - x - 12 = 0$ $(x-4)(x+3) = 0$ x = 4 or x = -3 **OR** $x(x-1) = 12$ $4(3) = 12$ $x^2 - x = 12$ By inspection x = 4 or x = -3	✓ standard form ✓ factors ✓ answers (3) ✓ factors ✓✓ answers (3)
1.1.2	$2x^2 + 3x - 7 = 0$ $x = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-7)}}{2(2)}$ $= \frac{-3 \pm \sqrt{65}}{4}$ x = 1,27 or x = – 2,77	✓ substitution into correct formula ✓ 65 ✓✓ answers (4)
1.1.3	$7x^2 + 18x - 9 > 0$ $(7x-3)(x+3) > 0$ + 0 – 0 + – 3 $\frac{3}{7}$ – 3 $\frac{3}{7}$ $x < -3$ or $x > \frac{3}{7}$ **OR** $x \in (-\infty; -3) \cup \left(\frac{3}{7}; \infty\right)$	✓ factors ✓ $\frac{3}{7}$ *and* – 3 ✓ correct intervals (4)
1.2	$2x - y = 7$ $y = 2x - 7$ Substitute $y = 2x - 7$ into $x^2 + xy = 21 - y^2$ $x^2 + x(2x-7) = 21 - (2x-7)^2$ $x^2 + 2x^2 - 7x = 21 - 4x^2 + 28x - 49$ $7x^2 - 35x + 28 = 0$ $x^2 - 5x + 4 = 0$ $(x-4)(x-1) = 0$ x = 4 or x = 1 y = 1 or y = – 5	✓ $y = 2x - 7$ ✓ substitution ✓ multiplication ✓ standard form ✓ factors ✓ *x*-answers ✓ *y*-answers (7)
	OR $2x - y = 7$ $x = \frac{7+y}{2}$ Substitute $x = \frac{7+y}{2}$ into $x^2 + xy = 21 - y^2$ $\left(\frac{7+y}{2}\right)^2 + \left(\frac{7+y}{2}\right)y = 21 - y^2$ $\frac{49 + 14y + y^2}{4} + \frac{7y + y^2}{2} = 21 - y^2$ $49 + 14y + y^2 + 2(7y + y^2) = 84 - 4y^2$ $49 + 14y + y^2 + 14y + 2y^2 = 84 - 4y^2$ $7y^2 + 28y - 35 = 0$ $y^2 + 4y - 5 = 0$ $(y+5)(y-1) = 0$ y = – 5 or y = 1 x = 1 x = 4	✓ $x = \frac{7+y}{2}$ ✓ substitution ✓ multiplication ✓ standard form ✓ factors ✓ *x*-answers ✓ *y*-answers (7)
1.3	$\left(\sqrt[5]{\sqrt{35} + \sqrt{3}}\right)\left(\sqrt[5]{\sqrt{35} - \sqrt{3}}\right)$ $\sqrt[5]{(\sqrt{35} + \sqrt{3})(\sqrt{35} - \sqrt{3})}$ $= \sqrt[5]{35 - 3}$ $= \sqrt[5]{32}$ $= 2$	✓ $\sqrt[5]{(\sqrt{35} + \sqrt{3})(\sqrt{35} - \sqrt{3})}$ ✓ $\sqrt[5]{35 - 3}$ ✓ answer (3) **[21]**

QUESTION 2

2.1	39	✓ answer (1)
2.2	3 9 17 27 6 8 10 2 2 Let $T_n = an^2 + bn + c$ $2a = 2$ $a = 1$ $3a + b = 6$ $3(1) + b = 6$ $b = 3$ $a + b + c = 3$ $1 + 3 + c = 3$ $c = -1$ $T_n = n^2 + 3n - 1$ **OR** $2a = 2$ $a = 1$ $c = 3 - 4 = -1$ $T_n = n^2 + bn - 1$ $3 = (1)^2 + b(1) - 1$ (using $T_1 = 3$) $b = 3$ $T_n = n^2 + 3n - 1$	✓ formula ✓ $a = 1$ ✓ $b = 3$ ✓ $c = -1$ (4) ✓ $a = 1$ ✓ $c = -1$ ✓ formula ✓ $b = 3$ (4)
2.3	$n^2 + 3n - 1 > 269$ $n^2 + 3n - 270 > 0$ $(n+18)(n-15) > 0$ The first value of n is 16 The term is $16^2 + 3(16) - 1 = 303$	✓ $n^2 + 3n - 1 > 269$ ✓ factors ✓ $n = 16$ ✓ answer

QUESTION 3

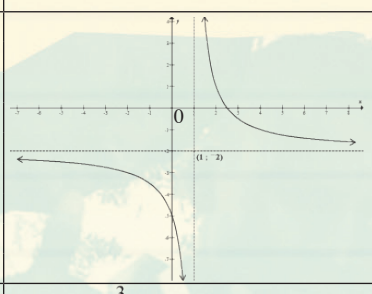
3.1	$S_\infty = 8 + \frac{8}{\sqrt{2}} + \ldots$ $r = \frac{1}{\sqrt{2}}$ and $s_\infty = \frac{a}{1-r}$ $= \frac{8}{1 - \frac{1}{\sqrt{2}}}$ $= \frac{8\sqrt{2}}{\sqrt{2} - 1}$ $= \frac{8\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$ $= 8\sqrt{2}\sqrt{2} + 8\sqrt{2}$ $= 16 + 8\sqrt{2}$ **OR** $S_\infty = 8 + \frac{8}{\sqrt{2}} + \ldots$ $r = \frac{1}{\sqrt{2}}$ and $s_\infty = \frac{a}{1-r}$ $= \frac{8}{1 - \frac{1}{\sqrt{2}}}$ $= \frac{8\left(1 + \frac{1}{\sqrt{2}}\right)}{\left(1 - \frac{1}{\sqrt{2}}\right)\left(1 + \frac{1}{\sqrt{2}}\right)}$ $= \frac{8\left(1 + \frac{1}{\sqrt{2}}\right)}{\frac{1}{2}}$ $= 16\left(1 + \frac{1}{\sqrt{2}}\right)$ $= 16 + \frac{16\sqrt{2}}{2}$	✓ $r = \frac{1}{\sqrt{2}}$ ✓ substitution ✓ substitution ✓ rationalisation ✓ simplification (4) ✓ $r = \frac{1}{\sqrt{2}}$ ✓ ✓ substitution ✓ rationalisation ✓ simplification (4)

3.2.1	$5 + 15 + 45 + \ldots + T_{20}$ $= \sum_{n=1}^{20} 5(3)^{n-1}$ **OR**	✓✓ answer (2)
	$5 + 15 + 45 + \ldots + T_{20}$ $= 5\sum_{n=0}^{19} (3)^{n}$ **OR**	✓✓ answer (2)
	$5 + 15 + 45 + \ldots + T_{20}$ $= 5\sum_{i=l}^{l+19} (3)^{i-l}$ for any $l \in Z$	✓✓ answer (2)
3.2.2	$5 + 15 + 45 + \ldots + T_{20}$ $= \frac{5(3^{20} - 1)}{3 - 1}$ $= 8\ 716\ 961\ 000$	✓ formula ✓ substitution ✓ answer (3) **[9]**

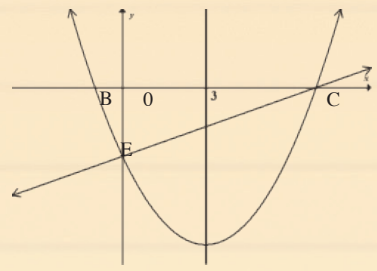
QUESTION 4

4.1.1	$S_{23} = \frac{23}{2}(5(23) + 9)$ $= 1426$	✓ substitution ✓ answer (2)
4.1.2	$T_{23} = S_{23} - S_{22}$ $= 1426 - \frac{22}{2}(5(22) + 9)$ $= 1426 - 1309$ $= 117$	✓ statement ✓ $S_{22} = 1309$ ✓ answer (3)
4.2	Arithmetic Sequence: 12; $12 + d$; $12 + 2d$ Geometric Sequence: 12 ; $12r$; $12r^2$ $12 + d = 12r$ $d = 12r - 12$ $12 + 12r + 12r^2 = 12 + 12 + d + 12 + 2d + 3$ $12r^2 = 12 + 2(12r - 12) + 3$ $12r^2 = 12 + 24r - 24 + 3$ $12r^2 - 24r + 9 = 0$ $4r^2 - 8r + 3 = 0$ $(2r - 3)(2r - 1) = 0$ $r = \frac{3}{2}$ or $r = \frac{1}{2}$ **OR** The 3rd term of GP = 3 + 3rd term of AP $12r^2 = 3 + 12 + 2d$ $12r^2 = 15 + 24r - 24$ $12r^2 - 24r + 9 = 0$ $4r^2 - 8r + 3 = 0$ $(2r - 3)(2r - 1) = 0$ $r = \frac{3}{2}$ or $r = \frac{1}{2}$	✓ equation ✓ equation ✓ standard form ✓ factors ✓ answer (6) ✓ equation ✓ equation ✓ standard form ✓ factors ✓ answer **[11]**

QUESTION 5

5.1	x = 1 y = – 2	✓✓ answers (2)
5.2	y-intercept: $y = \frac{3}{0 - 1} - 2 = -5$ x-intercept: $\left(\frac{5}{2} ; 0\right)$ $0 = \frac{3}{x - 1} - 2$ $2 = \frac{3}{x - 1}$ $2x - 2 = 3$ $2x = 5$ $x = \frac{5}{2}$	✓ y = -5 ✓ substitute y = 0 ✓ answer (3)
5.3		✓ asymptotes ✓ y–intercept ✓ shape (3)
5.4	$-f(x) = \frac{-3}{x - 1} + 2$ $y \in R - \{2\}$ **OR** $(-\infty ; 2) \cup (2 ; \infty)$ **OR** $y \in R; y \neq 2$	✓ answer (1)
5.5	$g(x) = \frac{-3}{x + 1} - 2$ $= \frac{3}{-x - 1} - 2$ Reflection of f about the y-axis. **OR** (i) horizontal shift 2 units to the left followed by (ii) reflection in x-axis, followed by (iii) vertical downward shift of 4 units	✓ manipulation ✓ answer (2) **[11]**

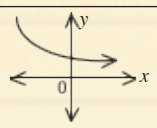
QUESTION 6



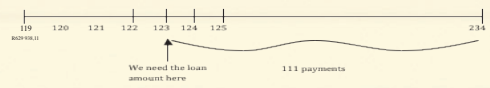
6.1	$\frac{x}{2} - \frac{7}{2} = 0$ $x = 7$ C(7 ; 0) **OR** $y = \frac{7}{2} - \frac{7}{2}$ $y = 0$ C(7 ; 0)	$\frac{x}{2} - \frac{7}{2} = 0$ (1) ✓ substitution ✓ answer (2)
6.2	x- coordinate of B is 3 – 4 = -1	✓ answer (1)
6.3	OPTION 1 $f(x) = a(x - 3)^2 + q$ At B and C: At E: $-\frac{7}{2} = 9a + q$ Solving simultaneously gives $a = \frac{1}{2}$ and	✓ substitution ✓ substitution ✓ substitution ✓ $a = \frac{1}{2}$ ✓ $q = -8$ (6)
	OPTION 2 $f(x) = a(x + 1)(x - 7)$ $y = a(x + 1)(x - 7)$ $-3{,}5 = a(0 + 1)(0 - 7)$ $-3{,}5 = -7a$ $a = \frac{1}{2}$ $f(x) = \frac{1}{2}(x + 1)(x - 7)$ $= \frac{1}{2}(x^2 - 6x - 7)$ $= \frac{1}{2}\left[(x - 3)^2 - 16\right]$ $= \frac{1}{2}(x - 3)^2 - 8$	✓ substitution ✓ substitution $a = \frac{1}{2}$ ✓ substitution ✓ simplification ✓ answer (6)
	OPTION 3 $a = \frac{1}{2}$ Axis of symmetry: $x = 3$ or $x = \frac{-1 + 7}{2} = 3$ $f(x) = \frac{1}{2}(x - 3)^2 + q$ $0 = \frac{1}{2}(7 - 3)^2 + q$ $q = -8$ $y = \frac{1}{2}(x - 3)^2 - 8$	✓✓✓ $a = \frac{1}{2}$ ✓ substitution ✓ substitution ✓ answer (6)
	OPTION 4 $a = \frac{1}{2}$ Axis of symmetry: $x = 3$ $f(x) = \frac{1}{2}(x - 3)^2 + q$ $q = f(3)$ $q = \frac{1}{2}(3 + 1)(3 - 7)$ $q = -8$ $y = \frac{1}{2}(x - 3)^2 - 8$	✓✓✓ $a = \frac{1}{2}$ ✓ substitution ✓ substitution ✓ answer (6)
6.4	$h(x) = -f(x) = \frac{-1}{2}(x - 3)^2 + 8$	✓ answer (1)
6.5	$1 - f(x) = -\frac{1}{2}(x - 3)^2 + 9$ ∴ Maximum value is 9.	✓ method ✓ answer (2)

	OR Maximum value $= 1-(-8)$ $= 9$ **OR** $t(x) = -\frac{1}{2}x^2 + 3x + \frac{9}{2}$ $t'(x) = -x + 3 = 0$ Max V *at* x=3= $-\frac{1}{2}(3)^2 + 3(3) + \frac{9}{2} = 9$	
6.6	$f(x^2-2) = 0$ $f(x) = 0$ *if* $x = -1$ or $x = 7$ $\therefore f(x^2-2) = 0$ if $x^2-2 = -1$ or $x^2-2 = 7$ $\therefore x^2 = 1$ or $x^2 = 9$ $\therefore x = 1$ or $x = -1$ or $x = 3$ or $x = -3$ $\frac{1}{2}(x^2-2-3)^2 - 8 = 0$ $\frac{1}{2}(x^2-5)^2 = 8$ $(x^2-5)^2 = 16$ $x^2-5 = 4$ or $x^2-5 = -4$ $x^2 = 9$ or $x^2 = 1$ $x = 3$ or $x = -3$ or $x = 1$ or $x = -1$ **OR** $f(x^2-2) = 0$ $\frac{1}{2}(x^2-2-3)^2 - 8 = 0$ $\frac{1}{2}(x^2-5)^2 = 8$ $(x^2-5)^2 - 16 = 0$ $(x^2-5-4)(x^2-5+4) = 0$ $(x^2-9)(x^2-1) = 0$ $(x-3)(x+3)(x-1)(x+1) = 0$ $x = 3$ or $x = -3$ or $x = 1$ or $x = -1$	✓ substitution ✓ simplification ✓ answer ✓ answer (4) ✓ substitution ✓ simplification ✓ factors ✓ answer ✓ substitution ✓ simplification ✓ factors ✓ answer (4) **[15]**

QUESTION 7

7.1	Decreasing function Since $0 < a < 1$ **OR** As x increases, f(x) decreases	✓ decreasing ✓ a < 1 (2)
7.2	f^{-1}: $x = \left(\frac{1}{3}\right)^y$ **OR**  **OR** f^{-1}: $x = \left(\frac{1}{3}\right)^y$ $y = -\log_3 x$	✓ $x = \left(\frac{1}{3}\right)^y$ ✓ $y = \log_{\frac{1}{3}} x$ $y = -\log_3 x$ (2)
7.3	$y = -5$	✓ answer (1)
7.4	Reflection about $y = x$. Reflection about the x-axis. **OR** Reflection about the y-axis. Then reflection about the line $y = x$. **OR** Reflection about the line $y = -x$ followed by reflection about the y-axis. **OR** Rotation through 90° in a clockwise direction. **OR** Rotation through 90° in an anti-clockwise direction. Reflection through the origin.	✓ reflection about $y = x$ ✓ reflection about y-axis (2) ✓ reflection about y-axis ✓ reflection about $y = x$ (2) ✓ rotation through 90° ✓ clockwise direction (2) ✓ answer ✓ answer (2) **[7]**

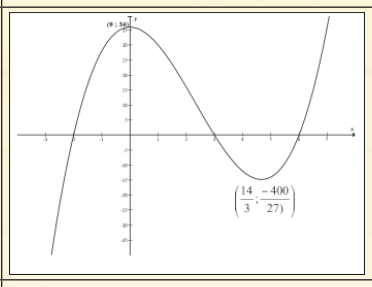
QUESTION 8

8.1	$A = P(1+i)^n$ $1711,41 = 1430,77\left(1+\frac{i}{12}\right)^{18}$ $\left(1+\frac{i}{12}\right)^{18} = 1,196146131...$ $1+\frac{i}{12} = 1,009999937...$ $i = 0,1199992431...$ **OR** $\left[\frac{1711,41}{1430,77}\right]^{\frac{1}{18}}$ $= 1,00999...$ $\therefore i = 12(1,01-1)$ $= 0,12$ $= 12\%$ Rate = 12, 00% p.a. compounded monthly.	✓ substitution ✓ $\left(1+\frac{i}{12}\right)^{18} = 1,196146131...$ ✓ $1+\frac{i}{12} = 1,009999937...$ ✓ answer (4)
8.2.1	$P_v = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $800000 = \frac{10000\left[1-\left(1+\frac{0,14}{12}\right)^{-n}\right]}{\frac{0,14}{12}}$ $1-\left(1+\frac{0,14}{12}\right)^{-n} = \frac{14}{15}$ $(= 0,933333)$ $\left(1+\frac{0,14}{12}\right)^{-n} = \frac{1}{15}$ $(= 0,06666666)$ $\log\left(1+\frac{0,14}{12}\right)^{-n} = \log\frac{1}{15}$ $-n\log\left(1+\frac{0,14}{12}\right) = \log\frac{1}{15}$ $\left(-n = \frac{\log\frac{1}{15}}{\log\left(1+\frac{0,14}{12}\right)} = -233,47\right)$ $n = 233,47$ $\therefore$ the loan will be paid off at the end of the 234th month **OR** Balance outstanding after 233rd month $= 800000\left(1+\frac{0,14}{12}\right)^{233} - \frac{10000\left[\left(1+\frac{0,14}{12}\right)^{233}-1\right]}{\frac{0,14}{12}}$ = R4 660,04 which is less than R10 000 Therefore the loan will be paid off after 234 months. **OR** Total value of the loan after 234 payments $= \frac{10000\left(1-\left(1+\frac{0,14}{12}\right)^{-234}\right)}{\frac{0,14}{12}}$ = R800 350,21 > R800 000 and the differences is less than R10 000 Therefore the loan will be paid off after 234 months.	✓ substitute into Pv ✓ $i = \frac{0,14}{12}$ ✓ using logs ✓ answer (4) ✓ substitution into P formula ✓ 234 ✓ answer ✓ argument (4) ✓ substitution into P formula ✓ 234 ✓ answer ✓ argument (4)
8.2.2	Balance Outstanding after 119 months $= 800000\left(1+\frac{0,14}{12}\right)^{119} - \frac{10000\left[\left(1+\frac{0,14}{12}\right)^{119}-1\right]}{\frac{0,14}{12}}$ = R629 938,11  Total Payable at the end of the 123rd month $629938,11\left(1+\frac{0,14}{12}\right)^4$ = R659 853,68 New instalment: $659\,853,68 = \frac{x\left[1-\left(1+\frac{0,14}{12}\right)^{-111}\right]}{\frac{0,14}{12}}$ x = R10 632,39	✓ $= 800000\left(1+\frac{0,14}{12}\right)^{119}$ ✓ $\frac{10000\left[\left(1+\frac{0,14}{12}\right)^{119}-1\right]}{\frac{0,14}{12}}$ ✓ R629 938,11 ✓ $629938,11\left(1+\frac{0,14}{12}\right)^4$ ✓ R659 853,68 ✓ substitution into P_v ✓ answer (7) **[15]**

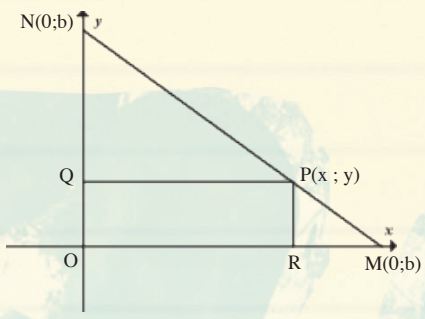
QUESTION 9

9.1	$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $= \lim_{h\to 0}\frac{1-3(x+h)^2-(1-3x^2)}{h}$ $= \lim_{h\to 0}\frac{1-3x^2-6xh-3h^2-1+3x^2}{h}$ $= \lim_{h\to 0}\frac{-6xh-3h^2}{h}$ $= \lim_{h\to 0}\frac{h(-6x-3h)}{h}$ $= \lim_{h\to 0}(-6x-3h)$ $= -6x$	✓ substitution into formula ✓ $1-3x^2-6xh-3h^2$ ✓ $h(-6x-3h)$ ✓ answer (4)
9.2	$D_x\left[4-\frac{4}{x^3}-\frac{1}{x^4}\right]$ $= D_x\left[4-4x^{-3}-x^{-4}\right]$ $= 12x^{-4}+4x^{-5}$	✓ simplification ✓✓ answer (3)
9.3	$y = (1+\sqrt{x})^2$ $y = 1+2\sqrt{x}+x$ $y = 1+2x^{\frac{1}{2}}+x$ $\frac{dy}{dx} = x^{-\frac{1}{2}}+1$	✓ expansion ✓ ✓ 1 (3) **[10]**

QUESTION 10

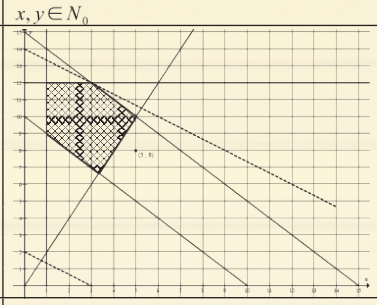
10.1	$(-6)(-3)(+2) = 36$ $x^{-\frac{1}{2}}$ y-intercept is 36 **OR** $g(x) = (x-6)(x^2-x-6)$ $g(x) = x^3-7x^2+36$ y-intercept : (0;36)	✓ $(-6)(-3)(+2)$ ✓ y-intercept is 36 (1) ✓ trinomial ✓ 36 (1)
10.2	$g(x) = 0$ $x = 6$ *or* $x = 3$ *or* $x = -2$ intercepts are (6 ; 0) and (3 ; 0) and (– 2; 0)	✓ $g(x) = 0$ ✓ all x-intercepts (2)
10.3	$g(x) = (x-6)(x^2-x-6)$ $= x^3-7x^2+36$ $g'(x) = 3x^2-14x$ $0 = x(3x-14)$ $x = 0$ or $x = \frac{14}{3}$ Turning points are (0 ; 36) and $\left(\frac{14}{3};-\frac{400}{27}\right)$	✓ x^3-7x^2+36 ✓ $g'(x) = 3x^2-14x$ ✓ $g'(x) = 0$ ✓ answers ✓✓ points (6)
10.4		✓ x-intercepts ✓✓ turning points ✓ shape (4)
10.5	$g(x).g'(x < 0$ $x < -2$ *or* $0 < x < 3$ *or* $\frac{14}{3} < x < 6$	1 mark for each inequality (3) **[16]**

QUESTION 11



11.1	$m = -\frac{b}{a}$ $y-b = \frac{-b}{a}(x-0)$ **OR** $y = mx+b$ $y = \frac{-b}{a}x+b$ $\quad 0 = ma+b$ $m = \frac{-b}{a}$ **OR** $y = -\frac{b}{a}x+b$ $\frac{x}{a}+\frac{y}{b} = 1$	✓ $m = -\frac{b}{a}$ ✓ answer ✓ answer (2)
11.2	A = xy $A = x\left(\frac{-bx}{a}+b\right)$ $= -\frac{b}{a}x^2+bx$ $\frac{dA}{dx} = -\frac{2b}{a}x+b$ $0 = -\frac{2b}{a}x+b$ $-ba = -2bx$ $x = \frac{a}{2}$ $y = -\frac{b}{a}\left(\frac{a}{2}\right)+b$ $= \frac{b}{2}$ P $\left(\frac{a}{2};\frac{b}{2}\right)$ which is the midpoint of MN **OR** $\frac{x}{a}+\frac{y}{b} = 1$ $\frac{y}{b} = 1-\frac{x}{a}$ To maximise xy, we maximise $\frac{xy}{ab} = \frac{x}{a}\left(\frac{y}{b}\right) = \frac{x}{a}\left(1-\frac{x}{a}\right)$ This is a maximum when $\frac{x}{a} = \frac{1}{2}$ i.e. $x = \frac{a}{2}$ By the midpoint theorem, P is then the midpoint of MN.	✓ area formula ✓ substitution ✓ $\frac{dA}{dx} = -\frac{2b}{a}x+b$ ✓ $\frac{dA}{dx} = 0$ ✓ x-value ✓ y-value (6) (6) **[8]**

QUESTION 12

12.1	$x \geq 1$ $y \leq 12$ $x+y \geq 10$ $x+y \leq 15$ $y \geq 2x$ $x, y \in N_0$	✓ $x \geq 1$ ✓ $y \leq 12$ ✓ $x+y \geq 10$ ✓ $x+y \leq 15$ ✓✓ $y \geq 2x$ (6)
12.2		✓ $x \geq 1$ ✓ $y \leq 12$ ✓ $x+y \leq 15$ ✓ $x+y \geq 10$ ✓ $y \geq 2x$ ✓✓ feasible region (7)
12.3	No. The point (5 ; 8) lies outside the feasible region **OR** 8 is not greater than 2(5) =10	✓ No ✓ Reason (2)
12.4	I = 600x + 900y $y = -\frac{2}{3}x+\frac{I}{900}$ Maximum Income at (3 ; 12) 3 single bedrooms and 12 double bedrooms **OR** To optimise profit, the group must build as many rooms as possible and then, as many double rooms as possible. So 15 rooms, 12 double rooms, hence 3 single rooms.	✓ objective function (3) **[18]**

TOTAL: 150