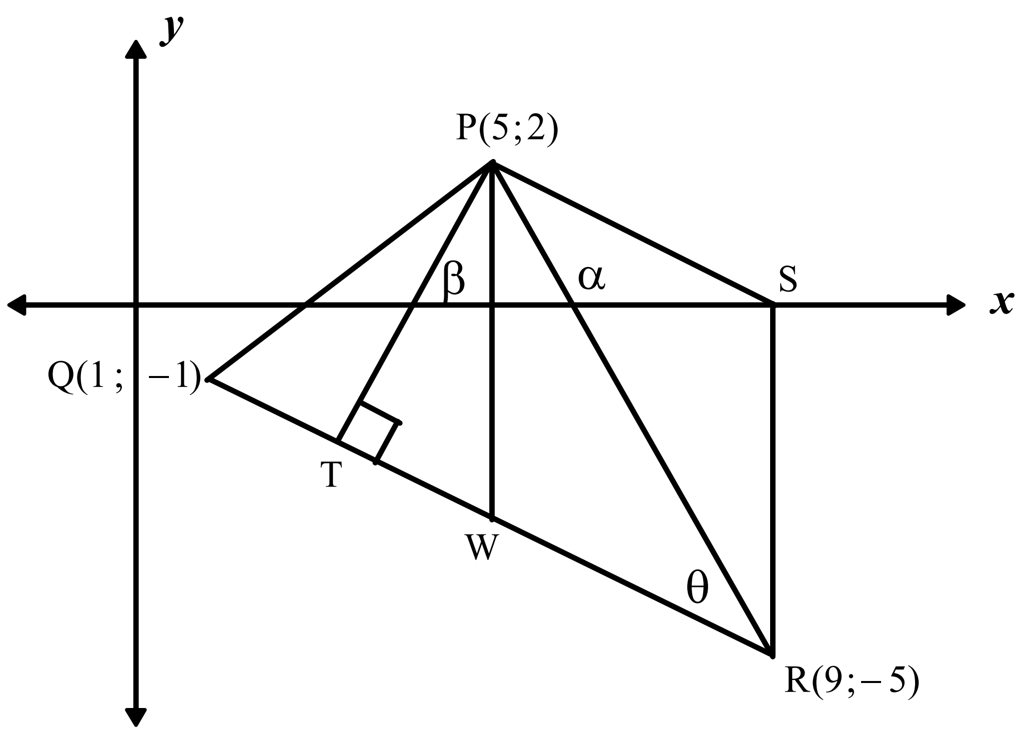
MATHEMATICS EXEMPLAR EXAMINATION
GRADE 12
PAPER 2

MEMORANDUM

QUESTION 1

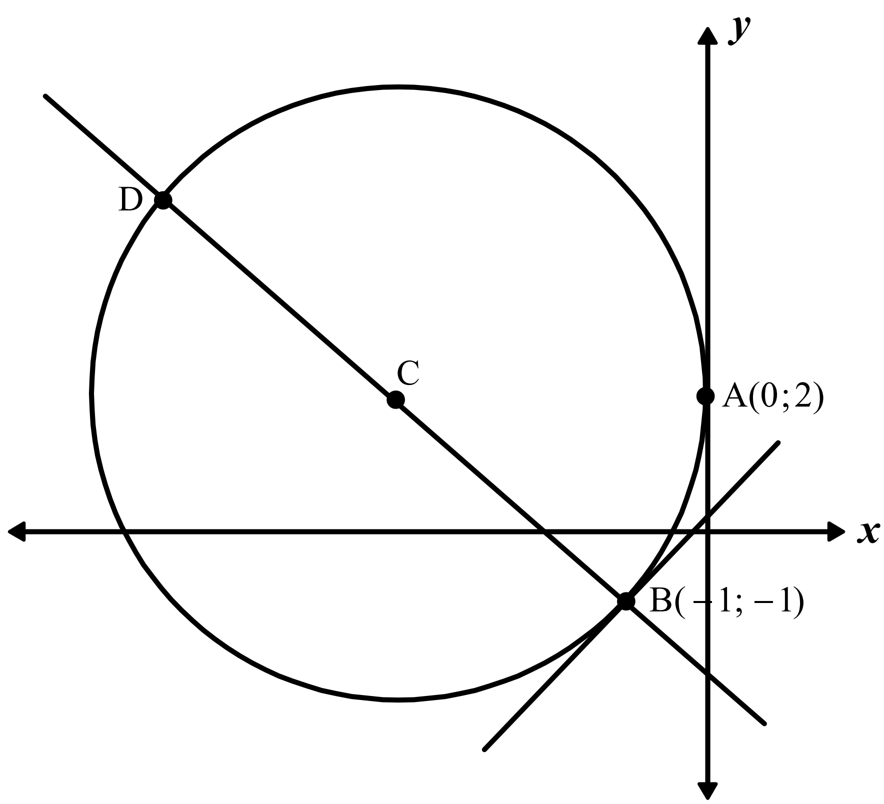


1.1	$W\left(\frac{1+(9)}{2}\ ;\ \frac{-1+(-5)}{2}\right)$ $= W(5\ ;\ -3)$ The equation of PW is $x = 5$	✓ midpoint ✓ $x = 5$ (2)
1.2	$m_{QR} = \frac{-5-(-1)}{9-1} = \frac{-4}{8} = -\frac{1}{2}$ $\therefore m_{PS} = -\frac{1}{2}$ (PS‖QR) $y-2 = -\frac{1}{2}(x-5)$ $\therefore y-2 = -\frac{1}{2}x + \frac{5}{2}$ $\therefore y = -\frac{1}{2}x + \frac{9}{2}$	✓ m_{QR} ✓ m_{PS} ✓ correct substitution into formula for equation ✓ $y = -\frac{1}{2}x + \frac{9}{2}$ (4)

1.3	$m_{PT} = 2 \quad (PT \perp QR)$ $y - 2 = 2(x - 5)$ $\therefore y - 2 = 2x - 10$ $\therefore y = 2x - 8$	✓ m_{PT} ✓ correct substitution into formula for equation ✓ $y = 2x - 8$ (3)
1.4	$m_{QR} = -\frac{1}{2}$ $y - (-1) = -\frac{1}{2}(x - 1)$ $\therefore y + 1 = -\frac{1}{2}x + \frac{1}{2}$ $\therefore y = -\frac{1}{2}x - \frac{1}{2}$ $\therefore -\frac{1}{2}x - \frac{1}{2} = 2x - 8$ $\therefore -x - 1 = 4x - 16$ $\therefore -5x = -15$ $\therefore x = 3$ $\therefore y = 2(3) - 8 = -2$ $\therefore T(3; -2)$	✓ correct substitution into formula for equation ✓ $y = -\frac{1}{2}x - \frac{1}{2}$ ✓ $-\frac{1}{2}x - \frac{1}{2} = 2x - 8$ ✓ $x = 3$ ✓ $T(3; -2)$ (5)
1.5	$QT^2 = (1-3)^2 + (-1-(-2))^2$ $\therefore QT^2 = 4 + 1$ $\therefore QT^2 = 5$ $\therefore QT = \sqrt{5}$ $TR^2 = (3-9)^2 + (-2-(-5))^2$ $\therefore TR^2 = 36 + 9$ $\therefore TR^2 = 45$ $\therefore TR = \sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$ $\therefore \frac{1}{3}TR = \sqrt{5} = QT$ $\therefore QT = \frac{1}{3}TR$	✓ correct substitution to get QT ✓ answer for QT ✓ correct substitution to get TR ✓ answer for TR ✓ establishing that $QT = \frac{1}{3}TR$ (5)

1.6	$\tan\alpha = m_{\text{PR}}$ $\therefore \tan\alpha = \dfrac{2-(-5)}{5-9}$ $\therefore \tan\alpha = -1$ $\therefore \alpha = 180° - 45°$ $\therefore \alpha = 135°$ $\tan\beta = m_{\text{PT}}$ $\therefore \tan\beta = 2$ $\therefore \beta = 63,43494882°$ Now $\text{T}\hat{\text{P}}\text{R} + \beta = \alpha$ $\therefore \text{T}\hat{\text{P}}\text{R} = \alpha - \beta$ $\therefore \text{T}\hat{\text{P}}\text{R} = 135° - 63,43494882°$ $\therefore \text{T}\hat{\text{P}}\text{R} = 71,56505118°$ $\theta + 90° + 71,56505118° = 180°$ $\therefore \theta = 18,43°$	✓ $\tan\alpha = -1$ ✓ $\alpha = 135°$ ✓ $\beta = 63,43494882°$ ✓ $\text{T}\hat{\text{P}}\text{R} = 71,56505118°$ ✓ $\theta = 18,43°$ (5)

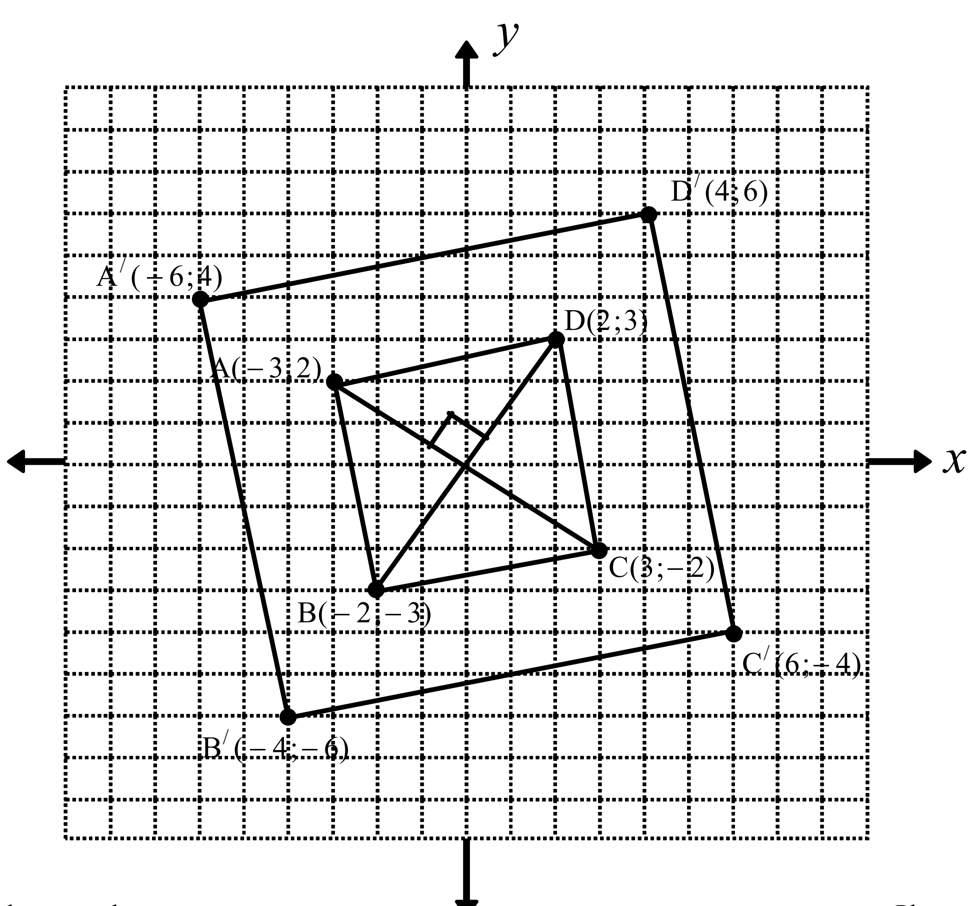
QUESTION 2



2.1	$m_{AB} = m_{BC}$ $\therefore \frac{2-5}{3-6} = \frac{k+4-2}{2k-3}$ $\therefore 1 = \frac{k+2}{2k-3}$ $\therefore 2k-3 = k+2$ $\therefore k = 5$	✓ $m_{AB} = m_{BC}$ ✓ working out gradients ✓ $k = 5$ (3)
2.2	$x^2 + y^2 - 4x + 6y + 4 = 0$ $\therefore x^2 - 4x + y^2 + 6y = -4$ $\therefore x^2 - 4x + \left[\frac{-4}{2}\right]^2 + y^2 + 6y + \left[\frac{6}{2}\right]^2 = -4 + \left[\frac{-4}{2}\right]^2 + \left[\frac{6}{2}\right]^2$ $\therefore (x-2)^2 + (y+3)^2 = -4+4+9$ $\therefore (x-2)^2 + (y+3)^2 = 9$ centre $= (2;-3)$ new centre after rotation of 90° clockwise: $(-3;-2)$ new centre after enlargement through origin: $(-6;-4)$ original radius: $r = 3$ new radius after enlargement through origin : $r = 3 \times 2 = 6$ new circle: $(x+6)^2 + (y+4)^2 = 36$	✓ $(x-2)^2$ ✓ $(y+3)^2$ ✓ $r^2 = 9$ ✓ new centre $= (-6;-4)$ ✓ new radius: $r = 6$ ✓ new circle: $(x+6)^2 + (y+4)^2 = 36$ (6)
2.3.1	$3x + 4y = -7$ $\therefore 4y = -3x - 7$ $\therefore y = -\frac{3}{4}x - \frac{7}{4}$ $\therefore m_{CB} = -\frac{3}{4}$ $\therefore m_{tangent} = \frac{4}{3}$ $y - (-1) = \frac{4}{3}(x - (-1))$ $\therefore y + 1 = \frac{4}{3}(x+1)$ $\therefore y + 1 = \frac{4}{3}x + \frac{4}{3}$ $\therefore y = \frac{4}{3}x + \frac{1}{3}$	✓ $m_{CB} = -\frac{3}{4}$ ✓ $m_{tangent} = \frac{4}{3}$ ✓ substitution into equation of line ✓ $y = \frac{4}{3}x + \frac{1}{3}$ (4)

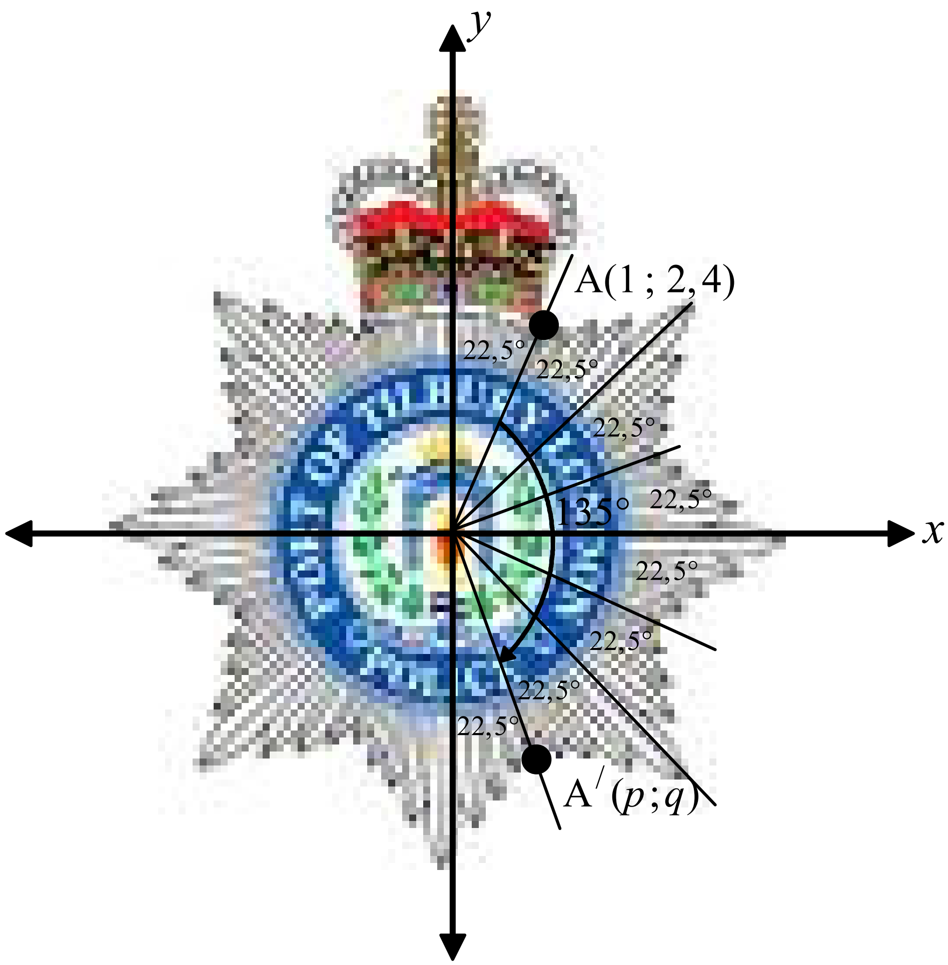
2.3.2	$C(x;2)$ Substitute $y=2$ into $3x+4y=-7$ $3x+4(2)=-7$ $\therefore 3x=-15$ $\therefore x=-5$ $\therefore C(-5;2)$ $\therefore (x+5)^2+(y-2)^2=r^2$ Now $r=5$ $\therefore (x+5)^2+(y-2)^2=25$	✓ $y_C=2$ ✓ $x=-5$ ✓ $C(-5;2)$ ✓ $r=5$ ✓ $(x+5)^2+(y-2)^2=25$ (5)
2.3.3	$C(-5;2)=\left(\dfrac{x_D+x_B}{2};\dfrac{y_D+y_B}{2}\right)$ $\therefore C(-5;2)=\left(\dfrac{x_D-1}{2};\dfrac{y_D-1}{2}\right)$ $\therefore -5=\dfrac{x_D-1}{2}$ and $2=\dfrac{y_D-1}{2}$ $\therefore -10=x_D-1$ and $4=y_D-1$ $\therefore x_D=-9$ and $y_D=5$ $\therefore D(-9;5)$	✓ correct substitution into midpoint formula ✓ $x_D=-9$ ✓ $y_D=5$ ✓ $D(-9;5)$ (4)

QUESTION 3



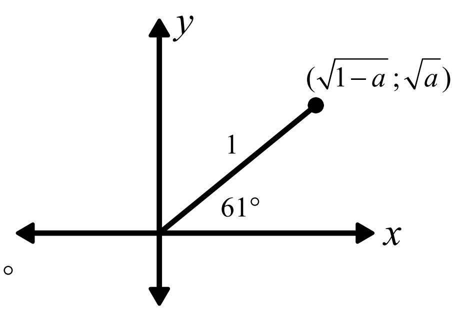
3.1	$B(-2;-3)$ $C(3;-2)$ $D(2;3)$	✓ $B(-2;-3)$ ✓ $C(3;-2)$ ✓ $D(2;3)$ (3)
3.2	ABCD is a square since: Diagonals are equal in length Diagonals bisect each other at right angles	✓ square ✓ properties (2)
3.3	$A^{/}(-6;4)$ $B^{/}(-4;-6)$ $C^{/}(6;-4)$ $D^{/}(4;6)$	✓ correct coordinates indicated ✓ joining points to form enlarged square (2)
3.4	$\dfrac{\text{Area ABCD}}{\text{Area A}^{/}\text{B}^{/}\text{C}^{/}\text{D}^{/}}=\dfrac{1}{2^2}=\dfrac{1}{4}$	✓ $\dfrac{1}{4}$ (1)
3.5.1	$E(3;2)$	✓ answer (1)
3.5.2	$\dfrac{\text{Perimeter ABCD}}{\text{Perimeter EFGH}}=\dfrac{4\times\text{side AB}}{4\times\text{side EF}}=1$ $(\text{since AB}=\text{EF})$	✓ answer (1)
3.6	$(x;y)\rightarrow\left(\frac{1}{2}x;\frac{1}{2}y\right)$ reduction by a factor of $\frac{1}{2}$ $\left(\frac{1}{2}x;\frac{1}{2}y\right)\rightarrow\left(\frac{1}{2}x;-\frac{1}{2}y\right)$ reflection about $x-$axis $\left(\frac{1}{2}x;-\frac{1}{2}y\right)\rightarrow\left(\frac{1}{2}x;-\frac{1}{2}y-1\right)$ translation of 1 unit downwards $\therefore(x;y)\rightarrow\left(\frac{1}{2}x;-\frac{1}{2}y-1\right)$	✓ reduction ✓ reflection ✓ translation (3)

QUESTION 4



4.1	$22,5° \times 6 = 135°$	✓ 22,5° ✓ 135° (2)
4.2	$x' = (1)\cos(-135°) - (2,4)\sin(-135°)$ $\therefore x' = 1$ $y' = (2,4)\cos(-135°) + (1)\sin(-135°)$ $y' = -2,4$ $\therefore A'(1; -2,4)$	✓ correct substitution into formula for x' ✓ $x' = 1$ ✓ correct substitution into formula for y' ✓ $y' = -2,4$ (4)

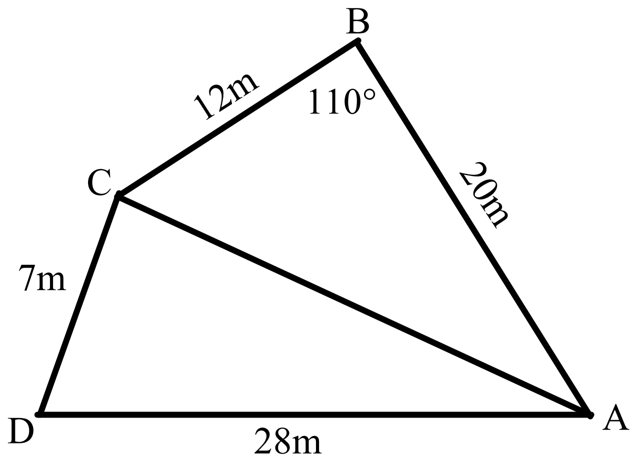
QUESTION 5

5.1	$\dfrac{\tan(-60°)\cos(-156°)\cos 294°}{\sin 492°}$ $= \dfrac{(-\tan 60°)(\cos 156°)(-\cos 66°)}{(\sin 132°)}$ $= \dfrac{(-\sqrt{3})(-\cos 24°)(-\sin 24°)}{(\sin 48°)}$ $= \dfrac{(-\sqrt{3})(-\cos 24°)(-\sin 24°)}{2\sin 24° \cos 24°}$ $= \dfrac{\sqrt{3}}{2}$	✓ $(-\tan 60°)(\cos 156°)$ ✓ $-\cos 66°$ ✓ $\sin 48°$ ✓ $-\sqrt{3}$ ✓ $-\sin 24°$ ✓ $2\sin 24° \cos 24°$ ✓ $\dfrac{\sqrt{3}}{2}$ (7)
5.2	$\cos^2(180° + x)\left[\tan(360° - x).\cos(90° + x) + \sin(x - 90°).\cos 180°\right]$ $= (-\cos x)^2\left[(-\tan x)(-\sin x) + (-\cos x)(-1)\right]$ $= (\cos^2 x)\left[\left(\dfrac{-\sin x}{\cos x}\right)(-\sin x) + \cos x\right]$ $= (\cos^2 x)\left[\dfrac{\sin^2 x}{\cos x} + \cos x\right]$ $= (\cos^2 x)\left[\dfrac{\sin^2 x + \cos^2 x}{\cos x}\right]$ $= (\cos^2 x)\left[\dfrac{1}{\cos x}\right]$ $= \cos x$	✓ $\cos^2 x$ ✓ $-\tan x$ ✓ $-\sin x$ ✓ $-\cos x$ ✓ -1 ✓ $\dfrac{-\sin x}{\cos x}$ ✓ $\dfrac{\sin^2 x + \cos^2 x}{\cos x}$ ✓ $\dfrac{1}{\cos x}$ ✓ $\cos x$ (9)
5.3	$\sin 61° = \dfrac{\sqrt{a}}{1}$ $x^2 + (\sqrt{a})^2 = (1)^2$ $\therefore x^2 = 1 - a$ $\therefore x = \sqrt{1-a}$  $\cos 73° \cos 15° + \sin 73° \sin 15°$ $= \cos(73° - 15°)$ $= \cos 58°$ $= 2\cos^2 29° - 1$ $= 2\sin^2 61° - 1$ $= 2(\sqrt{a})^2 - 1$ $= 2a - 1$	✓ diagram ✓ $x = \sqrt{1-a}$ ✓ $\cos 58°$ ✓ $2\cos^2 29° - 1$ ✓ $2\sin^2 61° - 1$ ✓ $2a - 1$ (6)

QUESTION 6

6.1.1	$\sin(45°+\theta).\sin(45°-\theta)$ $=\left[\sin 45°\cos\theta+\cos 45°\sin\theta\right]\left[\sin 45°\cos\theta-\cos 45°\sin\theta\right]$ $=\left[\frac{\sqrt{2}}{2}\cos\theta+\frac{\sqrt{2}}{2}\sin\theta\right]\left[\frac{\sqrt{2}}{2}\cos\theta-\frac{\sqrt{2}}{2}\sin\theta\right]$ $=\left[\frac{\sqrt{2}}{2}(\cos\theta+\sin\theta)\right]\left[\frac{\sqrt{2}}{2}(\cos\theta-\sin\theta)\right]$ $=\frac{2}{4}(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)$ $=\frac{1}{2}(\cos^2\theta-\sin^2\theta)$ $=\frac{1}{2}\cos 2\theta$	✓ expansion of $\sin(45°+\theta)$ ✓ expansion of $\sin(45°-\theta)$ ✓ $\sin 45°=\cos 45°=\frac{\sqrt{2}}{2}$ ✓ $(\cos^2\theta-\sin^2\theta)$ ✓ $\frac{1}{2}\cos 2\theta$ (5)
6.1.2	$\sin 75°.\sin 15°$ $=\sin(45°+30°).\sin(45-30°)$ $=\frac{1}{2}\cos 2(30°)$ $=\frac{1}{2}\cos 60°=\frac{1}{2}\left(\frac{1}{2}\right)=\frac{1}{4}$	✓ $45°+30°;\ 45°-30°$ ✓ $\frac{1}{2}\cos 60°$ ✓ $\frac{1}{4}$ (3)
6.2	$\sin 2x+2\sin x+\cos^2 x+\cos x=0$ $\therefore 2\sin x\cos x+2\sin x+\cos^2 x+\cos x=0$ $\therefore 2\sin x(\cos x+1)+\cos x(\cos x+1)=0$ $\therefore (\cos x+1)(2\sin x+\cos x)=0$ $\therefore \cos x=-1$ or $2\sin x=-\cos x$ $\therefore \frac{\sin x}{\cos x}=-\frac{1}{2}$ $\therefore \tan x=-0,5$ $x=0°+k360°$ $x=153,4°+k360°$ $x=180°+k360°$ $x=333,4°+k360°$ OR $x=0°+k180°$ $x=153,4°+k180°$ OR $x=180°+k180°$ $x=333,4°+k180°$ OR $x=\pm 180°+k360°$ $x=-45°+k180°$	✓ $2\sin x\cos x$ ✓ factorising by grouping ✓ $(\ \ )(\ \ )=0$ ✓ $\cos x=-1$ ✓ $\tan x=-0,5$ ✓✓ general solutions Deduct 1 mark if $k\in\mathbb{Z}$ is not stated. (7)

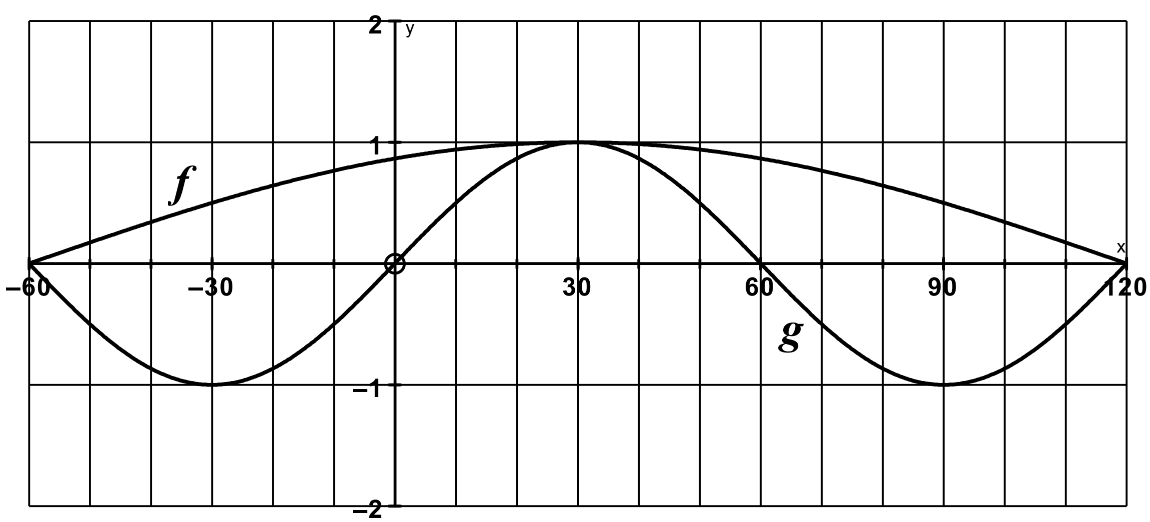
QUESTION 7



7.1	$\text{AC}^2 = (12m)^2 + (20m)^2 - 2(12m)(20m)\cos 110°$ $\therefore \text{AC}^2 = 708,1696688$ $\therefore \text{AC} = 26,6m$	✓ substitution into cosine rule ✓ answer (2)
7.2	$\frac{\sin \text{B}\hat{\text{A}}\text{C}}{12m} = \frac{\sin 110°}{26,6m}$ $\therefore \sin \text{B}\hat{\text{A}}\text{C} = \frac{12 \times \sin 110°}{26,6m}$ $\therefore \sin \text{B}\hat{\text{A}}\text{C} = 0,4239214831$ $\therefore \text{B}\hat{\text{A}}\text{C} = 25°$ OR $(12m)^2 = (20m)^2 + (26,6m)^2 - 2(20m)(26,6m)\cos \text{B}\hat{\text{A}}\text{C}$ $\therefore 1064\cos \text{B}\hat{\text{A}}\text{C} = 963,56m^2$ $\therefore \cos \text{B}\hat{\text{A}}\text{C} = 0,9056015038$ $\therefore \text{B}\hat{\text{A}}\text{C} = 25°$	✓ substitution into sine or cosine rule ✓ answer (2)
7.3	$(26,6m)^2 = (7m)^2 + (28m)^2 - 2(7m)(28m)\cos \hat{\text{D}}$ $\therefore 392\cos \hat{\text{D}} = 125,44$ $\therefore \cos \hat{\text{D}} = 0,32$ $\therefore \hat{\text{D}} = 71°$	✓ substitution into cosine rule ✓ $\cos \hat{\text{D}} = 0,32$ ✓ answer (3)
7.4	Area ABCD $= \frac{1}{2}(12m)(20m)\sin 110° + \frac{1}{2}(7m)(28m)\sin 71°$ $= 205,4m^2$	✓ $\frac{1}{2}(12m)(20m)\sin 110°$ ✓ $\frac{1}{2}(7m)(28m)\sin 71°$ ✓ answer (3)

QUESTION 8

8.1	$\cos(x-30°)=\sin 3x$ $\therefore \cos(x-30°)=\cos(90°-3x)$ $\therefore x-30°=90°-3x+k360°$ $x-30°=360°-(90°-3x)$ $\therefore 4x=120°+k360°$ $\therefore x-30°=270°+3x+k360°$ $\therefore x=30°+k90°$ $\therefore -2x=300°+k360°$ $\therefore x=-150°+k180°$ $x=30°$ or $x=120°$ or $x=-60°$	✓ $\cos(90°-3x)$ ✓ $4x=120°+k360°$ ✓ $-2x=300°+k360°$ ✓ $x=30°+k90°$ ✓ $x=-150°+k180°$ ✓ $x=30°$ ✓ $x=120°$ ✓ $x=-60°$ (8)
8.2	see diagram below	$f(x)=\cos(x-30°)$: ✓ shift of $30°$ right ✓ amplitude ✓ range $g(x)=\sin 3x$: ✓ period of $120°$ ✓ amplitude ✓ intercepts with axes (6)

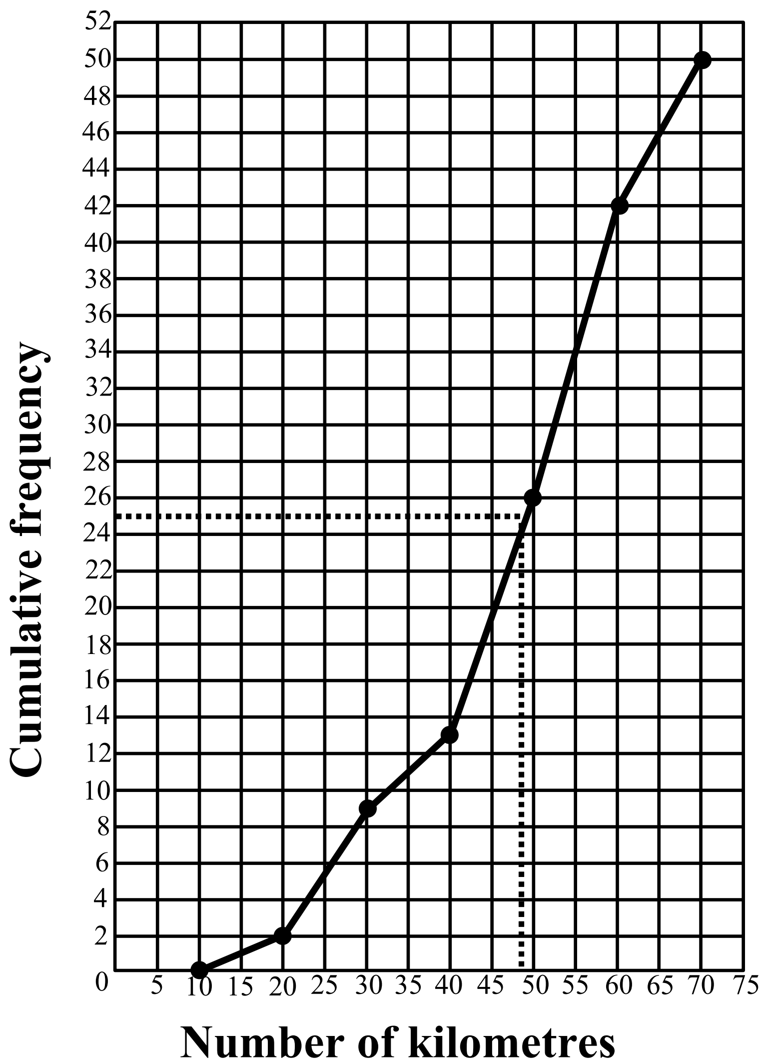


8.3	Points of intersection of the two graphs	✓ correct explanation (1)
8.4	$\cos(x-30°)>\sin 3x$ $\therefore -60°<x<120°$ where $x\neq 30°$ OR $x\in(-60°;30°)-\{30°\}$	✓ $-60°<x<120°$ ✓ $x\neq 30°$ (2)

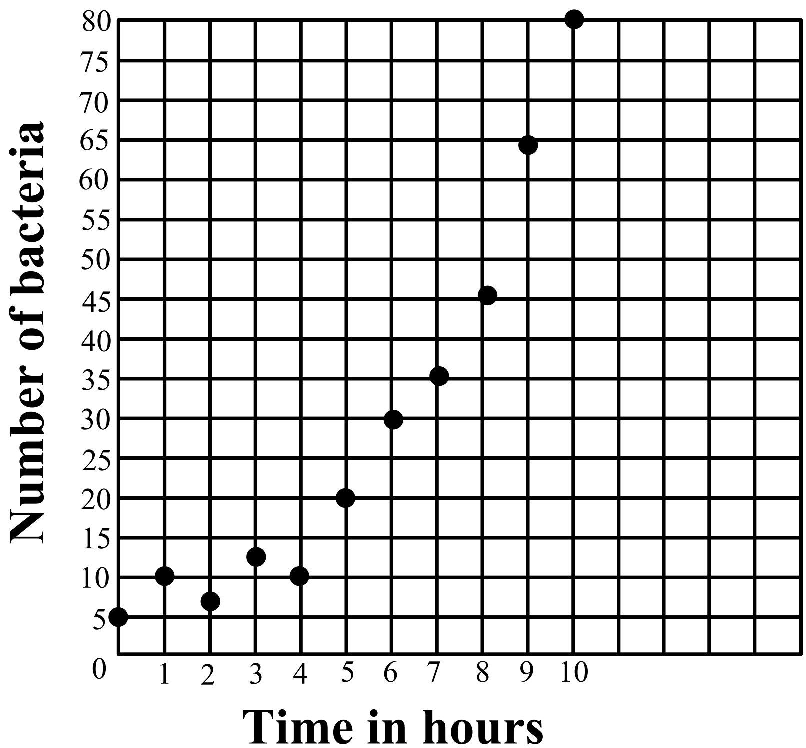
QUESTION 9

9.1	✓ correct second column (1)
9.2	✓ endpoints of class intervals ✓ cumulative frequencies ✓ joining points (3)
9.3 median lies in the interval $48 \leq x \leq 49$	✓ median in the allowable interval (1)

Number of kilometres	Number of motorists	Cumulative frequency
$10 < x \leq 20$	2	2
$20 < x \leq 30$	7	9
$30 < x \leq 40$	4	13
$40 < x \leq 50$	13	26
$50 < x \leq 60$	16	42
$60 < x \leq 70$	8	50



QUESTION 10

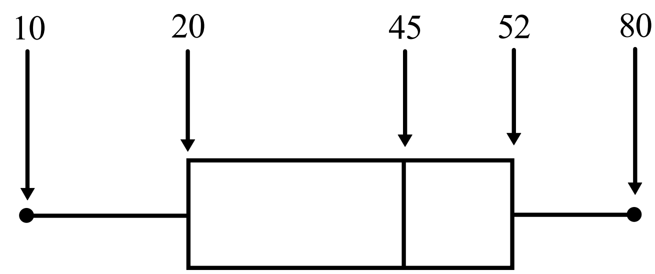
10.1 	✓✓ plotting of points (2)
10.2 quadratic or exponential	✓ answer (1)

QUESTION 11

23	**25**	**22**	**28**	**27**
20	**18**	**17**	**24**	**25**

11.1	$\overline{x} = 22,9$	✓✓ answer (2)
11.2	standard deviation$= 3,5$	✓✓ answer (2)
11.3	$\left(\overline{x} - s ; \overline{x} + s\right)$ $= \left(22,9 - 33,5 ; 22,9 + 33,5\right)$ $= (19,4 \ ; \ 26,4)$ 4 temperatures lie outside the first standard deviation interval	✓ $(19,4 \ ; \ 26,4)$ ✓ 4 temperatures (2)

QUESTION 12



10	**20**	**20**	***x***	**45**	***y***	**51**	**53**	**80**

minimum: 10 maximum: 80 median: 45 Lower quartile: $20 = \frac{20+20}{2}$ Upper quartile: $52 = \frac{51+53}{2}$ Mean: $\frac{10+20+20+x+45+y+51+53+80}{9} = 40$ $\therefore \frac{x+y+279}{9} = 40$ $\therefore x+y+279 = 360$ $\therefore x+y = 81$ Now $20 < x < 45$ and $45 < y < 51$ Therefore let $x = 34$ and $y = 47$ Therefore the set of nine numbers are: 10; 20; 20; 34; 45; 47; 51; 53; 80	✓ min, median and max ✓ $T_2 = T_3 = 20$ ✓ $T_7 = 51 \quad T_8 = 53$ ✓ working with the mean ✓ value for x and y ✓ nine set of numbers Accept variations for T_7, T_8, x and y BUT make sure that the mean of all nine numbers is 40. (6)