MATHEMATICS EXEMPLAR EXAMINATION
GRADE 12
PAPER 2

TIME: 3 HOURS **MARKS: 150**

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, etcetera that you have used in determining your answers.

3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

5. Diagrams are NOT necessarily drawn to scale.

6. FOUR diagram sheets for answering QUESTION 3, QUESTION 8.2, QUESTION 9.1 and 9.2, QUESTION 10.1 are included at the end of this question paper. Write your examination number on these sheets in the spaces provided and hand them in together with your ANSWER BOOK.

7. Number the answers correctly according to the numbering system used in this question paper.

8. It is in your own interest to write legibly and to present the work neatly.

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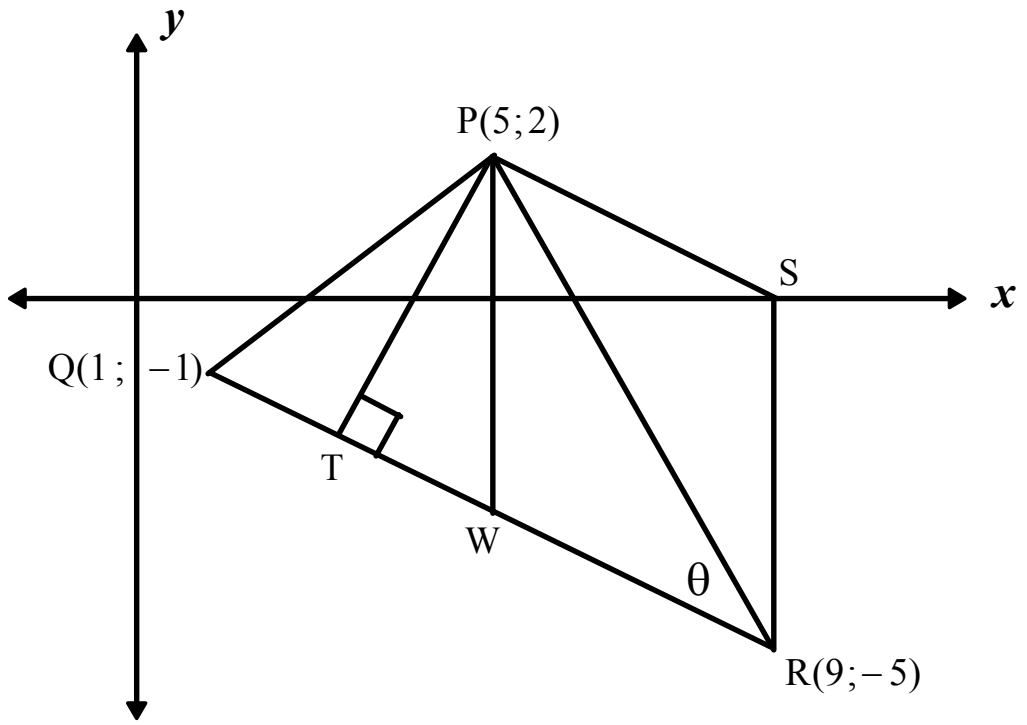
QUESTION 1

In the diagram, PQRS is a trapezium with vertices $P(5;2)$, $Q(1;-1)$, $R(9;-5)$ and S. PT is the perpendicular height of PQRS and W is the midpoint of QR. Point S lies on the *x*-axis and $P\hat{R}Q = \theta$.



1.1 Determine the equation of PW if W is the midpoint of QR. (2)

1.2 Determine the equation of PS. (4)

1.3 Determine the equation of PT. (3)

1.4 Determine the coordinates of T. (5)

1.5 Show that $QT = \frac{1}{3}TR$. (5)

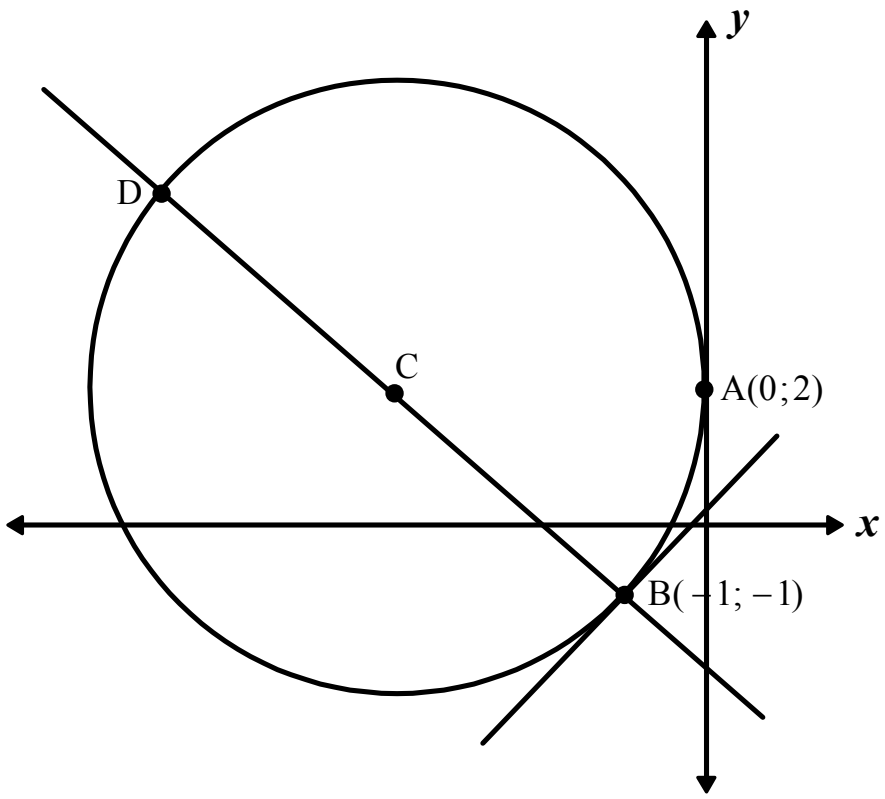
1.6 Calculate the size of θ rounded off to two decimal places. (5)
[24]

QUESTION 2

2.1 Calculate the value of k if the points A(6;5), B(3;2) and C(2k;k + 4) are collinear. (3)

2.2 A circle with equation $x^2 + y^2 - 4x + 6y + 4 = 0$ is rotated 90° clockwise about the origin and then enlarged through the origin by a scale factor of 2 units. Determine the equation of the new circle formed under this transformation. (6)

2.3 In the diagram below, a circle centre C touches the y-axis at A(0;2). A straight line with equation $3x + 4y = -7$ cuts the circle at B(−1;−1) and D.



2.3.1 Determine the equation of the tangent to the circle at B. (4)

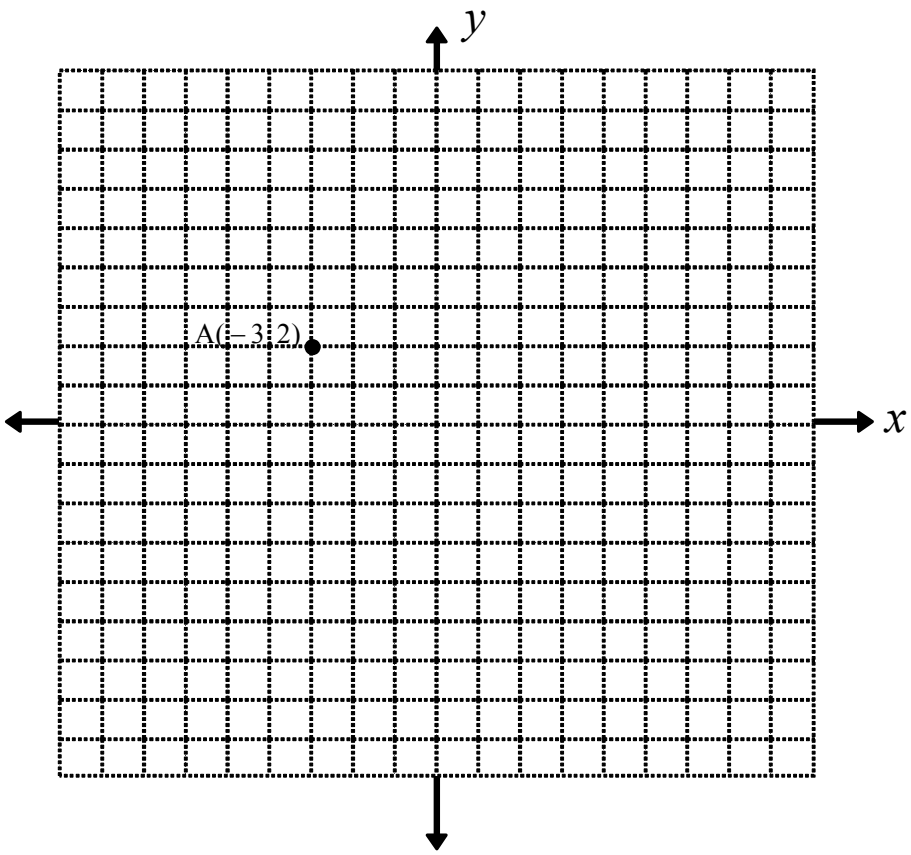
2.3.2 Determine the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$. (5)

2.3.3 Determine the coordinates of D. (4)

[22]

QUESTION 3

In the diagram below, the point $A(-3;2)$ is represented.



3.1 Represent the following points on the diagram provided on the attached diagram sheet:

- Point B, the rotation of point A, $90°$ anticlockwise about the origin.
- Point C, the rotation of point A, $180°$ about the origin.
- Point D, the rotation of point A, $90°$ clockwise about the origin. (3)

3.2 What type of quadrilateral is figure ABCD? Explain by referring to the properties of quadrilaterals. (3)

3.3 Figure ABCD is enlarged by a scale factor of 2 units through the origin to form its image $A'B'C'D'$. On the diagram provided on the diagram sheet, sketch the image $A'B'C'D'$ and indicate the coordinates of the vertices. (2)

3.4 Determine the ratio: $\dfrac{\text{Area ABCD}}{\text{Area } A'B'C'D'}$ (1)

3.5 ABCD is reflected about the y-axis to form its image EFGH.

3.5.1 Write down the coordinates of E. (1)

3.5.2 Determine the ratio: $\frac{\text{Perimeter ABCD}}{\text{Perimeter EFGH}}$ (1)

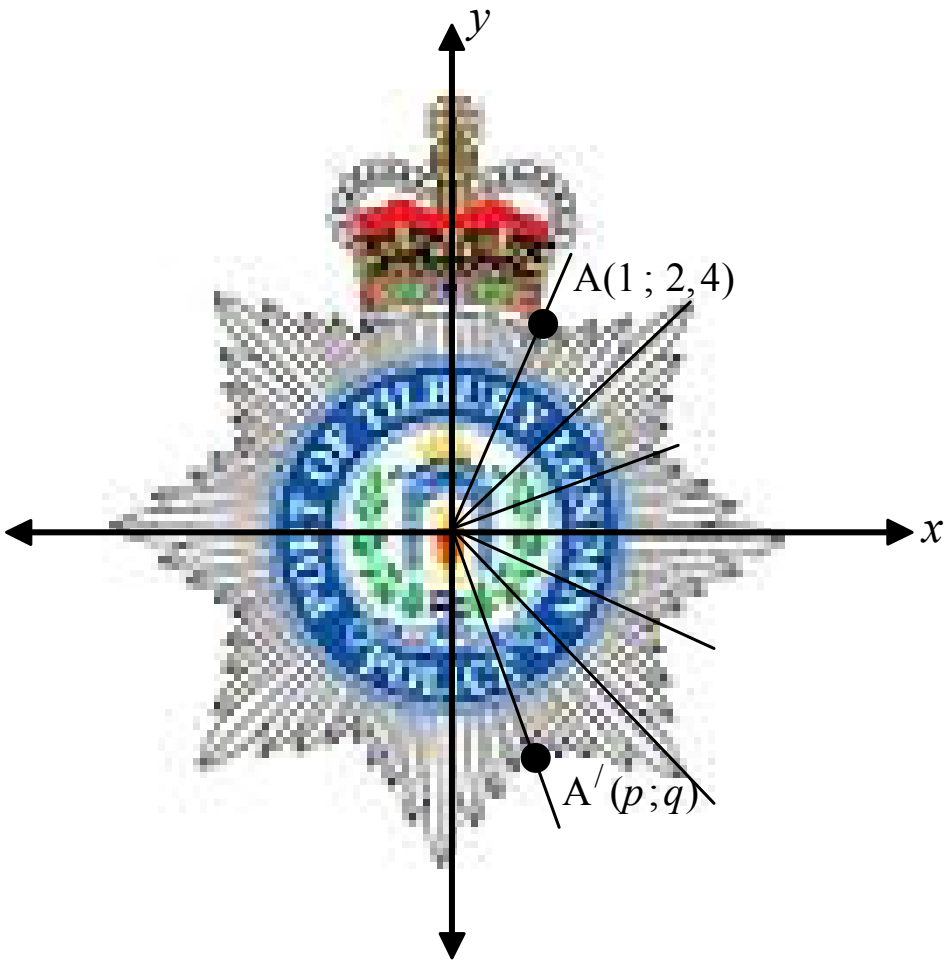
3.6 Describe, using words and algebraic notation, the single transformations involved if figure ABCD is transformed by the rule:

$$(x;y)\rightarrow\left(\frac{1}{2}x\ ;-\frac{1}{2}y-1\right)$$ (3)

[14]

QUESTION 4

The diagram below represents a London police emblem with its centre at the origin. Point $A(1;2,4)$ on the emblem is rotated clockwise about the origin.



The image of A is the point $A^{/}(p;q)$. The coordinates of this image point can be obtained by either reflecting point A about the *x*-axis, or by rotating the point A in a clockwise direction about the origin.

4.1 Calculate the size of the angle of rotation if point A is rotated clockwise about the origin to form its image point $A^{/}(p;q)$ (2)

4.2 Hence determine the coordinates of $A^{/}$, rounded off to one decimal place, by using the appropriate transformation rule. (4)
[6]

QUESTION 5

5.1 Simplify the following without using a calculator:

$$\frac{\tan(-60°)\cos(-156°)\cos 294°}{\sin 492°}$$ (7)

5.2 Simplify without using a calculator:

$$\cos^2(180°+x)\left[\tan(360°-x).\cos(90°+x)+\sin(x-90°).\cos 180°\right]$$ (9)

5.3 If $\sin 61° = \sqrt{a}$, determine the value of the following in terms of *a*:

$$\cos 73°\cos 15° + \sin 73°\sin 15°$$ (6)
[22]

QUESTION 6

6.1 6.1.1 Prove that $\sin(45°+\theta).\sin(45°-\theta) = \frac{1}{2}\cos 2\theta$ (5)

6.1.2 Hence determine the value of $\sin 75°.\sin 15°$ (3)

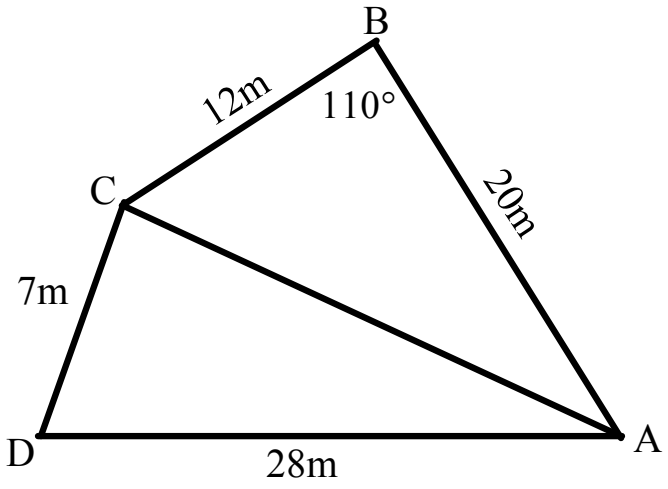
6.2 Determine the general solution of the following equation.

$$\sin 2x + 2\sin x + \cos^2 x + \cos x = 0$$

Round off your answers to one decimal place where appropriate. (7)
[15]

QUESTION 7

A piece of land has the form of a quadrilateral ABCD with $AB = 20\text{m}$, $BC = 12\text{m}$, $CD = 7\text{m}$ and $AD = 28\text{m}$. $\hat{B} = 110°$. The owner decides to divide the land into two plots by erecting a fence from A to C.



7.1 Calculate the length of the fence AC correct to one decimal place. (2)

7.2 Calculate the size of $B\hat{A}C$ correct to the nearest degree. (2)

7.3 Calculate the size of $\hat{D}$, correct to the nearest degree. (3)

7.4 Calculate the area of the entire piece of land ABCD, correct to one decimal place. (3)

[10]

QUESTION 8

8.1 Solve for x if $\cos(x - 30°) = \sin 3x$ where $x \in [-60°; 120°]$. (8)

8.2 Sketch the graphs of the following functions on the same set of axes for the interval $x \in [-60°; 120°]$. Use the diagram provided on the diagram sheet.
$f(x) = \cos(x - 30°)$ $g(x) = \sin 3x$ (6)

8.3 Explain graphically what the solutions to the equation $\cos(x - 30°) = \sin 3x$ represent. (1)

8.4 Determine graphically the values of x for which $\cos(x - 30°) > \sin 3x$ (2)

[17]

QUESTION 9

Fifty motorists were asked to record the number of kilometres travelled in one week. The following table shows the results:

Number of kilometres	Number of motorists	Cumulative frequency
$10 < x \leq 20$	2	2
$20 < x \leq 30$		9
$30 < x \leq 40$		13
$40 < x \leq 50$		26
$50 < x \leq 60$		42
$60 < x \leq 70$		50

9.1 Complete the second column of the table (use table provided on the diagram sheet) (1)

9.2 On the grid provided on the diagram sheet, draw the cumulative frequency polygon (ogive curve). (3)

9.3 Use your graph to estimate the median number of kilometres per week. (1)
[5]

QUESTION 10

A medical researcher recorded the growth in the number of bacteria over a period of 10 hours. The results are recorded in the following table:

Time in hours	0	1	2	3	4	5	6	7	8	9	10
Number of bacteria	5	10	7	13	10	20	30	35	45	65	80

10.1 On the diagram provided on the diagram sheet, draw a scatter plot to represent this data. (2)

10.2 State the type of relationship (linear, quadratic or exponential) that exists between the number of hours and the growth in the number of bacteria. (1)
[3]

QUESTION 11

The maximum daily temperatures in °C for Johannesburg for the first 10 days in July were recorded in the following table.

23	**25**	**22**	**28**	**27**
20	**18**	**17**	**24**	**25**

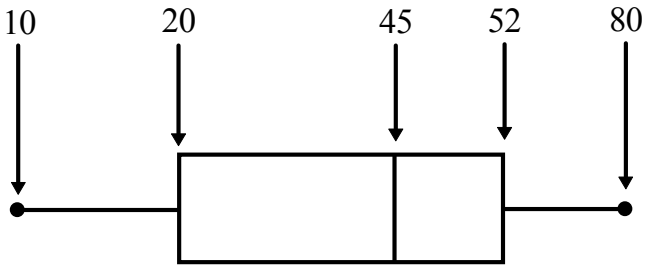
11.1 Calculate the mean for this data (one decimal place) (2)

11.2 Use a calculator to calculate the standard deviation for this data (one decimal place). (2)

11.3 How many temperatures lie outside one standard deviation of the mean? (2)
[6]

QUESTION 12

Consider the following box and whisker plot:



The data set contains a total of nine numbers. The second and third number of the data set are the same. The seventh and eighth numbers are different. The mean for the data set is 40.

Write down a possible list of nine numbers which will result in the above box and whisker plot. [**6**]

TOTAL MARKS: 150

NAME:

DIAGRAM SHEET 1

QUESTION 3.1, 3.3

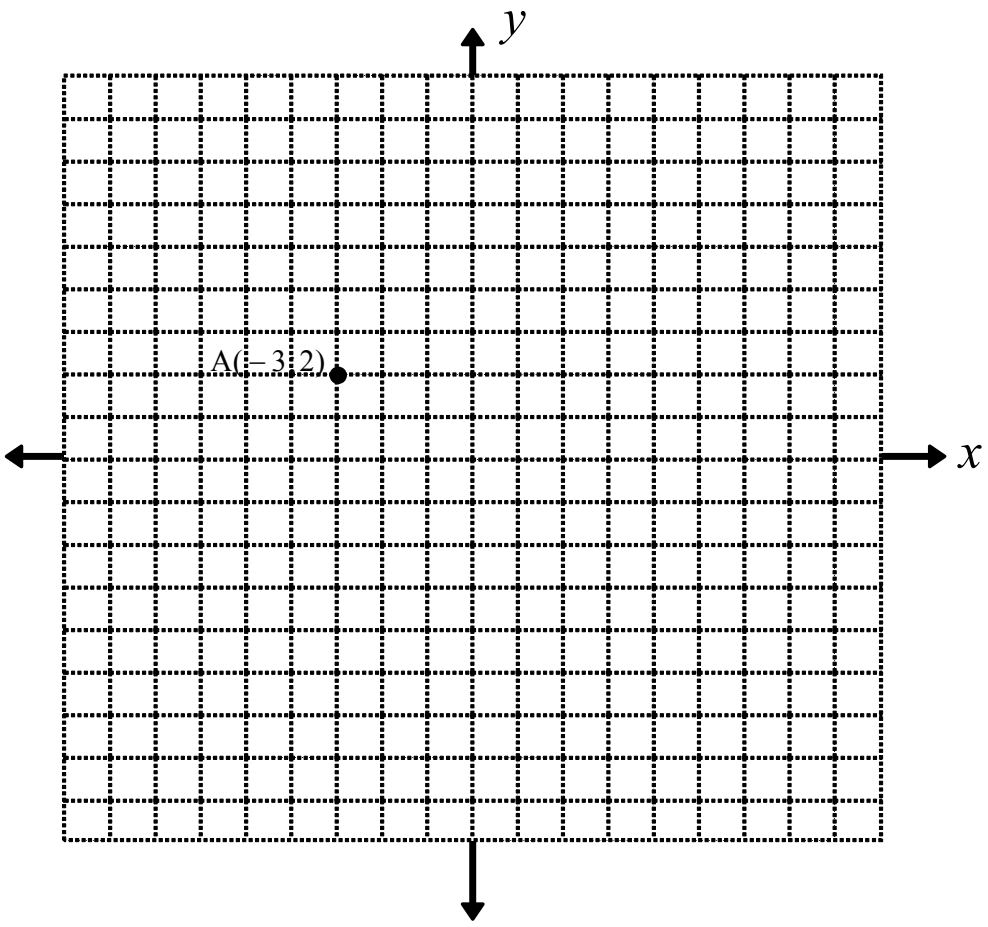


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DIAGRAM SHEET 2

QUESTION 8.2

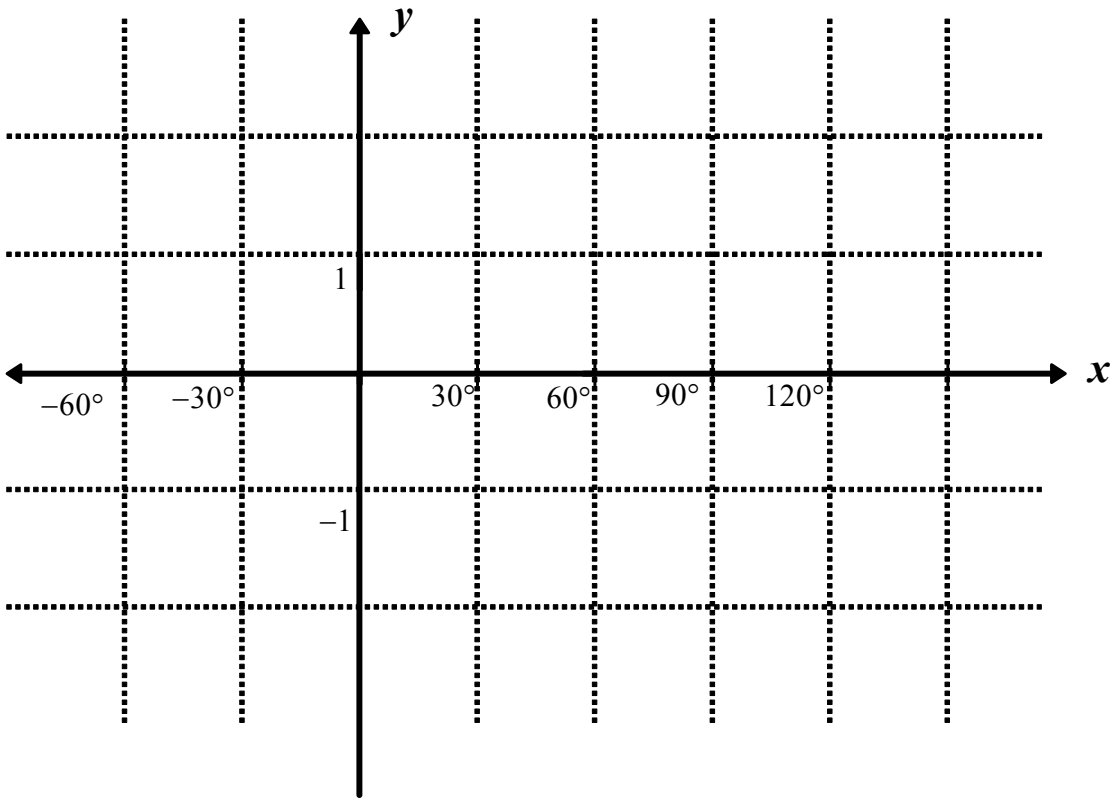

DIAGRAM SHEET 3

QUESTION 9

9.1

Number of kilometres	Number of motorists	Cumulative frequency
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$20 < x \leq 30$		9
$30 < x \leq 40$		13
$40 < x \leq 50$		26
$50 < x \leq 60$		42
$60 < x \leq 70$		50

9.2

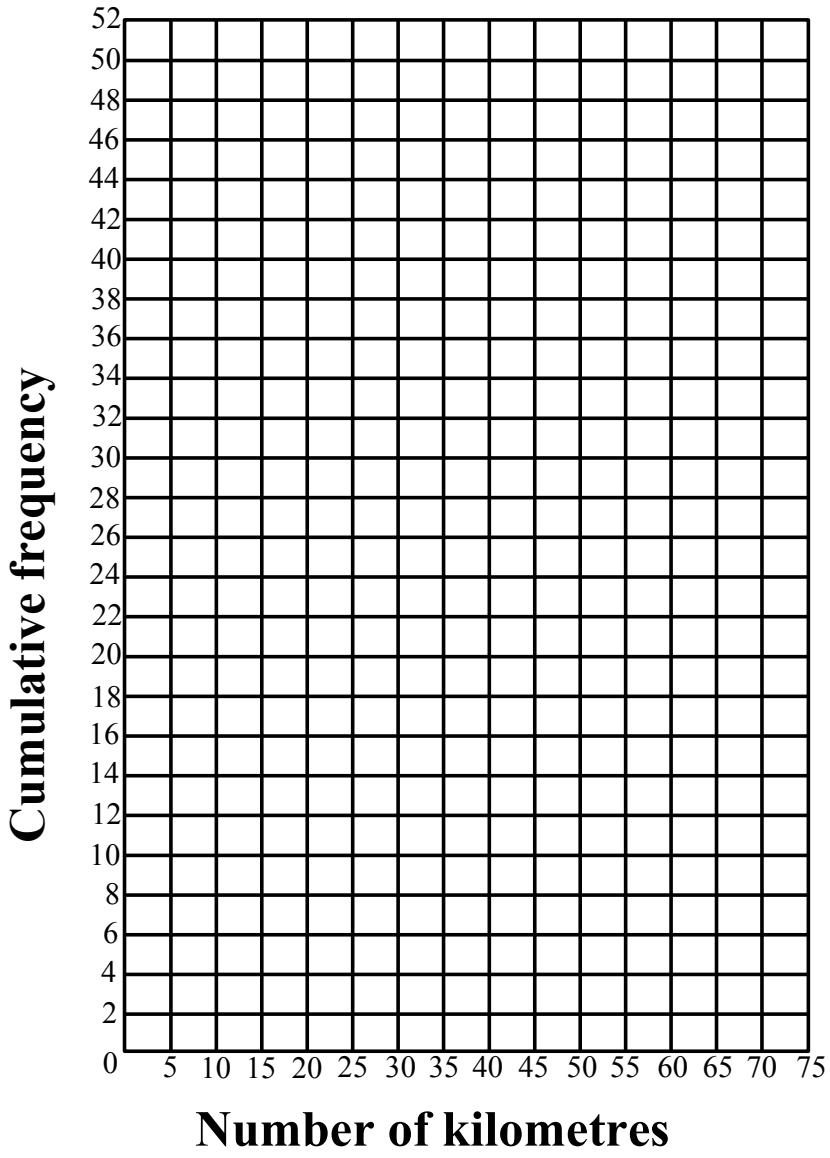
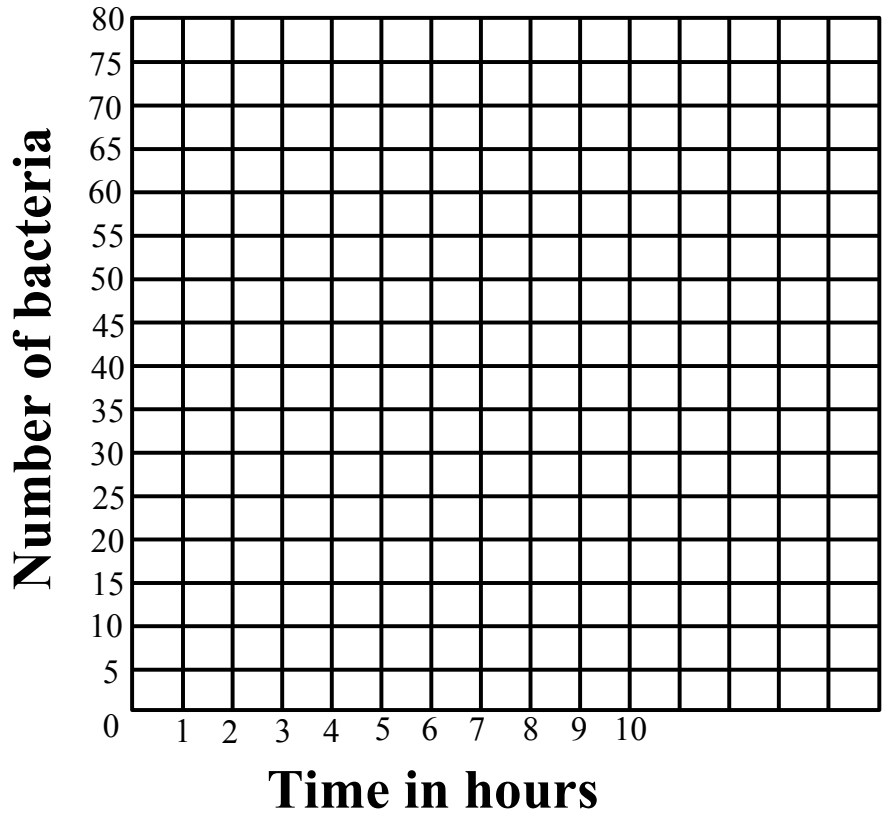


DIAGRAM SHEET 4

QUESTION 10

10.1