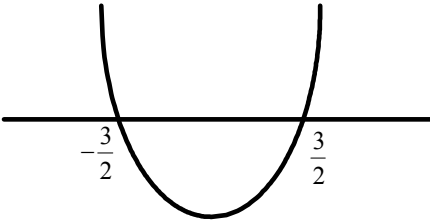


MATHEMATICS EXEMPLAR EXAMINATION
GRADE 12
PAPER 1

MEMORANDUM

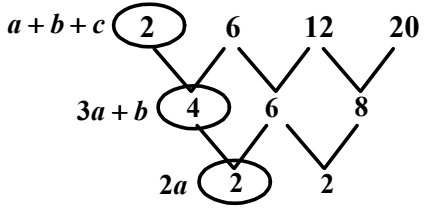
QUESTION 1

1.1.1	$\frac{x^2-1}{x+1} = 2$ $\therefore x^2-1 = 2(x+1)$ $\therefore x^2-1 = 2x+2$ $\therefore x^2-2x-3 = 0$ $\therefore (x-3)(x+1) = 0$ $\therefore x = 3 \quad \text{or} \quad x = -1$ But $x \neq -1$ $\therefore x = 3 \text{ is the solution}$ OR $\frac{x^2-1}{x+1} = 2$ $\therefore \frac{(x+1)(x-1)}{(x+1)} = 2$ $\therefore x-1 = 2 \quad \text{provided } x \neq -1$ $\therefore x = 3$	✓ standard form = 0 ✓ factorisation ✓ both answers ✓ excluding $x = -1$ (4)
1.1.2	$8 - (x-2)(x-3) = 0$ $\therefore 8 - (x^2 - 5x + 6) = 0$ $\therefore 8 - x^2 + 5x - 6 = 0$ $\therefore -x^2 + 5x + 2 = 0$ $\therefore x^2 - 5x - 2 = 0$ $\therefore x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-2)}}{2(1)}$ $\therefore x = \frac{5 \pm \sqrt{33}}{2}$ $\therefore x = 5,37 \quad \text{or} \quad x = -0,37$ – 1 for inaccurate rounding off for both answers.	✓ simplification ✓ standard form ✓ substitution into formula ✓ correct answers (4)

1.2.1	$4x^2 < 9$ $\therefore 4x^2 - 9 < 0$ $\therefore (2x+3)(2x-3) < 0$ $\therefore -\frac{3}{2} < x < \frac{3}{2}$ 	✓ factorisation ✓ endpoints ✓ inequality notation (3)
1.2.2	$x \in \{-1; 0; 1\}$	✓ correct answer (1)
1.3	$3x - y = 2$ $\therefore 3x - 2 = y$ $\therefore 3(3x - 2) + 9x^2 = 4$ $\therefore 9x - 6 + 9x^2 = 4$ $\therefore 9x^2 + 9x - 10 = 0$ $\therefore (3x+5)(3x-2) = 0$ $\therefore x = -\frac{5}{3} \quad \text{or} \quad x = \frac{2}{3}$ $\therefore y = -7 \quad \text{or} \quad y = 0$	✓ $3x - 2 = y$ ✓ substitution ✓ standard form ✓ factorisation ✓ both x-values ✓ $y = -7$ ✓ $y = 0$ (7)

QUESTION 2

2.1	Area of triangle 1: $\frac{1}{2}(2cm)(2cm) = (1)(2)cm^2$ Area of triangle 2: $\frac{1}{2}(4cm)(3cm) = (2)(3)cm^2$ Area of triangle 3: $\frac{1}{2}(6cm)(4cm) = (3)(4)cm^2$ Area of triangle 4: $\frac{1}{2}(8cm)(5cm) = (4)(5)cm^2$ The areas form the following pattern: $(1)(2); (2)(3); (3)(4); (4)(5); \dots$ Area of triangle n: $(n)(n+1)cm^2$ Area of triangle 100: $(100)(100+1)cm^2 = 10100cm^2$	✓ determining areas ✓ establishing pattern ✓ obtaining general term ✓ area of 100 th triangle (4)
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	 $2a = 2$ $3a + b = 4$ $a + b + c = 2$ $a = 1$ $\therefore 3(1) + b = 4$ $\therefore 1 + 1 + c = 2$ $\therefore b = 1$ $\therefore c = 0$ Area of triangle n: $(n^2 + n)cm^2$ Area of triangle 100: $\left[(100)^2 + 100\right]cm^2 = 10100cm^2$	
2.2	$n(n+1) = 240$ $\therefore n^2 + n - 240 = 0$ $\therefore (n+16)(n-15) = 0$ $\therefore n = -16$ or $n = 15$ But $n \neq -16$ $\therefore n = 15$ The 15th triangle will have an area of $240cm^2$	✓ equating general term to 240 ✓ factorising ✓ obtaining 15 triangles (3)

QUESTION 3

3.1.1	$\frac{1}{181} + \frac{2}{181} + \frac{3}{181} + \frac{4}{181} + \dots + \frac{180}{181}$ $a = \frac{1}{181} \quad d = \frac{1}{181} \quad n = 180$ $\therefore S_{180} = \frac{180}{2} \left[\frac{1}{181} + \frac{180}{181} \right] = 90[1] = 90$ OR $S_{180} = \frac{180}{2} \left[2 \left(\frac{1}{181} \right) + (179) \frac{1}{181} \right] = 90[1] = 90$	✓ correct a and d ✓ correct n ✓ S_n formula ✓ correct answer (4)
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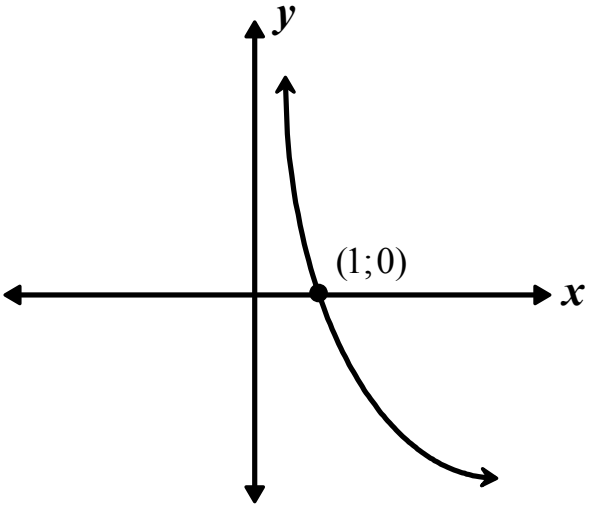
3.1.2	$\left(\frac{1}{2}\right) + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4}\right) + \dots + \left(\frac{1}{181} + \frac{2}{181} + \dots + \frac{180}{181}\right)$ $= \frac{1}{2} + 1 + 1\frac{1}{2} + 2 + \dots + 90$ $a = \frac{1}{2} \quad d = \frac{1}{2} \quad T_n = 90$ $\therefore \frac{1}{2} + (n-1)\frac{1}{2} = 90$ $\therefore 1 + n - 1 = 180$ $\therefore n = 180$ $\therefore S_{180} = \frac{180}{2} \left[\frac{1}{2} + 90 \right] = 90 \left[90\frac{1}{2} \right] = 8145$ OR $S_{180} = \frac{180}{2} \left[2\left(\frac{1}{2}\right) + (179)\left(\frac{1}{2}\right) \right] = 90 \left[90\frac{1}{2} \right] = 8145$	✓ simplifying fractions to get series ✓ $\frac{1}{2} + (n-1)\frac{1}{2} = 90$ ✓ $n = 180$ ✓ substitution into S_n formula to get 8145 (4)
3.2	$ar^5 = \sqrt{3}$ $ar^7 = \sqrt{27}$ $\therefore \frac{ar^7}{ar^5} = \frac{\sqrt{27}}{\sqrt{3}}$ $\therefore r^2 = \sqrt{\frac{27}{3}}$ $\therefore r^2 = \sqrt{9}$ $\therefore r^2 = 3$ $\therefore r = \sqrt{3} \quad (\text{terms are positive})$ $\therefore a(\sqrt{3})^5 = \sqrt{3}$ $\therefore a = \frac{\sqrt{3}}{(\sqrt{3})^5}$ $\therefore a = \frac{1}{(\sqrt{3})^4}$ $\therefore a = \frac{1}{(3^{\frac{1}{2}})^4}$ $\therefore a = \frac{1}{9}$	✓ $ar^5 = \sqrt{3}; ar^7 = \sqrt{27}$ ✓ dividing ✓ $r = \sqrt{3}$ ✓ correct working with surds ✓ $a = \frac{1}{9}$ (5)

3.3.1	$\sum_{n=1}^{\infty} 2\left(\frac{1}{2}x\right)^n$ $= 2\left(\frac{1}{2}x\right)^1 + 2\left(\frac{1}{2}x\right)^2 + 2\left(\frac{1}{2}x\right)^3 + 2\left(\frac{1}{2}x\right)^4 + \dots$ $= x + \frac{1}{2}x^2 + \frac{1}{4}x^3 + \frac{1}{8}x^4 + \dots$ The series converges for: $-1 < \frac{1}{2}x < 1$ $\therefore -2 < x < 2$	$\checkmark r = \frac{1}{2}x$ $\checkmark -1 < \frac{1}{2}x < 1$ $\checkmark -2 < x < 2$	(3)
3.3.2	$a = \frac{1}{2} \quad r = \frac{1}{2} \left(\frac{1}{2} \right) = \frac{1}{4}$ $\therefore S_{\infty} = \frac{\frac{1}{2}}{1 - \frac{1}{4}} = \frac{2}{3}$	$\checkmark a$ and r $\checkmark S_{\infty}$ formula $\checkmark \frac{2}{3}$	(3)
3.4	$4 + 6 + 9 + 13,5 + \dots$ $a = 4 \quad r = \frac{3}{2} \quad S_n = 2000\,000$ $(4) \left[\left(\frac{3}{2} \right)^n - 1 \right]$ $\therefore 2000\,000 = \frac{\left[\left(\frac{3}{2} \right)^n - 1 \right]}{\frac{3}{2} - 1}$ $\therefore 2000\,000 = 8 \left[\left(\frac{3}{2} \right)^n - 1 \right]$ $\therefore 250\,000 = \left(\frac{3}{2} \right)^n - 1$ $\therefore 250\,001 = \left(\frac{3}{2} \right)^n$ $\therefore \log_{\frac{3}{2}}(250\,001) = n$ $\therefore n = 30,65422881$ Malibongwe will be able to pay off the R2000 000 on the last day of March (31 days)	$\checkmark$ constant ratio $\checkmark$ correct substitution into the S_n formula $\checkmark$ use of logs $\checkmark n = 30,65422881$ $\checkmark$ 31 days	(5)

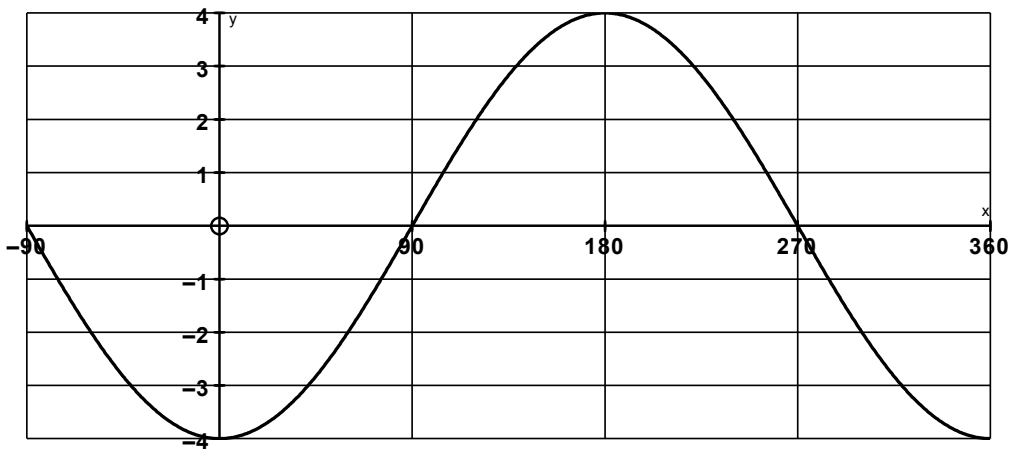
QUESTION 4

4.1.1	vertical: $x = -1$ horizontal: $y = 0$	✓ vertical asymptote ✓ horizontal asymptote (2)
4.1.2		✓ $x = -1$ ✓ $y = 0$ ✓ left branch ✓ coordinates on left branch ✓ right branch ✓ coordinates on right branch (6)
4.1.3	$y = \frac{2}{x+1-3} + 2$ $\therefore y = \frac{2}{x-2} + 2$	✓ denominator: $x - 2$ ✓ $+2$ (2)
4.1.4	Therefore $\frac{2}{x+1} \geq 1$ for $-1 < x \leq 1$	✓ $-1 < x$ ✓ $x \leq 1$ (2)

4.2.1	$f(x) = 2x^2$ where $x \geq 0$ OR $f(x) = 2x^2$ where $x \leq 0$	✓ $x \geq 0$ OR $x \leq 0$ (1)
4.2.2	 OR	✓ f ✓ f^{-1} (2)
4.2.3	$y = \left(\frac{1}{2}\right)^x$ $x = \left(\frac{1}{2}\right)^y$ $\therefore \log_{\frac{1}{2}} x = y$ $\therefore g^{-1}(x) = \log_{\frac{1}{2}} x$	✓ $x = \left(\frac{1}{2}\right)^y$ ✓ $g^{-1}(x) = \log_{\frac{1}{2}} x$ (2)

4.2.4		✓ shape ✓ (1;0) (2)
4.2.5	$\log_{\frac{1}{2}} x < 0$ for $x > 1$	✓ $x > 1$ (1)

QUESTION 5

5.1	$y = -2f(x)$ $\therefore y = -2(2 \cos x)$ $\therefore y = -4 \cos x$	
		✓ amplitude ✓ domain (2)

5.2	Amplitude is 4	✓ amplitude (1)
5.3	$y = f\left(\frac{x}{2}\right)$ $\therefore y = 2 \cos\left(\frac{1}{2}x\right)$ period is $\frac{360^\circ}{\frac{1}{2}} = 720^\circ$	✓ 720° (1)
5.4	$g(x) = f(x) - 2$ $g(x) = 2 \cos x - 2$ maximum is 0	✓ max (1)

QUESTION 6

6.1	$y = a(x + p)^2 + q$ $\therefore y = a(x + 1)^2 + 6$ Substitute (0 ; 2) : $\therefore 2 = a(0 + 1)^2 + 6$ $\therefore 2 = a + 6$ $\therefore 2 - 6 = a$ $\therefore a = -4$ $\therefore f(x) = -4(x + 1)^2 + 6$	✓ $y = a(x + 1)^2 + 6$ ✓ Substitute (0 ; 2) ✓ $a = -4$ ✓ $f(x) = -4(x + 1)^2 + 6$ (4)
6.2	Range: $y \in (-\infty; 6]$	✓ $y \in (-\infty; 6]$ (1)
6.3	$y = -4(x + 1)^2 + 6 \quad (f)$ $\therefore -y = -4(x + 1)^2 + 6 \quad (g)$ $\therefore g(x) = 4(x + 1)^2 - 6$	✓ $g(x) = 4(x + 1)^2 - 6$ (1)

QUESTION 7

7.1.1	$i_{eff} = \left(1 + \frac{0,08}{2}\right)^2 - 1$ $\therefore i_{eff} = 0,0816$	✓ formula ✓ 0,0816 (2)
7.1.2	$P = 100\,000(1,0816)^{-4}$ $\therefore P = R73\,069,02$ OR $P = 100\,000\left(1 + \frac{0,08}{2}\right)^{-8}$ $\therefore P = R73\,069,02$	✓ formula ✓ answer (2)
7.2	$90\,000 = 200\,000(1 - 0,08)^n$ $\therefore \frac{9}{20} = 0,92^n$ $\therefore \log_{0,92}\left(\frac{9}{20}\right) = n$ $\therefore n = 9,576544593$ 9 years and 7 months	✓ correct substitution into formula ✓ use of logs ✓ answer (3)

QUESTION 8

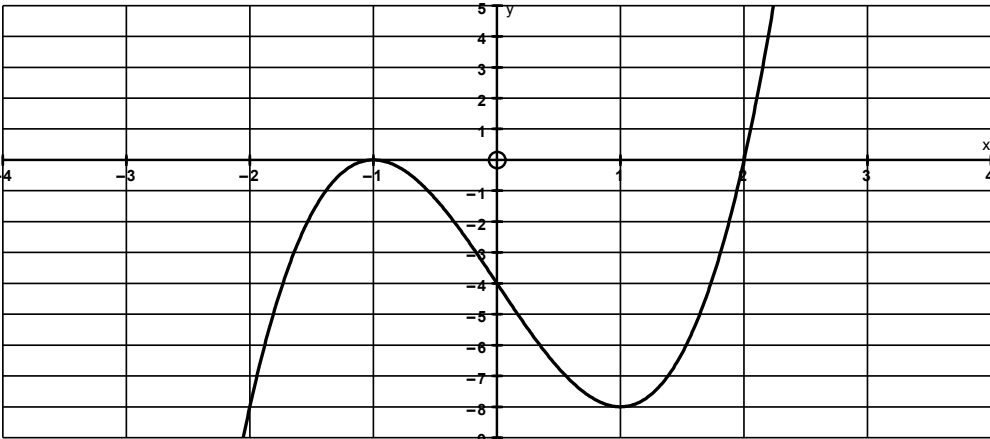
8.1	$2\,500\,000 = \frac{x[(1,0075)^{361} - 1]}{0,0075}$ $\therefore \frac{2\,500\,000 \times 0,0075}{[(1,0075)^{361} - 1]} = x$ $\therefore x = R1354,67$	✓ correct formula ✓ $n = 361$ ✓ $\frac{0,09}{12} = 0,0075$ ✓ $F = 2\,500\,000$ ✓ answer (5)
8.2	$2\,500\,000 = \frac{x\left[1 - \left(1 + \frac{0,07}{12}\right)^{-240}\right]}{\left(\frac{0,07}{12}\right)}$ $\therefore \frac{2\,500\,000 \times \left(\frac{0,07}{12}\right)}{\left[1 - \left(1 + \frac{0,07}{12}\right)^{-240}\right]} = x$ $\therefore x = R19\,382,47$	✓ correct formula ✓ $n = 240$ ✓ $\frac{0,07}{12}$ ✓ $P = 2\,500\,000$ ✓ answer (5)

QUESTION 9

9.1	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-2(x+h)^2 + 1 - (-2x^2 + 1)}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-2(x^2 + 2xh + h^2) + 1 + 2x^2 - 1}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-2x^2 - 4xh - 2h^2 + 1 + 2x^2 - 1}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{h(-4x - 2h)}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} (-4x - 2h)$ $\therefore f'(x) = -4x - 2(0)$ $\therefore f'(x) = -4x$ - 1 for inaccurate notation	$\checkmark -2(x+h)^2 + 1$ $\checkmark -(-2x^2 + 1)$ $\checkmark -2x^2 - 4xh - 2h^2$ $\checkmark \frac{h(-4x - 2h)}{h}$ $\checkmark -4x$ (5)
9.2	$y = \left(2\sqrt{x} - \frac{1}{3x}\right)^2$ $\therefore y = 4x - \frac{4\sqrt{x}}{3x} + \frac{1}{9x^2}$ $\therefore y = 4x - \frac{4x^{\frac{1}{2}}}{3x} + \frac{1}{9}x^{-2}$ $\therefore y = 4x - \frac{4}{3}x^{-\frac{1}{2}} + \frac{1}{9}x^{-2}$ $\therefore \frac{dy}{dx} = 4 - \frac{4}{3} \times -\frac{1}{2}x^{-\frac{3}{2}} + \frac{1}{9} \times -2x^{-3}$ $\therefore \frac{dy}{dx} = 4 + \frac{2}{3}x^{-\frac{3}{2}} - \frac{2}{9}x^{-3}$ $\therefore \frac{dy}{dx} = 4 + \frac{2}{3x^{\frac{3}{2}}} - \frac{2}{9x^3}$ - 1 for inaccurate notation	$\checkmark 4x - \frac{4}{3}x^{-\frac{1}{2}} + \frac{1}{9}x^{-2}$ $\checkmark \checkmark \checkmark 4 + \frac{2}{3x^{\frac{3}{2}}} - \frac{2}{9x^3}$ (4)

QUESTION 10

10.1	$f(x) = ax^3 + bx$ $\therefore f'(x) = 3ax^2 + b$ $\therefore f'(-1) = 3a(-1)^2 + b$ $\therefore f'(-1) = 3a + b$ Now $y = x + 4$ $m_t = 1$ $1 = 3a + b$ Now at $x = -1$ $y = -1 + 4 = 3$ $\therefore$ Substitute $(-1; 3)$ into the equation of f : $f(-1) = a(-1)^3 + b(-1)$ $\therefore 3 = -a - b$ $\therefore a + b = -3$ Solving simultaneously: $a = 2$ and $b = -5$	$\checkmark f'(x) = 3ax^2 + b$ $\checkmark m_t = 1$ $\checkmark 1 = 3a + b$ $\checkmark a + b = -3$ $\checkmark a = 2$ $\checkmark b = -5$ (6)
10.2.1	y -intercept: $(0; -4)$ x -intercepts: $0 = 2x^3 - 6x - 4$ $\therefore 0 = x^3 - 3x - 2$ $\therefore 0 = (x+1)(x^2 - x - 2)$ $\therefore 0 = (x+1)(x-2)(x+1)$ $\therefore x = -1$ or $x = 2$ $(-1; 0)$ $(2; 0)$	$\checkmark y$ -intercept $\checkmark 0 = 2x^3 - 6x - 4$ $\checkmark (x+1)(x^2 - x - 2) = 0$ $\checkmark (x+1)(x-2)(x+1)$ $\checkmark x$ -intercepts (5)
10.2.2	$f(x) = 2x^3 - 6x - 4$ $\therefore f'(x) = 6x^2 - 6$ $\therefore 0 = 6x^2 - 6$ $\therefore 0 = x^2 - 1$ $\therefore x = \pm 1$ $f(1) = -8$ $f(-1) = 0$ Turning points are $(1; -8)$ and $(-1; 0)$	$\checkmark f'(x) = 6x^2 - 6$ $\checkmark 0 = 6x^2 - 6$ $\checkmark x = \pm 1$ $\checkmark (1; -8)$ and $(-1; 0)$ (4)

10.2.3 	✓ intercepts with the axes ✓ turning points ✓ shape (3)
10.2.4 $f'(x) = 6x^2 - 6$ $\therefore f''(x) = 12x$ $\therefore 0 = 12x$ $\therefore x = 0$ $f(0) = -4$ Point of inflection at $(0; -4)$	✓ $f''(x) = 12x$ ✓ $x = 0$ ✓ $(0; -4)$ (3)
10.2.4 $p > 0 \quad \text{or} \quad p < -8$	✓ $p > 0$ ✓ $p < -8$ (2)

QUESTION 11

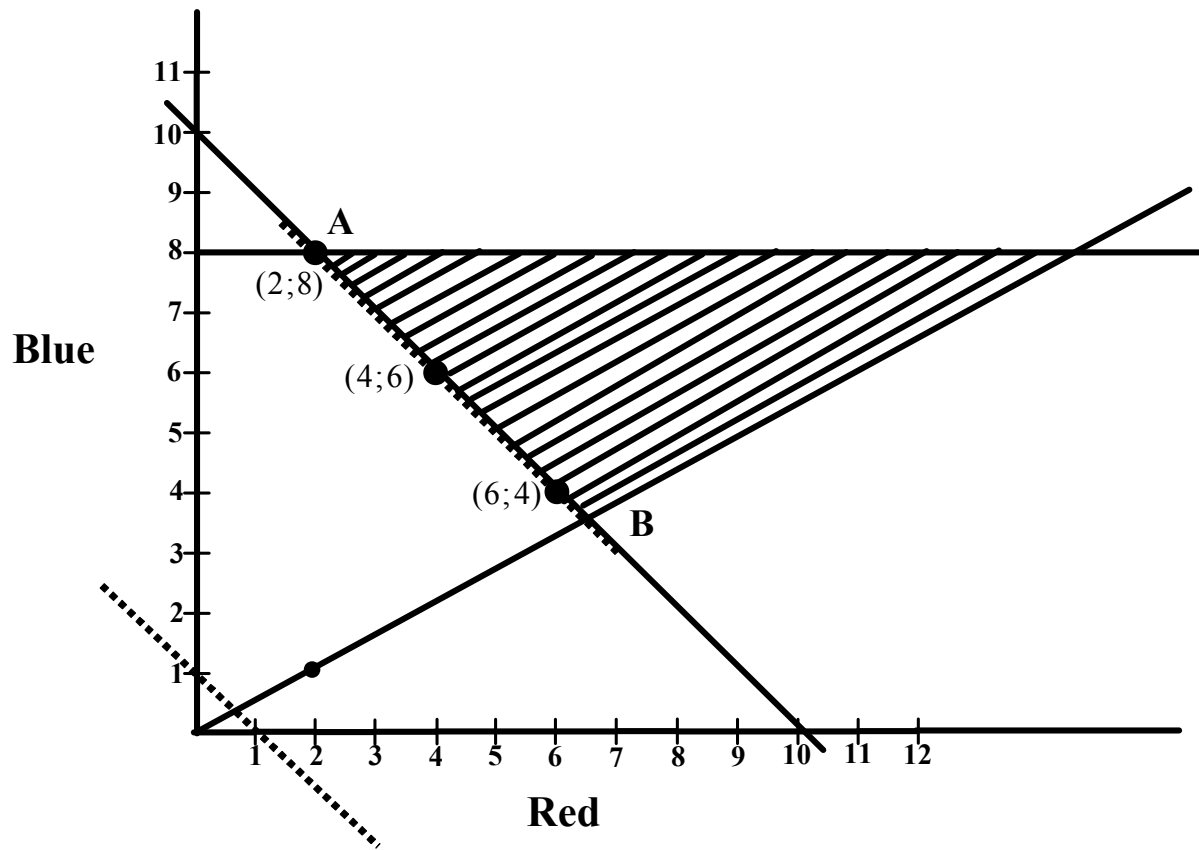
11.1	$A = (2x)(3x) + 2(y)(3x) + 2(y)(2x)$ $\therefore 200 = 6x^2 + 6xy + 4xy$ $\therefore 200 = 6x^2 + 10xy$ $\therefore 100 = 3x^2 + 5xy$ $\therefore 100 - 3x^2 = 5xy$ $\therefore \frac{100}{5x} - \frac{3x^2}{5x} = y$ $\therefore y = \frac{20}{x} - \frac{3x}{5}$	✓ $6x^2 + 6xy + 4xy$ ✓ $200 =$ ✓ arriving at answer (3)
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11.2	$V = (2x)(3x)(y)$ $\therefore V = (2x)(3x)\left(\frac{20}{x} - \frac{3x}{5}\right)$ $\therefore V = (6x^2)\left(\frac{20}{x} - \frac{3x}{5}\right)$ $\therefore V = 120x - \frac{18x^3}{5}$	$\checkmark V = (2x)(3x)(y)$ $\checkmark V = 120x - \frac{18x^3}{5}$ (2)
11.3	$V(x) = 120x - \frac{18}{5}x^3$ $\therefore V'(x) = 120 - \frac{18}{5} \times 3x^2$ $\therefore 0 = 120 - \frac{54}{5}x^2$ $\therefore 0 = 600 - 54x^2$ $\therefore 54x^2 = 600$ $\therefore x^2 = \frac{600}{54}$ $\therefore x^2 = \frac{100}{9}$ $\therefore x = \frac{10}{3}$	$\checkmark V'(x)$ $\checkmark V'(x) = 0$ $\checkmark x = \frac{10}{3}$ (3)

QUESTION 12

12.1	$x + y \geq 10$ $y \geq \frac{1}{2}x$ $y \leq 8$	$\checkmark x + y \geq 10$ $\checkmark y \geq \frac{1}{2}x$ $\checkmark y \leq 8$ (3)
12.2	see next page	
12.3	$C = 40x + 40y$ $\therefore 40x + 40y = C$ $\therefore 40y = -40x + C$ $\therefore y = -1x + \frac{C}{40}$	$\checkmark C = 40x + 40y$ $\checkmark$ search line on diagram $\checkmark (2;8)$ $\checkmark (4;6)$ $\checkmark (6;4)$ (5)

12.2



- ✓ $x + y \geq 10$
- ✓ $y \geq \frac{1}{2}x$
- ✓ $y \leq 8$
- ✓ feasible region

(4)