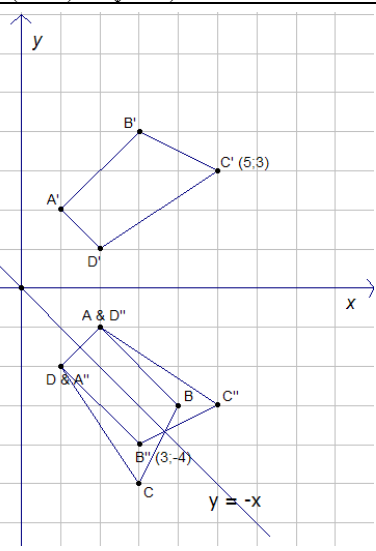
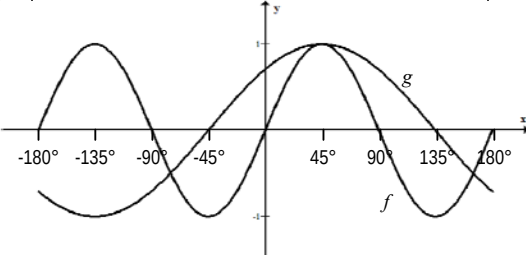
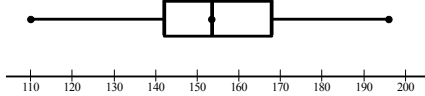




MATHEMATICS PAPER 2 MEMORANDUM

1.1	$M = \left(\frac{-1+3}{2}, \frac{3+(-1)}{2} \right) \checkmark$ $M = (1; 1) \checkmark$	(2)		(diagonals bisect) $\checkmark$ But $EH = EF \checkmark$ $\therefore EFGH$ is a rhombus.	
1.2	Midpoint $FH = \left(\frac{4+(-2)}{2}, \frac{4+(-2)}{2} \right) \checkmark$ $= (1; 1) \checkmark$ $\therefore$ midpoint $FH =$ midpoint EG $\therefore$ lines bisect each other.	(2)		1.4 $m_{EG} = -1$ from above $\checkmark$ $\therefore y = -x + c$ Substitute $(-1; 3)$: $\checkmark$ $3 = -(-1) + c$ $2 = c$ $\therefore y = -x + 2 \checkmark$	(3)
1.3	$m_{EG} = \frac{3-(-1)}{-1-3} = \frac{4}{-4} = -1 \checkmark$ $m_{FH} = \frac{4-(-2)}{4-(-2)} = \frac{6}{6} = 1 \checkmark$ $\therefore m_{EG} \times m_{FH} = -1$ $\Rightarrow EG \perp FH \checkmark$ $\therefore$ using 1.2, diagonals of $EFGH$ bisect at $90^\circ \checkmark$ $\therefore EFGH$ is a rhombus. OR $length_{EH} = \sqrt{(-1-(-2))^2 + (3-(-2))^2}$ $= \sqrt{1+25}$ $= \sqrt{26}$ $\checkmark$ $length_{EF} = \sqrt{(-1-4)^2 + (3-4)^2}$ $= \sqrt{25+1} \checkmark$ $= \sqrt{26}$ $\therefore$ using 1.2, $EFGH$ is a parallelogram	(4)		1.5 $y = -x + 2$ Let $x = \frac{5}{2}$. $y = -\frac{5}{2} + 2 \checkmark$ $y = -\frac{1}{2} \checkmark$ $\therefore \left(\frac{5}{2}; -\frac{3}{4} \right)$ does not lie on the line. $\checkmark$	(3)
				1.6 $m_{FH} = \frac{4-(-1)}{4-3} = \frac{5}{1} = 5 \checkmark$ $\therefore \tan \alpha = 5$ $A = \tan^{-1}(5) \checkmark$ $= 78,69^\circ \checkmark$	(3)
				1.7 $length_{EG} = \sqrt{(3-(-1))^2 + (-1-3)^2} \checkmark$ $= \sqrt{32}$ $length_{HM} = \sqrt{(1-(-2))^2 + (1-(-2))^2} \checkmark$ $= \sqrt{18}$ $\therefore \text{area } \Delta EGH = \frac{1}{2} \sqrt{18} \times \sqrt{32} \checkmark$	

	$= 12 \text{ units}^2 \checkmark$	(4)
1.8	$G \rightarrow H$ back 5 down 1 $\therefore E \rightarrow P$ is the same $\checkmark$ $\therefore P(-6; 2) \checkmark$ (or equivalent)	(2)
2.1	ΔEDC is right angled at C (tangent, radius) $ED^2 = EC^2 + DC^2 \checkmark$ $13^2 = 12^2 + DC^2 \checkmark$ $25 = DC^2 \checkmark$ $5 = DC \checkmark$	(2)
2.2	$DC^2 = (a-1)^2 + (2-(-1))^2 \checkmark$ $25 = a^2 - 2a + 1 + 9 \checkmark$ $0 = a^2 - 2a - 15 \checkmark$ $a = -3; a = 5 \checkmark$ By inspection, for this sketch $a = 5 \checkmark$	(5)
2.3	$m_{DC} = \frac{2-(-1)}{5-1} = \frac{3}{4} \checkmark$ $m_{\text{tangent}} = -\frac{4}{3} \checkmark$ $y = -\frac{4}{3}x + c$ Substitute $(1; -1)$: $-1 = -\frac{4}{3}x + c \checkmark$ $\frac{1}{3} = c \checkmark$ $y = -\frac{4}{3}x + \frac{1}{3}$	(4)
2.4	y -axis is tangent to circle at A. $\therefore AD$ is horizontal $\therefore A(0; 2) \checkmark \checkmark$ (inspection)	(2)
2.5	$(x-a)^2 + (y-b)^2 = c^2 \checkmark$ Substitute $(1; -1)$ and $(0; 2)$: $(0-1)^2 + (2-(-1))^2 = c^2 \checkmark \checkmark$ $10 = c^2 \checkmark$ $(x-1)^2 + (y+1)^2 = 10 \checkmark$	(5)
3.1 and 3.2		(10)
3.3	$P(x; y)$ $\rightarrow P'(-x; -y)$ $\rightarrow$ final image $(-\frac{4}{5}x; -\frac{4}{5}y) \checkmark \checkmark$	(2)
3.4	Linear factor $\frac{4}{5}$ $\Rightarrow$ area factor $\frac{16}{25} \checkmark$ $\therefore \text{area} = \frac{16}{25}p \text{ units}^2 \checkmark$	(2)
4.1	$(x \cos \theta - y \sin \theta; x \sin \theta + y \cos \theta) \checkmark$ $= (6 \cos 60^\circ - 3 \sin 60^\circ; 6 \sin 60^\circ + 3 \cos 60^\circ) \checkmark \checkmark$ $= \left(\frac{6-3\sqrt{3}}{2}; \frac{3+6\sqrt{3}}{2} \right) \checkmark \checkmark$ OR $= \left(3 - \frac{3\sqrt{3}}{2}; 3\sqrt{3} + \frac{3}{2} \right)$	(5)
4.2	$(6; 3) \rightarrow (3; 6) \rightarrow (3; -6) \checkmark \checkmark$	(2)
5.1.1	$\tan \theta = \frac{y}{x} = \frac{b}{a} \checkmark$	(1)
5.1.2	$\cos(-\theta) = \cos \theta = \frac{x}{r} \checkmark \checkmark$ $r = \sqrt{a^2 + b^2}$ (Pythagoras) $\checkmark$ $\therefore \cos(-\theta) = \frac{a}{\sqrt{a^2 + b^2}} \checkmark$	(4)
5.2.1	$\cos 53^\circ$ $= \sin(90^\circ - 53^\circ)$ $= \sin 37^\circ \checkmark$ $= k \checkmark$	(2)
5.2.2	$\sin(-74^\circ)$ $= -\sin 74^\circ$ $= -2 \sin 37^\circ \cos 37^\circ \checkmark \checkmark$ $= -2k \sqrt{1-k^2} \checkmark$	(4)
5.3.1	LHS $= \frac{\sin a \sin 2a}{\cos a} + \cos 2a \checkmark$ $= \frac{\sin a 2 \sin a \cos a}{\cos a} + 1 - 2 \sin^2 a \checkmark \checkmark$ $= 1 \checkmark$ $= \text{RHS}$	(4)
5.3.2	LHS $= \frac{\sin 234^\circ}{\cos 36^\circ} - \frac{\sin(x-90^\circ) \cos(90^\circ-2x)}{\sin(x-360^\circ)}$ $= \frac{-\sin 54^\circ}{\cos 36^\circ} - \frac{(-\cos x) \sin 2x}{\sin x} \checkmark \checkmark \checkmark \checkmark$ $= \frac{-\sin 54^\circ}{\sin 54^\circ} - \frac{\cos x \cdot 2 \sin x \cos x}{\sin x} \checkmark \checkmark$ $= -1 + 2 \cos^2 x \checkmark$ $= \cos 2x$ $= \text{RHS}$	(8)
5.4	$3 \cos^2 x + 5 \sin x = 3$ $3(1 - \sin^2 x) + 5 \sin x = 3 \checkmark$ $3 - 3 \sin^2 x + 5 \sin x = 3$ $0 = 3 \sin^2 x - 5 \sin x$ $0 = \sin x (3 \sin x - 5) \checkmark$ $\sin x = 0$ or $\sin x = \frac{5}{3} \checkmark \checkmark$ $\therefore x = 0^\circ + n180^\circ$ or x is undefined	(6)

	$(n \in \mathbb{Z}) \checkmark \checkmark$																												
6.1	$\hat{A}CB = 110^\circ - 50^\circ$ $= 60^\circ \checkmark$ $\therefore AB^2 = 150^2 + 260^2 - (2.150.260)\cos 60^\circ$ $\checkmark$ $AB^2 = 51\,100 \checkmark$ $\therefore AB = 226,05 \checkmark$ $AB = 226 \text{ km} \checkmark$	(5)																											
6.2.1	$\hat{C}DB = 180^\circ - (\theta + 30^\circ) \checkmark$	(1)																											
6.2.2	In $\triangle ABC$: $\tan \theta = \frac{p}{CB}$ $CB \tan \theta = p \dots\dots\dots(i) \checkmark$ In $\triangle CBD$: $\frac{CB}{\sin[180^\circ - (\theta + 30^\circ)]} = \frac{8}{\sin \theta} \checkmark$ $\frac{CB}{\sin(\theta + 30^\circ)} = \frac{8}{\sin \theta} \checkmark$ $CB = \frac{8\sin(\theta + 30^\circ)}{\sin \theta} \dots\dots\dots(ii) \checkmark$ Combining (i) and (ii): $p = \frac{8\sin(\theta + 30^\circ)}{\sin \theta} \cdot \tan \theta$ $p = \frac{8\sin(\theta + 30^\circ)}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta} \checkmark$ $p = \frac{8\sin(\theta + 30^\circ)}{\cos \theta} \checkmark$	(6)																											
7.1	$\sin 2x = \cos(x - 45^\circ)$ $\sin 2x = \sin[90^\circ - (x - 45^\circ)]$ $\sin 2x = \sin(135^\circ - x) \checkmark$ $\therefore 2x = 135^\circ - x + n360^\circ (n \in \mathbb{Z}) \checkmark$ $3x = 135^\circ + n360^\circ$ $x = 45^\circ + n120^\circ \checkmark$ or $2x = 180^\circ - (135^\circ - x) + n360^\circ (n \in \mathbb{Z}) \checkmark$ $2x = 45^\circ + x + n360^\circ$ $x = 45^\circ + n360^\circ \checkmark$ $\therefore \text{for } x \in [-180^\circ; 180^\circ]$ $x = 45^\circ; 165^\circ; -75^\circ \checkmark \checkmark \checkmark$	(8)																											
7.2		(6)																											
7.3.1	$g(x) \leq f(x) \text{ for } [-180^\circ; 90^\circ]$ $\Rightarrow -180^\circ \leq x \leq -75^\circ \checkmark \checkmark \checkmark$	(3)																											
7.3.2	$\frac{f(x)}{g(x)}$ undefined $\Rightarrow g(x) = 0 \checkmark$ $\therefore x = -45^\circ \text{ only for } [-180^\circ; 90^\circ] \checkmark$	(2)																											
8.1.1	$\bar{x} = 65,27 \checkmark \checkmark \checkmark$ Using stats mode on calculator or manually	(3)																											
8.1.2	$SD = 8,71 \checkmark \checkmark$	(2)																											
8.1.3	Upper boundary $= 65,27 + 8,71$ $= 73,98 \checkmark$ Lower boundary $= 65,27 - 8,71$ $= 56,56 \checkmark$ $\therefore \text{reject } 50; 45; 80$ i.e. 3 bags would be rejected $\checkmark$	(3)																											
8.2.1	Ordered list 11 000 12 600 14 200 $Q_1 = 14\,200$ 14 500 15 300 Median = 15 350 15 400 16 500 16 800 $Q_3 = 16\,800$ 18 600 19 600 $\therefore \text{Minimum} = 11\,000 \checkmark$ Lower quartile = 14 200 $\checkmark$ Median = 15 350 $\checkmark$ Upper quartile = 16 800 $\checkmark$ Maximum = 19 600 $\checkmark$	(5)																											
8.2.2	Scale in 100s 	(3)																											
8.2.3	Maximum per day = $\frac{19\,600}{28} = 700$ patients per day $\checkmark$ Minimum per day = $\frac{11\,000}{28} = 392$ patients per day $\checkmark$	(2)																											
9.1	<table border="1"> <thead> <tr> <th>Marks</th><th>F</th><th>CF</th></tr> </thead> <tbody> <tr> <td>$20 \leq x \leq 29$</td><td>4</td><td>4</td></tr> <tr> <td>$30 \leq x \leq 39$</td><td>12</td><td>16</td></tr> <tr> <td>$40 \leq x \leq 49$</td><td>30</td><td>46</td></tr> <tr> <td>$50 \leq x \leq 59$</td><td>82</td><td>128</td></tr> <tr> <td>$60 \leq x \leq 69$</td><td>55</td><td>183</td></tr> <tr> <td>$70 \leq x \leq 79$</td><td>35</td><td>218</td></tr> <tr> <td>$80 \leq x \leq 89$</td><td>24</td><td>242</td></tr> <tr> <td>$90 \leq x \leq 100$</td><td>3</td><td>245</td></tr> </tbody> </table>	Marks	F	CF	$20 \leq x \leq 29$	4	4	$30 \leq x \leq 39$	12	16	$40 \leq x \leq 49$	30	46	$50 \leq x \leq 59$	82	128	$60 \leq x \leq 69$	55	183	$70 \leq x \leq 79$	35	218	$80 \leq x \leq 89$	24	242	$90 \leq x \leq 100$	3	245	(2)
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9.2	✓ ✓ ✓ ✓	(4)
9.3	Median at $\frac{245 + 1}{2} = 123$ Median approximately 59 ✓ ✓	(2)
9.4	Data is grouped, so original raw data is lost. ∴ mean or median will be approximate. ✓ ✓	(2)