


MATHEMATICS PAPER 1 MEMORANDUM

$$\begin{aligned}
 1.1.1 \quad & 2x^2 + 7x = 30 \\
 & 2x^2 + 7x - 30 = 0 \quad \checkmark \\
 & (2x - 5)(x + 6) = 0 \quad \checkmark \\
 & \therefore x = \frac{5}{2} \text{ or } x = -6 \quad \checkmark \checkmark
 \end{aligned}$$

$$\begin{aligned}
 1.1.2 \quad & 2x(x - 2) - 5 = 0 \\
 & 2x^2 - 4x - 5 = 0 \quad \checkmark \\
 & x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 & x = \frac{4 \pm \sqrt{(-4)^2 - 4 \times (2) \times (-5)}}{2 \times 2} \quad \checkmark \\
 & \therefore x = \frac{4 + \sqrt{56}}{4} \text{ or } x = \frac{4 - \sqrt{56}}{4} \quad \checkmark \checkmark
 \end{aligned}$$

$$\begin{aligned}
 1.1.3 \quad & \text{Or } x = 2,87 \text{ or } x = -0,87 \\
 & 4x^2 + 7x - 2 < 0 \\
 & (4x - 1)(x + 2) < 0 \quad \checkmark \checkmark \\
 & \begin{array}{c} + \qquad \qquad \qquad - \qquad \qquad \qquad + \\ -2 \qquad \qquad \qquad \frac{1}{4} \end{array}
 \end{aligned}$$

$$-2 < x < \frac{1}{4} \quad \checkmark \checkmark$$

$$\begin{aligned}
 1.2.1 \quad & 2x + 6 - y = 0 \\
 & 2x + 6 = y \\
 & y + 3x^2 = 8x + 3 \\
 & y = -3x^2 + 8x + 3 \quad \checkmark
 \end{aligned}$$

(4)

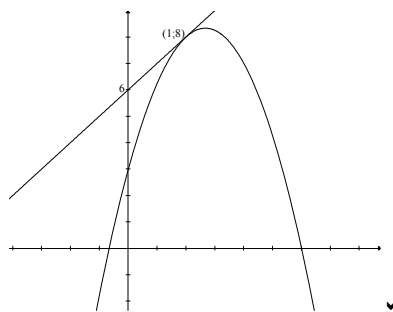
(4)

(4)

(5)

$$\begin{aligned}
 & \therefore 2x + 6 = -3x^2 + 8x + 3 \quad \checkmark \\
 & 3x^2 - 6x + 3 = 0 \\
 & x^2 - 2x + 1 = 0 \\
 & (x - 1)^2 = 0 \quad \checkmark \\
 & \therefore x = 1 \quad \checkmark \\
 & \therefore y = 2x + 6 = 8 \quad \checkmark
 \end{aligned}$$

1.2.2



(1; 8) is the point of contact between the straight line $y = 2x + 6$ and the parabola $y = -3x^2 + 8x + 3$. There is one point of contact.

$\Rightarrow$ line is a tangent to the curve. $\checkmark \checkmark$ (3)

2.1

$$P = \frac{x[1 - (1+i)^{-n}]}{i} \quad \checkmark$$

$$800\,000 = \frac{7500 \left[1 - \left(1 + \frac{9}{100} \right)^{-n} \right]}{\frac{9}{12}} \quad \checkmark \checkmark$$

$$800\,000 = \frac{7500 \left[1 - \left(1 + \frac{3}{400} \right)^{-n} \right]}{\frac{3}{400}}$$

$$800\,000 \times \frac{3}{400} \div 7\,500 = 1 - \left(\frac{403}{400} \right)^{-n}$$

$$\frac{20}{25} = 1 - \left(\frac{403}{400} \right)^{-n} \quad \checkmark$$

$$\left(\frac{403}{400} \right)^{-n} = \frac{5}{25} = \frac{1}{5}$$

$$-n = \left(\frac{\log \frac{1}{5}}{\log \frac{403}{400}} \right) \quad \checkmark$$

$$-n = -215,4 \text{ months} \quad \checkmark$$

$\therefore$ It will take 18 years to pay back the loan. $\checkmark$ (7)

(Simplifications at various stages are not necessary.)

$$2.2.1 \quad A = P(1+i)^n \quad \checkmark$$

$$A = 1\,500\,000 \left(1 - \frac{18}{100} \right)^5 \quad \checkmark$$

$$A = 556\,109,76 \quad \checkmark$$

$\therefore$ They are worth R556 109,76. (3)

$$2.2.2 \quad A = P(1+i)^n \quad \checkmark$$

$$A = 1\,500\,000 \left(1 + \frac{4.5}{100} \right)^5 \quad \checkmark$$

$$A = 1\,869\,272,91 \quad \checkmark$$

$\therefore$ Sinking fund required is

$$R1\,869\,272,91 - R556\,109,76$$

$$= R1\,313\,163,15 \quad \checkmark \quad (4)$$

$$2.2.3 \quad F = \frac{x \left[(1+i)^n - 1 \right]}{i} \quad \checkmark$$

$$1313163,15 = \frac{x \left[\left(1 + \frac{1}{12} \times 6\% \right)^{5 \times 12} - 1 \right]}{\frac{1}{12} \times 6\%}$$

$\checkmark \checkmark$

$$x = \frac{1313163,15 \times \frac{1}{12} \times 6\%}{\left[\left(1 + \frac{1}{12} \times 6\% \right)^{60} - 1 \right]} \quad \checkmark$$

$$x = 18\,821,30465 \quad \checkmark$$

$\therefore$ They will need to pay R18 821,30 per month.

(5)

3.1.1 Quadratic sequence $\checkmark$

(1)

3.1.2

T_0	T_1	T_2	T_3	T_4	T_5
9	9	11	15	21	29

Second difference = 2

$$\therefore 2a = 2$$

$$a = 1 \quad \checkmark$$

By inspection $T_0 = 9$

$$\therefore c = 9 \quad \checkmark$$

$$\therefore T_n = n^2 + bn + 9$$

$$T_1 = 1^2 + b + 9 = 9$$

$$\therefore b = -1 \quad \checkmark$$

$$\therefore T_n = n^2 - n + 9 \quad \checkmark$$

(or equivalent method)

(4)

3.2 Sammy:

$$a = 2 \quad T_4 = a + 3d \quad \checkmark$$

$$= 2 + 3d = 17$$

$$3d = 15$$

$$d = 5 \quad \checkmark$$

$$\therefore T_{20} = a + 19d$$

$$= 2 + 19 \times 5$$

$$= 97$$

$$\therefore x = 97 \quad \checkmark$$

Solly:

$$T_4 = a + 3d = 59$$

$$T_{20} = a + 19d = 107 \quad \checkmark$$

$$16d = 48$$

$$d = 3 \quad \checkmark$$

$$\therefore T_4 = a + 3d = 59$$

$$a + 9 = 59$$

$$a = 50 \quad \checkmark$$

$$\therefore y = 50$$

(6)

4.1.1 2010

$$109\% \text{ of } 85\,000 = R92\,650 \quad \checkmark$$

$$109\% \text{ of } 92\,650 = R100\,988,50 \quad \checkmark$$

(2)

4.1.2 GP with $a = 85\,000$

$$r = 109\% = 1,09 \quad \checkmark$$

$$T_n = ar^{n-1} \quad \checkmark$$

$$T_{15} = 85\,000(1,09)^{15} \quad \checkmark$$

$$= R309\,611,01 \quad \checkmark$$

(4)

4.2

$$\sum_{k=0}^3 x = \frac{2}{5^k}$$

$$= 2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} \quad \checkmark \checkmark$$

$$= \frac{312}{125} (2,496) \quad \checkmark$$

(3)

4.3.1 GP with $a = 20$

$$r = \frac{3}{4}$$

$$S_n = \frac{a[1-r^n]}{1-r} \quad \checkmark$$

$$S_n = \frac{20 \left[1 - \left(\frac{3}{4} \right)^{10} \right]}{1 - \frac{3}{4}} \checkmark$$

$$= 75,49 \checkmark$$

∴ Depth is 75,49 m.

4.3.2

$$S_\infty = \frac{a}{1-r} \checkmark$$

$$= \frac{20}{1 - \frac{3}{4}} \checkmark$$

$$= 80$$

$$S_\infty = 80 \text{ m} \checkmark$$

∴ Company will never reach the water. ✓ (4)

5.1.1

$$f(x) = \frac{-3}{x-1} + 2$$

asymptotes at $x = 1$; $y = 2$

$$y\text{-intercept: } x = 0 \quad \therefore f(0) = \frac{-3}{0-1} + 2 = 5$$

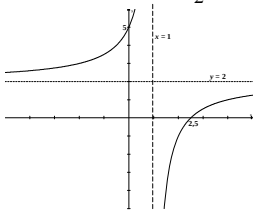
$$x\text{-intercept: } y = 0 \quad \therefore 0 = \frac{-3}{x-1} + 2$$

$$\frac{3}{x-1} = 2$$

$$3 = 2x - 2$$

$$5 = 2x$$

$$\frac{5}{2} = x$$



✓ ✓ ✓ ✓

5.1.2

Axes of symmetry:

$$y = x + c \text{ and } y = -x + c$$

Pass through (1; 2) ✓

$$\therefore 2 = 1 + c \quad 2 = -1 + c$$

$$1 = c \quad 3 = c$$

$$\therefore y = x + 1 \text{ and } y = -x + 3 \checkmark \checkmark$$

5.2.1

$$y = a(x - x_1)(x - x_2) \checkmark$$

$$y = a(x + 1)(x - 3) \checkmark$$

Substitute (0; -1):

$$-1 = a(0 + 1)(0 - 3) \checkmark$$

$$-1 = -3a$$

$$\frac{1}{3} = a \checkmark$$

$$\therefore p(x) = \frac{1}{3}(x + 1)(x - 3)$$

$$= \frac{1}{3}(x^2 - 2x - 3)$$

$$= \frac{1}{3}x^2 - \frac{2}{3}x - 1 \checkmark$$

5.2.2

$$q(x) = \frac{1}{2}b^x - 4$$

Substitute (3; 0):

$$0 = \frac{1}{2}b^3 - 4 \checkmark$$

$$4 = \frac{1}{2}b^3$$

$$8 = b^3 \checkmark$$

$$2 = b$$

5.2.3 Domain: $x \in \mathbb{R}$ ✓

Range: $y \in \mathbb{R}; y > -4$ ✓

5.2.4

$$q(x) = y = \frac{1}{2} \cdot 2^x - 4$$

$$q^{-1}(x): x = \frac{1}{2} \cdot 2^y - 4 \checkmark$$

$$2x = 2^y - 8$$

$$2x + 8 = 2^y \checkmark$$

$$\therefore y = \log_2(2x + 8) \checkmark$$

$$q^{-1}(x) = \log_2(2x + 8)$$

5.2.5 Domain of $q^{-1}(x)$ = Range of $q(x)$

$$= x \in \mathbb{R}; x > -4 \checkmark$$

5.3.1

$$a = -4 \checkmark$$

5.3.2

$$x = 180^\circ; x = -180^\circ \checkmark$$

$$x = 0^\circ \checkmark$$

5.3.3

$$y = \tan(x + 90^\circ + 45^\circ) + 2 \checkmark$$

$$y = \tan(x + 135^\circ) + 2 \checkmark$$

$$\text{or } y = \tan(x - 45^\circ) + 2$$

by inspection/reduction

6.1

$$f(x) = \frac{1}{2}x^2$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{2}(x+h)^2 - \frac{1}{2}x^2}{h} \checkmark$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{2}x^2 + xh + \frac{1}{2}h^2 - \frac{1}{2}x^2}{h} \checkmark$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{h(x + \frac{1}{2}h)}{h} \checkmark$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} x + \frac{1}{2}h \checkmark$$

$$f(x) = x \checkmark$$

6.2.1

$$f(x) = x^4 + \sqrt{x} - \frac{9}{x}$$

$$f(x) = x^4 + x^{\frac{1}{2}} - 9x^{-1} \checkmark$$

$$\therefore f'(x) = 4x^3 + \frac{1}{2}x^{-\frac{1}{2}} + 9x^{-2}$$

$$\checkmark \checkmark$$

6.2.2

$$y = t(t + 1)$$

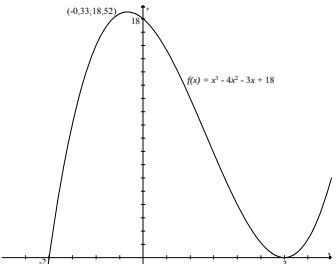
$$y = 3x(3x + 1) \checkmark$$

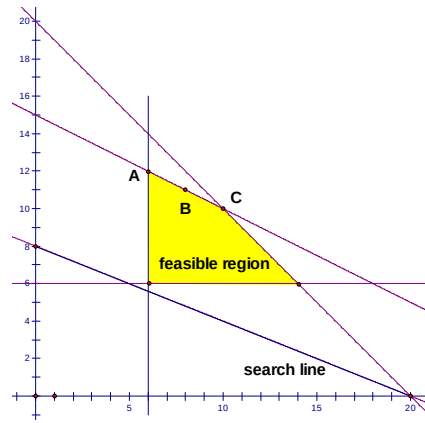
$$y = 9x^2 + 3x \checkmark$$

$$\therefore \frac{dy}{dx} = 18x + 3 \checkmark$$

6.3.1

$$A(x) = -\frac{1}{2}x^3 + 12x^2$$

- $A(x) = -\frac{3}{2}x^2 + 24x$ ✓
 $\therefore \text{let } 0 = -\frac{3}{2}x^2 + 24x$ ✓
 $0 = x^2 - 16x$ ✓
 $0 = x(x - 16)$ ✓
 $x = 0 \text{ or } x = 16$ ✓
 $\therefore$ Maximum area will be covered after 16 months. ✓
 6.3.2 $A'(x) = -\frac{3}{2}x^2 + 24x$ ✓
 $A'(1) = -\frac{3}{2}(1)^2 + 24(1)$ ✓
 $= 22\frac{1}{2}$ ✓
 $\therefore$ The rate of growth was $22\frac{1}{2} \text{ m}^2/\text{month}$ one month after the study had begun. (2)
 7.1 $f(x) = x^3 - 4x^2 - 3x + 18$ (2)
 $f(3) = 27 - (4 \times 9) - (3 \times 3) + 18 = 0$ ✓ (1)
 7.2 $0 = x^3 - 4x^2 - 3x + 18$ ✓
 $0 = (x - 3)(x^2 - x - 6)$ ✓
 $0 = (x - 3)(x - 3)(x + 2)$ ✓
 $\therefore$ x-intercepts at $x = 3$ and $x = 2$. ✓ ✓ (4)
 7.3 $f'(x) = 3x^2 - 8x - 3 = 0$ for turning points ✓
 $\therefore (3x + 1)(x - 3) = 0$ ✓
 $x = -\frac{1}{3}$ and $x = 3$ ✓ ✓
 $\therefore f(x) = (-\frac{1}{3})^3 - 4(-\frac{1}{3})^2 - 3(-\frac{1}{3}) + 18$ ✓
 $= \frac{500}{27} (= 18,52)$ ✓
 $f(3) = 0$ ✓
 $\therefore$ Turning points at $(-\frac{1}{3}; \frac{500}{27})$ and $(3; 0)$. ✓
 7.4 ✓

 $f(x) = x^3 - 4x^2 - 3x + 18$
 $\checkmark \checkmark$ for each of the points
 7.5 $f''(x) = 6x - 8 = 0$ ✓
 for point of inflection.
 $\therefore x = \frac{8}{6} = \frac{4}{3}$ ✓
 8.1 $V = \text{Area of base} \times \text{Height}$
 $2,5 = x^2 h$ ✓
 $\frac{2,5}{x^2} = h$ ✓
 8.2 $\text{Area} = 2(x^2 + 4xh)$ ✓
 $= 2x^2 + 8x \times \frac{2,5}{x^2}$ ✓ (3)

- $= 2x^2 + \frac{20}{x}$ ✓
 8.3 $A = 2x^2 + 20x^{-1}$ ✓
 $\frac{dA}{dx} = 4x - 20x^{-2}$ ✓
 $4x - \frac{20}{x^2} = 0$ (for minimum) ✓
 $\therefore 4x^3 - 20 = 0$ ✓
 $x^3 = 5$ ✓
 $x = \sqrt[3]{5} = 1,71$ ✓
 (5)
 9.1 $x = \text{no. of guitars of type A}$
 $y = \text{no. of guitars of type B}$
 $x + y \leq 20$ ✓
 $1\,500x + 3\,000y \leq 45\,000$ ✓
 $x \geq 6$ ✓
 $y \geq 6$ ✓ (4)
 9.2

 $\checkmark \checkmark \checkmark \checkmark \checkmark \checkmark$ (5)
 9.3 $P = 400x + 1\,000y$ ✓ (1)
 9.4 $-400x + P = 1\,000y$
 $-\frac{400}{1\,000}x + \frac{P}{1\,000} = y$
 $-\frac{2}{5}x + \frac{P}{1\,000} = y$ ✓
 $\therefore$ Use search line of slope $-\frac{2}{5}$... see sketch. ✓
 $\therefore$ Maximum profit for $x = 6$ and $y = 12$ at point A. ✓ (3)
 9.5 New profit equation:
 $P = 500x + 1\,000y$
 $-\frac{500}{1\,000}x + \frac{P}{1\,000} = y$
 $-\frac{1}{2}x + \frac{P}{1\,000} = y$ ✓
 This line is parallel to one of the borders of the feasible region. Therefore maximum profit occurs at any whole-number point on this line. ✓
 i.e. A(6; 12)
 B(8; 11) (3)

$$C(10; 10) \checkmark$$